Palestine Journal of Mathematics

Vol. 2(1) (2013) , 65–71

$\mathcal{B}-$TANGENT DEVELOPABLE OF BIHARMONIC $\mathcal{B}-$SLANT HELICES ACCORDING TO BISHOP FRAME IN THE $\widetilde{\mathcal{SL}_2}(\mathbb{R})$

Talat KÖRPINAR and Essin TURHAN

Communicated by Ayman Badawi

MSC 2010 Classifications: 53C41, 53A10.

Keywords and phrases: Biharmonic curve, $\widetilde{\mathcal{SL}_2}(\mathbb{R})$, curvature, torsion, tangent surface.

Abstract. In this paper, we study $\mathcal{B}-$tangent developable of biharmonic $\mathcal{B}-$slant helices in the $\widetilde{\mathcal{SL}_2}(\mathbb{R})$. We characterize $\mathcal{B}-$tangent developable of biharmonic $\mathcal{B}-$slant helices in terms of their curvature and torsion. Finally, we find out their explicit equations.

1 Introduction

Modeling developable surfaces through approximation is attractive as designers do not have to concern themselves with developability constraints during the modeling process. Ideally, they can freely utilize all sorts of modeling tools (e.g., blends, fillets) and then rely on an approximation algorithm to yield a developable result. In practice though, the approximation approach is highly restricted since the methods can only succeed if the original input surfaces already have fairly small Gaussian curvature. Moreover, in most cases the final result is not analytically developable. While this is not a problem for applications such as texture-mapping, it can be problematic for manufacturing, where the surfaces need to be realised from planar patterns (e.g., sewing).

This study is organised as follows: Firstly, we study tangent developable of biharmonic $\mathcal{B}-$slant helices in the $\widetilde{\mathcal{SL}_2}(\mathbb{R})$. Secondly, we characterize tangent developable of biharmonic $\mathcal{B}-$slant helices in terms of their curvature and torsion. Finally, we find out their explicit equations.

2 $\widetilde{\mathcal{SL}_2}(\mathbb{R})$

We identify $\widetilde{\mathcal{SL}_2}(\mathbb{R})$ with

$$\mathbb{R}^3_+ = \left\{(x,y,z) \in \mathbb{R}^3 : z > 0\right\}$$

endowed with the metric

$$g = ds^2 = (dx + \frac{dy}{z})^2 + \frac{dy^2 + dz^2}{z^2}.$$

The following set of left-invariant vector fields forms an orthonormal basis for $\widetilde{\mathcal{SL}_2}(\mathbb{R})$

$$\mathbf{e}_1 = \frac{\partial}{\partial x}, \mathbf{e}_2 = z\frac{\partial}{\partial y} - \frac{\partial}{\partial x}, \mathbf{e}_3 = z\frac{\partial}{\partial z}. \tag{2.1}$$

The characterizing properties of g defined by

$$\begin{aligned} g(\mathbf{e}_1,\mathbf{e}_1) &= g(\mathbf{e}_2,\mathbf{e}_2) = g(\mathbf{e}_3,\mathbf{e}_3) = 1, \\ g(\mathbf{e}_1,\mathbf{e}_2) &= g(\mathbf{e}_2,\mathbf{e}_3) = g(\mathbf{e}_1,\mathbf{e}_3) = 0. \end{aligned}$$

The Riemannian connection ∇ of the metric g is given by

$$\begin{aligned} 2g(\nabla_X Y, Z) &= Xg(Y,Z) + Yg(Z,X) - Zg(X,Y) \\ &\quad -g(X,[Y,Z]) - g(Y,[X,Z]) + g(Z,[X,Y]), \end{aligned}$$

which is known as Koszul's formula.

Using the Koszul's formula, we obtain

$$\nabla_{\mathbf{e}_1}\mathbf{e}_1 = 0, \quad \nabla_{\mathbf{e}_1}\mathbf{e}_2 = \frac{1}{2}\mathbf{e}_3, \quad \nabla_{\mathbf{e}_1}\mathbf{e}_3 = -\frac{1}{2}\mathbf{e}_2, \nabla_{\mathbf{e}_2}\mathbf{e}_1 = \frac{1}{2}\mathbf{e}_3, \quad \nabla_{\mathbf{e}_2}\mathbf{e}_2 = \mathbf{e}_3, \ \nabla_{\mathbf{e}_2}\mathbf{e}_3 = -\frac{1}{2}\mathbf{e}_1 - \mathbf{e}_2, \tag{2.2}$$

$$\nabla_{\mathbf{e}_3}\mathbf{e}_1 = -\frac{1}{2}\mathbf{e}_2, \quad \nabla_{\mathbf{e}_3}\mathbf{e}_2 = \frac{1}{2}\mathbf{e}_1, \quad \nabla_{\mathbf{e}_3}\mathbf{e}_3 = 0.$$

Moreover we put

$$R_{ijk} = R(\mathbf{e}_i, \mathbf{e}_j)\mathbf{e}_k, \quad R_{ijkl} = R(\mathbf{e}_i, \mathbf{e}_j, \mathbf{e}_k, \mathbf{e}_l),$$

where the indices i, j, k and l take the values $1, 2$ and 3

$$R_{1212} = R_{1313} = \frac{1}{4}, \ R_{2323} = -\frac{7}{4}. \tag{2.3}$$

3 Biharmonic $\mathcal{B}-$Slant Helices in $\widetilde{\mathcal{SL}_2}(\mathbb{R})$

Assume that $\{\mathbf{T}, \mathbf{N}, \mathbf{B}\}$ be the Frenet frame field along γ. Then, the Frenet frame satisfies the following Frenet–Serret equations:

$$\nabla_{\mathbf{T}}\mathbf{T} = \kappa\mathbf{N}, \nabla_{\mathbf{T}}\mathbf{N} = -\kappa\mathbf{T} + \tau\mathbf{B}, \tag{3.1}$$

$$\nabla_{\mathbf{T}}\mathbf{B} = -\tau\mathbf{N},$$

where κ is the curvature of γ and τ its torsion and

$$g_{\widetilde{\mathcal{SL}_2(\mathbb{R})}}(\mathbf{T},\mathbf{T}) = 1, \ g_{\widetilde{\mathcal{SL}_2(\mathbb{R})}}(\mathbf{N},\mathbf{N}) = 1, \ g_{\widetilde{\mathcal{SL}_2(\mathbb{R})}}(\mathbf{B},\mathbf{B}) = 1, \tag{3.2}$$

$$g_{\widetilde{\mathcal{SL}_2(\mathbb{R})}}(\mathbf{T},\mathbf{N}) = g_{\widetilde{\mathcal{SL}_2(\mathbb{R})}}(\mathbf{T},\mathbf{B}) = g_{\widetilde{\mathcal{SL}_2(\mathbb{R})}}(\mathbf{N},\mathbf{B}) = 0.$$

The Bishop frame or parallel transport frame is an alternative approach to defining a moving frame that is well defined even when the curve has vanishing second derivative. The Bishop frame is expressed as

$$\begin{aligned} \nabla_{\mathbf{T}}\mathbf{T} &= k_1\mathbf{M}_1 + k_2\mathbf{M}_2, \\ \nabla_{\mathbf{T}}\mathbf{M}_1 &= -k_1\mathbf{T}, \\ \nabla_{\mathbf{T}}\mathbf{M}_2 &= -k_2\mathbf{T}, \end{aligned} \tag{3.3}$$

where

$$g_{\widetilde{\mathcal{SL}_2(\mathbb{R})}}(\mathbf{T},\mathbf{T}) = 1, \ g_{\widetilde{\mathcal{SL}_2(\mathbb{R})}}(\mathbf{M}_1,\mathbf{M}_1) = 1, \ g_{\widetilde{\mathcal{SL}_2(\mathbb{R})}}(\mathbf{M}_2,\mathbf{M}_2) = 1, \tag{3.4}$$

$$g_{\widetilde{\mathcal{SL}_2(\mathbb{R})}}(\mathbf{T},\mathbf{M}_1) = g_{\widetilde{\mathcal{SL}_2(\mathbb{R})}}(\mathbf{T},\mathbf{M}_2) = g_{\widetilde{\mathcal{SL}_2(\mathbb{R})}}(\mathbf{M}_1,\mathbf{M}_2) = 0.$$

Here, we shall call the set $\{\mathbf{T}, \mathbf{M}_1, \mathbf{M}_1\}$ as Bishop trihedra, k_1 and k_2 as Bishop curvatures and $\mathfrak{Y}(s) = \arctan\frac{k_2}{k_1}$, $\tau(s) = \mathfrak{Y}'(s)$ and $\kappa(s) = \sqrt{k_1^2 + k_2^2}$.

Bishop curvatures are defined by

$$\begin{aligned} k_1 &= \kappa(s)\cos\mathfrak{Y}(s), \\ k_2 &= \kappa(s)\sin\mathfrak{Y}(s). \end{aligned}$$

With respect to the orthonormal basis $\{\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3\}$ we can write

$$\mathbf{T} = T^1 e_1 + T^2 e_2 + T^3 e_3, \mathbf{M}_1 = M_1^1\mathbf{e}_1 + M_1^2\mathbf{e}_2 + M_1^3\mathbf{e}_3, \tag{3.5}$$

$$\mathbf{M}_2 = M_2^1\mathbf{e}_1 + M_2^2\mathbf{e}_2 + M_2^3\mathbf{e}_3.$$

Theorem 3.1. ([9]) $\gamma : I \longrightarrow \widetilde{\mathcal{SL}_2}(\mathbb{R})$ *is a biharmonic curve according to Bishop frame if and only if*

$$k_1^2 + k_2^2 = \text{constant} \neq 0, k_1'' - \left[k_1^2 + k_2^2\right]k_1 = -k_1\left[\frac{15}{4}M_2^1 - \frac{1}{4}\right] - 2k_2M_1^1M_2^1, \tag{3.6}$$

$$k_2'' - \left[k_1^2 + k_2^2\right] k_2 = 2k_1 M_1^1 M_2^1 - k_2 \left[\frac{15}{4} M_1^1 - \frac{1}{4}\right].$$

Definition 3.2. *A regular curve $\gamma : I \longrightarrow \widetilde{\mathcal{SL}_2(\mathbb{R})}$ is called a slant helix provided the unit vector $\mathbf{M}_1$ of the curve γ has constant angle $\mathfrak{W}$ with some fixed unit vector u, that is*

$$g_{\widetilde{\mathcal{SL}_2(\mathbb{R})}}\left(\mathbf{M}_1(s), u\right) = \cos \mathfrak{W} \text{ for all } s \in I. \tag{3.7}$$

The condition is not altered by reparametrization, so without loss of generality we may assume that slant helices have unit speed. The slant helices can be identified by a simple condition on natural curvatures.

To separate a slant helix according to Bishop frame from that of Frenet- Serret frame, in the rest of the paper, we shall use notation for the curve defined above as $\mathcal{B}$-slant helix.

We shall also use the following lemma.

Lemma 3.3. *Let $\gamma : I \longrightarrow \widetilde{\mathcal{SL}_2(\mathbb{R})}$ be a unit speed curve. Then γ is a $\mathcal{B}$-slant helix if and only if*

$$k_1 = -k_2 \tan \mathfrak{W}. \tag{3.8}$$

Theorem 3.4. ([9]) *Let $\gamma : I \longrightarrow \widetilde{\mathcal{SL}_2(\mathbb{R})}$ be a unit speed non-geodesic biharmonic $\mathcal{B}-$slant helix. Then, the parametric equations of γ are*

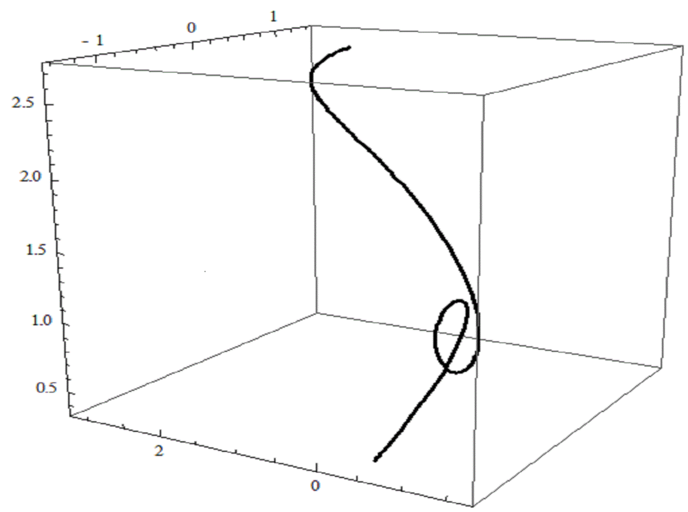
$$\boldsymbol{x}(s) = \frac{1}{\mathfrak{Q}_1} \cos \mathfrak{W} \sin\left[\mathfrak{Q}_1 s + \mathfrak{Q}_2\right] + \frac{1}{\mathfrak{Q}_1} \cos \mathfrak{W} \cos\left[\mathfrak{Q}_1 s + \mathfrak{Q}_2\right] + \mathfrak{Q}_4,$$

$$\boldsymbol{y}(s) = -\frac{\mathfrak{Q}_3}{\mathfrak{Q}_1^2 + \sin^2 \mathfrak{W}} \cos \mathfrak{W} e^{-\sin \mathfrak{W} s} \left(\mathfrak{Q}_1 \cos\left[\mathfrak{Q}_1 s + \mathfrak{Q}_2\right] + \sin \mathfrak{W} \sin\left[\mathfrak{Q}_1 s + \mathfrak{Q}_2\right]\right) + \mathfrak{Q}_5,$$

$$\boldsymbol{z}(s) = \mathfrak{Q}_3 e^{-\sin \mathfrak{W} s},$$

where $\mathfrak{Q}_1, \mathfrak{Q}_2, \mathfrak{Q}_3, \mathfrak{Q}_4, \mathfrak{Q}_5$ are constants of integration.

We can use Mathematica in above theorem, yields



4 $\mathcal{B}-$Tangent Developable Surfaces of $\mathcal{B}-$Slant Helices in $\widetilde{\mathcal{SL}_2(\mathbb{R})}$

To separate a tangent developable according to Bishop frame from that of Frenet- Serret frame, in the rest of the paper, we shall use notation for this surface as $\mathcal{B}-$tangent developable.

The purpose of this section is to study $\mathcal{B}-$tangent developable surfaces of $\mathcal{B}-$slant helices in $\widetilde{\mathcal{SL}_2(\mathbb{R})}$.

The $\mathcal{B}-$tangent developable of γ is a ruled surface

$$\mathcal{R}(s,u) = \gamma(s) + u\gamma'(s). \tag{4.1}$$

Theorem 4.1. *Let $\gamma : I \longrightarrow \widetilde{\mathcal{SL}_2(\mathbb{R})}$ is a unit speed non-geodesic biharmonic $\mathcal{B}-$slant helix in $\widetilde{\mathcal{SL}_2(\mathbb{R})}$. Then, the parametric equations of $\mathcal{B}-$tangent developable of γ are*

$$\begin{aligned} \boldsymbol{x}_{\mathcal{R}}(s,u) &= \frac{1}{\mathfrak{Q}_1}\cos\mathfrak{W}\sin[\mathfrak{Q}_1 s+\mathfrak{Q}_2] + \frac{1}{\mathfrak{Q}_1}\cos\mathfrak{W}\cos[\mathfrak{Q}_1 s+\mathfrak{Q}_2] \\ &\quad + u[\cos\mathfrak{W}\cos[\mathfrak{Q}_1 s+\mathfrak{Q}_2] - \cos\mathfrak{W}\sin[\mathfrak{Q}_1 s+\mathfrak{Q}_2]] + \mathfrak{Q}_4, \\ \boldsymbol{y}_{\mathcal{R}}(s,u) &= -\frac{\mathfrak{Q}_3}{\mathfrak{Q}_1^2+\sin^2\mathfrak{W}}\cos\mathfrak{W}e^{-\sin\mathfrak{W}s}(\mathfrak{Q}_1\cos[\mathfrak{Q}_1 s+\mathfrak{Q}_2] \end{aligned} \tag{4.2}$$

$+\sin\mathfrak{W}\sin[\mathfrak{Q}_1 s+\mathfrak{Q}_2]) + u\mathfrak{Q}_3 e^{-\sin\mathfrak{W}s}\cos\mathfrak{W}\sin[\mathfrak{Q}_1 s+\mathfrak{Q}_2] + \mathfrak{Q}_5$, $\boldsymbol{z}_{\mathcal{R}}(s,u) = \mathfrak{Q}_3 e^{-\sin\mathfrak{W}s} - \mathfrak{Q}_3 u e^{-\sin\mathfrak{W}s}\sin\mathfrak{W}$,
where $\mathfrak{Q}_1, \mathfrak{Q}_2, \mathfrak{Q}_3, \mathfrak{Q}_4, \mathfrak{Q}_5$ are constants of integration.

Proof. By the Bishop formula, we have the following equation

$$\mathbf{T} = \cos\mathfrak{W}\cos[\mathfrak{Q}_1 s+\mathfrak{Q}_2]\,\mathbf{e}_1 + \cos\mathfrak{W}\sin[\mathfrak{Q}_1 s+\mathfrak{Q}_2]\,\mathbf{e}_2 - \sin\mathfrak{W}\mathbf{e}_3. \tag{4.3}$$

Using (2.1) in (4.3), we obtain

$$\mathbf{T} = (\cos\mathfrak{W}\cos[\mathfrak{Q}_1 s+\mathfrak{Q}_2] - \cos\mathfrak{W}\sin[\mathfrak{Q}_1 s+\mathfrak{Q}_2], z\cos\mathfrak{W}\sin[\mathfrak{Q}_1 s+\mathfrak{Q}_2], -z\sin\mathfrak{W}). \tag{4.4}$$

In terms of (2.4) and (4.4), we may give:

$$\begin{aligned} \mathbf{T} = (&\cos\mathfrak{W}\cos[\mathfrak{Q}_1 s+\mathfrak{Q}_2] - \cos\mathfrak{W}\sin[\mathfrak{Q}_1 s+\mathfrak{Q}_2], \\ &\mathfrak{Q}_3 e^{-\sin\mathfrak{W}s}\cos\mathfrak{W}\sin[\mathfrak{Q}_1 s+\mathfrak{Q}_2], -\mathfrak{Q}_3 e^{-\sin\mathfrak{W}s}\sin\mathfrak{W}). \end{aligned} \tag{4.5}$$

Consequently, the parametric equations of $\mathcal{R}$ can be found from (4.1), (4.5). This concludes the proof of Theorem.

We can prove the following interesting main result.

Theorem 4.2. *Let $\gamma : I \longrightarrow \widetilde{\mathcal{SL}_2(\mathbb{R})}$ be a unit speed non-geodesic biharmonic $\mathcal{B}-$slant helix and $\mathcal{R}$ its $\mathcal{B}-$tangent developable surface in $\widetilde{\mathcal{SL}_2(\mathbb{R})}$. Then the equation of $\mathcal{B}-$tangent developable is*

$$\begin{aligned} \mathcal{R}(s,u) &= [\frac{1}{\mathfrak{Q}_1}\cos\mathfrak{W}\sin[\mathfrak{Q}_1 s+\mathfrak{Q}_2] + \frac{1}{\mathfrak{Q}_1}\cos\mathfrak{W}\cos[\mathfrak{Q}_1 s+\mathfrak{Q}_2] + \mathfrak{Q}_4 \\ &\quad -\frac{1}{\mathfrak{Q}_1^2+\sin^2\mathfrak{W}}\cos\mathfrak{W}(\mathfrak{Q}_1\cos[\mathfrak{Q}_1 s+\mathfrak{Q}_2] \\ &\quad +\sin\mathfrak{W}\sin[\mathfrak{Q}_1 s+\mathfrak{Q}_2]) + \frac{\mathfrak{Q}_5}{\mathfrak{Q}_3}e^{\sin\mathfrak{W}s} + u\cos\mathfrak{W}\cos[\mathfrak{Q}_1 s+\mathfrak{Q}_2]]\mathbf{e}_1 \\ &\quad [-\frac{1}{\mathfrak{Q}_1^2+\sin^2\mathfrak{W}}\cos\mathfrak{W}(\mathfrak{Q}_1\cos[\mathfrak{Q}_1 s+\mathfrak{Q}_2] \\ &\quad +\sin\mathfrak{W}\sin[\mathfrak{Q}_1 s+\mathfrak{Q}_2]) + \frac{\mathfrak{Q}_5}{\mathfrak{Q}_3}e^{\sin\mathfrak{W}s} + u\cos\mathfrak{W}\sin[\mathfrak{Q}_1 s+\mathfrak{Q}_2]]\mathbf{e}_2 \\ &\quad -u\sin\mathfrak{W}\mathbf{e}_3, \end{aligned} \tag{4.6}$$

where $\mathfrak{Q}_1, \mathfrak{Q}_2, \mathfrak{Q}_3, \mathfrak{Q}_4, \mathfrak{Q}_5$ *are constants of integration.*

Proof. We assume that γ is a unit speed $\mathcal{B}-$slant helix.
Substituting (2.4) to (4.2), we have (4.6). Thus, the proof is completed.

Thus, we proved the following:

Corollary 4.3. *Let* $\gamma : I \longrightarrow \widetilde{\mathcal{SL}_2(\mathbb{R})}$ *be a unit speed non-geodesic biharmonic* $\mathcal{B}-$*slant helix and* $\mathcal{R}$ *its* $\mathcal{B}-$*tangent developable surface in* $\widetilde{\mathcal{SL}_2(\mathbb{R})}$*. Then, normal of* $\mathcal{B}-$*tangent developable of* γ *is*

$$\mathbf{n}_{\mathcal{R}} = [uk_1 \sin[\mathfrak{Q}_1 s + \mathfrak{Q}_2] - uk_2 \sin \mathfrak{W} \cos[\mathfrak{Q}_1 s + \mathfrak{Q}_2]]\mathbf{e}_1 + [-uk_1 \cos[\mathfrak{Q}_1 s + \mathfrak{Q}_2] - uk_2 \sin \mathfrak{W} \sin[\mathfrak{Q}_1 s + \mathfrak{Q}_2]]\mathbf{e}_2 \quad (4.7)$$
$$-uk_2 \cos \mathfrak{W} \mathbf{e}_3.$$

where $\mathfrak{Q}_1, \mathfrak{Q}_2$ *are constants of integration.*

Proof. Assume that $\mathbf{n}_{\mathcal{R}}$ be the normal vector field on $\mathcal{B}-$tangent developable defined by

$$\mathbf{n}_{\mathcal{R}} = \mathcal{R}_s \wedge \mathcal{R}_u. \quad (4.8)$$

From Definition 3.2, we have the following equation

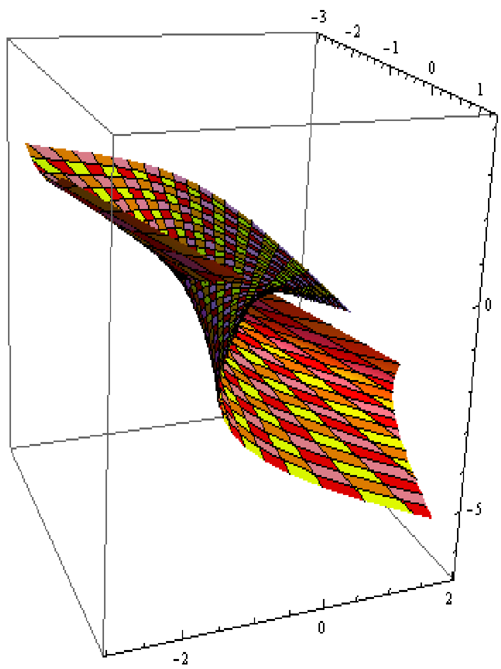
$$\mathbf{M}_1 = \sin \mathfrak{W} \cos[\mathfrak{Q}_1 s + \mathfrak{Q}_2]\, \mathbf{e}_1 + \sin \mathfrak{W} \sin[\mathfrak{Q}_1 s + \mathfrak{Q}_2]\, \mathbf{e}_2 + \cos \mathfrak{W} \mathbf{e}_3, \quad (4.9)$$

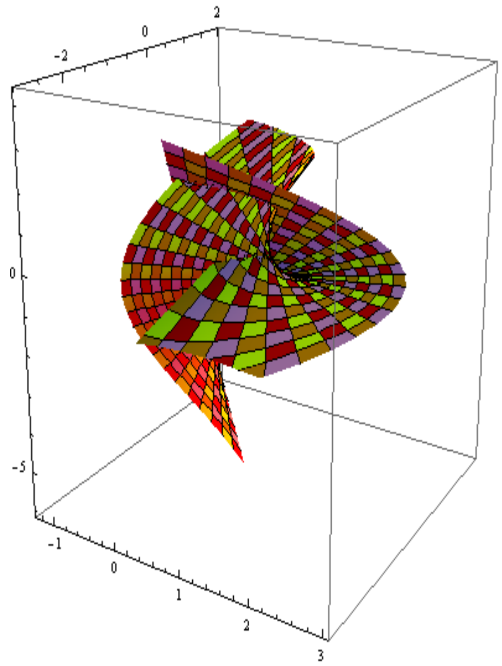
where $\mathfrak{Q}_1, \mathfrak{Q}_2$ are constants of integration.
On the other hand, using Bishop formulas (3.3) and (2.1), we have

$$\mathbf{M}_2 = \sin[\mathfrak{Q}_1 s + \mathfrak{Q}_2]\, \mathbf{e}_1 - \cos[\mathfrak{Q}_1 s + \mathfrak{Q}_2]\, \mathbf{e}_2.\mathbf{e}_2. \quad (4.10)$$

Substituting (4.9) and (4.10) to (4.8), we have (4.7). This concludes the proof of corollary.

Finally, the obtained parametric equations are illustrated in Figures 1 and 2:





References

[1] D. E. Blair: *Contact Manifolds in Riemannian Geometry*, Lecture Notes in Mathematics, Springer-Verlag 509, Berlin-New York, 1976.

[2] R. Caddeo and S. Montaldo: *Biharmonic submanifolds of* $\mathbb{S}^3$, Internat. J. Math. 12(8), 867–876 (2001).

[3] B. Y. Chen: *Some open problems and conjectures on submanifolds of finite type,* Soochow J. Math. 17, 169–188 (1991).

[4] I. Dimitric: *Submanifolds of* $\mathbb{E}^m$ *with harmonic mean curvature vector,* Bull. Inst. Math. Acad. Sinica 20, 53–65 (1992).

[5] J. Eells and L. Lemaire: *A report on harmonic maps,* Bull. London Math. Soc. 10, 1–68 (1978).

[6] J. Eells and J. H. Sampson: *Harmonic mappings of Riemannian manifolds,* Amer. J. Math. 86, 109–160 (1964).

[7] G. Y.Jiang: *2-harmonic isometric immersions between Riemannian manifolds,* Chinese Ann. Math. Ser. A 7(2), 130–144 (1986).

[8] G. Y. Jiang: *2-harmonic maps and their first and second variational formulas,* Chinese Ann. Math. Ser. A 7(4), 389–402 (1986).

[9] T. Körpınar, E. Turhan: *Biharmonic B-Slant Helices According To Bishop Frame In The* $\widetilde{\mathcal{SL}_2}(\mathbb{R})$, Bol. Soc. Paran. Mat. 31 (2), 39–45 (2013).

[10] E. Loubeau and S. Montaldo: *Biminimal immersions in space forms,* preprint, 2004, math.DG/0405320 v1.

[11] B. O'Neill: *Semi-Riemannian Geometry*, Academic Press, New York (1983).

[12] I. Sato: On a structure similar to the almost contact structure, Tensor, (N.S.), 30, 219-224 (1976).

[13] T. Takahashi: Sasakian ϕ-symmetric spaces, Tohoku Math. J., 29, 91-113 (1977).

[14] E. Turhan, T. Körpınar: *On Characterization Of Timelike Horizontal Biharmonic Curves In The Lorentzian Heisenberg Group Heis*^{3}, Zeitschrift für Naturforschung A- A Journal of Phys-

ical Sciences 65a , 641-648 (2010), 641-648.

[15] E. Turhan and T. Körpınar, *On Characterization Canal Surfaces around Timelike Horizontal Biharmonic Curves in Lorentzian Heisenberg Group Heis*^{3}, Zeitschrift für Naturforschung A- A Journal of Physical Sciences 66a, 441-449 (2011).

Author information

Talat KÖRPINAR and Essin TURHAN, Fırat University, Department of Mathematics, 23119, Elazığ, Turkey.
E-mail: talatkorpinar@gmail.com, essin.turhan@gmail.com

Received: February 2, 2012

Accepted: May 23, 2012