Palestine Journal of Mathematics

Vol. 2(1) (2013) , 46–49

Neutrino masses in the left-right mirror model at two loop level

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Communicated by Jose Luis Lopez-Bonilla

MSC 2010 Classifications: 81V25- Other Elementary Particle Theory.

PACS numbers: 12.60.Cn, 12.60.Fr, 13.35.Bv, 13.35.Dx, 13.35.Hb, 14.60.Pq.

Keywords and phrases: Neutrinos, Models Beyond de Standard Model.

All four authors wish to thank Sistema Nacional de Investigadores, SNI, México, A. H. G., EDI-IPN, COFAA-IPN, J. M. R. R. and COFAA-IPN for their generous support.

Abstract. In this work we use the left-right mirror model (LRMM) with a gauge group $SU(3)_C \otimes SU(2)_L \otimes SU(2)_R \otimes U(1)_{Y'}$ in a two loop graph to get neutrino masses by taking an approximate value for the rate $\frac{h}{m_{W_R}}$, where h es a free mixing parameter and $m_{W_R} \approx 1\ TeV$. The found neutrino masses are at most $1.8\ eV$, according to experiment, and $h \leq 80\ GeV$. One sees also that $h \leq 84\ GeV$, that is, less than roughly the W mass, for the present experimental bound $m_\nu < 2\ eV$.

1 Introduction

In a previous paper [1], we have worked the Left-Right Mirror Model with mirror fermions ([2]) to find bounds for mixing parameter and neutrino masses. These mirror fermions with *V* + *A* coupling leading to *P* conservation are "vector-like fermions" allowing left-handed (LH) and right-handed (RH) fermions in a gauge group *G* and representation *R*. In this work, we deal with a two loop graph to get improvement on those masses. Experiments such as Superkamiokande and others support neutrino oscillations ([3]-[9]). Neutrino masses can be explained with for instance the see saw mechanism ([10]-[19]) and with extra dimensions models ([20]). See saw models at TeV scale may give signatures in the CERN Large Hadron Collider LHC ([21]-[23]). The gauge group for this model is G = SU(3)$_C\otimes$SU(2)$_L\otimes$SU(2)$_R\otimes$U(1)$_{Y'}$. The mass matrix

$$M = \begin{pmatrix} K & \hat{\mu} \\ \mu & \hat{K} \end{pmatrix} \tag{1.1}$$

takes account of the Standard Model (SM) fermion masses K, its exotic counterpart $\hat{K}$, and their corresponding mixing, μ, $\hat{\mu}$. If m_l (m_h) is the light (heavy) diagonal mass matrix, the diagonal mass matrix M_d can be written as:

$$M_d = \begin{pmatrix} m_l & 0 \\ 0 & M_h \end{pmatrix} \tag{1.2}$$

In the gauge group $SU(2)_L \otimes SU(2)_R \otimes U(1)_{Y'}$ of the LRMM the fermionic gauge eigenstates have the form

$$\begin{aligned} l^0_{i\,L} &= \begin{pmatrix} \nu^0_i \\ e^0_i \end{pmatrix}_L , \; e^0_{i\,R} \,, \nu^0_{i\,R}, \qquad ; \qquad \widehat{l}^0_{i\,R} = \begin{pmatrix} \widehat{\nu}^0_i \\ \widehat{e}^0_i \end{pmatrix}_R , \; \widehat{e}^0_{i\,L} \,, \; \widehat{\nu}^0_{i\,L}, \\ Q^0_{i\,L} &= \begin{pmatrix} u^0_i \\ d^0_i \end{pmatrix}_L , \; u^0_{i\,R} \,, \; d^0_{i\,R}, \qquad ; \qquad \widehat{Q}^0_{i\,R} = \begin{pmatrix} \widehat{u}^0_i \\ \widehat{d}^0_i \end{pmatrix}_R , \; \widehat{u}^0_{i\,L} \,, \; \widehat{d}^0_{i\,L} \,, \end{aligned} \tag{1.3}$$

where the index i runs over the three fermion families and the superscripts ^{0} denote gauge eigenstates. The electric charge is defined as $Q = T_{3L} + T_{3R} + \frac{Y'}{2}$, where Y' is the hypercharge.
The corresponding scalar and interaction lagrangians for quarks and leptons are, respectively

$$\mathcal{L}_{sc} = (D_\mu \Phi)^+ (D^\mu \Phi) + (\hat{D}_\mu \hat{\Phi})^+ (\hat{D}^\mu \hat{\Phi}) \tag{1.4}$$

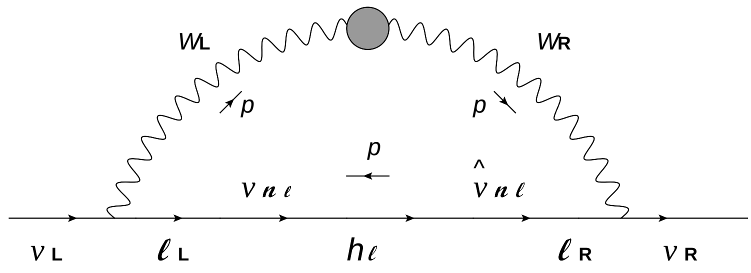

Figure 1. The two loop diagram inducing Dirac masses for neutrinos.

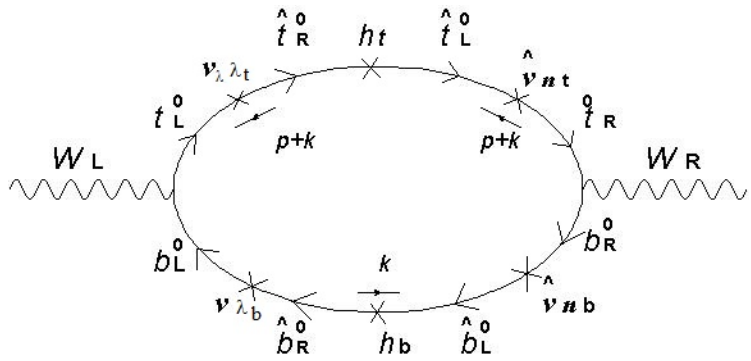

Figure 2. The one loop diagram from Fig. 1 corresponding to $W_L - W_R$ mixing.

$$\mathcal{L}^{int} = \bar{\psi} i\gamma^\mu D_\mu \psi + \bar{\hat{\psi}} i\gamma^\mu \hat{D}_\mu \hat{\psi} \tag{1.5}$$

where D_μ and $\hat{D}_\mu$ are the covariant derivatives for the SM and the mirror parts, respectively. Φ and $\hat{\Phi}$ are two doublets of scalar fields that induce the Spontaneous Symmetry breaking (SSB) in the model.

The gauge invariant Yukawa couplings for the neutral sectors are

$$h_{ij}\, \bar{\hat{\nu}}_{iL}\, \nu_{jR} + \lambda_{ij}\, \bar{l}_{iL}\, \tilde{\Phi}\, \nu_{jR} + \eta_{ij}\, \bar{\hat{l}}_{iR}\, \tilde{\hat{\Phi}}\, \hat{\nu}_{jL} \tag{1.6}$$

where $i, j = 1, 2, 3$, $\tilde{\Phi} = i\sigma_2 \Phi^*$, h_{ij} have dimensions of mass and λ_{ij} is dimensionless Yukawa coupling constant. When Φ and $\hat{\Phi}$ acquire VEV's, the Dirac neutrino mass terms are

$$h_{ij}\, \bar{\hat{\nu}}_{iL}\, \nu_{jR} + \frac{v}{\sqrt{2}}\, \lambda_{ij}\, \bar{\nu}_{iL}\, \nu_{jR} + \frac{\hat{v}}{\sqrt{2}}\, \eta_{ij}\, \bar{\hat{\nu}}_{iR}\, \hat{\nu}_{jL} \tag{1.7}$$

and similar terms for quarks instead of neutrinos. Eq. (1.7) is rewritten as

$$(\overline{\Psi}_{\nu L}, \overline{\Psi^c}_{\nu L}) \begin{pmatrix} 0 & M_D \\ M_D^T & 0 \end{pmatrix} \begin{pmatrix} (\Psi_\nu^c)_R \\ (\Psi_\nu)_R \end{pmatrix} \tag{1.8}$$

where

$$(\Psi_\nu)_{L,R} = \begin{pmatrix} \nu_i \\ \hat{\nu}_i \end{pmatrix}_{L,R} \quad , \quad (\Psi_\nu^c)_{L,R} = \begin{pmatrix} (\nu_i^c) \\ (\hat{\nu}_i^c) \end{pmatrix}_{L,R} \tag{1.9}$$

$$M_D = \begin{pmatrix} \frac{v}{\sqrt{2}}\,\lambda & 0 \\ & \\ h & \frac{\hat{v}}{\sqrt{2}}\,\eta \end{pmatrix} , \tag{1.10}$$

with λ , h and η unknown 3x3 matrices. From eq. (1.7), one has the corresponding Yukawa couplings for quarks $h_t\,\bar{\bar{t}}_L\,t_R$, $h_b\,\bar{\bar{b}}_L\,b_R$, etc.
Following Babu ([24]), we have from Fig. 2, with m_t, $m_b \ll h_{t,b}$:

$$\Pi_{\alpha\beta} = \frac{1}{(2\pi)^4}\int d^4k\, g_{\alpha\beta}\, g_L\, \frac{1}{(\not p + \not k)}\frac{1}{\sqrt{2}}\lambda_t v \frac{h_t}{(p+k)^2 - h_t^2}\frac{1}{\sqrt{2}}\lambda_t \hat{v}\frac{1}{(\not p + \not k)}$$
$$g_R \frac{1}{\not k}\frac{1}{\sqrt{2}}\lambda_b \hat{v}\frac{h_b}{k^2 - h_b^2}\frac{1}{\sqrt{2}}\lambda_b v \frac{1}{\not k} \tag{1.11}$$

or, with $g_L = g_R = g$:

$$\Pi_{\alpha\beta} = g_{\alpha\beta}\, g^2 \frac{1}{4}\lambda_t^2\lambda_b^2 v^2\hat{v}^2 h_t h_b \frac{1}{(2\pi)^4}\int d^4k \frac{1}{k^2(p+k)^2(k^2-h_b^2)[(p+k)^2 - h_t^2]} \tag{1.12}$$

Similarly, the amplitude for Fig. 1 is:

$$A = -\frac{1}{2} i g^2\, \lambda_l v\, \frac{1}{2}\, \lambda_l\, \hat{v}\, h_l\, [\bar{\nu}\gamma_\mu\gamma_\nu \frac{1}{2}(1-\gamma_5)\nu]\frac{1}{(2\pi)^4}$$
$$\int d^4k \Pi_{\alpha\beta}\frac{1}{p^2(p^2-h_l^2)(p^2-m_{W_R}^2)(p^2-m_{W_L}^2)}(g^{\mu\alpha} - \frac{1}{m_{W_R}^2}p^\mu p^\alpha)(g^{\nu\beta} - \frac{1}{m_{W_L}^2}p^\nu p^\beta)$$

that is:

$$A = -\frac{1}{2} g^4\, \lambda_t^2\, \lambda_b^2\, \lambda_l^2\, (\frac{1}{2}v)^3 (\frac{1}{2}\hat{v})^3\, \frac{h_t\, h_b\, h_l}{m_{W_L}^2 m_{W_R}^2}(\bar{\nu}_R\, \nu_L)\, I \tag{1.13}$$

where

$$I = \frac{1}{(2\pi)^8}\int d^4k d^4p \frac{1}{k^2(p+k)^2(k^2-p^2)[(p+k)^2 - h_t^2]p^2(p^2-h_l^2)} \tag{1.14}$$

Here we have used the fact that, calling

$$3m_{W_L}^2 m_{W_R}^2 = \alpha, \tag{1.15}$$

$$(p^2 - m_{W_L}^2)(p^2 - m_{W_R}^2) = \beta \tag{1.16}$$

and

$$f = \frac{1}{m_{W_R}^2} m_{W_L}^2 \tag{1.17}$$

then, for $m_{W_L}^2 \ll m_{W_R}^2$:

$$\alpha + \beta \approx 4m_{W_L}^2 m_{W_R}^2 + p^4 - m_{W_R}^2 p^2 \approx (1+f)\beta \tag{1.18}$$

To solve the integral I the Feynman parameters technique is needed. Then, after assuming for simplicity $h_t \sim h_b \sim h_l = h$, it results for the neutrino masses:

$$m_{\nu_l} \approx \frac{g^4}{512\pi^4}\frac{m_b m_t}{m_{W_L}^2}(\frac{h}{m_{W_R}^2})^2 m_l \tag{1.19}$$

Although the parameter h is free from (1.8) it takes account for the mixing among the standard and mirror fermions, so one hopes that it must be not so low; then, for estimation of neutrino masses we take $\frac{h}{m_{W_R}}$ as our new parameter, and since both quantities are unknown at all, we try a small value for its rate in the approximation $\frac{h}{m_{W_R}} \approx 0.08$ and gets

$$m_{\nu_e} \approx 5.14 \times 10^{-4}\ eV \tag{1.20}$$

$$m_{\nu_\mu} \approx 0.11\ eV \tag{1.21}$$

$$m_{\nu_\tau} \approx 1.8\ eV \tag{1.22}$$

which are in agreement with the present bounds for such masses [25]. This also tell us for instance that, $h \sim 80\ GeV$ when $m_{W_R} \sim 1\ TeV$.

2 Conclusions

In this work we have used the LRMM at two loop level to estimate neutrino masses working the corresponding amplitude and taking the mixing parameters approximately equal. Then the contribution of the model to the masses lies essentially in the single free parameter h. The results for the neutrino masses are roughly less than 2 eV, according to experimental limits. One can also use this small value to get the rather upper bound $h < 84\ GeV$. This is because the above neutrino bound does not take into account the neutrino type. Then we are confident of this model at two loop level within these approximations, the neutrino masses depending of only a single parameter.

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Received: July 4, 2012

Accepted: September 18, 2012