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On generalized Jordan *-derivations in semiprime rings with involution

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Abstract. Let R be an associative ring, and $F : R \longrightarrow R$ an additive mapping. F is called a Jordan triple *-derivation if $F(xyx) = F(x)y^*x^* + xF(y)x^* + xyF(x)$ for all $x, y \in R$ is fulfilled [1], and F is called a generalized Jordan triple *-derivation if $F(xyx) = F(x)y^*x^* + xf(y)x^* + xyf(x)$ with some Jordan *-derivation f for all $x, y \in R$ is fulfilled. In this note, we deal with generalized Jordan triple *-derivations of semiprime *-rings and generalize a Theorem of Daif and El-Sayiad [1].

1 Introduction

Throughout the present note, R will represent an associative ring. A ring R is said to be 2-torsion free if whenever $2x = 0$ with $x \in R$, implies $x = 0$. An additive mapping $F : R \longrightarrow R$ is called a *-derivation (resp. a Jordan *-derivation) if $F(xy) = F(x)y^* + xF(y)$(resp. $F(x^2) = F(x)x^* + xF(x)$) holds for all $x, y \in R$. An additive mapping $F : R \longrightarrow R$ is said to be a generalized *-derivation (resp. a generalized Jordan *-derivation) on R if there exists a *-derivation $d : R \longrightarrow R$ such that $F(xy) = F(x)y^* + xd(y)$(resp. $F(x^2) = F(x)x^* + xd(x)$) holds for all $x, y \in R$. An additive mapping $F : R \longrightarrow R$ is called a Jordan triple *-derivation if $F(xyx) = F(x)y^*x^* + xF(y)x^* + xyF(x)$ holds for all $x, y \in R$. An additive mapping $F : R \longrightarrow R$ is said to be a generalized Jordan triple *-derivation on R if there exists a Jordan *-derivation $f : R \longrightarrow R$ such that $F(xyx) = F(x)y^*x^* + xf(y)x^* + xyf(x)$ holds for all $x, y \in R$. In [1], Daif and El-Sayiad proved that any generalized Jordan triple *-derivation on a 6-torsion free semiprime *-ring is a generalized Jordan *-derivation. In this paper, we improve the result by Daif and El-Sayied as follows; Any generalized Jordan triple *-derivation on a 2-torsion free semiprime *-ring is a generalized Jordan *-derivation.

2 Generalized Jordan Triple *-Derivations

In [1], Daif and El-Sayiad proved that any generalized Jordan triple *-derivation on a 6-torsion free semiprime *-ring is a generalized Jordan *-derivation. Motivated by the result due to Daif and El-Sayiad [1], in the present note, we generalize the result as follows:Any generalized Jordan triple *-derivation on a 2-torsion free semiprime *-ring is a generalized Jordan *-derivation.

We state main theorem.

Theorem 2.1. Let R be a 2-torsion free semiprime *-ring, and $F : R \longrightarrow R$ an additive map such that $F(xyx) = F(x)y^*x^* + xf(y)x^* + xyf(x)$ for any $x, y \in R$ associated with a Jordan *-derivation f, then F is a generalized Jordan *-derivation.

Proof. Suppose that

$$F(xyx) = F(x)y^*x^* + xf(y)x^* + xyf(x)(x, y \in R) \tag{2.1}$$

Replacing x by $x + z(z \in R)$ in (2.1), we have

$$F(xyz + zyx) = F(x)y^*z^* + F(z)y^*x^* + xf(y)z^* + zf(y)x^* + xyf(z) + zyf(x)(x, y, z \in R) \tag{2.2}$$

Replacing z by x^2 in (2.2) and using (2.1), we have

$$\begin{aligned} F(x^2yx + x^2yx) = F(x)y^*(x^*)^2 + F(x^2)y^*x^* + xf(y)(x^*)^2 + x^2f(y)x^* + xyf(x)x^* \\ +xyxf(x) + x^2yf(x)(x, y \in R) \end{aligned} \tag{2.3}$$

On the other hand, substituting $xy + yx$ for y in (2.1), we have

$$\begin{aligned} F(x^2yx + x^2yx) &= F(x)y^*(x^*)^2 + F(x)x^*y^*x^* + xf(y)(x^*)^2 + x^2f(y)x^* \\ &+xf(x)y^*x^* + xyf(x)x^* + x^2yf(x) + xyxf(x)(x, y \in R) \end{aligned} \tag{2.4}$$

Comparing (2.3) and (2.4), we get

$$F(x^2)y^*x^* = F(x)x^*y^*x^* + xf(x)y^*x^* \text{ for all } x, y \in R.$$

And so, we have $\{F(x^2)-F(x)x^*-xf(x)\}y^*x^* = 0$ for all $x, y \in R$. Now, setting $A(x) = F(x^2)-F(x)x^*-xf(x)$, we can write the above equation as follows:

$$A(x)y^*x^* = 0 \tag{2.5}$$

Substituting y^* for y in (2.5), we have

$$A(x)yx^* = 0 \tag{2.6}$$

Since R is semiprime, using $x^*A(x)yx^*A(x) = 0$ for all $y \in R$, we get

$$x^*A(x) = 0 \tag{2.7}$$

On the other hand, replacing y by $x^*yA(x)$, we have $A(x)x^*yA(x)x^* = 0$ for all $y \in R$. Since R is semiprime, we get

$$A(x)x^* = 0 \tag{2.8}$$

Substituting $x + y$ for x in (2.8), we obtain

$$A(x)y^* + B(x, y)x^* + A(y)x^* + B(x, y)y^* = 0 \tag{2.9}$$

where $B(x, y) = F(xy + yx) - F(x)y^* - F(y)x^* - xf(y) - yf(x)$. Substituting $-x$ for x in (2.9), we have

$$A(x)y^* + B(x, y)x^* - A(y)x^* - B(x, y)y^* = 0 \tag{2.10}$$

By (2.9) and (2.10), we get $2A(x)y^* + 2B(x, y)x^* = 0$ Since R is 2-torsion free, we obtain that

$$A(x)y^* + B(x, y)x^* = 0 \tag{2.11}$$

Right multiplication of (2.11) by $A(x)$ gives $A(x)y^*A(x) = 0$. Substituting y^* for y
we get $A(x)yA(x) = 0$ for all $y \in R$. Since R is semiprime, we obtain that $A(x) = 0$ for all $x \in R$, that is, $F(x^2) = F(x)x^* + xf(x)$ for all $x \in R$. □

References

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