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EXISTENCE RESULTS FOR CONES UNDER INTIMATE CONDITION

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Abstract. Results on existence of fixed point in K-metric space (where K is cone) are established under intimate conditions for two sequences of single and multivalued mappings. Certain known results are obtained as corollaries.

1 Introduction

With the gradual strength gained by functional analysis, the theory of cones in Banach space is more thoroughly developed. The reason is that cones in Banach spaces can be studied in optimization theory [1]. Hence, the existence results which were proved in several metric space, one among them is abstract spaces whose neighbourhoods are conic segments. Eisenfeld and Lakshmikantham [2] have established existence of fixed point in such space. This was further extended by Som [13] for Jungck [6] contraction condition.

The object of this paper is three fold (i) to consider two sequences of mappings in orbitally K-complete space (ii) to consider intimate pair condition which is different from compatibility [6] (iii) to consider a much wider contraction inequality. With these objects we prove existence of fixed point in K-metric space. We claim that our results improve and extend the results of Eisenfeld and Lakshmikantham [2] and Som [13] as well.

2 Preliminaries

Definition 2.1. Let E be a real Banach space. A subset K of E is said to be a cone if (i) K is closed (ii) if $u, v \in K$ then $\alpha u + \beta v \in K$ for all $\alpha, \beta > 0$ (iii) $K \cap -K = \{\theta\}$, where θ is the zero of the space E. (iv) $K^o \neq \phi$ where K^o is the interior of K. We say $u \geq v$ if $u - v \in K$.

The cone is said to be normal if there is $\delta > 0$ such that $\|u + v\| > \delta$ for $u, v \in K$ and $\|u\| = \|v\| = 1$.

Definition 2.2. Let X be a nonempty set and K a cone. A function $\rho : X \times X \to K$ is said to be a K-metric on X if and only if (i) $\rho(x, y) = \theta$ if and only if $x = y$ (ii) $\rho(x, y) = \rho(y, x)$ (iii) $\rho(x, y) \leq \rho(x, z) + \rho(z, y)$.

Definition 2.3. A sequence $\{x_n\}$ in a K-metric space X is said to converge to $x_0 \in X$ if for each $u \in K^o$, there exists a positive integer N such that $\rho(x_n, x_0) \leq u$ for $n \geq N$.

A sequence $\{x_n\}$ in X is a Cauchy sequence if for each $u \in K^o$, there exists a positive integer N such that

$$\rho(x_n, x_m) \leq u \text{ for } n, m \geq N.$$

The K-metric space (X, ρ) is said to be complete if every Cauchy sequence in X converges.

Definition 2.4. Let $\{x_n\}$ be a monotonically nondecreasing sequence of the elements of E, which is partially ordered by means of the cone K, that is, $x_1 \leq x_2, \cdots \leq x_n \ldots$. Then the sequence $\{x_n\}$ is said to be bounded if there exists an element $y \in E$ such that $x_n \leq y$ (n = 1,2, . . .).

In the sequel, the spaces, in which every bounded monotonic sequence has a limit are called regularly partially ordered. A cone which generates a regular partial ordering is said to be regular.

Let K be a cone of a real Banach space E. Let ψ be a mapping from K into itself. The mapping ψ is said to be monotone if $\psi u > \psi v$ whenever $u > v$. The ψ is said to be upper semi-continuous from the right, if whenever $\{u_n\}$ and $\{\psi u_n\}$ are both decreasing and convergent sequence then $\lim \psi v_n \leq \psi \lim v_n$.

Definition 2.5. We define a mapping ψ from cone segment $0 < u < u_0$ into itself satisfying:

(i) $\psi^{n+1} u_0 < \psi^n u_0, u_0 \neq \theta, n = 0, 1, 2, \ldots$ and if $\bar{u} = \lim_{n\to\infty} \psi^n u_0$ exists, then $\psi \bar{u} \leq \bar{u}$,

(ii) ψ is upper semi-continuous from the right and K is regular,

(iii) $\psi u_0 < u_0$,

(iv) $\psi u = u$ iff $u = \theta$,

(v) ψ is monotone.

Definition 2.6. Let (X, ρ) be a K-metric space. A subset C of X is said to be proximinal if for each $x \in X$, there exists a point $y \in C$ such that $\rho(x, y) = D(x, C)$.

We shall use the following notations.

$P(X) = \Big\{A : A$ is nonempty proximinal subset of $X\Big\}$,

$CB(X) = \Big\{A : A$ is nonempty closed and bounded subset of $X\Big\}$,

$C(X) = \Big\{A : A$ is nonempty compact subset of $X\Big\}$.

For nonempty subsets A and B of X and $x \in X$:

$D(X, A) = \inf\Big\{\rho(x, a) : a \in A\Big\}$,

$H(A, B) = \max\Big\{\sup\big\{D(a, B) : a \in A\big\}, \sup\big\{D(A, b) : b \in B\big\}\Big\}$.

Remark 2.7. It is well known that $CB(X)$ is a metric space with distance function H. Thus $(CB(X), H)$ is complete metric space in the event that (X, ρ) is K-complete.

It is obvious that $P(X) \subset CB(X)$. Therefore $P(X)$ is also complete induced by distance function H.

Definition 2.8. Let $\{A_i\}_{i\in N} : X \to P(X)$ and $\{S_i\}_{i\in N} : X \to X$ be two sequences of mappings.

If, for $x_0 \in X$, there exist sequences $\{x_n\}$ and $\{y_n\}$ in X such that

$$y_{2n+1} = S_j x_{2n+1} \in A_i x_{2n}$$
$$y_{2n+2} = S_i x_{2n+2} \in A_j x_{2n+1}$$

for fixed $i, j \in N$ and all $n = 0, 1, 2, \ldots$. Then $O(A_i, A_j, S_i, S_j; x_0) = \{y_n : n \in N\}$ is said to be orbit at x_0 for the mappings $\{A_i, A_j, S_i, S_j\}$. Further $O(A_i, A_j, S_i, S_j; x_0)\}$ is said to be regular orbit for mappings $\{A_i, A_j, S_i, S_j\}$ if

$$\rho(y_n, y_{n+1}) \leq \begin{cases} H(A_i x_{n-1}, A_j x_n) & \text{when n is odd} \\ H(A_j x_{n-1}, A_i x_n) & \text{when n is even} \end{cases}$$

Definition 2.9. If, for $x_0 \in X$ there exist sequences $\{x_n\}$ and $\{y_n\}$ in X such that every sequence of the form $O(A_i, A_j, S_i, S_j; x_0)\}$ converges in X, then X is said to be $\{A_i, A_j, S_i, S_j\}-$orbitally K-complete with respect to x_0 or simply $\{A_i, A_j, S_i, S_j\}-$orbitally K-complete.

Definition 2.10. [11] Let A and S be two self maps of K-metric space (X, ρ). The pair $\{A, S\}$ is said to be S-intimate iff

$$\alpha\rho(SAx_n, Sx_n) \leq \alpha\rho(AAx_n, Ax_n),$$

where $\alpha = \limsup$ or $\liminf$ and $\{x_n\}$ is a Cauchy sequence in X such that

$$\lim_{n\to\infty} Sx_n = \lim_{n\to\infty} Ax_n = t \text{ for some } t \in X.$$

Definition 2.11. Let (X, ρ) be a K-metric space and $\{A_i\}_{i\in N} : X \to P(X)$ and $\{S_i\}_{i\in N} : X \to X$ be two sequences of multivalued and single valued mappings respectively. Then the pair $\{A_i, S_i\}$ is said to be S_i-intimate iff

$$\alpha\rho(S_i y_n, S_i x_n) \leq \alpha D(A_i y_n, y_n).$$

For all sequences $\{x_n\}$ and $\{y_n\}$ in X such that $y_n \in A_i x_n$, $S_i x_n \in X$ and $\lim_{n\to\infty} \rho(A_i x_n, S_i x_n) = 0$ for fixed $i \in N$, where $\alpha = \lim sup$ or $\liminf$.

Remark 2.12. If $A_i = A$ and $S_i = S$ are single valued mappings, then Definition 2.11 reduces to those of Sahu et al. [11].

Example 2.13. Let $X = [0, 1]$ with $\rho(x, y) = |x - y|$ and A, S are self mappings of X defined as follows:

$$Ax = \frac{1}{1+x} \text{ and } Sx = \frac{2}{x+2} \text{ for all } x \in [0, 1].$$

Now the sequence $\{x_n\}$ in X is defined as $x_n = \frac{1}{n}, n \in N,$. Then we have

$$\lim_{n\to\infty} Sx_n = \lim_{n\to\infty} \frac{2}{1/n+2} = 1$$

and

$$\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} \frac{2}{1+1/n} = 1.$$

Now,

$$|SAx_n - Sx_n| \to \frac{1}{3} \text{ and } |AAx_n - Ax_n| \to \frac{1}{2} \text{ as } n \to \infty.$$

Thus, we have

$$\lim_{n\to\infty} |SAx_n - Sx_n| < \lim_{n\to\infty} |AAx_n - Ax_n|.$$

Hence $\{A, S\}$ is S-intimate.

3 Main Results

Theorem 3.1. *Let (X, ρ) be a K-metric space. Let $\{A_i\}_{i\in N} : X \to P(X)$ and $\{S_i\}_{i\in N} : X \to X$ be two sequences of multivalued and single valued mappings respectively such that the following conditions hold:*

(i)

$$A_i x \subset S_j(X) \text{ and } A_j x \subset S_i(X), \text{ for all } x \in X; i, j \in N, \tag{3.1}$$

where i is even when j is odd and i is odd when j is even,

(ii)

$$H(A_i x, A_j y) \leq \psi\Big(\max\Big\{\rho(S_i x, S_j y), \frac{D(A_i x, S_i x) + D(A_j y, S_j x)}{2}, \frac{D(A_i x, S_j y) + D(A_j y, S_i x)}{2}\Big\}\Big) \tag{3.2}$$

for all $x, y \in X$ and ψ is defined as Definition 2.5,

(iii)

$$\text{orbit } O(A_i, A_j, S_i, S_j, x_0) \text{ is regular and for fixed } i \in N \tag{3.3}$$

$S_i(X)$ is $(A_i, A_j, S_i, S_j, x_0)-$orbitally K-complete,

(iv)

$$\text{the pairs } \{A_i, S_i\} \text{ and } \{A_j, S_j\} \text{are } S_i \text{ and } S_j - \text{intimate respectively.} \tag{3.4}$$

Then there exists a unique point z in X such that

$$z = S_i z = S_j z \in A_i z \cap A_j z \text{ for } i, j \in N.$$

Proof. Let for $x_0 \in X$ and fixed $i, j \in N$, there exists $x_1 \in X$ such that $y_1 = S_j x_1 \in A_i x_0$ and for $x_1 \in X$, there exists $x_2 \in X$ such that $y_2 = S_i x_2 \in A_j x_1$. Thus continuing this process, there exist two sequences $\{x_n\}$ and $\{y_n\}$ in X such that

$$y_{2n+1} = S_j x_{2n+1} \in A_i x_{2n}$$
$$y_{2n+2} = S_i x_{2n+2} \in A_j x_{2n+1}$$

for $n = 0, 1, 2, \dots$ and fixed $i, j \in N$, which is possible by virtue of (3.1). If $x_0 \in X$ is such that

$$\rho(y_2, y_1) \le H(A_j x_1, A_i x_0) < \psi(u_0).$$

Then we shall show that the sequence $\{y_n\}$ generated in X is a Cauchy sequence. Now for this,

$$\begin{aligned}
\rho(y_3, y_2) &\le H(A_i x_2, A_j x_1)\\
&\le \psi\Big(\max\Big\{\rho(S_i x_2, S_j x_1), \frac{D(A_i x_2, S_i x_2) + D(A_j x_1, S_j x_1)}{2},\\
&\qquad \frac{D(A_i x_2, S_j x_1) + D(A_j x_1, S_i x_2)}{2}\Big\}\Big)\\
&\le \psi\Big(\max\Big\{\rho(y_2, y_1), \frac{\rho(y_3, y_2) + \rho(y_2, y_1)}{2}, \frac{\rho(y_3, y_1)}{2}\Big\}\Big)\\
&\le \psi\Big(\max\Big\{\rho(y_2, y_1), \frac{\rho(y_3, y_2) + \rho(y_2, y_1)}{2}, \frac{\rho(y_3, y_2) + \rho(y_2, y_1)}{2}\Big\}\Big)\\
&\le \psi\big(\max\big\{\rho(y_2, y_1), \rho(y_3, y_2)\big\}\big).
\end{aligned}$$

Suppose that $\rho(y_3, y_2) > \rho(y_2, y_1)$ in above inequality, then we obtain

$$\rho(y_3, y_2) \le \psi(\rho(y_3, y_2)),$$

which is a contradiction by definition of ψ. Therefore,

$$\rho(y_3, y_2) \le \psi(\rho(y_2, y_1)).$$

Again for,

$$\begin{aligned}
\rho(y_{2n+1}, y_{2n}) &\le H(A_i x_{2n}, A_j x_{2n-1})\\
&\le \psi\Big(\max\Big\{\rho(S_i x_{2n}, S_j x_{2n-1}), \frac{D(A_i x_{2n}, S_i x_{2n}) + D(A_j x_{2n-1}, S_j x_{2n-1})}{2},\\
&\qquad \frac{D(A_i x_{2n}, S_j x_{2n-1}) + D(A_j x_{2n-1}, S_i x_{2n})}{2}\Big\}\Big)\\
&\le \psi\Big(\max\Big\{\rho(y_{2n}, y_{2n-1}), \frac{\rho(y_{2n+1}, y_{2n}) + \rho(y_{2n}, y_{2n-1})}{2}, \frac{\rho(y_{2n+1}, y_{2n-1})}{2}\Big\}\Big)\\
&\le \psi\big(\max\big\{\rho(y_{2n}, y_{2n-1}), \rho(y_{2n+1}, y_{2n})\big\}\big).
\end{aligned}$$

Suppose that $\rho(y_{2n+1}, y_{2n}) > \rho(y_{2n}, y_{2n-1})$ in above inequality, then we obtain

$$\rho(y_{2n+1}, y_{2n}) \le \psi(\rho(y_{2n+1}, y_{2n}))$$

which is a contradiction by definition of ψ.

Hence $\rho(y_{2n+1}, y_{2n}) \le \psi(\rho(y_{2n}, y_{2n-1}))$. So, in general $\rho(y_{n+1}, y_n) \le \psi(\rho(y_n, y_{n-1}))$ for all $n \ge 1$.

Consider $u_n = \psi^n u_0$. Thus in view of Lemma 3.1, 3.2 of [2], $\{u_n\}$ decreases and converges to a fixed point of ψ, which is, θ by Definition 2.5. Since by the monotinicity of ψ and $H(A_j x_i, A_i x_0) < \psi(u_0)$, we obtain

$$\rho(y_2, y_1) < u_1$$

and

$$\rho(y_3, y_2) \le \psi(\rho(y_2, y_1)) < \psi u_1 = u_2.$$

Similarly, one can show further that

$$\rho(y_{n+1}, y_n) < u_n; n = 1, 2, 3, \ldots$$

letting $n \to \infty$, we obtain

$$\lim_{n\to\infty} \rho(y_{n+1}, y_n) = \theta.$$

Now, for $m > n$,

$$\begin{aligned}\rho(y_n, y_m) &\leq \rho(y_n, y_{n+1}) + \rho(y_{n+1}, y_{n+2}) + \cdots + \rho(y_{m-1}, y_m)\\ &< u_n + u_{n+1} + \cdots + u_{m-1}\end{aligned}$$

which tends to θ as $n \to \infty$.

Hence $\{y_n\}$ is a Cauchy sequence in X. As the subsequence $\{y_{2n}\}$ is in $S_i(X)$, it is also Cauchy. Since $S_i(X)$ is $\{A_i, A_j, S_i, x_0\}$-orbitally K-complete, $\{y_{2n}\}$ converges to a point z in $S_i(X)$. Let $z = S_i u$ for some $u \in X$ and fixed $i \in N$. Therefore $y_n \to z$.

Now (3.2) implies that

$$\begin{aligned}D(z, A_i u) &\leq \rho(z, y_{2n+2}) + D(y_{2n+2}, A_i u)\\ &\leq \rho(z, y_{2n+2}) + H(A_i u, A_j x_{2n+1}) \leq \rho(z, y_{2n+2}) + \psi\Big(\max\Big\{\rho(S_i u, S_j x_{2n+1}),\\ &\qquad \frac{D(A_i u, S_i u) + D(A_j x_{2n+1}, S_j x_{2n+1})}{2}, \frac{D(A_i u, S_j x_{2n+1}) + D(A_j x_{2n+1}, S_i u)}{2}\Big\}\Big)\\ &\leq \rho(z, y_{2n+2}) + \psi\Big(\max\Big\{\rho(z, y_{2n+1}), \frac{D(A_i u, z) + \rho(y_{2n+2}, y_{2n+1})}{2},\\ &\qquad \frac{D(A_i u, y_{2n+1}) + \rho(y_{2n+1}, z)}{2}\Big\}\Big)\end{aligned}$$

letting $n \to \infty$, we get

$$\begin{aligned}D(z, A_i u) &\leq \psi\Big(\max\Big\{0, \frac{D(A_i u, z)}{2}\Big\}\Big)\\ D(z, A_i u) &\leq \psi\Big(\frac{D(A_i u, z)}{2}\Big),\end{aligned}$$

which is a contradiction, so that $z \in A_i u$ for fixed $i \in N$.

Since $z \in A_i u \subset S_j(X)$ for fixed $i, j \in N$, then there exists a point v in X such that

$$z = S_j v \in A_i u.$$

Again (3.2) implies that

$$\begin{aligned}D(z, A_j v) &\leq H(A_i u, A_j v)\\ &\leq \psi\Big(\max\Big\{\rho(S_i u, S_j v), \frac{D(A_i u, S_i u) + D(A_j v, S_j v)}{2},\\ &\qquad \frac{D(A_i u, S_j v) + D(A_j v, S_i u)}{2}\Big\}\Big)\\ &\leq \psi\Big(\max\Big\{0, \frac{D(A_j v, z)}{2}\Big\}\Big)\\ &\leq \psi\Big(\frac{D(A_j v, z)}{2}\Big),\end{aligned}$$

which is a contradiction, so that $z \in A_j v$.

Thus, we obtained $z = S_i u \in A_i u$ and $z = S_j v \in A_j v$. Since $\{A_i, S_i\}$ is S_i-intimate, then

$$\rho(S_i z, z) \leq D(A_i z, z). \tag{3.5}$$

Now from (3.2)

$$\begin{aligned}
D(z,A_iz) &\le H(A_jv,A_iz) = H(A_iz,A_jv)\\
&\le \psi\Big(\max\Big\{\rho(S_iz,S_jv),\frac{D(A_iz,S_iz)+D(A_jv,S_jv)}{2},\frac{D(A_iz,S_jv)+D(A_jv,S_iz)}{2}\Big\}\Big)\\
&\le \psi\Big(\max\Big\{\rho(z,S_iz),\frac{D(A_iz,S_iz)}{2},\frac{D(A_iz,z)+\rho(z,S_iz)}{2}\Big\}\Big)\\
&\le \psi\Big(\max\Big\{D(z,A_iz),\frac{D(A_iz,z)+\rho(z,S_iz)}{2},\frac{D(A_iz,z)+\rho(z,S_iz)}{2}\Big\}\Big)\\
&\le \psi\Big(D(z,A_iz)\Big)
\end{aligned}$$

which is a contradiction, therefore $z \in A_iz$. From (3.5) $z = S_iz \in A_iz$.

Similarly, when the pair $\{A_j, S_j\}$ is S_j-intimate. We obtain $z = S_jz \in A_jz$. Therefore,

$$z = S_iz = S_jz \in A_iz \cap A_jz \text{ for all } i, j \in N.$$

UNIQUENESS. Let we consider w as another common fixed point of A_i, A_j, S_i and S_j such that $z \neq w$. Therefore,

$$\begin{aligned}
\rho(z,w) &\le H(A_iz,A_jw)\\
&\le \psi\Big(\max\Big\{\rho(S_iz,S_jw),\frac{D(A_iz,S_iz)+D(A_jw,S_jw)}{2},\\
&\qquad \frac{D(A_iz,S_jw)+D(A_jw,S_iz)}{2}\Big\}\Big)\\
&\le \psi\Big(\max\Big\{\rho(z,w),0,\frac{\rho(z,w)+\rho(z,w)}{2}\Big\}\Big)\\
&\le \psi(\rho(z,w))
\end{aligned}$$

which is a contradiction. Therefore, $z = w$.

Remark 3.2. In light of above fact, generalization and improvement of the results of Fisher [3], Hadzic [4], Jungck [5], Kang et al. [8], Sessa et al. [12] can be obtained in abstract spaces from our Theorem 3.1 by only simple modification in condition (3.2).

Remark 3.3. It is well known that if A is multivalued mapping from X into $C(X)$, then for every $x, y \in X$ and $u \in Ax$, there exists a point $v \in Ay$ such that

$$\rho(u, v) \le H(Ax, Ay).$$

Hence, if A_i and A_j are multivalued mappings form X into $C(X)$ then orbital regularity in Theorem 3.1 can be dropped.

Corollary 3.4. *Let (X, ρ) be a K-metric space. Let $A, B : X \to P(X)$ and $S, T : X \to X$ such that the following conditions hold:*

(i)

$$Ax \subset T(X) \text{ and } Bx \subset S(X) \text{ for all } x \in X, \tag{3.6}$$

(ii)

$$\begin{aligned}
H(Ax,By) \le \psi\Big(\max\Big\{\rho(Sx,Ty),&\frac{D(Ax,Sx)+D(By,Ty)}{2},\\
&\frac{D(Ax,Ty)+D(By,Sx)}{2}\Big\}\Big)
\end{aligned} \tag{3.7}$$

for all $x, y \in X$,

(iii)

$$\text{orbit } O(A,B,S,T,x_0) \text{ is regular and } S(X) \text{ is } \{A,B,S,T,x_0) - orbitally\ K\text{-complete}, \quad (3.8)$$

(iv)

$$\text{the pairs } \{A,S\} \text{ and } \{B,T\} \text{ are S and T-intimate respectively.} \quad (3.9)$$

Then there exists a point z in X such that

$$z = Sz = Tz \in Az \cap Bz.$$

Proof. If we consider

$$A_i = A, A_j = B, S_i = S \text{ and } S_j = T.$$

Then Corollary 3.4 follows from Theorem 3.1.

Corollary 3.5. *Let A, B, S and T be self mappings of K-metric space (X,ρ) such that the following conditions hold:*

(i)

$$A(X) \subset T(X) \text{ and } B(X) \subset S(X) \text{ for all } x \in X, \quad (3.10)$$

(ii)

$$\rho(Ax,By) \leq \psi\rho(Sx,Ty) \text{ for all } x,y \in X, \quad (3.11)$$

(iii)

$$S(X) \text{ is K-complete}, \quad (3.12)$$

(iv)

$$\text{the pairs } \{A,S\} \text{ and } \{B,T\} \text{ are S and T-intimate respectively.} \quad (3.13)$$

Then there exists a point z in X such that $z = Sz = Tz = Az = Bz$.

Proof. If we define

$$\max\left\{\rho(Sx,Ty), \frac{\rho(Ax,Sx)+\rho(By,Ty)}{2}, \frac{\rho(Ax,Ty)+\rho(By,Tx)}{2}\right\} = \rho(Sx,Ty)$$

in corollary 3.4. Then we have the corollary 3.5.

Remark 3.6. If we consider $A = B$ and $S = T$ in corollary 3.5. Then the Theorem 1 of Som [13] is same as our corollary 3.5.

Remark 3.7. If we take $S = T = I$ (identity map) then the Theorem 3 of Som [13] is same as our corollary 3.5.

Remark 3.8. In case $A = B$ and $S = T = I$ (identity map). Then Theorem 4.1 of [2] is same as our corollary 3.5.

Theorem 3.9. *Let (X,ρ) be a K-metric space. Let $\{A_i\}_{i\in N} : X \to P(X)$ and $\{S_i\}_{i\in N} : X \to X$ be two sequences of mappings such that the conditions (3.1), (3.3), (3.4) are satisfied and*

$$\begin{aligned} H(A_ix,A_jy) \leq \psi\Big(\sqrt{\Big\{}\max\Big\{\rho^2(S_ix,S_jy), \quad (3.14)\\ D(S_ix,A_ix).D(S_jy,A_jy), D(S_ix,A_jy).D(S_jy,A_ix),\\ D(S_ix,A_ix).D(S_jy,A_ix), D(S_ix,A_jy).D(S_jy,A_jy)\Big\}\Big\}\Big) \end{aligned}$$

for all $x,y \in X$. Then there exists a point z in X such that

$$z = S_iz = S_jz \in A_iz \cap A_jz.$$

Proof. If we define $\max\{t_1^2, t_1t_2, 0, 0, (t_1+t_2).t_2\} = t_1^2$ or $\max\{0,0,0,t_1^2,0\} = t_1^2$ or $\max\{t_1^2, 0, t_1^2, 0, t_1t_2\} = t_1^2$, where t_1, t_2 and $t_1 + t_2 < u_0$.

Then Theorem 3.9 can be obtained by proceeding in the same fashion as in Theorem 3.1.

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