Palestine Journal of Mathematics

Vol. 4(2) (2015) , 348–359

ON RANDOM COINCIDENCE POINT AND RANDOM COUPLED FIXED POINT THEOREMS

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Communicated by Ayman Badawi

MSC 2010 Classifications: Primary 46T99, 47H10; Secondary 54H25, 54E50.

Keywords and phrases: Coupled random coincidence point, Common Fixed Point, Mixed g- monotone, Measurable mapping, Random operator.

The authors thank the referees for their careful reading of the manuscript and for their suggestions.

Abstract In this article we review the notion of random coupled coincidence and prove a random coupled fixed point theorem under a certain set of conditions.

1 Introduction and Preliminaries

Probabilistic functional analysis is one of the most crucial research fields in mathematics because, it has applications to probabilistic models in real world problems. In particular, probabilistic functional analysis is needed for the study of various classes of random equations and random operators which form the center of this field. Random fixed point theorems are stochastic generalizations of classical fixed point theorems for random operators. Because of their importance, the study of random fixed points and random approximations attracted considerable attention from a lot of authors, see e.g. [4]-[8], [14, 11],[16, 17] [21, 22],[27]-[30],[37]-[38].

One of the recent trends, initiated by Ran and Reurings [28], in fixed point theory is to study the existence and uniqueness of certain operators in the context of partially ordered metric spaces (see e.g. [1]-[3],[5, 9, 12, 25, 26, 34]). In 1987, Guo and Lakshmikantham [15] introduced the concept of coupled coincidence point of a mapping $F : X \times X \to X$ where X is a metric space. In 2006, Gnana-Bhaskar and Lakshmikantham [9] introduced the mixed monotone property for F and studied fixed point theorems in partially ordered metric spaces. Moreover, the authors [9] discussed the existence and uniqueness of solution for a periodic boundary value problem as an application. Later Karapınar [18, 19] extended the results of Bhaskar and Lakshmikantham to cone metric spaces. In [20] Lakshmikantham and Ćirić studied common/coincidence coupled fixed point theorems for nonlinear contractions in partially ordered metric spaces. Coupled fixed point theorems is one of the active research areas in fixed point theory (see e.g. [10, 13, 18, 19, 23, 33, 34, 32] and the references there in). The same authors, Ciric and Lakshmikantham [12], also considered random coupled fixed point and random coupled coincidence point theorems for a pair of random mappings $F : \Omega \times (X \times X) \to X$ and $g : \Omega \times X \to X$ under certain contractive conditions.

We recall some basic notions and fundamental results in the literature. A triple $(X, d, \preceq)$ is called a partially ordered metric space if the pair $(X, \preceq)$ is a partially ordered set endowed with a metric d on X. A partially ordered metric space $(X, d, \preceq)$ is called ordered complete, if for each convergent sequence $\{x_n\}_{n=0}^{\infty} \subset X$, the following condition holds: either

- if $\{x_n\}$ is a non-increasing sequence in X such that $x_n \to x^*$ implies $x^* \preceq x_n$ $\forall n \in \mathbb{N}$, or
- if $\{x_n\}$ is a non-decreasing sequence in X such that $x_n \to x^*$ implies $x_n \preceq x^*$ $\forall n \in \mathbb{N}$.

Recall that if $(X, \preceq)$ is partially ordered set and $F : X \to X$ is a mapping such that for $x, y \in X$, $x \preceq y$ implies $F(x) \preceq F(y)$ then F is said to be nondecreasing.

Definition 1.1. (See [9]) Let $(X, \preceq)$ be an ordered set and $F : X \times X \to X$ be a mapping. Then F is said to has the mixed monotone property if F is monotone non-decreasing in its first argument and is monotone non-increasing in its second argument, that is, for any $x, y \in X$,

$$x_1, x_2 \in X, x_1 \preceq x_2 \Rightarrow F(x_1, y) \preceq F(x_2, y) \quad \text{for all } y \in X,$$

and

$$y_1, y_2 \in X, y_1 \preceq y_2 \Rightarrow F(y_2, x) \preceq F(y_1, x) \quad \text{for all } x \in X. \tag{1.1}$$

Definition 1.2. (See [9]) An element $(x, y) \in X$ is called a coupled fixed point of the mapping $F : X \times X \to X$ if,

$$F(x, y) = x, \, F(y, x) = y.$$

Definition 1.3. (See [12]) Let $(X, \preceq)$ be an ordered set, $F : X \times X \to X$ and $g : X \to X$. The mapping F is said to have mixed g-monotone property if $F(x, y)$ is monotone g-nondecreasing in x and is monotone g-nonincreasing in y; that is, for any $x, y \in X$,

$$x_1, x_2 \in X, gx_1 \preceq gx_2 \Rightarrow F(x_1, y) \preceq F(x_2, y) \quad \text{for all } y \in X,$$

and

$$y_1, y_2 \in X, gy_1 \preceq gy_2 \Rightarrow F(y_2, x) \preceq F(y_1, x) \quad \text{for all } x \in X.$$

Definition 1.4. (See [12]) An element $(x, y) \in X$ is called a coupled fixed point of the mapping $F : X \times X \to X$ if,

$$F(x, y) = gx, \, F(y, x) = gy.$$

Definition 1.5. (See [12]) Let X be a non empty set. Then we say that the mappings $F : X \times X \to X$ and $g : X \to X$ are commutative if

$$gF(x, y) = F(gx, gy).$$

For the sake of consistency, throughout the paper, we follow the notations of Ćirić, V. Lakshmikantham [12]: Let (Ω, Σ) be a measurable space, where Σ is a sigma algebra of subsets of Ω. Let (X, d) be a metric space. A mapping $T : \Omega \to X$ is called σ-measurable if for any open subset U of X, the set $T^{-1}(U) = \{\omega : T(\omega) \in U\} \in \Sigma$. Notice that when we say that a set A is "measurable" we mean that A is σ-measurable. A mapping $T : \Omega \to X$ is called a random operator if $T(\cdot, x)$ is measurable for any $x \in X$. A measurable mapping $\zeta : \Omega \to X$ is called a random fixed of $T : \Omega \to X$ if $\zeta(\omega) = T(\omega, \zeta(\omega))$ for all $\omega \in \Omega$. A measurable mapping $\zeta : \Omega \to X$ is called random coincidence of $T : \Omega \to X$ and $g : \Omega \to X$ if $g(\omega, \zeta(\omega)) = T(\omega, \zeta(\omega))$ for all $\omega \in \Omega$.

Ćirić and Lakshmikantham [12] proved the following theorem:

Theorem 1.6. *Let $(X, \preceq)$ be a partially ordered set, (X, d) be a complete separable metric space, and (Ω, Σ) be a measurable space. Let $F : \Omega \times (X \times X) \to X$ and $g : \Omega \times X \to X$ be mappings such that there is a non negative real number k with*

$$d(F(\omega, (x, y)), F(\omega, (u, v))) \leq \frac{k}{2}[d(g(\omega, x), g(\omega, u)) + d(g(\omega, y), g(\omega, v))] \tag{1.2}$$

for all $x, y, u, v \in X$ with $g(\omega, x) \preceq g(\omega, u)$ and $g(\omega, v) \preceq g(\omega, y)$ for all $\omega \in \Omega$. Assume that F and g satisfy the following conditions:

(i) $F(\omega, \cdot)$ and $g(\omega, \cdot)$ are continuous for all $\omega \in \Omega$,

(ii) $F(\cdot, v), g(.\cdot, x)$ are measurable for all $v \in X \times X$ and $x \in X$, respectively,

(iii) $F(\omega \times X) \subseteq X$ for each $\omega \in \Omega$

(iv) g is continuous and commutes with F and also suppose either

(a) F is continuous or

(b) X is ordered complete

If there exist measurable mappings $\eta_0, \theta_0 \in X$ such that $g(\omega, \eta_0(\omega) \preceq F(\omega, (\eta_0, \theta_0))$ and $F(\omega, (\eta_0(\omega), \theta_0(\omega))) \preceq g(\omega, \theta_0(\omega))$, then there are measurable mappings $\eta, \theta : \Omega \to X$ such that $g(\omega, \eta(\omega)) = F(\omega, (\eta(\omega), \theta(\omega)))$ and $F(\omega, (\eta(\omega), \theta(\omega))) = g(\omega, \theta(\omega))$, for all $\omega \in \Omega$, that is, F and g have a random coupled coincidence.

The aim of this paper is to prove some random coupled fixed point theorems for mixed g-monotone operator in the context of ordered metric spaces involving following functions.

2 Main Result

In this section, we study random version of a random coupled fixed point theorem for a pair of random mappings $F : \Omega \times (X \times X) \to X$ and $g : \Omega \times X \to X$ under the set of conditions.

Let $(X, \preceq)$ be a partially ordered set and d a metric on X such that (X, d) is a complete metric space. Furthermore, we consider the product space $X \times X$ with the following partial order:

$$if \ \ (x, y), (u, v) \in X \times X, (x, y) \preceq_2 (u, v) \Longleftrightarrow x \preceq u \text{ and } y \geq v.$$

Let Δ denote the class of those functions $\beta : [0, \infty) \to [0, \infty)$ which satisfy the following condition

$$\beta(t_n) \to 1 \quad \text{implies} \quad t_n = 0.$$

Now we present our main result which is given as follows.

Theorem 2.1. *Let $(X, \preceq)$ be a partially ordered set, (X, d) be a complete separable metric space, and (Ω, Σ) be a measurable space. Let $F : \Omega \times (X \times X) \to X$ and $g : \Omega \times X \to X$ be mappings such that,*

$$d(F(\omega, (x, y)), F(\omega, (u, v))) \leq \beta(M_\omega(x, y, u, v,))M_\omega(x, y, u, v) \tag{2.1}$$

where

$$M_\omega(x, y, u, v) = \max\{d(g(\omega, x), g(\omega, u)), d(g(\omega, y), g(\omega, v))\}$$

for all $x, y, u, v \in X$ and $\beta \in \Delta$ with $g(\omega, x) \preceq g(\omega, u)$ and $g(\omega, v) \preceq g(\omega, y)$ for all $\omega \in \Omega$. Assume that F and g satisfy the following conditions:

(i) $F(\omega, \cdot), g(\omega, \cdot)$ are continuous for all $\omega \in \Omega$,

(ii) $F(\cdot, v), g(.\cdot, x)$ are measurable for all $v \in X \times X$ and $x \in X$, respectively,

(iii) $F(\omega \times X) \subseteq X$ for each $\omega \in \Omega$

(iv) g is continuous and commutes with F and also suppose either

(a) F is continuous or

(b) X is ordered complete

If there exist measurable mappings $\eta_0, \theta_0 \in X$ such that $g(\omega, \eta_0(\omega)) \preceq F(\omega, (\eta_0(\omega), \theta_0(\omega)))$ and $F(\omega, (\eta_0(\omega), \theta_0(\omega))) \preceq g(\omega, \theta_0(\omega))$, then there are measurable mappings $\eta, \theta : \Omega \to X$ such that $g(\omega, \eta(\omega)) = F(\omega, (\eta(\omega), \theta(\omega)))$ and $F(\omega, (\eta(\omega), \theta(\omega))) = g(\omega, \theta(\omega))$, for all $\omega \in \Omega$, that is, F and g have a random coupled coincidence.

Proof. Let $\Theta = \{\eta : \Omega \to X\}$ be a family of measurable mappings. We define a function $h : \Omega \times X \to R^+$ as follows,

$$h(\omega, x) = d(x, g(\omega, x)).$$

Since $x \to g(\omega, x)$ is continuous for all $\omega \in \Omega$, we find that $h(\omega, \cdot)$ is continuous for all $\omega \in \Omega$. Also, due to the fact that $\omega \to g(\omega, x)$ is measurable for $x \in X$, we derive that $h(\cdot, x)$ is measurable for all $\omega \in \Omega$ (in [38], pp. 868]). Hence, $h(\cdot, x)$ is a Caratheodory function. Thus, if $\eta : \Omega \to X$ is measurable, then $\omega \to h(\omega, \eta(\omega))$ is also measurable. Further, for each $\theta \in \Theta$ the function $\eta : \Omega \to X$ defined by $\eta(\omega) = g(\omega, \theta(\omega))$ is measurable; that is, $\eta \in \Theta$.

We shall construct two sequences of measurable mappings $\{\zeta_n\}$ and $\{\eta_n\}$ in Θ and two sequences $g(\omega, \zeta_n(\omega)))$ and $g(\omega, \eta_n(\omega)))$ in X as follows: Let $\zeta_0, \eta_0 \in \Theta$ be such that

$$g(\omega, \zeta_0(\omega)) \preceq F(\omega, (\zeta_0(\omega), \eta_0(\omega))) \ \ and \ \ g(\omega, \eta_0(\omega)) \geq F(\omega, (\eta_0(\omega), \zeta_0(\omega)))$$

for all $\omega \in \Omega$.

Since $F(\omega, (\zeta_0(\omega), \eta_0(\omega))) \in X = g(\omega \times X)$, (by a sort of Filippov's measurable implicit function theorem (see [5, 16, 17, 24])), there is $\zeta_1 \in \Theta$ such that

$$g(\omega, \zeta_1(\omega)) = F(\omega, (\zeta_0(\omega), \eta_0(\omega))).$$

Similarly, as $F(\omega, (\eta_0(\omega), \zeta_0(\omega))) \in X = g(\omega \times X)$, there is $\eta_1 \in \Theta$ such that

$$g(\omega, \eta_1(\omega)) = F(\omega, (\eta_0(\omega), \zeta_0(\omega))).$$

Thus $F(\omega, (\zeta_0(\omega), \eta_0(\omega)))$ and $F(\omega, (\eta_0(\omega), \zeta_0(\omega)))$ are well defined. Since

$$F(\omega, (\eta_1(\omega), \zeta_1(\omega))), \ \ F(\omega, (\zeta_1(\omega), \eta_1(\omega))) \in g(\omega \times X) \tag{2.2}$$

there are $\eta_2, \zeta_2 \in \Theta$ such that

$$g(\omega, \zeta_2(\omega)) = F(\omega, (\zeta_1(\omega), \eta_1(\omega))) \text{ and } g(\omega, \eta_2(\omega)) = F(\omega, (\eta_1(\omega), \zeta_1(\omega))).$$

Continuing this process we can construct sequences $\{\zeta_n(\omega)\}$ and $\{\eta_n(\omega)\}$ in X such that

$$g(\omega, \zeta_{n+1}(\omega)) = F(\omega, (\zeta_n(\omega), \eta_n(\omega))) \text{ and } g(\omega, \eta_{n+1}(\omega)) = F(\omega, (\eta_n(\omega), \zeta_n(\omega)))$$

for all $n \in N \cup \{0\}$.

In order to cover the proof more comprehensively we will divide it in several steps.
Step 1. First we will show that,

$$\begin{aligned} g(\omega, \zeta_n(\omega)) \preceq g(\omega, \zeta_{n+1}(\omega)), \text{ and,} \\ g(\omega, \eta_n(\omega)) \geq g(\omega, \eta_{n+1}(\omega)), \end{aligned} \tag{2.3}$$

for all $n \in N \cup \{0\}$.

For this purpose we use mathematical induction. Let $n = 0$. By the assumption we have

$$g(\omega, \zeta_0(\omega)) \preceq F(\omega, (\zeta_0(\omega), \eta_0(\omega))) \text{ and } g(\omega, \eta_0(\omega)) \geq F(\omega, (\eta_0(\omega), \zeta_0(\omega))).$$

Since

$$g(\omega, \zeta_1(\omega)) = F(\omega, (\zeta_0(\omega), \eta_0(\omega))) \text{ and } g(\omega, \eta_1(\omega)) = F(\omega, (\eta_0(\omega), \zeta_0(\omega))),$$

we have

$$g(\omega, \zeta_0(\omega)) \preceq g(\omega, \zeta_1(\omega)) \text{ and } g(\omega, \eta_0(\omega)) \geq g(\omega, \eta_1(\omega)).$$

Therefore (2.3) holds for $n = 0$. Suppose (2.3) holds for some fixed $n \geq 0$. Then, since

$$g(\omega, \zeta_n(\omega)) \preceq g(\omega, \zeta_{n+1}(\omega)) \text{ and } g(\omega, \eta_n(\omega)) \geq g(\omega, \eta_{n+1}(\omega))$$

and F is monotone g-nonincreasing in its first argument, we have

$$F(\omega, (\zeta_n(\omega), \eta_n(\omega))) \preceq F(\omega, (\zeta_{n+1}(\omega), \eta_n(\omega))) \tag{2.4}$$

and

$$F(\omega, (\eta_{n+1}(\omega), \zeta_n(\omega))) \preceq F(\omega, (\eta_n(\omega), \zeta_n(\omega))). \tag{2.5}$$

Also, since

$$g(\omega, \zeta_n(\omega)) \preceq g(\omega, \zeta_{n+1}(\omega)) \text{ and } g(\omega, \eta_n(\omega)) \geq g(\omega, \eta_{n+1}(\omega))$$

and F is monotone g-nondecreasing in its second argument, we have

$$F(\omega, (\zeta_{n+1}(\omega), \eta_{n+1}(\omega))) \geq F(\omega, (\zeta_{n+1}(\omega), \eta_n(\omega))) \tag{2.6}$$

and

$$F(\omega, (\eta_{n+1}(\omega), \zeta_n(\omega))) \geq F(\omega, (\eta_{n+1}(\omega), \zeta_{n+1}(\omega))). \tag{2.7}$$

Thus, from (2.4) - (2.7), we get

$$g(\omega, \zeta_{n+1}(\omega)) \preceq g(\omega, \zeta_{n+2}(\omega)) \text{ and } g(\omega, \eta_{n+1}(\omega)) \geq g(\omega, \eta_{n+2}(\omega)).$$

Thus by mathematical induction we conclude that (2.3) holds for all $n \in N \cup \{0\}$.
Step 2. In this step we prove

$$\lim_{n\to\infty} d(g(\omega, \zeta_n(\omega)), g(\omega, \zeta_{n+1}(\omega))) = 0$$

and

$$\lim_{n\to\infty} d(g(\omega, \eta_n(\omega)), g(\omega, \eta_{n+1}(\omega))) = 0.$$

For this, let $n \in N \cup \{0\}$. Then by (2.1) - (2.3), we have

$$\begin{aligned} d(g(\omega, \zeta_n(\omega)), g(\omega, \zeta_{n+1}(\omega))) &= d(F(\omega, (\zeta_{n-1}(\omega), \eta_{n-1}(\omega))), F(\omega, (\zeta_n(\omega), \eta_n(\omega)))) \\ &\leq \beta(t_1)t_1 \leq t_1 \end{aligned} \tag{2.8}$$

where

$$t_1 = M_\omega(\zeta_{n-1}(\omega), \zeta_n(\omega), \eta_{n-1}(\omega), \eta_n(\omega))$$

and

$$\begin{aligned} d(g(\omega, \eta_{n+1}(\omega)), g(\omega, \eta_n(\omega))) &= d(F(\omega, (\eta_n(\omega), \zeta_n(\omega))), F(\omega, (\zeta_{n-1}(\omega), \zeta_{n-1}(\omega)))) \\ &\leq \beta(t_2)t_2 \leq t_2 \end{aligned} \tag{2.9}$$

where

$$t_2 = M_\omega(\eta_{n-1}(\omega), \eta_n(\omega), \zeta_{n-1}(\omega), \zeta_n(\omega)).$$

By the use of two inequalities above, we have

$$M_\omega(\zeta_{n+1}(\omega), \zeta_n(\omega), \eta_{n+1}(\omega), \eta_n(\omega) \leq M_\omega(\eta_n(\omega), \eta_{n-1}(\omega), \zeta_n(\omega), \zeta_{n-1}(\omega))$$

and the sequence $\{M_\omega(\zeta_{n+1}(\omega), \zeta_n(\omega), \eta_{n+1}(\omega), \eta_n(\omega))\}$ is nonnegative nonincreasing. This implies that there exists $r \geq 0$ such that

$$\lim_{n\to\infty} M_\omega(\zeta_{n+1}(\omega), \zeta_n(\omega), \eta_{n+1}(\omega), \eta_n(\omega)) = r \tag{2.10}$$

We shall show that $r = 0$. Suppose, to the contrary, that $r > 0$. Taking (2.8) and (2.9) into account, we get

$$\frac{M_\omega(\zeta_{n+1}(\omega), \zeta_n(\omega), \eta_{n+1}(\omega), \eta_n(\omega))}{M_\omega(\zeta_n(\omega), \zeta_{n-1}(\omega), \eta_n(\omega), \eta_{n-1}(\omega))} \leq \beta(M_\omega(\zeta_n(\omega), \zeta_{n-1}(\omega), \eta_n(\omega), \eta_{n-1}(\omega))). \tag{2.11}$$

Letting $n \to \infty$ in the last inequality and by (2.10), we get

$$\lim_{n\to\infty} \beta(M_\omega(\zeta_n(\omega), \zeta_{n-1}(\omega), \eta_n(\omega), \eta_{n-1}(\omega))) = 1$$

and since $\beta \in \Delta$ this implies $r = 0$ and, consequently,

$$\lim_{n\to\infty} M_\omega(\zeta_n(\omega), \zeta_{n-1}(\omega), \eta_n(\omega), \eta_{n-1}(\omega)) = 0,$$

which shows our claim.

Step 3. In this step we prove $\{\zeta_n\}$ and $\{\eta_n\}$ are Cauchy sequences. On the contrary, assume that at least one of the sequences $\{\zeta_n\}$ or $\{\eta_n\}$ is not a Cauchy sequence. This implies that

$$\lim_{n\to\infty} d(g(\omega, \zeta_m(\omega)), g(\omega, \zeta_n(\omega))) \nrightarrow 0,$$

and

$$\lim_{n\to\infty} d(g(\omega, \eta_m(\omega)), g(\omega, \eta_n(\omega))) \nrightarrow 0.$$

Consequently,

$$\lim_{n,m\to\infty} M_\omega(\zeta_n(\omega), \zeta_m(\omega), \eta_n(\omega), \eta_m(\omega)) \nrightarrow 0.$$

This means that there exist $\epsilon > 0$ for which we can find subsequences $\{\zeta_n(k)\}$, $\{\eta_n(k)\}$, $\{\zeta_m(k)\}$ and $\{\eta_m(k)\}$ with $n(k) > m(k) > k$ such that

$$\lim_{n,m\to\infty} M_\omega(\zeta_{n(k)}(\omega), \zeta_{m(k)}(\omega), \eta_{n(k)}(\omega), \eta_{m(k)}(\omega)) \geq \epsilon. \tag{2.12}$$

Furthermore, corresponding to $m(k)$ we can choose $n(k)$ in such way that it is the smallest integer with $n(k) > m(k)$ satisfying (2.12).
Then

$$M_\omega(\zeta_{n(k)}(\omega), \zeta_{n(k)-1}(\omega), \eta_{n(k)}(\omega), \eta_{m(k)-1}(\omega)) < \epsilon. \tag{2.13}$$

Since $g(\omega, \zeta_{n(k)-1}(\omega)) \geq g(\omega, \zeta_{m(k)-1}(\omega))$ and $g(\omega, \eta_{n(k)-1}(\omega)) \preceq g(\omega, \eta_{m(k)-1}(\omega))$, using the contractive condition we can obtain

$$\begin{aligned} &\frac{M_\omega(\zeta_{n(k)}(\omega), \zeta_{m(k)}(\omega), \eta_{n(k)}(\omega), \eta_{m(k)}(\omega))}{M_\omega(\zeta_{n(k)-1}(\omega), \zeta_{m(k)-1}(\omega), \eta_{n(k)-1}(\omega), \eta_{m(k)-1}(\omega))} \\ &\qquad \leq \beta(M_\omega(\zeta_{n(k)-1}(\omega), \zeta_{m(k)-1}(\omega), \eta_{n(k)-1}(\omega), (\eta_{m(k)-1}(\omega))). \end{aligned} \tag{2.14}$$

On the other hand, the triangle inequality and (2.13) give us

$$\begin{aligned} d(g(\omega,\zeta_{m(k)}(\omega)),g(\omega,\zeta_{n(k)}(\omega))) &\le d(g(\omega,\zeta_{m(k)}(\omega)),g(\omega,\zeta_{n(k)-1}(\omega))) \\ &\quad +d(g(\omega,\zeta_{m(k)-1}(\omega)),g(\omega,\zeta_{m(k)}(\omega))) \\ &< d(g(\omega,\zeta_{n(k)}(\omega)),g(\omega,\zeta_{n(k)-1}(\omega)))+\epsilon \end{aligned} \tag{2.15}$$

and

$$\begin{aligned} d(g(\omega,\eta_{m(k)}(\omega)),g(\omega,\eta_{n(k)}(\omega))) &\le d(g(\omega,\eta_{m(k)}(\omega)),g(\omega,\eta_{n(k)-1}(\omega))) \\ &\quad +d(g(\omega,\eta_{m(k)-1}(\omega)),g(\omega,\eta_{m(k)}(\omega))) \\ &< d(g(\omega,\eta_{n(k)}(\omega)),g(\omega,\eta_{n(k)-1}(\omega)))+\epsilon. \end{aligned} \tag{2.16}$$

From (2.12), (2.15) and (2.16) we get

$$\begin{aligned} \epsilon &\le M_\omega(\zeta_{n(k)}(\omega),\zeta_{m(k)}(\omega),\eta_{n(k)}(\omega),\eta_{m(k)}(\omega)) \\ &\le M_\omega(\zeta_{n(k)}(\omega),\zeta_{n(k)-1}(\omega),\eta_{n(k)}(\omega),\eta_{n(k)-1}(\omega))+\epsilon. \end{aligned} \tag{2.17}$$

Letting $k\to\infty$ in the last inequality and taking Step 2 into the account we have

$$\lim_{k\to\infty}(M_\omega(\zeta_{n(k)}(\omega),\zeta_{m(k)}(\omega),\eta_{n(k)}(\omega),g(\omega,(\eta_{m(k)}(\omega))=\epsilon. \tag{2.18}$$

Again, the triangle inequality and (2.13) give us

$$\begin{aligned} d(g(\omega,\zeta_{n(k)-1}(\omega)),g(\omega,\zeta_{m(k)-1}(\omega))) &\le d(g(\omega,\zeta_{n(k)-1}(\omega)),g(\omega,\zeta_{m(k)}(\omega))) \\ &\quad +d(g(\omega,\zeta_{m(k)}(\omega)),g(\omega,\zeta_{m(k)-1}(\omega))) \\ &< d(g(\omega,\zeta_{m(k)}(\omega)),g(\omega,\zeta_{m(k)-1}(\omega)))+\epsilon \end{aligned} \tag{2.19}$$

and

$$\begin{aligned} d(g(\omega,\eta_{n(k)-1}(\omega)),g(\omega,\eta_{m(k)-1}(\omega))) &\le d(g(\omega,\eta_{n(k)-1}(\omega)),g(\omega,\eta_{m(k)}(\omega))) \\ &\quad +d(g(\omega,\eta_{m(k)}(\omega)),g(\omega,\eta_{m(k)-1}(\omega))) \\ &< d(g(\omega,\eta_{m(k)}(\omega)),g(\omega,\eta_{m(k)-1}(\omega)))+\epsilon \end{aligned} \tag{2.20}$$

and by the last inequality and (2.12) we get

$$\begin{aligned} \epsilon &\le M_\omega(\zeta_{n(k)}(\omega),\zeta_{m(k)}(\omega),\eta_{n(k)}(\omega),\eta_{m(k)}(\omega)) \\ &\le M_\omega(\zeta_{n(k)}(\omega),\zeta_{n(k)-1}(\omega),\eta_{n(k)}(\omega),\eta_{n(k)-1}(\omega)) \\ &\quad +M_\omega(\zeta_{n(k)-1}(\omega),\zeta_{m(k)-1}(\omega),\eta_{n(k)-1}(\omega),\eta_{m(k)-1}(\omega)) \\ &\quad +M_\omega(\zeta_{m(k)-1}(\omega),\zeta_{m(k)}(\omega),\eta_{m(k)-1}(\omega),\eta_{m(k)}(\omega)). \end{aligned} \tag{2.21}$$

Letting $k\to\infty$ in the last inequality and using Step 2 we obtain

$$\lim_{k\to\infty} M_\omega(\zeta_{n(k)-1}(\omega),\zeta_{m(k)-1}(\omega),\eta_{n(k)-1}(\omega),g(\omega,(\eta_{m(k)-1}(\omega))=\epsilon. \tag{2.22}$$

Finally, letting $k\to\infty$ in (2.14) and using (2.18) and (2.22) we get

$$\epsilon\le\beta(M_\omega(\zeta_{n(k)-1}(\omega),\zeta_{m(k)-1}(\omega)),\eta_{n(k)-1}(\omega),\eta_{m(k)-1}(\omega)))\epsilon$$

or

$$\epsilon\le\beta(\epsilon)\epsilon.$$

We obtain

$$\lim_{n,m\to\infty}\beta(M_\omega(\zeta_{n(k)}(\omega),\zeta_{m(k)}(\omega),\eta_{n(k)}(\omega),\eta_{m(k)}(\omega)))=1.$$

But since $\beta\in\Delta$, we get

$$\lim_{n,m\to\infty}M_\omega(\zeta_{n(k)}(\omega),\zeta_{m(k)}(\omega),\eta_{n(k)}(\omega),\eta_{m(k)}(\omega))=0.$$

This is a contradiction. Then our claim follows. Since X is complete and $g(\omega\times X)=X$, there exist $\zeta_0,\eta_0\in\Theta$ such that

$$\lim_{n\to\infty}g(\omega,\zeta_n(\omega))=g(\omega,\zeta_0(\omega))$$

and

$$\lim_{n\to\infty}g(\omega,\eta_n(\omega))=g(\omega,\eta_0(\omega)).$$

Define $\zeta, \eta : \Omega \to X$ by $\zeta(\omega) = g(\omega, \zeta_0(\omega))$ and $\eta(\omega) = g(\omega, \eta_0(\omega))$. Since $\zeta(\omega) = g(\omega, \zeta_0(\omega))$ and $\zeta(\omega) = g(\omega, \eta_0(\omega))$ are measurable, the functions $\zeta(\omega)$ and $\eta(\omega)$ are also measurable. Thus, we have

$$\lim_{n\to\infty} g(\omega, \zeta_n(\omega)) = \zeta(\omega) \tag{2.23}$$

and

$$\lim_{n\to\infty} g(\omega, \eta_n(\omega)) = \eta(\omega). \tag{2.24}$$

From (2.21) and (2.22) and the continuity of g, we have

$$\lim_{n\to\infty} g(\omega, g(\omega, \zeta_n(\omega))) = g(\omega, \zeta(\omega))$$

and

$$\lim_{n\to\infty} g(\omega, g(\omega, \eta_n(\omega))) = g(\omega, \eta(\omega)).$$

By using the fact that F and g are commutative, from (2.2) we have

$$F(\omega, (g(\omega, \zeta_n(\omega)), g(\omega, \eta_n(\omega))) = g(\omega, F(\omega, \zeta_n(\omega), \eta_n(\omega))) = g(\omega, g(\omega, \zeta_{n+1}(\omega)))$$

and

$$F(\omega, (g(\omega, \eta_n(\omega)), g(\omega, \zeta_n(\omega))) = g(\omega, F(\omega, \eta_n(\omega), \zeta_n(\omega))) = g(\omega, g(\omega, \eta_{n+1}(\omega))).$$

Suppose F is continuous. Then

$$\begin{aligned} g(\omega, \zeta(\omega)) &= \lim_{n\to\infty} g(\omega, g(\omega, \zeta_{n+1}(\omega))) \\ &= \lim_{n\to\infty} F(\omega, (g(\omega, \zeta_n(\omega)), g(\omega, \eta_n(\omega))) \\ &= F(\omega, (\lim_{n\to\infty} g(\omega, \zeta_n(\omega)), \lim_{n\to\infty} g(\omega, \eta_n(\omega))) \\ &= F(\omega, (\zeta(\omega), \eta(\omega))) \end{aligned} \tag{2.25}$$

and

$$\begin{aligned} g(\omega, \eta(\omega)) &= \lim_{n\to\infty} g(\omega, g(\omega, \eta_{n+1}(\omega))) \\ &= \lim_{n\to\infty} F(\omega, (g(\omega, \eta_n(\omega)), g(\omega, \zeta_n(\omega))) \\ &= F(\omega, (\lim_{n\to\infty} g(\omega, \eta_n(\omega)), \lim_{n\to\infty} g(\omega, \zeta_n(\omega))) \\ &= F(\omega, (\eta(\omega), \zeta(\omega))). \end{aligned} \tag{2.26}$$

From the inequalities above, we deduce that $(\zeta(\omega), \eta(\omega)) \in X \times X$ is a coupled coincidence of F and g.

Suppose that (b) holds. From (2.3), the sequence $(g(\omega, \zeta_n(\omega))$ is non-decreasing and the sequence $(g(\omega, \eta_n(\omega))$ is non-increasing. Since X satisfies (b), we have $(g(\omega, \zeta_n(\omega)) \preceq (g(\omega, \zeta(\omega))$ and $(g(\omega, \eta_n(\omega)) \succeq (g(\omega, \eta(\omega))$. Thus from (2.1) and as $n \to \infty$, we conclude that

$$d(g(\omega, \zeta(\omega)), F(\omega, (\zeta(\omega), \eta(\omega)))) = 0.$$

Hence

$$g(\omega, \zeta(\omega)) = F(\omega, (\zeta(\omega), \eta(\omega))).$$

Similarly we can show that

$$g(\omega, \eta(\omega)) = F(\omega, (\eta(\omega), \zeta(\omega))).$$

Thus we prove that $(\zeta(\omega), \eta(\omega)) \in X \times X$ is a random coupled fixed point of F and g.

□

Example 2.2. Let $X = \mathbb{R}$ with the usual ordering and usual metric. Let $\Omega = [0, 1]$ and let σ be the sigma algebra of Lebesgue?s measurable subset of $[0, 1]$. Define $g : \Omega \times X \to X$,$F : \Omega \times X \times X \to X$ and $\beta : [0, \infty) \to [0, \infty)$ as follows.

$$g(\omega, x) = \frac{3}{4}\left(1 - \omega^2\right) x,$$

and

$$F(\omega, x, y) = \frac{1}{8}\left(1 - \omega^2\right)(x - y),$$

$\omega \in \Omega$, and $\beta(t) = (2/3)t$ for all $t \in [0, \infty)$. We will check that the contraction 2.1 is satisfied for all $x, y, u, v \in X$ satisfying $g(\omega, x) \leq g(\omega, y)$, $g(\omega, y) \geq g(\omega, v)$, for all $\omega \in \Omega$. Then from 2.1 we have,

$$\begin{aligned}
& d(\omega,(x,y)), F(\omega,(u,v))) \\
= \;& d\left(\frac{1}{8}\left(1-\omega^2\right)(x-y), \frac{1}{8}\left(1-\omega^2\right)(u-v)\right) \\
= \;& \frac{1}{8}\left(1-\omega^2\right)[(x-y)-(u-v)] \\
= \;& \frac{1}{8}\left(1-\omega^2\right)[(x-u)+(y-v)] \\
\leq \;& \frac{2}{3}\left[\max\{d(g(\omega,x),g(\omega,u)), d(g(\omega,y),g(\omega,v))\}\right] \max\{d(g(\omega,x),g(\omega,u)), d(g(\omega,y),g(\omega,v))\}
\end{aligned}$$

that is, contraction 2.1 is satisfied. It is obvious that the other hypotheses of Theorem-2.1 are satisfied. We deduce that (0,0) is the unique couple common random fixed point of F and g.

Example 2.3. Let $X = \mathbb{R}$ with the usual ordering and usual metric. Let $\Omega = [0, 1]$ and let σ be the sigma algebra of Lebesgue's measurable subset of $[0, 1]$. Define $g : \Omega \times X \to X$,$F : \Omega \times X \times X \to X$ and $\beta : [0, \infty) \to [0, \infty)$ as follows.

$$g(\omega, x) = \frac{n+1}{n}\left(1-\omega^2\right)x,$$

and

$$F(\omega, x, y) = \frac{1}{4}\left(1-\omega^2\right)x$$

$\omega \in \Omega$, and $\beta(t) = \frac{n}{n+1}t$ for all $t \in [0, \infty)$. We will check that the contraction 2.1 is satisfied for all $x, y, u, v \in X$ satisfying $g(\omega, x) \leq g(\omega, y)$, $g(\omega, y) \geq g(\omega, v)$, for all $\omega \in \Omega$. Then from 2.1 we have,

$$\begin{aligned}
& d(\omega,(x,y)), F(\omega,(u,v))) \\
= \;& d\left(\frac{1}{4}\left(1-\omega^2\right)x, \frac{1}{4}\left(1-\omega^2\right)u\right) \\
= \;& \frac{\left(1-\omega^2\right)}{4}\left(1-\omega^2\right)(x-y) \\
\leq \;& \frac{n}{n+1}\left[\max\{d(g(\omega,x),g(\omega,u)), d(g(\omega,y),g(\omega,v))\}\right] \max\{d(g(\omega,x),g(\omega,u)), d(g(\omega,y),g(\omega,v))\}
\end{aligned}$$

that is, contraction 2.1 is satisfied. It is obvious that the other hypotheses of Theorem-2.1 are satisfied. We deduce that (0,0) is the unique couple common random fixed point of F and g.

Example 2.4. Let $X = \mathbb{R}$ be ordered by the following relation

$$x \leq y \Longleftrightarrow x = y \quad or \quad (x, y \in [0, 1] \;\; and \;\; x \leq y).$$

Let $g : \Omega \times X \to X$ and $F : \Omega \times X \times X \to X$ be defined by

$$g(\omega, x) = \begin{cases} \frac{1}{20}(1-\omega)x & \text{if } x < 0 \\ \frac{1}{2}(1-\omega)(x) & \text{if } x \in [0,1] \\ \frac{1}{20}(1-\omega)x + \frac{9}{20} & \text{if } x > 1, \end{cases} \quad and \quad F(\omega, x, y) = \frac{x+y}{20}(1-\omega)$$

Take $\beta(t) = \frac{1}{10}t$. Then we found that all the conditions of Theorem 2.1 and Theorem 2.1 are satisfied. Obviously, the mappings g and F have a unique common coupled fixed point $(0, 0)$.

Example 2.5. Let $X = [0, 1]$, with the usual partial ordered $\leq$. Let $g : \Omega \times X \to X$ and $F : \Omega \times X \times X \to X$ be defined by

$$F(x, y) = \frac{x+y}{24}(1-\omega^2) \quad g(x) = \tfrac{x}{3}(1-\omega^2) \tag{2.27}$$

Take $\beta(t) = \frac{t}{8}$. Then we found that all the conditions of Theorem 2.1 and Theorem 2.1 are satisfied. Obviously, the mappings g and F have a unique common coupled fixed point $(0, 0)$.

Example 2.6. Let $X = [0,1]$, with the usual partial ordered $\leq$. Let $g : \Omega \times X \to X$ and $F : \Omega \times X \times X \to X$ be defined by

$$F(x,y) = \frac{1}{20}[\sin x + \sin y](1-\omega^2) \quad g(x) = \tfrac{x(1-\omega^2)}{5} \tag{2.28}$$

for all $x, y \in X$. Since $(1-\omega^2) \mid \sin x - \sin y \mid \leq (1-\omega^2) \mid x - y \mid$ holds for all $x, y \in X$. Then we have $\beta = \frac{t}{4}$. So all the conditions of Theorem 2.1 and Theorem 2.1 are satisfied. Then there exists a common coupled fixed point of F and g. In this case $(0,0)$ is a common coupled fixed point of F and g.

In what follows, we give a sufficient condition for the uniqueness of coupled fixed point in Theorem 2.1. This condition is, if

$$(\zeta(\omega), \eta(\omega)), (\zeta^1(\omega), \eta^1(\omega)) \in X \times X$$

then there exists

$$(\zeta^*(\omega), \eta^*(\omega)) \in X \times X \tag{2.29}$$

which is comparable to $(\zeta(\omega), \eta(\omega))$ and $(\zeta^1(\omega), \eta^1(\omega))$.

Notice that in $X \times X$ we consider the partial order relation given by

$$(\zeta(\omega), \eta(\omega)) \preceq (\zeta^1(\omega), \eta^1(\omega)) \Leftrightarrow \zeta(\omega) \preceq \zeta^1(\omega) \ \ and \ \ \eta(\omega) \succeq \eta^1(\omega).$$

Theorem 2.7. *Adding the above condition to the hypothesis of Theorem 2.1, we obtain uniqueness of random coupled fixed point of F and g.*

Proof. Suppose that $(\zeta(\omega), \eta(\omega)) \ \ and \ \ (\zeta^1(\omega), \eta^1(\omega))$ are random coupled fixed points of F and g, that is,

$$\zeta(\omega) = g(\omega, \zeta(\omega)) = F(\omega, (\zeta(\omega), \eta(\omega))),$$

$$\eta(\omega) = g(\omega, \eta(\omega)) = F(\omega, (\eta(\omega), \zeta(\omega))),$$

$$\zeta^1(\omega) = g(\omega, \zeta^1(\omega)) = F(\omega, (\zeta^1(\omega), \eta^1(\omega)))$$

and

$$\eta^1(\omega) = g(\omega, \eta^1(\omega)) = F(\omega, (\eta^1(\omega), \zeta^1(\omega))).$$

Let $(\zeta^*(\omega), \eta^*(\omega)) \in X \times X$ be an element comparable to $(\zeta(\omega)$ and $\eta(\omega))$ and $(\zeta^1(\omega), \eta^1(\omega))$. Suppose that

$$(\zeta(\omega), \eta(\omega)) \succeq (\zeta^*(\omega), \eta^*(\omega))$$

(the proof is similar in other cases).

We construct the sequences $\{\zeta_n^*(\omega)\}$ and $\{\eta_n^*(\omega)\}$ in X defined by

$$g(\omega, \zeta_{n+1}^*(\omega)) = F(\omega, (\zeta_n^*(\omega), \eta_n^*(\omega))) \text{ and } g(\omega, \eta_{n+1}^*(\omega)) = F(\omega, (\eta_n^*(\omega), \zeta_n^*(\omega))).$$

We claim that

$$(\zeta(\omega), \eta(\omega)) \succeq (\zeta_n^*(\omega), \eta_n^*(\omega)) \text{ for each } n \in N.$$

In fact, we will use the mathematical induction. For $n = 0$ as

$$(\zeta(\omega), \eta(\omega)) \succeq (\zeta_n^*(\omega), \eta_n^*(\omega)) \text{ for each } n \in N$$

this means

$$\zeta^*(\omega) \preceq \zeta(\omega) \text{ and } \eta^*(\omega) \succeq \eta(\omega)$$

and consequently,

$$(\zeta(\omega), \eta(\omega)) \succeq (\zeta_0^*(\omega), \eta_0^*(\omega))$$

Suppose that

$$(\zeta(\omega), \eta(\omega)) \succeq (\zeta_n^*(\omega), \eta_n^*(\omega))$$

then using mixed g-monotone property of F and g, we get

$$\zeta_{n+1}^*(\omega) = F(\zeta_n^*(\omega), \eta_n^*(\omega)) \preceq F(\zeta(\omega), \eta_n^*(\omega)) \preceq F(\zeta(\omega), \eta(\omega)) = \zeta(\omega)$$

and

$$\eta_{n+1}^*(\omega) = F(\eta_n^*(\omega), \zeta_n^*(\omega)) \succeq F(\eta(\omega), \zeta_n^*(\omega)) \succeq F(\eta(\omega), \zeta(\omega)) = \eta(\omega)$$

and this proves our claim.

Now, since $\zeta_n^*(\omega) \preceq \zeta(\omega)$ and $\eta_n^*(\omega) \succeq \eta(\omega)$, using the contractive condition we have

$$\begin{aligned} d(g(\omega, \zeta(\omega), g(\omega, \zeta_n^*(\omega)) &= d(F(\zeta(\omega, \eta(\omega)), F(\zeta_n^*(\omega, \eta_n^*(\omega))) \\ &\leq M_\omega(\zeta(\omega), \zeta_{n-1}^*(\omega), \eta(\omega), \eta_{n-1}^*(\omega)) \end{aligned} \tag{2.30}$$

and analogously

$$\begin{aligned} d(g(\omega, \eta(\omega), g(\omega, \eta_n^*(\omega)) &= d(F(\eta(\omega, \zeta(\omega)), F(\eta_n^*(\omega, \zeta_n^*(\omega))) \\ &\leq M_\omega(\eta(\omega), \eta_{n-1}^*(\omega), \zeta(\omega), \zeta_{n-1}^*(\omega)) \end{aligned} \tag{2.31}$$

From (2.30) and (2.31), we obtain

$$\begin{aligned} &M_\omega(\zeta(\omega), \zeta_n^*(\omega), \eta(\omega), \eta_n^*(\omega)) \\ &\leq M_\omega(\zeta(\omega), \zeta_{n-1}^*(\omega), \eta(\omega), \eta_{n-1}^*(\omega)) \end{aligned} \tag{2.32}$$

The sequence $\{M_\omega(\zeta(\omega), \zeta_n^*(\omega), \eta(\omega), \eta_n^*(\omega))\}$ is decreasing and non negative, and so,

$$\lim_{n\to\infty} M_\omega(\zeta(\omega), \zeta_n^*(\omega), \eta(\omega), \eta_n^*(\omega)) = r \tag{2.33}$$

for certain $r \geq 0$.

Now we show that $r = 0$. Assume on the contrary that $r > 0$,
Using (2.33) and letting $n \to \infty$ in (2.32) we have

$$r \leq \beta(r)r.$$

Consequently $\lim\limits_{n\to\infty} \beta(r) = 1$, since $\beta \in \Delta$, then

$$\lim_{n\to\infty} M_\omega(\zeta(\omega), g(\omega, \zeta_n^*(\omega), \eta(\omega), \eta_n^*(\omega)) = 0. \tag{2.34}$$

This is a contradiction. Therefore, $r = 0$. This gives us $\zeta_n^*(\omega) \to \zeta(\omega)$ and $\eta_n^*(\omega) \to \eta(\omega)$ using a similar argument for $(\zeta^1(\omega), \eta^1(\omega))$ we obtain $\zeta_n^*(\omega) \to \zeta^1(\omega)$ and $\eta_n^*(\omega) \to \eta^1(\omega)$, and the uniqueness of the limit gives $\zeta(\omega) = \zeta^1(\omega)$ and $\eta(\omega) = \eta^1(\omega)$. This finishes the proof. □

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Received: June 15, 2014.

Accepted: October 23, 2014.