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Rings in which every ideal is v-finite

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Dedicated to Patrick Smith and John Clark on the occasion of their 70th birthdays.

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Abstract. In this paper, we introduce and study the notion of "v-Noetherian rings" which is a notion between Noetherian rings and v-coherent rings. Then we establish the transfer of this notion to finite direct products, trivial ring extensions, localization, and amalgamated algebras along an ideal. These results provide examples of non-Noetherian v-Noetherian rings and examples of non-v-Noetherian v-coherent rings. The article includes a brief discussion of the scope and precision of our results.

1 Introduction

All rings considered below are commutative with unit and all modules are unital. Let R be a commutative ring and let $Q(R)$ denote the total ring of quotients of R. A ring R is called a total ring of quotients if $R = Q(R)$, that is every element of R is invertible or zero-divisor.

We review some terminology related to basic operations on fractional ideals in an arbitrary ring (i.e., not necessarily a domain). Let R be a commutative ring and let $Q(R)$ denote the total ring of quotients of R. By an ideal of R we mean an integral ideal of R. Let I and J be two nonzero fractional ideals of R. We define the fractional ideal $(I : J) = \{x \in \mathrm{Q}(R) \mid xJ \subset I\}$. We denote $(R : I)$ by I^{-1} and $(I^{-1})^{-1}$ by I_v (called the v-closure of I). A nonzero fractional ideal I is said to be invertible if $II^{-1} = R$, divisorial (or a v-ideal) if $I_v = I$, and v-finite if $I_v = J_v$ (or, equivalently, if $I^{-1} = J^{-1}$) for some finitely generated fractional ideal J of A.

A ring R is coherent if every finitely generated ideal of R is finitely presented; equivalently, if $(0 : a)$ and $I \cap J$ are finitely generated for every $a \in R$ and any two finitely generated ideals I and J of R. Examples of coherent rings are Noetherian rings, Boolean algebras, von Neumann regular rings, valuation rings, and Prüfer/semihereditary rings. See for instance [8, 9, 10, 12].

A ring R is v-coherent if $(0 : a)$ and $\bigcap_{1\leq i\leq n} Ra_i$ are v-finite ideals of R for any finite set of elements a and $a_1, ..., a_n$ of R. This concept was introduced in integral domain by Nour El Abidine in [17, 18] and generalized to a ring with zero-divisors by Kabbaj and Mahdou in [12]. Examples of v-coherent rings are coherent rings and total rings of quotients. See for instance [10, 12, 17, 18].

A ring R is Noetherian if every ideal of R is finitely generated. In this paper, we introduce and investigate a new class of rings called v-Noetherian rings which is between two notions: Noetherian rings and v-coherent rings. A ring R is called a v-Noetherian ring if any ideal of R is v-finite.

Let A be a ring, E be an A-module and $R := A \propto E$ be the set of pairs (a, e) with pairwise addition and multiplication given by $(a, e)(b, f) = (ab, af + be)$. R is called the trivial ring extension of A by E. Considerable work has been concerned with trivial ring extensions. Part of it has been summarized in Glaz's book [8] and Huckaba's book (where R is called the idealization of E by A) [11]. See for instance [1, 8, 11, 12, 13, 14].

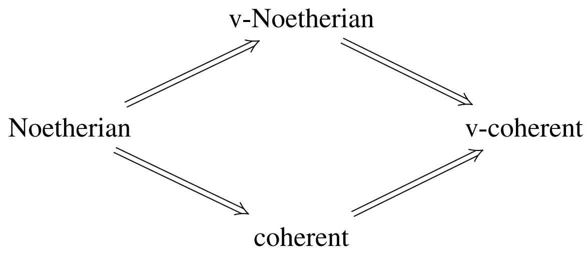
The amalgamation algebras along an ideal, introduced and studied by D'Anna, Finocchiaro and Fontana in [4, 5] and defined as follows:

Let A and B be two rings with unity, let J be an ideal of B and let $f : A \to B$ be a ring homomorphism. In this setting, we can consider the following subring of $A \times B$:

$$A \bowtie^f J := \{(a, f(a) + j) \mid a \in A, j \in J\}$$

called the amalgamation of A and B along J with respect to f. In particular, they have studied amalgmations in the frame of pullbacks which allowed them to establish numerous (prime) ideal and ring-theoretic basic properties for this new construction. This construction is a generalization of the amalgamated duplication of a ring along an ideal (introduced and studied by D'Anna and Fontana in [3, 6, 7]). See for instance [4, 5].

In this work, we introduce and study the notion of v-Noetherian rings. It is clear that the following diagram of implications hold:



We examine the transfer of the v-Noetherian property to finite direct products, trivial ring extensions, and amalgamated algebras along an ideal. These results provide examples of non-Noetherian v-Noetherian rings and examples of non-v-Noetherian v-coherent rings.

2 Main Results

This section develops a result of the transfer of the v-Noetherian property to finite direct products, trivial ring extensions, and amalgamated algebras along an ideal. These results provide examples of non-Noetherian v-Noetherian rings and examples of non-v-Noetherian v-coherent rings.

First, we will construct a wide class of v-Noetherian rings.

Proposition 2.1. Any total ring of quotients is v-Noetherian.

Proof. Let R be a total ring of quotients and let I be an ideal of R. Then, $I^{-1} = \{x \in R/xI \subseteq R\} = R$ since R is a total ring. Hence, $I^{-1} = R^{-1}$ and so I is v-finite, as desired. □

Examples of non-Noetherian v-Noetherian rings may stem from Proposition 2.1 as shown by the following trivial ring extension.

Proposition 2.2. Let (A, M) be a local ring and E an A-module with $ME = 0$. Let $R := A \propto E$ be the trivial ring extension of A by E. Then:
1) R is a v-Noetherian ring.
2) R is a coherent ring if and only if A is a coherent ring, M is a finitely generated ideal of A, and E is an (A/M)-vector space of finite rank.

Proof. 1) By Proposition 2.1, it suffices to show that R is a total ring of quotients. Let (a, e) be an element of R. Two cases are then possibles:
If $a \notin M$ (that is a is invertible in A), then (a, e) is invertible in R by [11, Theorem 25.1].
Now, assume that $a \in M$. Then $(a, e)(0, f) = (0, 0)$ for all $f \in E$ and so (a, e) is a zero-divisor, as desired.
2) By [12, Theorem 2.6 (2)]. □

Now, we are able to construct examples of non-coherent (and so non-Noetherian) v-Noetherian rings.

Example 2.3. Let (A, M) be a local non-coherent ring and E an A-module with $ME = 0$. Let $R := A \propto E$ be the trivial ring extension of A by E. Then:
1) R is a v-Noetherian ring by Proposition 2.2(1).
2) R is a non-coherent ring by Proposition 2.2 (2) since A is non-coherent ring. In particular, R is non-Noetherian ring.

Next, we explore a different context, namely, the trivial ring extension of a domain by its quotient field.

Theorem 2.4. Let A be a domain which is not a field, $K = \mathrm{qf}(A)$, E be a K-vector space, and let $R := A \propto E$ be the trivial ring extension of A by E. Then:
(1) R is v-Noetherian if and only if so is A.
(2) R is a non-coherent ring. In particular, R is a non-Noetherian ring.

Proof. Remark that the total ring of quotients of R is $Q(R) = K \propto E$. Also, it is clear that any ideal of R has the form $0 \propto F$ for some A-submodule of E or $I \propto E$ for some nonzero ideal I of A. And we can easily verify that $(0 \propto F)^{-1} = Q(R)$ and $(I \propto E)^{-1} = I^{-1} \propto E$ if I is a nonzero ideal of A. Finally, remark that a nonzero ideal $I \propto E$ of R is finitely generated if and only if I is a finitely generated ideal of A.

(1) Assume that A is a v-Noetherian ring and let I be a nonzero ideal of A. Then $J := I \propto E (= I \propto IE)$ is an ideal of R and easily we have $J^{-1} (= (I \propto E)^{-1}) = I^{-1} \propto E$ since $Q(R) = K \propto E$. But $J^{-1} = I_0^{-1} \propto E$ for some nonzero finitely generated ideal I_0 of A since R is v-Noetherian. Therefore, $I^{-1} = I_0^{-1}$ and so I is v-finite, as desired.
Conversely, assume that A is v-Noetherian and let J be a proper ideal of R. Two cases are then possibles:
Case 1: $J = 0 \propto F$ for some A-submodule F of E. Then $J^{-1} = K \propto E = (0 \propto F_0)^{-1}$ for any finitely generated A-submodule of E, as desired since $0 \propto F_0$ is a finitely generated ideal of R.
Case 2: $J = I \propto E$ for some nonzero ideal I of A. There exists a nonzero finitely generated ideal I_0 of A such that $I_0^{-1} = I^{-1}$ since A is v-Noetherian. Hence, $J^{-1} = (I \propto E)^{-1} = I^{-1} \propto E = I_0^{-1} \propto E = (I_0 \propto E)^{-1}$ and $I_0 \propto E$ is a finitely generated ideal of R (since I_0 is a nonzero finitely generated ideal of A), as desired.
Hence, in all cases, $J^{-1} = J_0^{-1}$ for some finitely generated ideal of R. Therefore, R is v-Noetherian.

(2) R is a non-coherent ring by [12, Theorem 2.8(1)]. □

We know that a Noetherian ring is v-Noetherian and coherent. The following two examples show that the class of v-Noetherian rings and the class of coherent rings are not comparable.

Now, we may construct new examples of non-coherent v-Noetherian ring.

Example 2.5. Let $R := \mathbb{Z} \propto \mathbb{Q}$ be the trivial ring extension of $\mathbb{Z}$ by $\mathbb{Q}$. Then:
(1) R is v-Noetherian by Theorem 2.4(1) since $\mathbb{Z}$ is Noetherian.
(2) R is a non-coherent ring by Theorem 2.4(2).

Now, we construct an example of a coherent domain (so a v-coherent domain) which is not a v-Noetherian domain.

Example 2.6. Let $T = K((X))[[Y]] = K((X)) + M$, where X is an indeterminate over a field K, Y is an indeterminate over a field $K((X))$, and $M = YK((X))[[Y]] = YT$. Set $R = K[[X]] + M$. Then:
(1) R is a coherent domain by [8, Theorem 5.2.3, p.162]. In particular, R is a v-coherent domain.
(2) R is a non-v-Noetherian ring since M is not v-finite in R by [10, Proposition 2.3] (since M is invertible in T).

Any von Neumann regular ring R, that is $wdim(R) = 0$ is a v-Noetherian ring since it is a total ring. Now we construct a non-Noetherian v-Noetherian ring (with zerodivosors) with $wdim(R) = 1$.

Example 2.7. Let E be a countable direct sum of copies of $\mathbb{Z}/2\mathbb{Z}$ with addition and multiplication defined component wise, where $\mathbb{Z}$ is the ring of integers. Let $R = \mathbb{Z} \times E$ with multiplication defined by $(a, e)(b, f) = (ab, af + be + ef)$. Then:
(1) $wdim(R) = 1$.
(2) R is a v-Noetherian ring.
(3) R is a non-coherent ring.

Proof. (1) That $wdim(R) = 1$ this is handled in [19, Example 1.3, page 10].

(2) Notice first that an element $s \in R$ is regular if and only if $s = (a, 0)$ with $a \in \mathbb{Z} \setminus 2\mathbb{Z}$. This easily follows from the four basic facts: E is Boolean; $2E = 0$; $ae = e$ for any $a \in \mathbb{Z} \setminus 2\mathbb{Z}$ and $e \in E$; and for any $e \neq 0 \in E$, there exists $f \neq 0 \in E$ such that $ef = 0$.

Next, we wish to show that each ideal of R is v-finite. Let J be an ideal of R and let $I = \{a \in \mathbb{Z}/(a, e) \in J$ for some $e \in E\}$. Assume $I = 0$. Let s be any regular element of R. Clearly, $(0, e) = s(0, e)$ for any $e \in E$. It follows that $sJ = J$ and hence $J^{-1} = Q(R) = (R(0, e))^{-1}$ for any $e \neq 0 \in E$. Now, assume $I = x\mathbb{Z}$, where x is a nonzero integer. We claim that $J^{-1} = (R(x, 0))^{-1}$. Indeed, let $y/s \in Q(R)$, where $y = (a, e) \in R$ and $s = (b, 0)$ is a regular element. It can easily be seen that $sR = b\mathbb{Z} \times E$. Then $y/s \in J^{-1} \Leftrightarrow yJ \subseteq sR \Leftrightarrow (a, e)J \subseteq b\mathbb{Z} \times E \Leftrightarrow aI \subseteq b\mathbb{Z} \Leftrightarrow ax \in b\mathbb{Z} \Leftrightarrow (a, e)(R(x, 0)) \subseteq sR \Leftrightarrow y/s \in (R(x, 0))^{-1}$. Thus, in both cases, J is v-finite, as asserted.

(3) Let $x = (2, 0) \in R$. Then $(0 : x) = \{(a, e) \in R/(a, e)(2, 0) = 0\} = \{(a, e) \in R/(2a, 0) = 0\} = 0 \times E$ which is not a finitely generated ideal of R. Therefore, R is not a coherent ring., which completes the proof of Example 2.6. □

Now, we study the transfer of the v-Noetherian property to the direct product.

Theorem 2.8. Let $(R_i)_{i=1,\dots,n}$ be a family of rings. Then $\prod_{i=1}^{n} R_i$ is a v-Noetherian ring if and only if so is R_i for each $i = 1, \dots, n$.

Proof. We will prove the result for $i = 1, 2$, and the Theorem will be established by induction on n. Remark that the total ring of quotients of $R_1 \times R_2$ $Q(R_1 \times R_2) = Q(R_1) \times Q(R_2)$ and easily we have $(I_1 \times I_2)^{-1} = (I_1)^{-1} \times (I_2)^{-1}$ for every ideal I_1 (resp., I_2) of R_1 (resp., R_2).

Assume that $R_1 \times R_2$ is a v-Noetherian ring and we show that R_1 is a v-Noetherian ring (it is the same for R_2). Let I_1 be an ideal of R_1 and set $I := I_1 \times R_2$ which is an ideal of $R_1 \times R_2$. Then, there exists a finitely generated ideal $J := J_1 \times J_2$ of $R_1 \times R_2$ such that $I^{-1} = J^{-1}$. Hence, $(I_1)^{-1} \times (R_2)^{-1} = (I_1 \times R_2)^{-1} = I^{-1} = J^{-1} = (J_1 \times J_2)^{-1} = (J_1)^{-1} \times (J_2)^{-1}$ and so $(I_1)^{-1} = (J_1)^{-1}$, as desired since J_1 is a finitely generated ideal of R_1.

Conversely, assume that R_1 and R_2 are v-Noetherian rings and we show that $R_1 \times R_2$ is a v-Noetherian ring. Let $I := I_1 \times I_2$ be an ideal of $R_1 \times R_2$, where I_1 (resp., I_2) is an ideal of R_1 (resp., R_2). Hence, there exists a finitely generated ideal J_1 of R_1 (resp., J_2 of R_2) such that $I_1^{-1} = J_1^{-1}$ (resp., $I_2^{-1} = J_2^{-1}$). Therefore, $I^{-1} = (I_1 \times I_2)^{-1} = (I_1)^{-1} \times (I_2)^{-1} = (J_1)^{-1} \times (J_2)^{-1} = (J_1 \times J_2)^{-1}$ and so I is v-finite since $J_1 \times J_2$ is a finitely generated ideal of $R_1 \times R_2$, and this completes the proof of Theorem 2.5. □

We know that the localization of a v-coherent ring is v-coherent ring. Now, we show that the localization of a v-Noetherian ring is not always a v-Noetherian ring.

Example 2.9. Let $T = K((X, Y))[[Z]] = K((X, Y)) + M$, where X and Y are indeterminates over a field K, Z is an indeterminate over a field $K((X, Y))$, and $M = ZK((X, Y))[[Z]] = ZT$. Let $A = K[[X, Y]] + M$, $R := A \propto (A/(XA + YA))$ be the trivial ring extension of A by $(A/(XA + YA))$, and let $S = \{X^n/n \in \mathbb{N}\}$ which is a multiplicative set of both $K[[X, Y]]$, A, R and T. Then:
(1) R is a v-Noetherian ring.
(2) $S^{-1}R$ is not a v-Noetherian ring.

Proof. 1) R is a v-Noetherian ring by Proposition 2.2(1) since A is a local ring with maximal ideal $XA + YA$.

2) Since X is invertible in T, then $S^{-1}T = T$. Also, we have $S^{-1}K[[X,Y]] = S^{-1}K[[X]][[Y]] = K((X))[[Y]]$ since $S^{-1}K[[X]] = K((X))$. Therefore, $S^{-1}R = (S^{-1}A) \propto (S^{-1}(A/(XA + YA)) = (S^{-1}A) \propto 0 \cong (S^{-1}A) = K((X))[Y]] + M$ (since $S^{-1}(A/(XA+YA)) = 0$ (since $X(A/(XA+YA)) = 0$)) is not a v-Noetherian ring since M is not v-finite in R by [10, Proposition 2.3] (since M is invertible in T), as desired. □

Finally, we study the transfer of v-Noetherian property between a ring R and his amalgamated algebras along some ideals I of R.

Theorem 2.10. Let (A, m) be a local ring, B be a ring, $f : A \to B$ be a ring homomorphism and J be a proper ideal of B. Assume that one of the following statements holds:
1) A is a total ring of quotients, $J \subseteq Rad(B)$, $J \subseteq Z(B)$, f is injective and $f(A) \cap J \neq (0)$.
2) A is a total ring of quotients, $J \subseteq Rad(B)$, $J \subseteq Z(B)$, and f is not injective.
3) $f(M) \subseteq J$ and $J^2 = 0$.

Then $A \bowtie^f J$ is a v-Noetherian ring.

Proof. By Proposition 2.2, it suffices to show that $A \bowtie^f J$ is a total ring of quotients in all three cases.

1) Assume that $f(A) \cap J \neq (0)$. We claim that $A \bowtie^f J$ is a total ring of quotients. Indeed, let $(a, f(a) + j) \in A \bowtie^f J$, we prove that $(a, f(a) + j)$ is invertible or zero-divisor element. If $a \notin m$, then $(a, f(a) + j) \notin m \bowtie^f J$. And so $(a, f(a) + j)$ is invertible in $A \bowtie^f J$. Assume that $a \in m$. So, $(a, f(a) + j) \in m \bowtie^f J$. Since A is a total ring of quotients, there exists $0 \neq b \in A$ such that $ab = 0$. We have $(a, f(a) + j)(b, f(b)) = (0, jf(b))$. Using the fact that $f(A) \cap J \neq (0)$ and $J \subseteq Z(B)$, there exists some $0 \neq f(c) \in J$ and $0 \neq k \in J$ such that $jk = 0$ and so $(c, k) \in A \bowtie^f J$. It follows that $(a, f(a) + j)(bc, f(b)k) = (0, 0)$. Hence, there exists $(0, 0) \neq (bc, f(b)k) \in A \bowtie^f J$ such that $(a, f(a) + j)(bc, f(b)k) = (0, 0)$. Thus, $(A \bowtie^f J, m \bowtie^f J)$ is local total ring of quotients.

2) Assume that f is not injective. Our aim is to show that $A \bowtie^f J$ is a total ring of quotients. We prove that for each element $(a, f(a) + j)$ of $A \bowtie^f J$ is invertible or zero-divisor element. Indeed, if $a \notin m$, then $(a, f(a) + j) \notin m \bowtie^f J$. And so $(a, f(a) + j)$ is invertible in $A \bowtie^f J$. Assume that $a \in m$. So, $(a, f(a) + j) \in m \bowtie^f J$. Since A is a total ring of quotients, there exists $0 \neq b \in A$ such that $ab = 0$. We have $(a, f(a) + j)(b, f(b)) = (0, jf(b))$. Using the fact that f is not injective and $J \subseteq Z(B)$, there exist some $0 \neq c \in \mathrm{Ker}(f)$ and $0 \neq k \in J$ such that $jk = 0$ and $(c, k) \in A \bowtie^f J$. It follows that $(a, f(a) + j)(bc, f(b)k) = (0, 0)$. Hence, there exists $(0, 0) \neq (bc, f(b)k) \in A \bowtie^f J$ such that $(a, f(a) + j)(bc, f(b)k) = (0, 0)$. Thus, $(A \bowtie^f J, m \bowtie^f J)$ is a local total ring of quotients.

3) Assume that $f(M) \subseteq J$ and $J^2 = 0$. We prove that for each element $(a, f(a) + j)$ of $A \bowtie^f J$ is invertible or zero-divisor element. Indeed, if $a \notin m$, then $(a, f(a) + j) \notin m \bowtie^f J$. And so $(a, f(a) + j)$ is invertible in $A \bowtie^f J$. Assume that $a \in m$. So, $(a, f(a) + j) \in m \bowtie^f J$. Hence, $(a, f(a) + j)(0, k) = (0, 0)$ for every $k \in J$ since $f(a) + j \in J$ (since $f(M) \subseteq J$ and $a \in M$) and $J^2 = 0$. Thus, $(A \bowtie^f J, m \bowtie^f J)$ is a local total ring of quotients, completing the proof. □

Now, we are able to construct examples of non-coherent (and so non-Noetherian) v-Noetherian rings.

Example 2.11. Let A be a non-coherent total ring of quotients (See Proposition 2.2), $I \subseteq Nil(A)$, and set $R := A \bowtie I$. Then:
1) $R := A \bowtie I$ is a v-Noetherian ring by Theorem 2.10(1).
2) $R := A \bowtie I$ is a non-coherent ring by [8, Theorem 4.1.5, page 111] since A is a non-coherent ring.

Example 2.12. Let (A, M) be a non-coherent local ring, $B := A/M^2$ be a local ring with a maximal ideal $J := M/M^2$, and $f : A \to B$ be a canonical rings homomorphism, and set $R := A \bowtie^f$. Then:
1) R is a v-Noetherian ring by Theorem 2.10(3) since $f(M) = J$ and $J^2 = 0$.
2) R is a non-coherent ring by [8, Theorem 4.1.5, page 111] since A is a non-coherent ring.

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