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Image Thresholding Using Discrete Wavelet Transform with Retention of Possible Edge Contour

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Abstract. Image thresholding is the classical problem in the area of image processing. The noise of the image usually assume to be Gaussian can be removed using threshold methods, without disturbing the significant features of the image. One of the important features of the image is its edge contour, which is well illustrated by the ability to recognize objects from the drawing that only outlines edges. In this article, we introduced a novel threshold value used to estimate the true image from its noisy counterpart that considerably avoids the loss of edge contour. The standard of the image has been derived by peak signal to noise ratio $(psnr)$ and investigate with the established threshold parameters.

1 Introduction

De-noising of an image is the process of undoing noise from the image manipulated by the noise simultaneously keeping the significant features of the image. The most important feature of the image is its edge contour. Edge contours are vital visual clues and particularly important for image understanding. The non-linear method is applied as proposed by Donoho and Johnstone[2] and Donoho [3] to undo the noise. This method is set upon thresholding the wavelet coefficients derived from Discrete Wavelet Transform (DWT) of the data. Wavelet thresholding rely on the selection of threshold factor. Since the work of Donoho and Johnstone[2],[3]and [4] there has been a fair amount of work on the choice of the threshold level([5],[7],[8]). A small threshold value may yield the image with an edge contour close to the edge contour of the original image and the resultant may be noisy. The largest threshold value gives a smooth image at the expense of significant features of the image. In this paper, we are using Orthogonal DWT and we have described an optimal threshold parameter that keeps the image features as much as possible and improves the quality of the image.

2 Discrete wavelet transform

Given discrete set of values $y = (y_1, y_2, \ldots, y_n)^T$, $w = Wy$ is the DWT of y; W is the $n \times n$ DWT matrix, rely on the choice of the wavelets. The resultant $w = (c_{(0,0)}, d_{(0,0)}, \cdots, d_{(J-1,2^j-1)})^T$ is an empirical wavelet coefficient at level j and position $k2^{-j}$ for $j = 1, 2, \cdots, J-1$ and $k = 1, 2, \cdots, 2^j - 1$. Since W is orthogonal matrix the inverse discrete wavelet transform (IDWT) is given by $y = W^T w$. In case of two dimension the data $\{y_{ij}, 1 \le i, j \le n\}$ form a square matrix called a data set. In image processing a matrix data set represent a digital image. The two dimensional DWT can be computed by Mallats pyramid algorithm which applied many one dimensional DWT (Duabechies 1992, Mallat 1989). The outcome of the transformation is the diagonal details, horizontal details, vertical details and sub-band approximate coefficients.

3 Wavelet thresholding

Suppose, an image $\{x_{ij}, 1 \leq i, j \leq n\}$, with $N = n \times n$ pixels manipulated by white Gaussian noise. The noisy image is given by:

$$y_{ij} = x_{ij} + \epsilon_{ij} \tag{3.1}$$

ϵ is the iid as $N(0, \sigma^2)$; noise has $\sigma-$ standard deviation. Now, we have to find x_{ij} by undoing noise, thus mean square error (mse) is minimum.
Here, non-linear estimation is given by

(i) 2-dimensional DWT is applied to noisy data y_{ij} and we get the sub-band coefficients. The orthogonality of the transform assure that the noise is of the Gaussian nature.

(ii) Use of hard or soft threshold rule to obtain the detail coefficients. A particular threshold $\lambda > 0$, have the soft threshold value

$$\delta^s(w, \lambda) = Sign(w)(|w| - \lambda)I(|w| > \lambda) \tag{3.2}$$

a "shrink or kill" rule. The hard threshold value is

$$\delta^H(w, \lambda) = wI(|w| > \lambda) \tag{3.3}$$

a "keep or kill" rule. The thresholded wavelet coefficient attained using the above threshold rule $\delta(w, \lambda)$ from 3.2 and 3.3 we attain the selective reconstruction.

(iii) The IDWT from thresholded wavelet coefficients applied to rebuild the image to achieve de-noised image $\hat{x}_{ij}$.

4 The selection of threshold

The selection of the threshold is important to succeed in the above procedure. Various research activities have been undertaken on finding the threshold value and some of the threshold values are particularly defined for images. In this section, we discussed some of the established thresholding techniques along with the proposed technique.

4.1 Visushrink

A thresholding technique obtained by using the universal threshold put forward by Donoho and Johnstone [2] is called Visushrink and is presented by $\lambda_{univ} = \hat{\sigma}\sqrt{2logN}$; N cardinality of the pixels and $\hat{\sigma}$ is the estimated standard deviation of the noise. The threshold value is quite large. Therefore, produces an exceedingly smoothed estimate of the image. It is due to the universal the threshold is obtained beneath the restriction having larger probability and the estimate must be smooth as original signal.

4.2 Minimax estimation

Minimax estimation is the thresholding technique which uses the threshold parameter which rely on the size of data N, given by $\lambda^M = \hat{\sigma}\lambda_N^*$; λ_N^* is the value of λ attains

$$\Lambda_N^* = \inf_{\lambda} \sup_{d} \{R_\lambda(d)/(N^{-1} + R_o(d))\} \tag{4.1}$$

with $R_\lambda(d) = E[\delta_\lambda(\hat{d}) - d]^2$ and $R_o(d)$ is the ideal risk obtained by oracle. This threshold value obtained from Donoho and Johnstone [2] approaches the the universal threshold value for large data and hence resulting to yield smoothed estimate of the image.

4.3 Sure shrink

Sure shrink is the thresholding technique that uses the threshold obtained by Steins unbiased risk (Sure)[4]. The level-dependent thresholds are obtained by the different level j of the wavelet transformation an independent multivariate normal estimation problem. For fixed j, if $\{y_{jk}\}$ is the noisy wavelet coefficient obtained by applying DWT to the noisy data. Steins unbiased estimate of risk to $\hat{\theta_{j,k}} = \delta^s(y_{jk}, \lambda)$ allow an estimate of the risk to the given threshold value λ and minimized λ offer a selection of the threshold for j^{th} level.

4.4 Proposed estimate

As a try to preserve the edge features of the restored image, we put forward to threshold the image considering the threshold value

$$\lambda_p = \hat{\sigma}(\sqrt{2J} - \sqrt{J/2}) \tag{4.2}$$

with $N = 2^J \times 2^J$ as the size of data. This threshold value is proposed based on the experiments done on several test images applying soft threshold rule. Soft thresholding applied over hard thresholding since it offers visually rich image where as the latter gives rough artifacts in the retrieved image. Since the continuity of the soft threshold the algorithms can be made more tractable [3]. The proposed threshold value which depends on the size of the data is asymptotically ideal and easy to use. Compared to the universal and minimax, this threshold value is low and hence includes some important features in the estimated image. From the above estimation processes, the noise standard deviation $\hat{\sigma}$ is calculated by following equation $\hat{\sigma} = \dfrac{median(|Y_{ij}|)}{0.6745}$ where Y_{ij} belongs to the subband in the first level of decomposition, which gives the diagonal details of the image [4].

5 Results and discussion

To test the performance we oversee experiments on different Gray scale images of different sizes like Lena(512 × 512pixels), house(256 × 256 pixels) and cameraman(128 × 128 pixels) at various noise levels $\sigma = 10, 15, 20, 25$ and 30 applying Daubechies least symmetric compactly supported wavelets with eight vanishing moments at level three [1]. Here, outcomes are related to sure, minimax and the universal threshold applying soft threshold rule given by equation 3.3. The aimed trait of the restored image is given by peak signal to noise ratio

$$psnr = 10log_{10}\frac{256^2}{mse}. \tag{5.1}$$

with mse is the mean square error of the original and the de-noised image of size $N = n \times n = 2^J \times 2^J$. The mse is calculated by applying the relation

$$mse = \frac{1}{N}\sum_{i=1}^{n}\sum_{j=1}^{n}[x(ij) - \hat{x}(ij)]^2. \tag{5.2}$$

Table 1. compare the sure, universal, minimax and the proposed threshold in terms of $psnr$ with soft thresholding rule. The resultant gives the $psnr$ of the proposed threshold value is significant for large value of σ and hence yields a quality reconstruction of the image. *Table 2* compare mse between the edges of the estimated and the original image by above threshold values. For various values of σ, the proposed threshold value provides far better performance compare to other threshold parameters. *Figure 1* exhibit the comparison of sure, universal, minimax and proposed threshold values of the image house(256 × 256 pixels) at $\sigma = 25$ considering soft threshold rule. *Figure 2* gives the comparison of sure, minimax, universal and proposed threshold value for the image Lena(512 × 512 pixels) at $\sigma = 25$ with soft threshold rule. The proposed threshold value retains the edge features far better compare to minimax and universal threshold values.

The above method is also applicable for rectangluar images of various sizes for different noisy images, here we are using speckle noise. In general, the establishment of threshold values

Table 1. $psnr$ values for various noise level σ for the images of Lena, house and cameraman of pixels 512×512, 256×256 and 128×128 respectively as follows.

σ	sure	minimax	universal	proposed
10	32.9245	30.0161	28.9805	31.9988
15	30.2887	28.7022	27.8085	30.4680
20	28.2871	27.8163	27.0453	29.3558
25	26.6714	27.1629	26.4963	28.4835
30	25.3169	26.6507	26.0720	27.7699
10	32.7193	29.9687	28.7099	31.8314
15	30.1258	28.6096	27.4867	30.2671
20	28.1358	27.6846	26.6593	29.1214
25	26.5284	26.9731	26.0355	28.2200
30	25.1768	26.3954	25.5420	27.4688
10	30.9705	26.7469	25.0016	28.8998
15	28.6670	25.2444	23.6432	27.1929
20	26.7782	24.1849	22.1050	25.9721
25	25.3254	23.4215	22.7380	25.0500
30	24.1056	22.8037	21.6274	24.2863

Table 2. mse between the edge contours of the original and estimated images of Lena, house and cameraman for various noise level σ respectively.

σ	sure	Minimax	universal	proposed
10	0.0170	0.0285	0.0313	0.0231
15	0.0220	0.0320	0.0339	0.0270
20	0.0266	0.0340	0.0355	0.0303
25	0.0306	0.0351	0.0361	0.0328
30	0.0346	0.0358	0.0367	0.0350
10	0.0145	0.0267	0.0309	0.0203
15	0.0187	0.0306	0.0347	0.0247
20	0.0224	0.0335	0.0386	0.0282
25	0.0257	0.0371	0.0398	0.0305
30	0.0298	0.0383	0.0410	0.0330
10	0.0129	0.0276	0.0356	0.0182
15	0.0177	0.0347	0.0410	0.0240
20	0.0204	0.0386	0.0453	0.0292
25	0.0245	0.0408	0.0502	0.0333
30	0.0275	0.0438	0.0519	0.0369

Table 3. $psnr$ values for various noise level σ for the boat image pixels 256×512 as follows.

σ	sure	Minimax	universal	proposed
10	31.14	30.92	31.18	29.83
15	30.60	31.08	30.63	30.77
20	30.09	30.72	30.02	31.13
25	29.72	30.26	29.7	31.13
30	29.5	29.93	29.05	30.88

of existing methods depends on number of pixels N which can be given interms of J in the following relation.

$$J = \frac{logN}{2log2} + \frac{r}{2}; 1 \leq r < J$$

Table 3. compare the sure, universal, minimax and the proposed threshold in terms of $psnr$ with soft thresholding rule. The resultant gives the $psnr$ of the proposed threshold value is significant for large value of σ and hence yields a quality reconstruction of the image. *Figure 3.* exhibit the comparison of sure, universal, minimax and proposed threshold values of the image boat(256×512 pixels) at $\sigma = 25$ considering soft threshold rule.

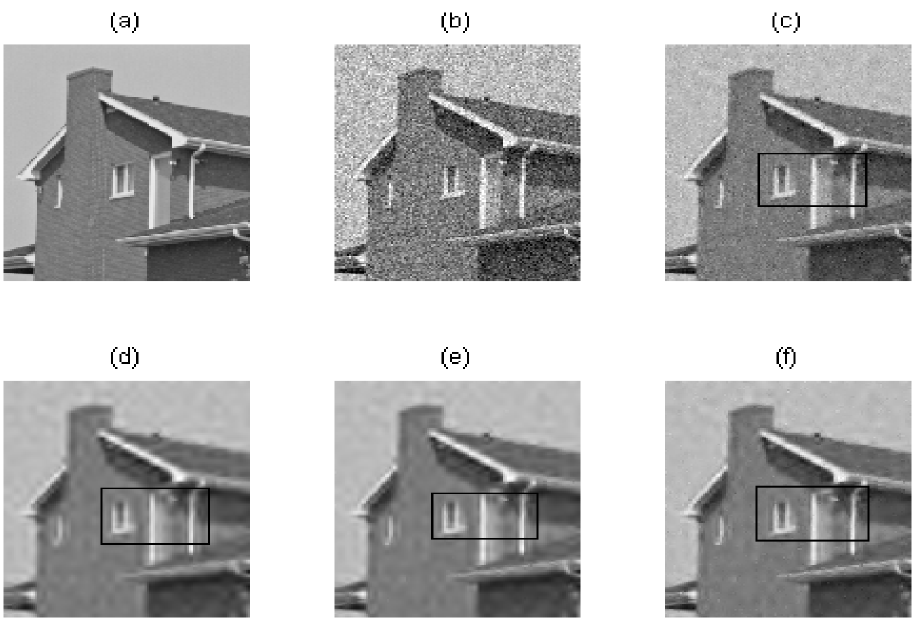

Figure 1. Images: (a) Original house (a) noisy image with $\sigma = 25$(c) de-noised using sure shrink (d) using universal (e) using minimax (f) using proposed threshold with soft threshold rule.

6 Comments and Conclusion:

This article, provides a novel threshold value obtained to undo the noise from the noise-contaminated image with the retention of edge features by the use of wavelets. This threshold value has been applied for undoing the noise from different test images of different sizes and calculated the peak signal-to-noise ratio of various values of noise level σ to interpret the performance of this threshold value. By comparing the results we can analyze that the proposed threshold value gives more improved values applied in visushrink and minimax estimation. We also measured the mse performance between the edges of original and estimated images and found very close to sure shrink.

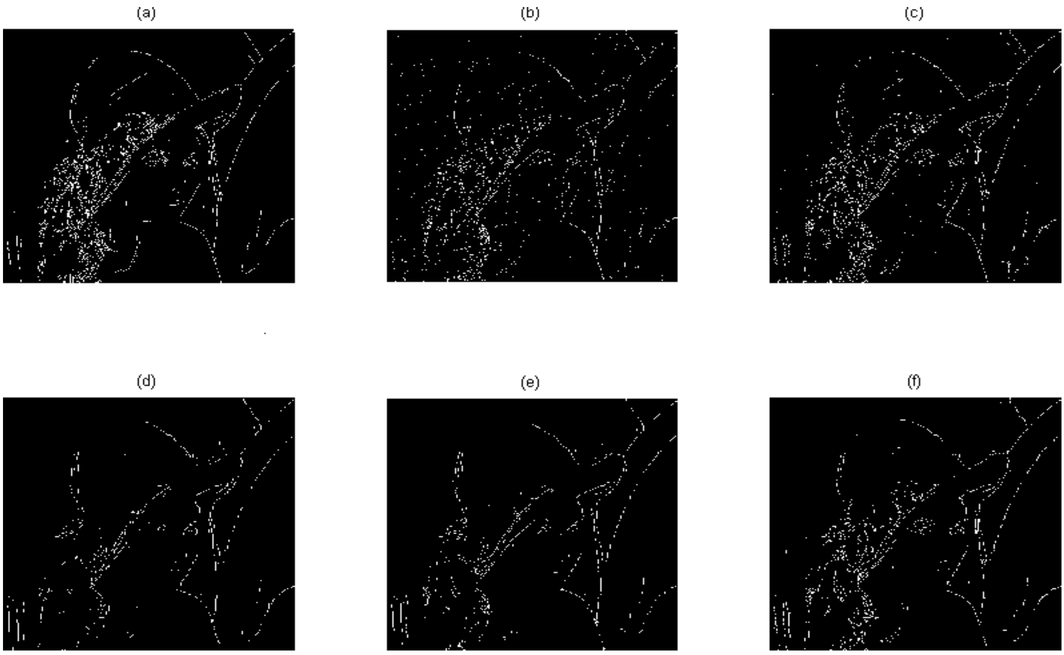

Figure 2. (a) Original edge contour (b)noisy edge contour (c) Edge contour of de-noised image with $\sigma = 25$ using sure shrink (d) Applying universal (e) Applying minimax (f) Applying proposed threshold with soft threshold rule.

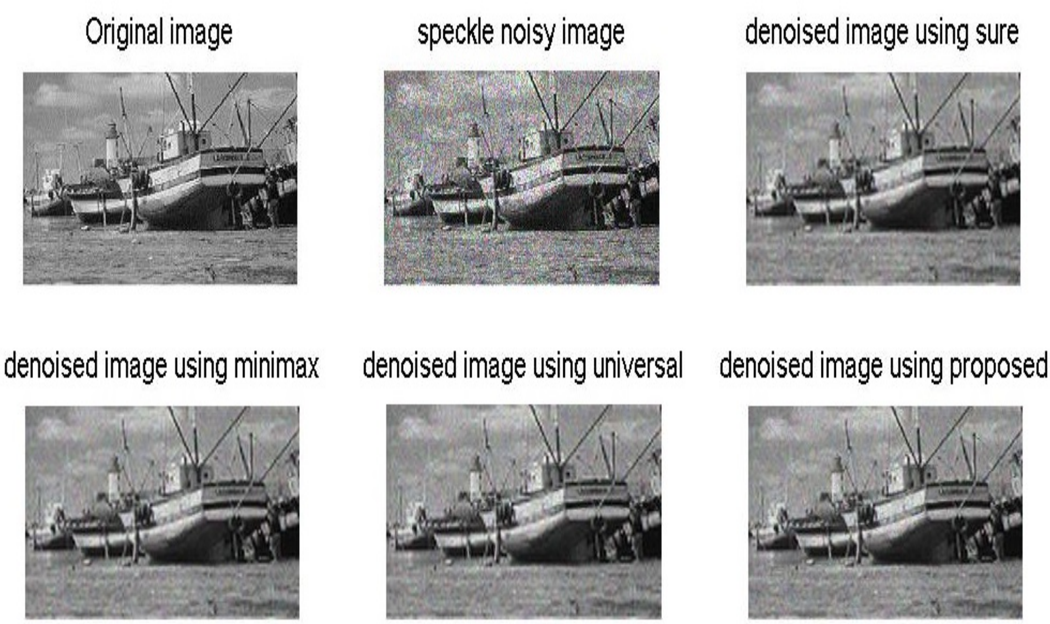

Figure 3. Images of boat with soft threshold rule.

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