HOMOMORPHISMS BETWEEN RESTRICTED GENUS GROUPS

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Abstract For two finitely generated groups with finite commutator subgroup $G_1$ and $G_2$, homomorphisms between genera of groups $G(G_1)$ and $G(G_2)$ have been established in the literature, especially when $G_2$ is some quotient group obtained from $G_1$ or when $G_2$ is some power of $G_1$. These groups $G_i, i = 1, 2$ are called $\chi_0$-groups. For $\chi_0$-groups under a given finite group $F$, we establish a homomorphism between the restricted genera. In the case, the homomorphism is surjective, it provides a computational method of the restricted genus.

1 Introduction

The theory of $\pi$-localization of groups, where $\pi$ is a family of primes, appears to have been first discussed in [7, 8] by Mal’cev and Lazard. In their work emphasis was placed on the explicit construction of the localization and properties of the localization $G_\pi$ of the nilpotent group $G$ were deduced from the construction, utilizing nilpotent group theory. Baumslag in [1] has given a comprehensive treatment of the main properties of nilpotent groups as they relate to the problem of localization. He has explicitly shown in [2] how to construct $G_\pi$ in the case of an arbitrary nilpotent group $G$ and an arbitrary family of primes $\pi$. Thus extending the generality of Malcev’s original construction. Bousfield-Kan [3] exploit this general Malcev construction in their study of completion and localization.

In the 1970s, Hilton and Mislin became interested, through their work on the localization of nilpotent spaces, in the localization of nilpotent groups. Mislin [10] defines the genus $G(N)$ of a finitely generated nilpotent group $N$ to be the set of isomorphism classes of finitely generated nilpotent groups $M$ such that the localizations $M_p$ and $N_p$ are isomorphic at every prime $p$. This version of genus became known as the Mislin genus, and other very useful variations of this concept came into being. In [6] Hilton and Mislin define an abelian group structure on the genus set $G(N)$ of a finitely generated nilpotent group $N$ with finite commutator subgroup. For nilpotent groups which belong to class $K$ (of semidirect products of the form $T \times \mathbb{Z}^k$, where $T$ is a finite abelian group and $k$ is a positive integer), many computations of the genus groups appear in the literature.

The concept of cancellation is closely related to those of genus and localization of groups. When localizing non-nilpotent groups, it may happen that the kernel of the localizing homomorphism is bigger than what we would require. So, for a non-nilpotent finitely generated group $G$ with finite commutator subgroup, rather than considering localization, the idea of the genus is generalized through non-cancellation. For a group $G$, the non-cancellation set, denoted by $\chi(G)$, is the set of isomorphism classes of groups $H$ such that $G \times \mathbb{Z} \cong H \times \mathbb{Z}$. For a nilpotent $\chi_0$-group $H$ it was shown by Warfield [14] that $\chi(H) = G(H)$, where $G(H)$ is the Mislin genus of $H$. In O’Sullivan’s paper [11, theorem 4.2] there are further equivalent definitions for the set $\chi(G)$. It is proved in [11] that for two $\chi_0$-groups $H$ and $G$, $H \times \mathbb{Z} \cong G \times \mathbb{Z}$ if and only if for every finite set $\pi$ of primes, we have $H_\pi \cong G_\pi$ (the $\pi$-localizations are isomorphic). Thus for a $\chi_0$-group $H$, the set $\chi(H)$ coincides with the restricted genus $\Gamma_\pi(H)$ of $H$.

Aspects of localization as in groups and related categories have been studied in a unified way in a categorical setting, see [12] for instance. In [9], Mba and Witbooi introduced a category $\text{Grp}_F$ of groups under a finite group $F$. An object of $\text{Grp}_F$ is a group homomorphism $h : F \rightarrow G$, and
is denoted by $(G, h)$. They defined a group structure on the restricted genus set $\Gamma_f(h) = \chi(h)$ of such object. We will $\chi(G, h)$ for the notation of this restricted genus. This genus group coincides with the non-cancellation group $\chi(G)$ of the underlying group $G$, in the case that $F$ is the trivial group.

Constructing a map that preserves algebraic structure is a natural exercise when dealing with objects having interesting algebraic structure, and presents computations advantages. For example, for a semidirect product $H = \mathbb{Z}_m \rtimes_\omega \mathbb{Z}$, the authors in [5] showed that there is a well-defined surjective homomorphism $\Gamma : \chi(H) \to \chi(H')$ given by $[K] \to [K \times H'^{-1}]$ where $K$ is a group such that $K \times \mathbb{Z} \cong H \times \mathbb{Z}$ and $r$ is a natural number. Thus, in order to compute the group $\chi(H')$ one needs only to compute the kernel of the homomorphism $\Gamma$. For a fixed morphism $h : F \to G$, the restricted genus $\chi(G, h)$ is the set of isomorphism classes of morphisms $F \to H$, which are $\pi$-equivalent to $h$ at every finite set of primes $\pi$. For a well-defined integer $n$ depending on $G$, in [9] an epimorphism $\zeta : (\mathbb{Z}/n)^* / \pm 1 \to \chi(G, h)$ is established and it is shown that there exist natural epimorphisms $\chi(G, h) \to \chi(G/h(F))$ (provided $h(F)$ is normal in $G$) and $\chi(G, h) \to \chi(G, h \circ i)$ (provided $\chi(F_0) = F$ is a morphism), which are compatible with the various involved maps $\zeta$.

Having such homomorphisms is not always given. In [9], computation methods of $\chi(G, h)$ in the special case $G$ is a semidirect product $T \rtimes_\omega \mathbb{Z}^k$ are used in a very particular example to provide a concrete computation of $\chi(G, h)$. It is used to show that there doesn’t exist any natural morphism $\chi(G) \to \chi(G/h(F))$ under $\chi(G, h)$.

In this paper, we focus on the class $\mathcal{X}_0$ of all finitely generated groups with finite commutator subgroup. Given two such groups $G_1$ and $G_2$ for which $n_1 = n(G_1)$ and $n_2 = n(G_2)$ are relatively prime, we aim at establishing a homomorphism between localization of such groups under a given finite group $F$.

The remainder of this paper is organized as follows: Section 2 presents some preliminaries and notations. In Section 3, we establish a homomorphism $\chi(G_1, h) \to \chi(G_2, h)$ where $(n_1, n_2) = 1$ and provide the conditions under which it is an epimorphism. Under this condition, it follows from the first Isomorphism Theorem that $\chi(G_1, h_1)/\text{Ker}\varphi \cong \chi(G_2, h_2)$. This gives a computational method to obtain $\chi(G, h)$ from $\chi(G_1, h_1)$ when $n_1$ and $n_2$ are relatively prime.

## 2 Preliminaries and Notation

Let us fix a finite group $F$ (and throughout this chapter $F$ will denote some finite group). Let $\text{Grp}_F$ be the category of groups under $F$. Here we mean that the objects of $\text{Grp}_F$ are group homomorphisms $\varphi : F \to G$. Given another object $\varphi_1 : F \to G_1$, a morphism in $\text{Grp}_F$ corresponds to a group homomorphism $\alpha : G \to G_1$ such that $\alpha \circ \varphi = \varphi_1$.

For a set of primes $\pi$, let us regard the $\pi$-localization of an object $\varphi : F \to G$ as being the object $\varphi_\pi : F \to G_\pi$. Then localization is an endofunctor of $\text{Grp}_F$. Let $\mathcal{X}_F$ be the full subcategory of $\mathcal{X}_0$-groups under $F$. Thus we can define the restricted genus $\Gamma_f(\varphi)$ as the set of all isomorphism classes $[\psi]$ of objects $\psi \in \mathcal{X}_F$ such that for every finite set $\pi$ of primes, we have $\psi_\pi$ isomorphic to $\varphi_\pi$. If $F$ is the trivial group, then $\mathcal{X}_F$ can be identified with the class $\mathcal{X}_0$ of groups. In line with [15] and in analogy with $\mathcal{X}_0$-groups we shall write $\Gamma_f(\varphi) = \chi(\varphi)$.

For a specified homomorphism $\beta : E \to F$, there is a functor $\beta_* : \text{Grp}_F \to \text{Grp}_E$. Henceforth we shall also specify the codomain when referring to an object $\varphi : F \to G$ of $\mathcal{X}_F$, and denote the object by $(G, \varphi)$.

For any $\mathcal{X}_0$-group $G$, the torsion elements form a characteristic subgroup of $G$, which we shall denote by $T_G$.

**Remark 2.1.** (a) Recall from [15, Section 2] how we assign to a $\mathcal{X}_0$-group $G$ a natural number $n(G)$: Let $n_1$ be the exponent of $T_G$, let $n_2$ be the exponent of the group $\text{Aut}(T_G)$, and let $n_3$ be the exponent of the torsion subgroup of the centre of $G$. Now we take $n(G) = n_1n_2n_3$. Then $n = n(G)$ has the property that the subgroup $G^{(n)} = \langle g^n : g \in G \rangle$ of $G$ belongs to the centre of $G$ and $G/G^{(n)}$ is a finite group.

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1For the special case $F$ is trivial $(F = *)$, $\chi(G, *) \to G = \chi(G)$
(b) Let $\pi = \{p : p \text{ is a prime and } p|n(G)\}$. Then the short exact sequence $1 \to G^{(n)} \to G \to G/G^{(n)} \to 1$ determines $G$ as an extension of a $\pi'$-torsion-free finitely generated abelian group $G^{(n)}$ by a $\pi$-torsion group $G/G^{(n)}$. From [4, Proposition 3.1], it follows that the $\pi$-localization homomorphism $G \to G_\pi$ is injective.

(c) In [15] it is shown that $\chi(G)$ is a group and there is an epimorphism $\zeta : \mathbb{Z}_n^* \to \chi(G)$.

(d) Throughout the rest of Section 2, $G$ will denote an infinite $\chi_0$-group, $n = n(G)$, $T_G$ a torsion subgroup of $G$, $F$ a finite group and $h : F \to G$ a homomorphism.

We note that [15, Theorem 4.3] can be formulated in a stronger form.

**Proposition 2.2.** Let $G$ be a $\chi_0$-group and let $n = n(G)$. Let $H$ and $L$ be subgroups of finite index in $G$. If the index $[G : H]$ of $H$ in $G$ is relatively prime to $n$ and $[G : L] \equiv \pm [G : H] \bmod n$, then there is an isomorphism $\phi : L \to H$ such that $\phi(x) = x$ whenever $x \in T_G$.

**Proof.** A proof of Proposition 2.2 can be obtained from the proof of [15, Theorem 4.3], noting that the latter proof starts off with the inclusion map of $T_G$ into $G$ and we consistently work under the inclusions of $T_G$ into $H$, $L$ and $G$.

Note that if $X \leq G$ and $T_G \subseteq X$, then $h$ induces a homomorphism $h_X : F \to X$. We use this notation henceforth. □

**Proposition 2.3.** [9, Theorem 2.3] Let $(L, l)$ be an object representing a member of $\chi(G, h)$. Then there exists a subgroup $J$ of $G$ with $[G : J]$ finite and $[G : J]$ relatively prime to $n$, such that in $\text{Grp}_{\mathbb{F}}$ the object $F \to J$ is isomorphic to $(L, l)$.

**Proof.** Suppose that $(L, l)$ is an object representing a member of $\chi(G, h)$.

Let $\pi$ be the set consisting of all the prime divisors of $n$ and all the prime divisors of $|T_G|$. Then there is an isomorphism $\alpha : L_\pi \to G_\pi ([L_\pi, l'] \to (G_\pi, h'))$ such that the following diagram is commutative:

$$
\begin{array}{ccc}
F & \xrightarrow{i} & L \\
\downarrow h & & \downarrow \alpha \\
G & \xrightarrow{\alpha} & G_\pi
\end{array}
$$

We observe that $G_\pi$ contains an isomorphic copy of $T_G$ and every torsion element of $G_\pi$ is contained in this subset, which we shall denote by $T_G$.

Now $T_G$ is normal in $G_\pi$ and $G_\pi/T_G$ is a $\pi$-local torsion-free abelian group.

Let $\eta : G_\pi \to G_\pi/T_G$ be the canonical epimorphism.

We now set out to find a subgroup $M$ of $G_\pi$ which has the following properties:

1. $M \supseteq \exists(L) \cup \exists(G)$,
2. $[M : \exists(G)] \equiv 1 \bmod n$.

First we choose $M_0 = \langle \exists(G) \cup \exists(L) \rangle < G_\pi$.

Then $r = [M_0 : G] < \infty$ and $r$ is relatively prime to $n$. Choose $r'$ such that $(r', n) = 1$ and $r'r \equiv 1 \bmod n$.

Let $M_1 = \exists[M_0 \to G_\pi/T_G] = \eta M_0$. Then $M_1$ is finitely generated and torsion-free. Take any $\mathbb{Z}$-basis $\delta_1, \cdots, \delta_k$ for $M_1$. Then since $\mathbb{Z}_\pi$ is $\pi$-local and $(r', q) = 1 \forall q \in \pi$, we can find $\delta_0 \in G_\pi/T_G$ such that $r'\delta_0 = \delta_1$.

Then $\{\delta_0, \delta_2, \delta_3, \cdots, \delta_k\}$ is linearly independent.

Let $M_2 = \langle \delta_0, \delta_2, \delta_3, \cdots, \delta_k \rangle$ and take $M = \eta^{-1}(M_2)$.

It follows that $[M : M_2] = r'$ and therefore $[M : G] = [M : M_0] \cdot [M_0 : G] = r'r \equiv 1 \bmod n$.

Then by Theorem 2.2, $(M, h')$ is isomorphic to $(G, h)$ (here $h'$ is $h$ followed by the inclusion $G \to M$). Furthermore we have a morphism $\beta : (L, l) \to (M, h')$ such that $\beta : L \to M$ is a monomorphism. The theorem follows. □
Notation Fix any \( m \in \mathbb{N} \). Let \( X(m) = \{u \in \mathbb{N} \mid (u, m) = 1\} \). Now consider any \( G \in \mathcal{X}_2 \) and let \( n = n(G) \). Let \( Y(G, h) \) be the set of all \( u \in X(n) \) for which there exists a subgroup \( J \) of \( G \) with \( [G : J] = u \) and such that the object \((J, h_J)\) represents a member of \( \chi(G, h) \). Here \( h_J \) is the induced homomorphism obtained from \( h \) with \( J \).

(b) If \( \xi \) is a function of the type defined in 2, then \( \xi \) induces a (well-defined) function \( \zeta: Y^*(G, h) / \pm 1 \rightarrow \chi(G, h) \).

(c) The fibre \( \zeta^{-1}[G, h] \) of \( \zeta \) over \([G, h]\) is a subgroup of \( Y^*(G, h) / \pm 1 \).

(d) For any \([K, k] \in \chi(G, h)\), \( \zeta^{-1}[K, k] \) is a coset of \( \zeta^{-1}[G, h] \).

Proof. (a) Since \( 1 \in Y^*(G, h) \), the subset in non-empty. It suffices to prove multiplicative closure. Now given any \( u, v \in Y(G, h) \), choose a subgroup \( G_u \). We obtain a subgroup of the form \( G_{uv} \) by taking any subgroup \( K \) of \( G_u \) such that \((K, h_K)\) represents a member of \( \chi(G_u, h_u) \) and with \([G_u : K] = v \). This proves (a).

(b) This follows from Theorem 2.2.

(c) We first prove that \( \zeta^{-1}(G, h) \) is closed with respect to inversion. Pick any \( u \in X(n) \) such that \( u \in \zeta^{-1}(G, h) \), and let \( v \in X(n) \) be such that \( uv \equiv \pm 1 \) mod \( n \). Let \( K \) and \( L \) be subgroups of indices \( u \) and \( v \) in \( G \). Since \((K, h_K) \cong (G, h)\) there is an isomorphism \( \alpha: G \rightarrow K \) such that \( \alpha(x) = x \) for each \( x \in h(F) \). Let \( \alpha(L) = K \). Now we not that \((L, h_L) \cong (J, h_J)\). On the other hand, \([G : J] = [G : K] \cdot [K : J] = u \cdot [G : L] = uv \equiv \pm 1 \) mod \( m \), and thus by Theorem 2.2, we have \((J, h_J) \cong (G, h)\) and it follows that \( \zeta^{-1}(G, h) \). So we have proved that \( \zeta^{-1}(G, h) \) is closed with respect to inversion. It is easy to prove that \( \zeta^{-1}(G, h) \) is multiplicatively closed.

(d) Let \( u, v \in X(n) \). Suppose that \( \zeta(v) = [G, h] \), and let \( K \subset G \) be such that \([G : K] = u \). Now we prove that \( \zeta(uv) = [K, h_K] \). Since \( \zeta(v) = [G, h] \), there is an embedding \( \alpha: G \rightarrow G \) such that \( \alpha(x) = x \) for each \( x \in h(F) \), and such that \([G : \alpha(G)] = v \).

Let \( L = \alpha(K) \). Then \( [L, h_L] = \zeta(uv) \) because \([G : L] = [G : \alpha(G)] \cdot [\alpha(G) : \alpha(K)] = vu \). Moreover, \([L, h_L] = [K, h_K] \). Thus \( v \zeta^{-1}[G, h] \subset \zeta^{-1}[K, h_K] \). The inclusion \( v \zeta^{-1}[G, h] \supset \zeta^{-1}[K, h_K] \) can be proved in a similar manner. This completes the proof of (c). \( \square \)

Remark 2.5. Suppose that \( h: F \rightarrow G \) is a group homomorphism and \( h(F) \trianglelefteq K \subset G \). If \( h(F) \trianglelefteq G \) and \( h(F) \subset K \), then \( h(F) \trianglelefteq K \). In this case, the quotient group \( K/h(F) \) is defined and we shall denote it by \( \bar{K} \).

3 Homomorphisms between localization genera
Let \( G_1 \) and \( G_2 \) be \( \mathcal{X}_2 \)-groups and let \( h_1: F \rightarrow G_1 \) and \( h_2: F \rightarrow G_2 \) be fixed morphisms for a given finite group \( F \).

Consider the non-cancellation groups \( \chi(G_1, h_1) \) and \( \chi(G_2, h_2) \). Let \( n = n(G) \) be as defined above. In the following, we assume that \( n_1 = n(G_1) \) and \( n_2 = n(G_2) \) are relatively prime. We have the following proposition which is a consequence of Euclid’s Lemma.

Proposition 3.1. Given two integers \( n \) and \( m \) such that \((n, m) = 1\), it follows that for any \( u, k \in \mathbb{N} \), \((n^u, m^k) = 1\), that is any powers of \( n \) and \( m \) are also relatively prime.
Remark 3.7. Consider the following diagram:

\[ \begin{array}{ccc}
  Y(G_1, h_1) & \xrightarrow{\gamma_{12}} & Y(G_2, h_2) \\
  \gamma_1 & & \gamma_2 \\
  Y^*(G_1, h_1)/\pm 1 & \xrightarrow{\alpha} & Y^*(G_2, h_2)/\pm 1 \\
  \chi(G_1, h_1) & \xrightarrow{\varphi} & \chi(G_2, h_2)
\end{array} \]

\[ \begin{array}{c}
  \gamma_{12} = \gamma_2 \circ \gamma_1
\end{array} \]

Proof. The existence and well-definedness of \( \gamma_{12} \) follows directly from the proof of Proposition 3.1.

Proposition 3.6. Suppose that we have groups \( A, B \) and \( C \) together with a homomorphism \( \beta : A \to C \) and a surjective group homomorphism \( \gamma : A \to B \). If \( \alpha : B \to C \) is a function (between sets) such that \( \alpha \circ \gamma = \beta \), then \( \alpha \) is a homomorphism. Moreover, if \( \beta \) is surjective, then \( \alpha \) is also surjective.

Remark 3.7. Consider the following diagram:
Notice that if $u_i \neq \pm 1$, $u_i \in Y(G_i, h_i)$, and since the units of $\mathbb{Z}_{n_i}$ are relatively prime, we have that $\gamma_i(u_i) = u_i$. Therefore $\alpha \circ \gamma_1(u_i) = \alpha(u_i)$ and $\gamma_2(\gamma_1(u_i)) = \gamma_2(\gamma_1(u_i))$. This implies that $\alpha \circ \gamma_1 = \gamma_2(\gamma_1(u_i))$, that is the diagram is commutative. We can therefore conclude that the map $\alpha$ can be defined as $\alpha : u \mapsto n_1^{\ln(u)}$.

Now we state and prove our main result.

**Theorem 3.8.** Let $\alpha : Y^*(G_1, h_1)/\pm 1 \to Y^*(G_2, h_2)/\pm 1$ be as defined above. Then we have the following:

(i) $\alpha$ is a well-defined homomorphism for which we have the following diagram, with $\alpha \circ \gamma_1 = \gamma_2 \circ \gamma_1$:

$$Y(G_1, h_1) \xrightarrow{\gamma_1} Y(G_2, h_2)$$

$$Y^*(G_1, h_1)/\pm 1 \xrightarrow{\alpha} Y^*(G_2, h_2)/\pm 1$$

(ii) $\alpha$ is the homomorphism such that there exist a unique homomorphism $\varphi : \chi(G_1, h_1) \to \chi(G_2, h_2)$ such that the following diagram is commutative:

$$Y^*(G_1, h_1)/\pm 1 \xrightarrow{\alpha} Y^*(G_2, h_2)/\pm 1$$

$$\chi(G_1, h_1) \xrightarrow{\varphi} \chi(G_2, h_2)$$

**Proof.**

(i) We first prove that $\alpha$ is well-defined.

Let $u_1$ and $u_2$ be in the domain of $\alpha$ such that $u_1 = u_2$. Then $\alpha(u_1) = n_1^{\ln(u_1)} = n_1^{\ln(u_2)} = \alpha(u_2)$. Next we show that $\alpha$ is a homomorphism, that is, for any $u_1, u_2$ in the domain of $\alpha$, $\alpha(u_1 u_2) = \alpha(u_1) \alpha(u_2)$.

**Case 1:** $\ln(u_1) \mod 1 + \ln(u_2) \mod 1 < 1$. $\alpha(u_1 u_2) = n_1^{\ln(u_1) + \ln(u_2)} = n_1^{\ln(u_1) + \ln(u_2)} = n_1^{\ln(u_1) + \ln(u_2)} = \alpha(u_1) \alpha(u_2)$.

**Case 2:** $\ln(u_1) \mod 1 + \ln(u_2) \mod 1 \geq 1$.

$\alpha(u_1 u_2) = n_1^{\ln(u_1) + \ln(u_2) - 1} = n_1^{\ln(u_1) + \ln(u_2) - 1} = n_1^{\ln(u_1) + \ln(u_2) - 1} = \alpha(u_1) \alpha(u_2)$.

Hence $\alpha$ is a homomorphism.

(ii) Let $u \in Y^*(G_1, h_1)/\pm 1$. Then, $\theta(u) = [J_u, h_u]$ such that $[G : J_u] = u$ and $\alpha(u) = n_1^{\ln(u)} \in Y^*(G_2, h_2)$.

$\gamma(n_1^{\ln(u)}) = [J_1^{\ln(u)}, h_1^{\ln(u)}]$ such that $[G : J_1^{\ln(u)}] = n_1^{\ln(u)}$.

Therefore $\varphi : \chi(G_1, h_1) \to \chi(G_2, h_2)$ is such that $\varphi([J_u, h_u]) = [J_1^{\ln(u)}, h_1^{\ln(u)}]$.

Thus $(\varphi \circ \theta)(u) = \varphi(\theta(u)) = \varphi([J_u, h_u]) = [J_1^{\ln(u)}, h_1^{\ln(u)}]$ and $(\gamma \circ \alpha)(u) = \gamma(\alpha(u)) = \gamma(n_1^{\ln(u)}) = [J_1^{\ln(u)}, h_1^{\ln(u)}]$.

Lastly, we prove that $\varphi$ is a homomorphism. Consider the following diagram which is the top half of the above diagram:

$$Y^*(G_1, h_1)/\pm 1 \xrightarrow{\alpha} \chi(G_1, h_1)$$

$$\chi(G_2, h_2)$$
since $\varphi \circ \theta = \beta$, it follows from Proposition 3.6 that $\varphi$ is a homomorphism.

$\square$

**Remark 3.9.** Given the following diagram,

$$
\begin{array}{ccc}
Y^*(G_1, h_1) / \pm 1 & \xrightarrow{\theta} & \chi(G_1, h_1) \\
& \searrow_{\varphi \circ \theta = \beta} & \\
& & \chi(G_2, h_2)
\end{array}
$$

it follows from Proposition 3.6 that if $\beta$ is surjective then $\varphi$ is surjective. Hence, from the first Isomorphism Theorem, $\chi(G_1, h_1)/\text{Ker}\varphi \cong \chi(G_2, h_2)$. This gives a computational method to obtain $\chi(G_2, h_2)$ from $\chi(G_1, h_1)$ when $n_1$ and $n_2$ are relatively prime.

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