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Some results on k-step Hamiltonian graph

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Abstract Let $G(V, E)$ be a simple graph with $|V| = n$ vertices and $|E| = m$ edges. A graph G is said to admit an $AL(k)$-traversal if there exists a sequence of vertices $\{v_1, v_2, \ldots, v_n\}$ such for each $i = 1, 2, \ldots, n-1$, the distance for v_i and v_{i+1} is equal to k. The graph G is called a k-step Hamiltonian graph if it has an $AL(k)$-traversal in G and $d(v_1, v_n) = k$. In this paper, we obtain results of the investigation of property k-step Hamiltonian on some graph operations such that line graph, join of two graphs and jump graph of certain graphs. We investigate k-step Hamiltonicity for some graph families.

1 Introduction

All graphs in this paper are simple, finite, connected and undirected. Let $G = (V, E)$ be a simple graph with n vertices and the size of m. The distance $d(u, v)$ is denoted by the distance between two vertices u and v of a graph that is the minimum length of the paths connecting them [1]. Throughout this paper, K_n, C_n and P_n denote a complete graph, the cycle and the path of order n, respectively [1].

Hamiltonicity is a very important topic in graph theory. A Hamiltonian path(cycle) is a path(cycle) that visits each vertex of the graph exactly once. A graph that contains a Hamiltonian cycle is called a Hamiltonian graph [2]. In [3] is defined as a new concept of the Hamiltonian graph as a k-step Hamiltonian graph.

For $k \geq 1$, a graph G is said to have an $AL(k)$–traversal if there exists a sequence $\{v_1, v_2, \ldots, v_n\}$ such for each $i = 1, 2, \ldots, n-1$, the distance for v_i and v_{i+1} is equal to k. Graph G is called a k-step Hamiltonian if it has an $AL(k)$–traversal and $d(v_n, v_1) = k$. Note that a 1-step Hamiltonian graph is a graph with a Hamiltonian cycle.

For example, the cubic graph in Figure 1 is 2-step Hamiltonian [4]. A Hamiltonian graph need not be a k-step Hamiltonian. The simplest examples are cycles C_n with $n \equiv 0$ (mod k) which are not $AL(k)$–traversable, hence cannot be k-step Hamiltonian.

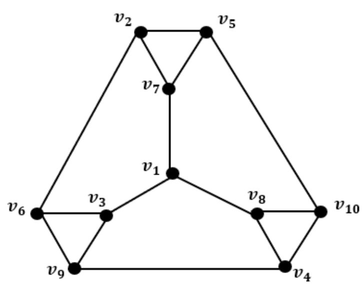

Figure 1. A 2-step Hamiltonian cubic graph.

Lau et al. investigated 2-step Hamiltonicity of tripartite graphs [3]. In [4] is considered k-step Hamiltonicity of bipartite and tripartite graphs. In [5] is proposed several constructions of 3-step Hamiltonian trees from smaller 3-step Hamiltonian trees. More results on the k-step Hamiltonian graph are obtained in [6, 7, 8]. In this paper, we study the k-step Hamiltonicity of some operations of graphs.

The join of two graphs G_1 and G_2, denoted by $G_1 + G_2$, is the graph with vertex set $V(G_1) \cup V(G_2)$ and edge set $E(G_1) \cup E(G_2) \cup \{uv \,|\, u \in V(G_1)\, and\, v \in V(G_2)\}$ [1]. In this paper, the k-step Hamiltonicity of $G_1 + G_2$ is investigated.

The line graph of a graph G, denoted by $L(G)$, has E as its vertex set, where two vertices in $L(G)$ are adjacent if and only if the corresponding edges in G have at least one vertex in common. The complement of a graph G is denoted by $\bar{G}$ with the same vertices set of G such that two distinct vertices of $\bar{G}$ are adjacent if and only if they are not adjacent in G [1]. The complement of line graph $L(G)$ is called the jump graph of G and is denoted by $J(G)$ [9]. We study the property of k-step Hamiltonicity of the complement of some graphs and line graphs.

2 Main results

In this section, we propose main results of this paper to investigate the k-step Hamiltonicity of some graphs. First, we recall some results that are used in this paper. For this purpose, let $D_k(G)$, for a graph G, denote the generated graph from G such that $V(D_k(G)) = V(G)$ and $E(D_k(G)) = \{uv \,|\, d(u,v) = k\}$ in G [3].

Lemma 2.1. [3] *A graph G is a k-step Hamiltonian if and only if $D_k(G)$ is a Hamiltonian graph.*

Lemma 2.2. [3] *Cycle C_n for odd $n \geq 5$ is 2-step Hamiltonian graph.*

Lemma 2.3. [4] *For integers $n \geq 3$ and $k \geq 2$, the cycle C_n is a k-step Hamiltonian if and only if $n \geq 2k+1$ and $gcd(n,k) = 1$.*

Theorem 2.4. *If G_1 and G_2 are k-step Hamiltonian graphs for $k \geq 1$, then $D_k(G_1) + D_k(G_2)$ is Hamiltonian.*

Proof. Let G_1 and G_2 be k-step Hamiltonian graphs of orders n and m, respectively. By Lemma 2.1, $D_k(G_1)$ and $D_k(G_2)$ are Hamiltonian. Assume that C_i is a Hamiltonian cycle on graph C_i for $i = 1, 2$ which $u_1u_2 \ldots u_nu_1$ is a cycle C_1 and $v_1v_2 \ldots v_mv_1$ is cycle C_2.

According to the structure of graph $D_k(G_1) + D_k(G_2)$, the vertex u_i is adjacent to v_j for every $1 \leq i \leq n$ and $1 \leq j \leq m$. Thus, one can consider the following Hamiltonian cycle for graph $D_k(G_1) + D_k(G_2)$.

First, on the cycle C_1 from the vertex u_1 go to v_1 in the graph $D_k(G_2)$ and so, path $v_1v_2 \ldots v_m$ is passed in the graph $D_k(G_2)$. Thus, by edge v_mu_2 go back to $D_k(G_1)$. So, with passing the path $u_2 \ldots u_nu_1$, the Hamiltonian cycle in graph $D_k(G_1) + D_k(G_2)$ is the complete. This cycle is as following

$$u_1v_1v_2 \ldots v_mu_2 \ldots u_nu_1.$$

Therefore, the proof completes. $\square$

Theorem 2.5. *For any two connected graphs G_1 and G_2, then $G_1 + G_2$ is not k-step Hamiltonian graph for $k \geq 2$.*

Proof. According to the structure of graph $G = G_1 + G_2$, all of the vertices from G_1 and G_2 are adjacent to each other. The vertex of G_1 is not adjacent to the vertex of G_2, vice versa, in $D_k(G)$ for $k \geq 2$. Therefore, $D_k(G)$ is disconnected with at least two components consisting of $V_1 \subseteq V(G_1)$ and $V_2 \subseteq V(G_2)$, respectively. Thus, $D_k(G)$ is not Hamiltonian and by Lemma 2.1, $G_1 + G_2$ is not the k-step Hamiltonian graph for $k \geq 2$. $\square$

According to Lemma 2.2 and since the line graph of a cycle is isomorphic to the same cycle, it is clear to have the following result.

Proposition 2.6. *For cycle C_n of order odd n, $L(C_n)$ is a 2-step Hamiltonian graph.*

Theorem 2.7. *For $n \geq 5$, graph $\bar{P}_n$ has $AL(2)$-traversal. But it is not a 2-step Hamiltonian graph.*

Proof. Let P_n be the path of order n with vertex set $\{u_1, \ldots, u_n\}$. We consider the graph structure of the complement of P_n as following.

On the path P_n, the edges of consecutive vertices $u_i u_{i+1}$ are deleted for $i = 1, \ldots, n-1$. We have two cases.

Case 1) If n is even, then any vertex u_i connect to any vertex u_j for $i = 1, \ldots, \frac{n}{2}+1$ and $j = i+2, \ldots, n$. Therefore, we consider the path $u_i u_{i+3} u_{i+1}$ for any two consecutive vertices u_i and u_{i+1} where $i = 1, \ldots, \frac{n}{2}$ and the path $u_i u_{i-2} u_{i+1}$ for $i = \frac{n}{2}+1, \ldots, n-1$. Thus, $d(u_i, u_{i+1}) = 2$ for $1 \leq i \leq n-1$. So, graph $\bar{P}_n$ is $AL(2)$-traversable. On the other hand, since two vertices u_1 and u_n are adjacent in $\bar{P}_n$, $d(u_1, u_n) = 1$, then $\bar{P}_n$ is not a 2-step Hamiltonian graph.

Case 2) If n is odd, then every vertex u_i connect to any vertex u_j for $i = 1, \ldots, \frac{n-1}{2}+1$ and $j = i+2, \ldots, n$. Similar to case 1, we consider the path $u_i u_{i+3} u_{i+1}$ for any two consecutive vertices u_i and u_{i+1} which $i = 1, \ldots, \frac{n-1}{2}$ and the path $u_i u_{i-2} u_{i+1}$ for $i = \frac{n+1}{2}, \ldots, n-1$. Therefore, the distance between two vertices u_i and u_{i+1} is 2 for $1 \leq i \leq n-1$. Also, since two vertices u_1 and u_n are adjacent in $\bar{P}_n$, $d(u_1, u_n) = 1$. Therefore, in this case the result completes too. □

Since the line graph of path P_n is isomorphic to the path of order $n-1$, $J(P_n) = \bar{P}_{n-1}$. Let vertices on path P_{n-1} be $u_1, \ldots, u_{n-1}$. Using Theorem 2.7, we have the following result.

Corollary 2.8. *For $n \geq 4$, the jump graph of P_n has $AL(2)$-traversal. But, it is not a 2-step Hamiltonian graph.*

Theorem 2.9. *Graph of $\bar{P}_n$, for $n > 4$ is Hamiltonian.*

Proof. Let the vertices set of $\bar{P}_n$ be $\{u_1, \ldots, u_n\}$. There is an edge between any two vertices u_i and v_j for $1 \leq i, j \leq n-1$ such that the following edges set are not in the graph $\bar{P}_n$.

$$\{u_1 u_2, u_2 u_3, \ldots, u_{n-2} u_{n-1}, u_{n-1} u_n\}.$$

We have two the following cases.

Case 1) If n is even, then we consider the Hamiltonian cycle in the graph $\bar{P}_n$ as following,

$$u_1 u_3 u_5 \ldots u_{n-1} u_2 u_4 \ldots u_n u_1.$$

Case 2) If n is odd, then the Hamiltonian cycle in the graph $\bar{P}_n$ is as following,

$$u_1 u_3 u_5 \ldots u_n u_2 u_4 \ldots u_{n-1} u_1.$$

Therefore, the proof completes. □

Corollary 2.10. *$J(P_n)$, for $n > 4$ has a Hamiltonian cycle.*

Proof. Since $L(P_n) = P_{n-1}$, the jump graph of P_n is $\bar{P}_{n-1}$. According to the proof of Theorem 2.9, the result holds. □

Theorem 2.11. *For $n \geq 5$, the graph $\bar{C}_n$ is a 2-step Hamiltonian graph.*

Proof. Let C_n be the cycle of order n with vertex set $\{u_1, \ldots, u_n\}$. The edge set of the complement of C_n is $E(K_n \setminus C_n)$. With the same discussion in the proof of Theorem 2.7, $\bar{C}_n$ includes the vertices $\{u_1, \ldots, u_n\}$ that every vertex u_i is adjacent to other vertices except for u_{i-1} and u_{i+1} for any $2 \leq i \leq n-1$. Also, vertex u_1 is adjacent to other vertices in $\bar{C}_n$ except for two vertices u_2 and u_n.

Note that $d(u_i, u_{i+1}) = 2$ for all $i = 1, \ldots, n$ by considering the path $u_i u_k u_{i+1}$ for $k \neq i, i \pm 1, i+2$. On the other hand, by selecting the path $u_1 u_k u_n$ that $k \neq 1, n$, we have $d(u_1, u_n) = 2$. Therefore, $\bar{C}_n$ is a 2-step Hamiltonian graph and a 2-step Hamiltonian cycle of $\bar{C}_n$ is given by $u_1, u_2, \ldots, u_n, u_1$. □

Corollary 2.12. *For $n \geq 5$, the jump graph of C_n is a 2-step Hamiltonian graph.*

Proof. The jump graph of C_n is the complement of the line graph of C_n. Since the line graph of C_n is isomorphic to C_n itself, $J(C_n) = \bar{C}_n$. Using Theorem 2.11, the result holds. □

We consider some families of graphs and investigate the property of k-step Hamiltonian for $k \geq 1$ about these families. For a graph G, the subdivision graph $S(G)$ is a graph obtained from G by replacing each edge of G by a path of length 2. The t-subdivision graph $S_t(G)$ of G is a graph obtained from G by replacing each edge of G by a path of length $t+1$. Obviously, $S_1(G) = S(G)$ [10].

Theorem 2.13. *For $n \geq 4$ that $n \equiv 1$ (mod 3) or $n \equiv 2$ (mod 3), the subdivision graph of C_n is a 3-step Hamiltonian graph.*

Proof. Let $S(C_n)$ be the subdivision graph of C_n. According to the definition, $S(C_n)$ is obtained from C_n by replacing each edge of C_n with a path of length 2. Thus, the subdivision graph of C_n is a cycle of order $2n$. Since $n \equiv 1$ (mod 3) or $n \equiv 2$ (mod 3), we have $2n \equiv 2$ (mod 3) or $2n \equiv 1$ (mod 3). Therefore, using Lemma 2.3 the result complements. □

Theorem 2.14. *Let C_n be a cycle graph with odd n. Then $S_t(C_n)$ is a 2-step Hamiltonian graph for any even t.*

Proof. According to the definition of the t-subdivision graph, $S_t(G)$ of G is a graph obtained from G by replacing each edge of G with a path of length $t+1$. If t is even, then every edge in C_n is replaced with a path of length odd $t+1$.
So, the number of vertices of $S_t(C_n)$ is equal to $(t+1)n$ which the cycle of order odd. Therefore, by Lemma 2.2 graph $S_t(G)$ is a 2-step Hamiltonian graph. □

The helm graph H_n is the graph obtained from a wheel graph with n vertices by adjoining a pendant edge at each vertex of the cycle. The web graph is obtained by joining the pendant vertices of a helm graph to form a cycle and then adding a single pendant edge to each vertex of this outer cycle. We consider the kind of generalized web graph that is without a center vertex [11]. First, we recall the following results.

Lemma 2.15. [1] *A graph is bipartite if and only if it has no odd cycle.*

Lemma 2.16. [3] *All bipartite graphs are not k-step Hamiltonian for even $k \geq 2$.*

Theorem 2.17. *Let H_n be a helm graph with a wheel graph of order $n \geq 6$. Then H_n is 2-step Hamiltonian graph for any n.*

Proof. Let H_n be the helm graph of order $2n-1$ that n is the number of vertices of the wheel graph in H_n. It is easy to show that for $n = 4, 5$, the helm graph is not 2-step Hamiltonian graph. For $n \geq 6$, we consider two cases.

Case 1) If n is even, then we consider the sequence of vertices on H_n as in Figure 2(a) such that the distance of any two consecutive vertices v_i and v_{i+1} is 2 for $i = 1, 2, \ldots, 2n-2$ and $d(v_1, v_{2n-1}) = 2$.

Case 2) If n is odd, then we consider the sequence of vertices on graph H_n as in Figure 2(b). According labeling vertices in Figure 2, $d(v_i, v_{i+1}) = 2$ for $i = 1, 2, \ldots, 2n-1$ and $d(v_1, v_{2n-1}) = 2$.
Therefore, for the above two cases, graph H_n is a 2-step Hamiltonian graph. □

Theorem 2.18. *Let Wb_n be a web graph. Then Wb_n is a 2-step Hamiltonian graph of order odd.*

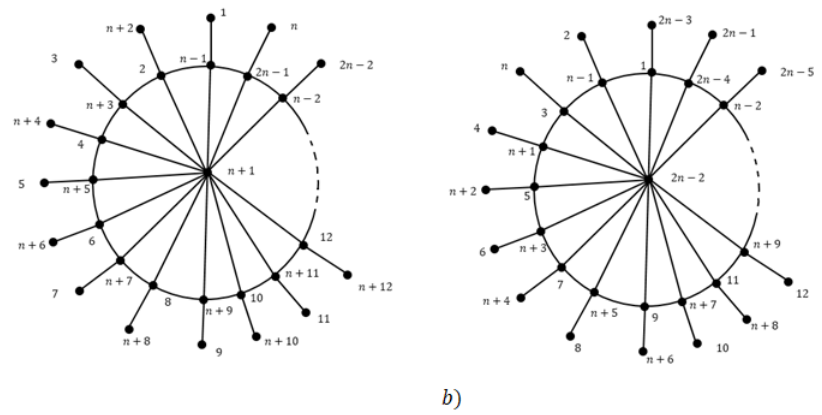

Figure 2. a) Labeling for 2-step Hamiltonian Helm graph with even n, b) Labeling for 2-step Hamiltonian Helm graph with odd n.

Proof. Let Wb_n be the web graph of order $3n$. If n is even, then the order of Wb_n is even. Using Lemma 2.15, graph Wb_n is a bipartite graph. According to Lemma 2.16, graph Wb_n is not a 2-step Hamiltonian graph.

If n is odd, then assuming C_1 and C_2 are the inner and outer cycles, respectively. Let $\{v_1, \ldots, v_n\}$ be the set of vertices of C_1, $\{u_1, \ldots, u_n\}$ and $\{l_1, \ldots, l_n\}$ be the sets of vertices of C_2 and the leaves in H_n, respectively (see Figure 3(a)). Note that the vertex v_i in C_1 is adjacent to the vertices u_i and vertex u_i is adjacent to leaf l_i for $i = 1, \ldots, n$.

First, we consider Wb_3. Figure 3(b) is shown the labeling on vertices for 2-step Hamiltonicity.

For $n \geq 4$, we determine the sequence $\{v_1, v_3, v_5, \ldots, v_n, v_2, \ldots, v_{n-1}\}$ on cycle C_1 such that the distance of any two vertices is 2. So, we select the sequence of vertices on the cycle C_2 and the set of leaves as following

$$\{l_{n-1}, u_{n-2}, l_{n-3}, u_{n-4}, \ldots, l_2, u_1, l_n, u_{n-1}, \ldots, l_3, u_2, l_1, u_n\}.$$

To merge the first sequence and second sequence, the distance of any two consecutive vertices is 2 and $d(v_1, u_2) = 2$. Thus, Wb_n is a 2-step Hamiltonian graph. □

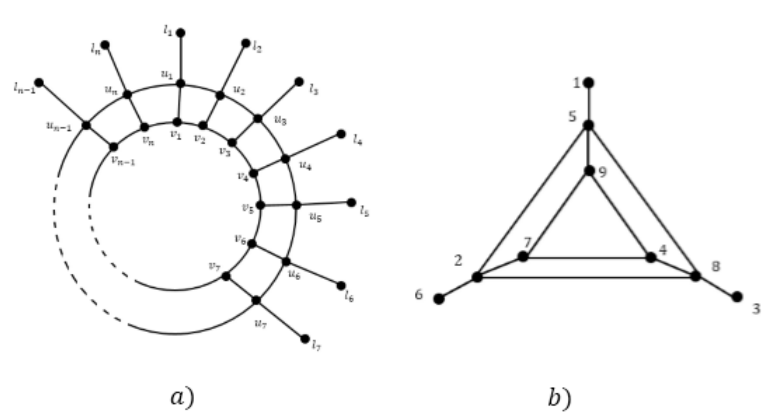

Figure 3. a) A web graph with labeling on vertices, b) A labeling for 2-step Hamiltonian Wb_3.

A polar grid graph $P_{m,n}$ is a graph contains m cycles with a common center as vertex such that any cycle has n vertices [12]. We investigate the 2-step Hamiltonicity on graph $P_{m,n}$ for

$m = 2, 3$ in the following Theorems.

Theorem 2.19. *Let $P_{2,n}$ be a polar grid graph. Then $P_{2,n}$ is a 2-step Hamiltonian graph for any odd n.*

Proof. Assume that $P_{2,n}$ is the polar grid graph of order $2n + 1$ for odd n. If $n = 3$, then Figure 4(a) is shown the labeling on vertices for $Al(2)$-traversal in graph $P_{2,3}$. It is clear to see that $P_{2,3}$ is not a 2-step Hamiltonian graph.

For odd $n \geq 5$, we consider the sequence of vertices of $P_{2,n}$ as in Figure 4(b) such that the distance of any two consecutive vertices i and $i+1$ is 2 for $i = 1, \ldots, 2n-2$ and $d(1, 2n+1) = 2$. Therefore, graph $P_{2,n}$ is a 2-step Hamiltonian for any odd $n \geq 5$. □

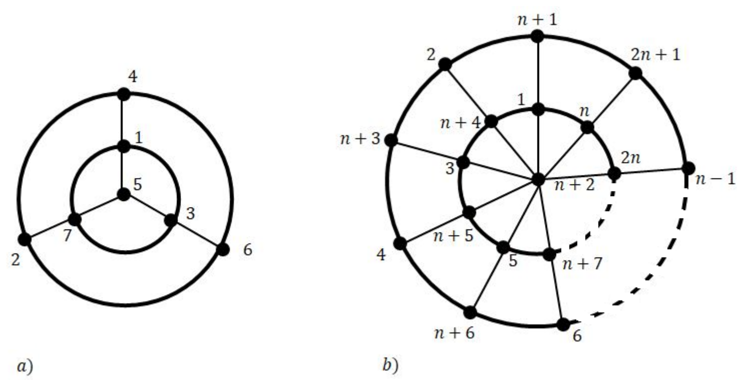

Figure 4. a) Labeling for 2-step Hamiltonian $P_{2,3}$, b) Labeling for 2-step Hamiltonian on a polar grid graph $P_{2,n}$ for any odd $n \geq 5$.

Theorem 2.20. *Let $P_{3,n}$ be a polar grid graph. Then $P_{3,n}$ is a 2-step Hamiltonian graph for any odd n.*

Proof. Let $P_{3,n}$ be the polar grid graph of the order $3n+1$ for odd n. If n is odd, then suppose that C_1, C_2 and C_3 are the cycles with the vertices sets $\{v_1, \ldots, v_n\}$, $\{u_1, \ldots, u_n\}$ and $\{w_1, \ldots, w_n\}$, respectively (see Figure 5). Also, the vertex o is the center of inner cycle C_3.
According to labeling on vertices in Figure 5, we determine the sequence as following.

$$\{v_1, u_2, v_3, u_4, v_5, u_6, v_7, u_8, \ldots, v_{n-2}, u_{n-1}, v_n, u_1,$$

$$v_2, u_3, v_4, u_5, v_6, u_7, v_8, \ldots, u_{n-2}, o, u_n, v_{n-1}, w_1\},$$

on graph $P_{3,n}$ such that the distance of any two vertices is 2. So, we select vertices on cycle C_3 with the condition $AL(2)$-traversal as following

$$\{w_1, w_3, w_5, \ldots, w_{n-2}, w_n, w_2, w_4, \ldots, w_{n-3}, w_{n-1}\}.$$

To merge the first sequence and the second sequence, the distance of any two consecutive vertices is 2 and $d(w_{n-1}, v_1) = 2$. □

The Gear graph G_n is the graph obtained from the wheel W_n by inserting a vertex between any two adjacent vertices in its cycle C_n [12].

Theorem 2.21. *Let G_n be a gear graph. Then G_n is not a k-step Hamiltonian graph for any even $k \geq 2$.*

Proof. Let G_n be the gear graph of order $2n + 1$ for $n \geq 3$. According to the structure of graph G_n and using Lemma 2.15, graph G_n is a bipartite graph. So, by Lemma 2.16 graph G_n is not k-step Hamiltonian for any even $k \geq 2$. □

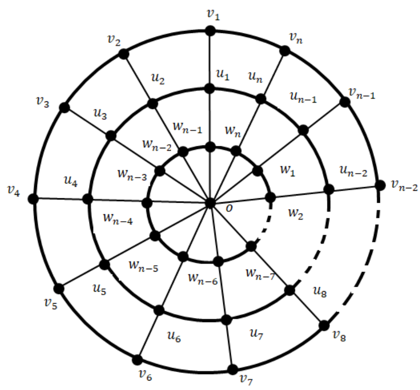

Figure 5. A labeling on vertices in the polar grid graph $P_{3,n}$.

The graph lotus inside a circle LC_n is obtained from the cycle C_n with the vertices set $\{u_1, \ldots, u_n\}$ and a star graph $K_{1,n}$ with central vertex o and the end vertices $\{v_1, \ldots, v_n\}$ by joining each v_i to u_i and u_{i+1} (mod n) [13].

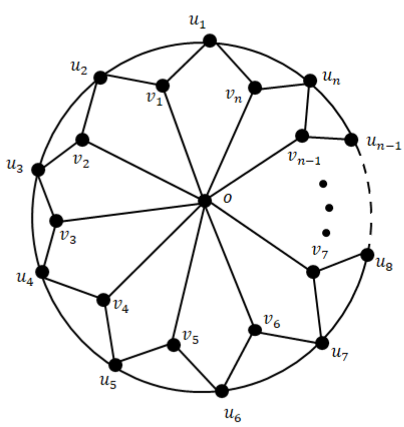

Figure 6. Labeling on vertices in the graph lotus inside a circle LC_n.

Theorem 2.22. *Let LC_n be a graph lotus inside a circle. Then LC_n is a 2-step Hamiltonian graph for odd n.*

Proof. Let LC_n be the graph lotus inside a circle that the vertices are labeled as Figure 6. We determine the sequence on vertices as following

$$\{u_1, o, u_3, u_5, \ldots, u_n, u_2, u_4, \ldots, u_{n-1}, v_n, v_1, v_2, v_3, v_4, \ldots, v_{n-1}\},$$

such that the distance of any two vertices is 2.
Also, $d(v_{n-1}, u_1) = 2$. So, graph LC_n is 2-step Hamiltonicity. $\square$

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