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A UNIFIED CLASS OF SPIRAL-LIKE FUNCTIONS INCLUDING KOBER FRACTIONAL OPERATORS IN QUANTUM CALCULUS

Sunil Dutt Purohit, Gangadharan Murugusundaramoorthy, Kaliappan Vijaya, Daya Lal Suthar and Kamlesh Jangid

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Abstract Under this article, we familiarize a combined class of spiral-like functions $\mathcal{P}_{\wp}^{\gamma}(\beta, \varpi, \omega)$ using the convolution product and acquire the coefficient bounds and extreme points. In addition, the findings of an integral transform, integral means inequalities and subordination are discussed.

1 Introduction

The q-series hypothesis is being used to a variety of areas in mathematics and physics, covering optimal control issues, fractional calculus, q-transform analysis, geometric function theory, and solving to the q-difference equations. Influenced by q-calculus applications, researchers investigated and studied multiple creative classes of analytic functions employing q-calculus operators. In fact, the first author with Raina, in [12], employed fractional q-calculus operators initially to investigate specific analytic function classes inside the open disk $\mathbb{D}$. In addition, researchers constructed novel q-integral operators over certain analytic functions and examined some basic features within [8, 14, 15] and fractional Askey-Wilson integrals [3], Ramanujan's integrals[1] and Mittag–Leffler functions [2], see also the references cited therein. In [19] Srivastava explored the outstanding work on q-calculus and fractional q-calculus operators, that are very much appreciated by academics researching on these topics for functions designated by q-calculus operators. Aside from subjects with relation towards this works (see [19, 20] and [26, 27]), lately Wang et al. [22] and Yan [24] explored the properties of sub classes of multivalent analytic or meromorphic functions articulated with q-difference operators. Inspired by earlier study on analytic functions utilizing q-calculus operators, here we create a novel subclass of spiral-like functions and explore the coefficient limits and extreme points for this function class. Furthermore, the outcomes of an integral transform, integral means inequalities, and subordination are explored.

Now we customize the primary symbolizations and concepts in the q-calculus that are appropriate to our study.

The q-shifted factorials for each complex number υ are outlined as

$$(\upsilon;q)_{\mathrm{k}} := \prod_{j=0}^{\mathrm{k}-1}(1-q^j\upsilon), \ \ \mathrm{k}\in\mathbb{N} \quad \text{and} \ \ (\upsilon;q)_0 = 1, \tag{1.1}$$

and in relation to the gamma function's basic surveys

$$(q^\upsilon;q)_{\mathrm{k}} := \frac{\Gamma_q(\upsilon+k)(1-q)^{\mathrm{k}}}{\Gamma_q(\upsilon)}, \qquad (\mathrm{k}>0), \tag{1.2}$$

where q-gamma is established to ([6])

$$\Gamma_q(\upsilon) = \frac{(q;q)_\infty(1-q)^{1-\upsilon}}{(q^\upsilon;q)_\infty}\ ; \quad (0<q<1). \tag{1.3}$$

Gaspar and Rahman [5], enumerated the recurrence bond for q-gamma function as

$$\Gamma_q(1+\upsilon) = \frac{(1-q^{\upsilon})\Gamma_q(\upsilon)}{1-q}. \tag{1.4}$$

For |q| < 1, the equation (1.1) intend to take part $k = \infty$ as a convergent infinite product

$$(\upsilon; q)_\infty = \prod_{j=0}^{\infty}(1-\upsilon q^j),$$

and we necessity

$$(\upsilon; q)_k = \frac{(\upsilon; q)_\infty}{(\upsilon q^k; q)_\infty}; \quad \big(k \in \mathbb{N} \cup \{\infty\}\big).$$

The q-binomial expansion stands as below:

$$(a-b)_\nu = a^\nu \Big(\frac{-b}{a}; q\Big)_\nu = a^\nu \, {}_1\phi_0 \Big[q^{-\nu}; -; q, \frac{b\,q^\nu}{a}\Big]. \tag{1.5}$$

In [5], Jackson's q-integral and q-derivative of a function

$$f(\tau) = \tau + \sum_{n=2}^{\infty} a_n \tau^n,$$

are described on a subset of $\mathbb{C}$, inferred as

$$\mathfrak{D}_q f(\tau) = \frac{f(\tau) - f(q\tau)}{\tau(1-q)}; \quad (\tau \neq 0,\, q \neq 0), \tag{1.6}$$

through

$$\mathfrak{D}_q \tau^{\wp} = \frac{(1-q^{\wp})\tau^{\wp-1}}{1-q},$$

and

$$\int_0^{\tau} f(y)\, d_q y \;= \tau(1-q)\sum_{n=0}^{\infty} q^n f(\tau q^n). \tag{1.7}$$

Garg and Chanchalani [4] defined the Kober fractional q-integral operator for a real-valued function $f(x)$ as

$$I_q^{\ell,\upsilon} f(x) = \frac{x^{-\ell-\upsilon}}{\Gamma_q(\upsilon)} \int_0^x (x-yq)_{\upsilon-1}\, y^{\ell} f(y)\, d_q y, \tag{1.8}$$

where ℓ denotes real or complex number and υ denotes an absolute order of integration with $\Re(\upsilon) > 0$. As q $\to 1$ the operator (1.8) reduces to Kober operator $I^{\ell,\upsilon} f(x)$ as demarcated in [7]. The above operator transforms to a Riemann-Liouville fractional q-integral operator with a power weight function when $\ell = 0$: $I_q^{0,\upsilon} f(x) = x^{-\upsilon} I_q^{\upsilon} f(x)$.

Garg and Chanchalani [4] widespread the Kober fractional q-derivative operator for a real-valued function $f(x)$ as

$$\mathfrak{D}_q^{\ell,\upsilon} f(x) = \prod_{j=1}^{k} \big([\ell+j]_q + x q^{\ell+j}\mathfrak{D}_q\big)\big(I_q^{\ell+\upsilon,\, k-\upsilon} f(x)\big), \tag{1.9}$$

where υ denotes an order of derivative with $\Re(\upsilon) > 0$ and $k = [\Re(\upsilon)] + 1,\; k \in \mathbb{N}$. As $q \to 1$ the operator (1.9) condense to Kober operator $\mathfrak{D}^{\ell,\upsilon} f(x)$ as defined in [7].

We now evoke Kober fractional q-calculus operators owed to Purohit et al. [11] as specified below:

Definition 1.1. (**Kober Fractional** q**-Integral Operator**) (see [11]):
The Kober fractional q-integral operator customary as

$$I_q^{\ell,\upsilon} f(\tau) = \frac{x^{-\ell-\upsilon}}{\Gamma_q(\upsilon)} \int_0^\tau (\tau - yq)_{\upsilon-1}\, y^\ell\, f(y)\, d_q y, \tag{1.10}$$

where ℓ being real or complex, υ $(\Re(\upsilon) > 0)$ denotes an absolute order of integration with $\Re(\upsilon) > 0$ and the q-binomial $(\tau - yq)_{\upsilon-1}$ be specified as

$$(z - yq)_{\upsilon-1} = \tau^{\upsilon-1}{}_1\phi_0\Big[q^{1-\upsilon}; -; q, \frac{yq^\upsilon}{\tau}\Big], \tag{1.11}$$

where ${}_1\phi_0[\upsilon; -; q, \tau]$ is exceptional case of basic hypergeometric series ${}_2\phi_1[\upsilon; \mathfrak{J}; \ell; q, \tau]$ for $\ell = \mathfrak{J}$ is single valued for $|arg(\tau)| < \pi$ and $|\tau| < 1$ (see [5] and Purohit et al. [12, 13]).

Definition 1.2. (**Kober Fractional** q**-Derivative Operator**) (see [11]):
The Kober fractional q-derivative operator is reputable as

$$\mathfrak{D}_q^{\ell,\upsilon} f(\tau) = \prod_{j=1}^{k} \big([\ell + j]_q + \tau q^{\ell+j} D_q\big)\big(I_q^{\ell+\upsilon,\, k-\upsilon} f(\tau)\big), \tag{1.12}$$

where the order of derivative υ $(\Re(\upsilon) > 0)$ with $k = [\Re(\upsilon)] + 1,\ k \in \mathbb{N}$.
For $q \to 1$ the operator given in (1.12) condense to $\mathfrak{D}^{\ell,\upsilon} f(\tau)$, the Kober derivative operator defined in [7]. and the operator (1.10) reduce to Kober integral operator $I^{\ell,\upsilon} f(\tau)$ as defined in [7]. Now we present the ensuing image formulae for $\tau^\wp$ based on Kober q-integral and q-derivative operators assumed as in (1.10) and (1.12), as below.

Remark 1.3. If $\upsilon,\ \ell,\ \wp \in \mathbb{C},\ \ \Re(\ell + \upsilon + \wp + 1) > 0$, and $\Re(\ell + \wp + 1) > 0$, then

$$\mathfrak{D}_q^{\ell,\upsilon} \tau^\wp = \frac{\Gamma_q(\ell + \upsilon + \wp + 1)}{\Gamma_q(\ell + \wp + 1)} \tau^\wp$$

and a function $\hbar(\tau) = \left(\frac{\tau}{1-\tau}\right) = \tau + \sum\limits_{n=2}^{\infty} \tau^n$, are described on a subset of $\mathbb{C}$, inferred as

$$\begin{aligned} \mathfrak{D}_q^{\ell,\upsilon} \hbar(\tau) &= \mathfrak{D}_q^{\ell,\upsilon}(\tau) + \sum_{n=2}^{\infty} \mathfrak{D}_q^{\ell,\upsilon} \tau^n \\ &= \frac{\Gamma_q(\ell + \upsilon + 2)}{\Gamma_q(\ell + 2)} \tau + \sum_{n=2}^{\infty} \frac{\Gamma_q(\ell + \upsilon + n + 1)}{\Gamma_q(\ell + n + 1)} \tau^n. \end{aligned} \tag{1.13}$$

Remark 1.4. If $\upsilon,\ \ell,\ \wp \in \mathbb{C}$, $\Re(\ell + \wp + 1) > 0$, and $\Re(\ell + \upsilon + \wp + 1) > 0$, then

$$I_q^{\ell,\upsilon} \tau^\wp = \frac{\Gamma_q(\ell + \wp + 1)}{\Gamma_q(\ell + \upsilon + \wp + 1)} \tau^\wp.$$

and a function $\hbar(\tau) = \left(\frac{\tau}{1-\tau}\right) = \tau + \sum\limits_{n=2}^{\infty} \tau^n$, are described on a subset of $\mathbb{C}$, inferred as

$$\begin{aligned} I_q^{\ell,\upsilon} \hbar(\tau) &= I_q^{\ell,\upsilon}(\tau) + + \sum_{n=2}^{\infty} I_q^{\ell,\upsilon} \tau^n \\ &= \frac{\Gamma_q(\ell + 2)}{\Gamma_q(\ell + \upsilon + 2)} \tau + \sum_{n=2}^{\infty} \frac{\Gamma_q(\ell + n + 1)}{\Gamma_q(\ell + \upsilon + n + 1)} \tau^n \end{aligned} \tag{1.14}$$

2 New Classes of Spiral-Like Functions

Let $\mathcal{A}$ represent a class of analytic and univalent functions in the open disc

$$\mathcal{U} = \{\tau : \tau \in \mathbb{C},\ |\tau| < 1\}$$

and comprising of functions of the form

$$f(\tau)=\tau+\sum_{n=2}^{\infty}a_n\tau^n. \tag{2.1}$$

We correspondingly consider $\mathcal{T}$ be a subclass of $\mathcal{A}$ comprising functions such as the one

$$f(\tau)=\tau-\sum_{n=2}^{\infty}|a_n|\tau^n, \quad \tau\in\mathcal{U} \tag{2.2}$$

considered by Silverman [16]. For functions $f\in\mathcal{A}$ given by (2.1) and $g\in\mathcal{A}$ given by $g(\tau)=\tau+\sum\limits_{n=2}^{\infty}b_n\tau^n$, we describe the Hadamard product (or convolution) of f and g by

$$(f\star g)(\tau)=\tau+\sum_{n=2}^{\infty}a_nb_n\tau^n, \quad \tau\in\mathcal{U}. \tag{2.3}$$

In this article we familiarize a new linear operator:

$$\Omega_q^{\ell,\upsilon}:\mathcal{A}\to\mathcal{A}$$

demarcated in footings of (1.13), the Hadamard product(2.3) and normalized as below

$$\Omega_q^{\ell,\upsilon}f(\tau)=\frac{\Gamma_q(\ell+2)}{\Gamma_q(\ell+\upsilon+2)}\left(\mathfrak{D}_q^{\ell,\upsilon}\hbar(\tau)\star f(\tau)\right) \quad (\tau\in\mathcal{U}), \tag{2.4}$$

which trail easily

$$\Omega_q^{\ell,\upsilon}f(\tau)=z+\sum_{n=2}^{\infty}\frac{\Gamma_q(\ell+2)\Gamma_q(\ell+\upsilon+n+1)}{\Gamma_q(\ell+\upsilon+2)\Gamma_q(\ell+n+1)}a_n\tau^n.$$

For conciseness we let

$$\Omega_q^{\ell,\upsilon}f(\tau)=\tau+\sum_{n=2}^{\infty}\Psi_q^{\upsilon,\ell}(n)a_n\tau^n \tag{2.5}$$

where

$$\Psi_q^{\upsilon,\ell}(n)=\frac{\Gamma_q(\ell+2)\Gamma_q(\ell+\upsilon+n+1)}{\Gamma_q(\ell+\upsilon+2)\Gamma_q(\ell+n+1)}, \tag{2.6}$$

and (during this paper unless otherwise stated) the constraints $0<q<1$, $Re(\upsilon)>0$; $Re(\ell+2)>0$, $\tau\in\mathcal{U}$, and $\mathfrak{D}_q^{\ell,\upsilon}f(\tau)$ signify a fractional q-derivative of $f(\tau)$ of order υ.

For ϖ real, $|\varpi|<\frac{\pi}{2}$ a function $f\in\mathcal{A}$ is stated in $\mathcal{S}_p^{\varpi}$ the class of ϖ-spiral-like functions sustaining the analytic condition

$$Re\left(e^{i\varpi}\frac{\tau f'(\tau)}{f(\tau)}\right)>0, z\in\mathcal{U}$$

was presented such as Spacek [21]. Later, Zamorski [25] found sharp coefficient bounds along with $f\in\mathcal{S}_p^{\varpi}$. Inspired by the earlier works on analytic functions and erection of the operator $\Omega_q^{\ell,\upsilon}$, we define a new subclass of spiral-like functions and talk about a certain typical properties of the geometric function theory of this comprehensive function class.

Definition 2.1. For $0\le\gamma\le 1,\ 0\le\sigma<1,\ \ 0\le\omega<1,$ and $|\varpi|<\frac{\pi}{2},$ let $\mathcal{P}_q^{\gamma}(\sigma,\varpi,\omega)$ be the subclass of $\mathcal{A}$ comprising functions of the form (2.1) and sustaining the inequality

$$\left|\frac{\mathcal{F}(\tau)-1}{(2\sigma-1)[\mathcal{F}(\tau)-1]+2\sigma(1-\omega)e^{-i\varpi}\cos\varpi}\right|<1 \tag{2.7}$$

where

$$\mathcal{F}(\tau)=\frac{\tau\mathfrak{D}_q(\Omega_q^{\ell,\upsilon}f(\tau))+\gamma\tau^2\mathfrak{D}_q[\mathfrak{D}_q(\Omega_q^{\ell,\upsilon}f(\tau))]}{(1-\gamma)\Omega_q^{\ell,\upsilon}f(\tau)+\gamma\tau\mathfrak{D}_q(\Omega_q^{\ell,\upsilon}f(\tau))} \tag{2.8}$$

and $\tau\in\mathcal{U}$, $\Omega_q^{\ell,\upsilon}f(\tau)$ is given by (2.5) .

In precise, for $0 \leq \gamma \leq 1$, the class $\mathcal{P}_q^{\gamma}(\sigma,\varpi,\omega)$ describe a transition from starlike functions to convex functions as illustrated below.

Definition 2.2. For $0 \leq \sigma < 1$, $0 \leq \omega < 1$, and $|\varpi| < \frac{\pi}{2}$, let $\mathcal{S}_q(\sigma,\varpi,\omega)$ be the subclass of $\mathcal{A}$ entailing of functions of the form (2.1) and satiating the inequality

$$\left| \frac{\frac{\tau\mathfrak{D}_q(\Omega_q^{\ell,\upsilon}f(\tau))}{\Omega_q^{\ell,\upsilon}f(\tau)} - 1}{(2\sigma-1)[\frac{\tau\mathfrak{D}_q(\Omega_q^{\ell,\upsilon}f(\tau))}{\Omega_q^{\ell,\upsilon}f(\tau)} - 1] + 2\sigma(1-\omega)e^{-i\varpi}\cos\varpi} \right| < 1$$

where $\tau \in \mathcal{U}$, $\Omega_q^{\ell,\upsilon}f(\tau)$ is assumed by (2.5) .

Definition 2.3. For $0 \leq \sigma < 1$, $0 \leq \omega < 1$, and $|\varpi| < \frac{\pi}{2}$, we let $\mathcal{C}_q(\sigma,\varpi,\omega)$ be the subclass of $\mathcal{A}$ involving of functions of the form (2.1) and sustaining the inequality

$$\left| \frac{\frac{\tau\mathfrak{D}_q[\mathfrak{D}_q(\Omega_q^{\ell,\upsilon}f(\tau))]}{\mathfrak{D}_q(\Omega_q^{\ell,\upsilon}f(\tau))}}{(2\sigma-1)[\frac{\tau\mathfrak{D}_q[\mathfrak{D}_q(\Omega_q^{\ell,\upsilon}f(\tau))]}{\mathfrak{D}_q(\Omega_q^{\ell,\upsilon}f(\tau))}] + 2\sigma(1-\omega)e^{-i\varpi}cos\varpi} \right| < 1$$

where $\tau \in \mathcal{U}$, $\Omega_q^{\ell,\upsilon}f(\tau)$ is given by (2.5) .

Further by appropriately specifying the values of ω, σ, ϖ and the class $\mathcal{P}_q^{\gamma}(\sigma,\varpi,\omega)$ condenses to several new subclasses. For functions $f \in \mathcal{T}$ set by (2.2), we let

$$\mathcal{PT}_q^{\gamma}(\sigma,\varpi,\omega) \equiv \mathcal{PT}_q^{\gamma}(\sigma,\varpi,\omega) \cap \mathcal{T},$$

$$\mathcal{ST}_q(\sigma,\varpi,\omega) \equiv \mathcal{S}_q(\sigma,\varpi,\omega) \cap \mathcal{T}$$

and

$$\mathcal{CT}_q(\sigma,\varpi,\omega) \equiv \mathcal{C}_q(\sigma,\varpi,\omega) \cap \mathcal{T}.$$

The foremost purpose of this article is to figure out the coefficient bounds, for $f \in \mathcal{PT}_q^{\gamma}(\sigma,\varpi,\omega)$. Furthermore, to discuss an integral transform results, integral means inequalities and subordination results.

3 Coefficient Bounds

This part presents a condition for functions $f \in \mathcal{PT}_q^{\gamma}(\sigma,\varpi,\omega)$ that is both necessary and sufficient.

Theorem 3.1. *A function $f(\tau)$ defined by (2.2), belongs to the class $\mathcal{PT}_q^{\gamma}(\sigma,\varpi,\omega)$ iff*

$$\begin{aligned} &\sum_{n=2}^{\infty}[1+\gamma([n]_q-1)]\{([n]_q-1)(1-\sigma)+\sigma(1-\omega)|e^{-i\varpi}\cos\varpi|\}\Psi_q^{\upsilon,\ell}(n)|a_n| \\ \leq\ & \sigma(1-\omega)|e^{-i\varpi}\cos\varpi|, \end{aligned} \tag{3.1}$$

$0 \leq \gamma \leq 1$, $0 \leq \sigma < 1$, $0 \leq \omega < 1$, *and* $|\varpi| < \frac{\pi}{2}$ *where* $\Psi_q^{\upsilon,\ell}(n)$ *is assumed by (2.6).*

Proof. For $|\tau| = 1$, we have

$$\begin{aligned} & |\mathcal{F}(\tau)-1| - |(2\sigma-1)[\mathcal{F}(\tau)-1] + 2\sigma(1-\omega)e^{-i\varpi}\cos\varpi| \\ =\ & |\sum_{n=2}^{\infty}[1-\gamma+[n]_q\gamma]([n]_q-1)\Psi_q^{\upsilon,\ell}(n)a_n\tau^n| \\ -\ & \Big|2\sigma(1-\omega)e^{-i\varpi}\cos\varpi z \\ -\ & \sum_{n=2}^{\infty}[1-\gamma+[n]_q\gamma][([n]_q-1)(2\sigma-1)-2\sigma(1-\omega)e^{-i\varpi}\cos\varpi]\Psi_q^{\upsilon,\ell}(n)a_n\tau^n\Big| \end{aligned}$$

$$\begin{aligned}
&\le \sum_{n=2}^{\infty}[1-\gamma+[n]_q\gamma]([n]_q-1)\Psi_q^{\upsilon,\ell}(n)|a_n| - 2\sigma(1-\omega)|e^{-i\varpi}\cos\varpi| \\
&- \sum_{n=2}^{\infty}[1-\gamma+[n]_q\gamma][([n]_q-1)(2\sigma-1)-2\sigma(1-\omega)|e^{-i\varpi}\cos\varpi|]\Psi_q^{\upsilon,\ell}(n)|a_n| \\
&= \sum_{n=2}^{\infty}[1+\gamma([n]_q-1)]\{([n]_q-1)(1-\sigma)+\sigma(1-\omega)|e^{-i\varpi}\cos\varpi|\}\Psi_q^{\upsilon,\ell}(n)|a_n| \\
&- \sigma(1-\omega)|e^{-i\varpi}\cos\varpi| \le 0, \qquad \text{(by hypothesis)}.
\end{aligned}$$

Hence $f \in \mathcal{PT}_q^{\gamma}(\sigma,\varpi,\omega)$ by the use of maximum modulus theorem. Conversely, assume

$$\left|\frac{\mathcal{F}(\tau)-1}{(2\sigma-1)[\mathcal{F}(\tau)-1]+2\sigma(1-\omega)e^{-i\varpi}\cos\varpi}\right|$$

$$= \left|\frac{\frac{\tau\mathfrak{D}_q(\Omega_q^{\ell,\upsilon}f(\tau))+\gamma\tau^2\mathfrak{D}_q[\mathfrak{D}_q(\Omega_q^{\ell,\upsilon}f(\tau))]}{(1-\gamma)\Omega_q^{\ell,\upsilon}f(\tau)+\gamma\tau\mathfrak{D}_q(\Omega_q^{\ell,\upsilon}f(\tau))}-1}{(2\sigma-1)[\frac{\tau\mathfrak{D}_q(\Omega_q^{\ell,\upsilon}f(\tau))+\gamma\tau^2\mathfrak{D}_q[\mathfrak{D}_q(\Omega_q^{\ell,\upsilon}f(\tau))]}{(1-\gamma)\Omega_q^{\ell,\upsilon}f(\tau)+\gamma\tau\mathfrak{D}_q(\Omega_q^{\ell,\upsilon}f(\tau))}-1]+2\sigma(1-\omega)e^{-i\varpi}\cos\varpi}\right| < 1.$$

Substituting the values of $\Omega_q^{\ell,\upsilon}f(\tau), \mathfrak{D}_q(\Omega_q^{\ell,\upsilon}f(\tau))$ $\tau^2\mathfrak{D}_q[\mathfrak{D}_q(\Omega_q^{\ell,\upsilon}f(\tau))$ and by simple computation,

$$\left|\frac{\sum_{n=2}^{\infty}[1-\gamma+[n]_q\gamma]([n]_q-1)\Psi_q^{\upsilon,\ell}(n)a_n\tau^n}{2\sigma(1-\omega)e^{-i\varpi}\cos\varpi\tau-\sum_{n=2}^{\infty}[1-\gamma+[n]_q\gamma][([n]_q-1)(1-2\sigma)-2\sigma(1-\omega)e^{-i\varpi}cos\varpi]\Psi_q^{\upsilon,\ell}(n)a_n\tau^n}\right| < 1.$$

$$\Re\left(\frac{\sum_{n=2}^{\infty}[1-\gamma+[n]_q\gamma]([n]_q-1)\Psi_q^{\upsilon,\ell}(n)a_n\tau^n}{2\sigma(1-\omega)e^{-i\varpi}\cos\varpi\tau-\sum_{n=2}^{\infty}[1-\gamma+[n]_q\gamma][([n]_q-1)(1-2\sigma)-2\sigma(1-\omega)e^{-i\varpi}cos\varpi]\Psi_q^{\upsilon,\ell}(n)a_n\tau^n}\right) < 1.$$

By choosing the value of $\tau \in R$, so that $\Omega_q^{\ell,\upsilon}f(\tau) \in R$ and as $\tau \to 1^-$, we get

$$\sum_{n=2}^{\infty}(1+\gamma[[n]_q-1])\{([n]_q-1)(1-\sigma)+\sigma(1-\omega)|e^{-i\varpi}\cos\varpi|\}\Psi_q^{\upsilon,\ell}(n)|a_n| \le \sigma(1-\omega)|e^{-i\varpi}\cos\varpi|.$$

□

For simplicity let

$$\Upsilon_q(\gamma,\sigma,\omega,n) = (1+\gamma[[n]_q-1])\{([n]_q-1)(1-\sigma)+\sigma(1-\omega)|e^{-i\varpi}cos\varpi|\}\Psi_q^{\upsilon,\ell}(n) \quad (3.2)$$

$$\Upsilon_q(\gamma,\sigma,\omega,2) = (1+q\gamma)\{q(1-\sigma)+\sigma(1-\omega)|e^{-i\varpi}cos\varpi|\}\Psi_q^{\upsilon,\ell}(2) \quad (3.3)$$

$$\Psi_q^{\upsilon,\ell}(2) = \frac{\Gamma_q(\ell+2)\Gamma_q(\ell+\upsilon+3)}{\Gamma_q(\ell+\upsilon+2)\Gamma_q(\ell+3)} > 0 \quad (3.4)$$

unless otherwise stated.

Corollary 3.2. *Let* $f(\tau) \in \mathcal{PT}_q^{\gamma}(\sigma,\varpi,\omega)$ *then*

$$\begin{aligned}
a_n &\le \frac{\sigma(1-\omega)|e^{-i\varpi}cos\varpi|}{(1+\gamma[[n]_q-1])\{([n]_q-1)(1-\sigma)+\sigma(1-\omega)|e^{-i\varpi}cos\varpi|\}\Psi_q^{\upsilon,\ell}(n)} \\
&= \frac{\sigma(1-\omega)|e^{-i\varpi}cos\varpi|}{\Upsilon_q(\gamma,\sigma,\omega,n)\Psi_q^{\upsilon,\ell}(n)} \quad (3.5)
\end{aligned}$$

and it is sharp for given

$$\begin{aligned} f(\tau) &= \tau - \frac{\sigma(1-\omega)|e^{-i\varpi}cos\varpi|}{(1+\gamma[[n]_q-1])\{([n]_q-1)(1-\sigma)+\sigma(1-\omega)|e^{-i\varpi}cos\varpi|\}\Psi_q^{\upsilon,\ell}(n)}\tau^n \\ &= \tau - \frac{\sigma(1-\omega)|e^{-i\varpi}cos\varpi|}{\Upsilon_q(\gamma,\sigma,\omega,n)\Psi_q^{\upsilon,\ell}(n)}\tau^n. \end{aligned} \tag{3.6}$$

Without providing evidence, we define the following conclusion:

Theorem 3.3. *The extreme points of* $f \in \mathcal{PT}_q^{\gamma}(\sigma,\varpi,\omega)$ *are*

$$f_1(\tau) = \tau \text{ and } f_n(\tau) = \tau - \frac{\sigma(1-\omega)|e^{-i\varpi}cos\varpi|}{\Upsilon_q(\gamma,\sigma,\omega,n)\Psi_q^{\upsilon,\ell}(n)}\tau^n, \quad \text{for } n = 2,3,\cdots. \tag{3.7}$$

where $\Upsilon_q(\gamma,\sigma,\omega,n)$ *defined by (3.2). Then* $f \in \mathcal{PT}_q^{\gamma}(\sigma,\varpi,\omega)$ *iff it can be uttered in the form*

$$f(\tau) = \sum_{n=0}^{\infty}\omega_n f_n(\tau), \qquad \omega_n \geq 0, \quad \sum_{n=0}^{\infty}\omega_n = 1. \tag{3.8}$$

4 Integral Transform of the class $\mathcal{PT}_q^{\gamma}(\sigma,\varpi,\omega)$

We show in this section that the class $\mathcal{PT}_q^{\gamma}(\sigma,\varpi,\omega)$ is closed under integral transform.

For $f \in \mathcal{A}$ we let the integral transform

$$\mathcal{V}_\nu(f)(\tau) = \int_0^1 \nu(\varsigma)\frac{f(\varsigma z)}{\varsigma}d\varsigma,$$

where $0 < \nu \in R$ is weight function normalized as $\int_0^1 \nu(\varsigma)d\varsigma = 1$. Fixing of $\wp(\varsigma)$ as $\nu(\varsigma) = (1+c)\varsigma^c, c > -1$, $\mathcal{V}_\nu$ denotes the Bernardi operator, and

$$\nu(\varsigma) = \frac{(c+1)^\epsilon}{\Gamma(\epsilon)}\varsigma^c\left(log\frac{1}{\varsigma}\right)^{\epsilon-1}, c > -1, \epsilon \geq 0$$

$\mathcal{V}_\nu$ is known as Komatu operator [9].

We now demonstrate that the class $\mathcal{PT}_q^{\gamma}(\sigma,\varpi,\omega)$ is closed under $\mathcal{V}_\nu(f)(z)$.

Theorem 4.1. *Let* $f(\tau) \in \mathcal{PT}_q^{\gamma}(\sigma,\varpi,\omega)$. *Then* $\mathcal{V}_\nu(f)(z) \in \mathcal{PT}_q^{\gamma}(\sigma,\varpi,\omega)$.

Proof. By definition

$$\begin{aligned} \mathcal{V}_\nu(f)(\tau) &= \frac{(c+1)^\epsilon}{\nu(\epsilon)}\int_0^1(-1)^{\epsilon-1}\varsigma^c(\log\varsigma)^{\epsilon-1}\left(\tau - \sum_{n=2}^{\infty}a_n\tau^n\varsigma^{n-1}\right)d\varsigma \\ &= \frac{(-1)^{\epsilon-1}(c+1)^\epsilon}{\nu(\epsilon)}\lim_{r\to 0^+}\left[\int_r^1\varsigma^c(\log\varsigma)^{\epsilon-1}\left(\tau - \sum_{n=2}^{\infty}a_n\tau^n\varsigma^{n-1}\right)d\varsigma\right]. \end{aligned}$$

After simplification, we obtain

$$\mathcal{V}_\nu(f)(\tau) = \tau - \sum_{n=2}^{\infty}\left(\frac{c+1}{c+n}\right)^\epsilon a_n\tau^n.$$

We must demonstrate that

$$\sum_{n=2}^{\infty}\frac{\Upsilon_q(\gamma,\sigma,\omega,n)}{\sigma(1-\omega)|e^{-i\varpi}\cos\varpi|}\left(\frac{c+1}{c+n}\right)^\epsilon a_n \leq 1. \tag{4.1}$$

By Theorem 3.2, $f \in \mathcal{PT}_q^{\gamma}(\sigma, \varpi, \omega)$ iff

$$\sum_{n=2}^{\infty} \frac{\Upsilon_q(\gamma,\sigma,\omega,n)}{\sigma(1-\omega)|e^{-i\varpi}\cos\varpi|} a_n \leq 1,$$

where $\Upsilon_q(\gamma,\sigma,\omega,n)$ is defined in (3.2). Hence $\frac{c+1}{c+n} < 1$, thus (4.1) retains and it completes the proof. □

The preceding theorem corresponds two special cases.

Theorem 4.2. *Let* $f \in \mathcal{PT}_q^{\gamma}(\sigma, \varpi, \omega)$. *Then* $\mathcal{V}_\nu(f)(\tau)$ *is starlike of order* $0 \leq \delta < 1$ *in* $|\tau| < R_1$ *where*

$$R_1 = \inf_n \left[\left(\frac{c+n}{c+1}\right)^{\epsilon} \frac{(1-\delta)\Upsilon_q(\gamma,\sigma,\omega,n)}{(n-\delta)\sigma(1-\omega)|e^{-i\varpi}\cos\varpi|} \right]^{\frac{1}{n-1}} \quad (n \geq 2),$$

where $\Upsilon_q(\gamma,\sigma,\omega,n)$ *belongs to (3.2).*

Proof. This is sufficient to prove

$$\left| \frac{z(\mathcal{V}_\nu(f)(\tau))'}{\mathcal{V}_\nu(f)(\tau)} - 1 \right| < 1 - \delta. \tag{4.2}$$

For LHS of (4.2), we have

$$\left| \frac{\tau(\mathcal{V}_\nu(f)(\tau))'}{\mathcal{V}_\nu(f)(\tau)} - 1 \right| = \left| \frac{\sum_{n=2}^{\infty}(n-1)\left(\frac{c+1}{c+n}\right)^{\epsilon} a_n \tau^{n-1}}{1 - \sum_{n=2}^{\infty}\left(\frac{c+1}{c+n}\right)^{\epsilon} a_n \tau^{n-1}} \right| \leq \frac{\sum_{n=2}^{\infty}(n-1)\left(\frac{c+1}{c+n}\right)^{\epsilon} a_n |\tau|^{n-1}}{1 - \sum_{n=2}^{\infty}\left(\frac{c+1}{c+n}\right)^{\epsilon} a_n |\tau|^{n-1}}.$$

The last activation is confined above with $1-\delta$ since,

$$|\tau|^{n-1} < \left(\frac{c+n}{c+1}\right)^{\epsilon} \frac{(1-\delta)\Upsilon_q(\gamma,\sigma,\omega,n)}{(n-\delta)\sigma(1-\omega)|e^{-i\varpi}\cos\varpi|}.$$

Therefore, the proof is complete. □

Using the fact that $f(\tau)$ is convex if and only if $zf'(z)$ is starlike, we attain the following.

Theorem 4.3. *Let* $f \in \mathcal{PT}_q^{\gamma}(\sigma, \varpi, \omega)$. *Then* $\mathcal{V}_\nu(f)(z)$ *is convex of order* $0 \leq \delta < 1$ *in* $|\tau| < R_2$ *where*

$$R_2 = \inf_n \left[\left(\frac{c+n}{c+1}\right)^{\epsilon} \frac{(1-\delta)\Upsilon_q(\gamma,\sigma,\omega,n)}{n(n-\delta)\sigma(1-\omega)|e^{-i\varpi}cos\varpi|} \right]^{\frac{1}{n-1}} \quad (n \geq 2),$$

where $\Upsilon_q(\gamma,\sigma,\omega,n)$ *belongs to (3.2).*

5 Integral Means

In [16], Silverman initiate that the function $f_2(\tau) = \tau - \frac{\tau^2}{2}$ is frequently extremal over the class $\mathcal{T}$. He used this function to solve his integral means inequality, speculated in [17] and established in [18], that

$$\int_0^{2\pi} \left|f(r\,e^{i\varphi})\right|^{\eta} d\varphi \leq \int_0^{2\pi} \left|f_2(r\,e^{i\varphi})\right|^{\eta} d\varphi,$$

for all $f \in \mathcal{T}$, $\eta > 0$ and $0 < r < 1$. In [18], he also evidenced his conjecture for the class of starlike functions and convex functions with negative coefficients of order $\tau(0 \leq \tau < 1)$.

To show our result on Integral means inequality, we recall the succeeding definition and the lemma .

Definition 5.1. *(Subordination Principle)*[10]:

For analytic functions $\mathfrak{f}_1$ and $\mathfrak{f}_2$ with $\mathfrak{f}_1(0) = \mathfrak{f}_2(0)$, $\mathfrak{f}_1$ is said to be subordinate to $\mathfrak{f}_2$, denoted by $\mathfrak{f}_1 \prec \mathfrak{f}_2$, if $\exists$ an analytic function w such that $w(0) = 0$, $|w(z)| < 1$ and $\mathfrak{f}_1(z) = \mathfrak{f}_2(w(z))$, $\forall z \in \mathcal{U}$.

Lemma 5.2. *[10] If f and g are analytic in $\mathcal{U}$ with $g \prec f$, then for $\eta > 0$, and $0 < r < 1$,*

$$\int_0^{2\pi} \left|g(re^{i\varphi})\right|^{\eta} d\varphi \leq \int_0^{2\pi} \left|f(re^{i\varphi})\right|^{\eta} d\varphi. \tag{5.1}$$

Applying Lemma 5.2, Theorem 3.2 and Theorem 3.3, we prove the Silverman's conjecture for $f \in \mathcal{PT}_q^{\gamma}(\sigma, \varpi, \omega)$.

Theorem 5.3. *Suppose $f \in \mathcal{PT}_q^{\gamma}(\sigma, \varpi, \omega)$, $\eta > 0$, $0 \leq \ell < 1$, $0 \leq \gamma \leq 1$, and $f_2(\tau)$ is defined by*

$$f_2(\tau) = \tau - \frac{\sigma(1-\omega)|e^{-i\varpi}\cos\varpi|}{\Upsilon_q(\gamma, \sigma, \omega, 2)}\tau^2,$$

where $\Upsilon_q(\gamma, \sigma, \omega, 2)$ is given by (3.3). Then for $\tau = re^{i\varphi}$, $0 < r < 1$, we have

$$\int_0^{2\pi} |f(\tau)|^{\eta} d\varphi \leq \int_0^{2\pi} |f_2(z)|^{\eta} d\varphi. \tag{5.2}$$

Proof. For $f(\tau) \in \mathcal{T}$, (5.2) is comparable to evidencing that

$$\int_0^{2\pi} \left|1 - \sum_{n=2}^{\infty} |a_n| z^{n-1}\right|^{\eta} d\varphi \leq \int_0^{2\pi} \left|1 - \frac{\sigma(1-\omega)|e^{-i\varpi}\cos\varpi|}{\Upsilon_q(\gamma, \sigma, \omega, 2)} z\right|^{\eta} d\varphi.$$

By Lemma 5.2, it suits to show that

$$1 - \sum_{n=2}^{\infty} |a_n| \tau^{n-1} \prec 1 - \frac{\sigma(1-\omega)|e^{-i\varpi}\cos\varpi|}{\Upsilon_q(\gamma, \sigma, \omega, 2)}\tau.$$

Setting

$$1 - \sum_{n=2}^{\infty} |a_n| \tau^{n-1} = 1 - \frac{\sigma(1-\omega)|e^{-i\varpi}\cos\varpi|}{\Upsilon_q(\gamma, \sigma, \omega, 2)} w(\tau), \tag{5.3}$$

making use of (3.1), we obtain

$$\begin{aligned} |w(\tau)| &= \left|\sum_{n=2}^{\infty} \frac{\Upsilon_q(\gamma, \sigma, \omega, n)}{\sigma(1-\omega)|e^{-i\varpi}\cos\varpi|} |a_n| \tau^{n-1}\right| \\ &\leq |\tau| \sum_{n=2}^{\infty} \frac{\Upsilon_q(\gamma, \sigma, \omega, n)}{\sigma(1-\omega)|e^{-i\varpi}\cos\varpi|} |a_n| \\ &\leq |\tau|. \end{aligned}$$

□

6 Subordination Results

We use the below stated results (see [23]) to prove our findings.

Definition 6.1. *(Subordinating Factor Sequence)*

A sequence $\{b_n\}_{n=1}^{\infty}$ of complex numbers is said to be a subordinating sequence, if whenever $f(\tau) = \sum\limits_{n=1}^{\infty} a_n \tau^n$, $a_1 = 1$ is regular, univalent and convex in $\mathcal{U}$, we have

$$\sum_{n=1}^{\infty} b_n a_n \tau^n \prec f(\tau), \quad \tau \in \mathcal{U}. \tag{6.1}$$

Lemma 6.2. *The sequence* $\{b_n\}_{n=1}^{\infty}$ *is a subordinating factor sequence iff*

$$\Re\left(1+2\sum_{n=1}^{\infty} b_n\tau^n\right) > 0, \quad \tau \in \mathcal{U}. \tag{6.2}$$

Theorem 6.3. *Let* $f \in \mathcal{PT}_q^{\gamma}(\sigma,\varpi,\omega)$ *and* $g(\tau)$ *be any function in the usual class of convex functions* $\mathcal{C}$, *then*

$$\frac{\Upsilon_q(\gamma,\sigma,\omega,2)}{2[\sigma(1-\omega)|e^{-i\varpi}\cos\varpi| + \Upsilon_q(\gamma,\sigma,\omega,2)]}(f*g)(\tau) \prec g(\tau) \tag{6.3}$$

where $0 \le \ell < 1$ *and* $0 \le \gamma \le 1$, *and*

$$\Re\{f(\tau)\} > -\frac{[\sigma(1-\omega)|e^{-i\varpi}\cos\varpi| + \Upsilon_q(\gamma,\sigma,\omega,2)]}{\Upsilon_q(\gamma,\sigma,\omega,2)}, \quad \tau \in \mathcal{U}. \tag{6.4}$$

The constant factor $\frac{\Upsilon_q(\gamma,\sigma,\omega,2)}{2[\sigma(1-\omega)|e^{-i\varpi}\cos\varpi|+\Upsilon_q(\gamma,\sigma,\omega,2)]}$ *in (6.3) cannot be superseded by a higher number.*

Proof. Consider $f \in \mathcal{PT}_q^{\gamma}(\sigma,\varpi,\omega)$ and suppose that $g(\tau) = \tau + \sum_{n=2}^{\infty} b_n\tau^n \in \mathcal{C}$. Then

$$\begin{aligned}&\frac{\Upsilon_q(\gamma,\sigma,\omega,2)}{2[\sigma(1-\omega)|e^{-i\varpi}\cos\varpi| + \Upsilon_q(\gamma,\sigma,\omega,2)]}(f*g)(\tau)\\ &\qquad = \frac{\Upsilon_q(\gamma,\sigma,\omega,2)}{2[\sigma(1-\omega)|e^{-i\varpi}\cos\varpi| + \Upsilon_q(\gamma,\sigma,\omega,2)]}\left(\tau + \sum_{n=2}^{\infty} b_n a_n \tau^n\right).\end{aligned} \tag{6.5}$$

Consequently, through Definition 6.1, the subordination result grips exact where

$$\left\{\frac{\Upsilon_q(\gamma,\sigma,\omega,2)}{2[\sigma(1-\omega)|e^{-i\varpi}\cos\varpi| + \Upsilon_q(\gamma,\sigma,\omega,2)]}\right\}_{n=1}^{\infty}$$

is a subordinating factor sequence, with $a_1 = 1$. In sight of Lemma 6.2, this would be the case similar to

$$\Re\left(1+\sum_{n=1}^{\infty}\frac{\Upsilon_q(\gamma,\sigma,\omega,2)}{[\sigma(1-\omega)|e^{-i\varpi}\cos\varpi| + \Upsilon_q(\gamma,\sigma,\omega,2)]}a_n\tau^n\right) > 0, \quad \tau \in \mathcal{U}. \tag{6.6}$$

By viewing the fact that $\frac{\Upsilon_q(\gamma,\sigma,\omega,n)}{\sigma(1-\omega)|e^{-i\varpi}\cos\varpi|}$ is increasing function for $n \ge 2$ and in precise

$$\frac{\Upsilon_q(\gamma,\sigma,\omega,2)}{\sigma(1-\omega)|e^{-i\varpi}\cos\varpi|} \le \frac{\Upsilon_q(\gamma,\sigma,\omega,n)}{\sigma(1-\omega)|e^{-i\varpi}\cos\varpi|}, \quad n \ge 2,$$

thus, for $|\tau| = r < 1$, we get

$$\begin{aligned}&\Re\left\{1+\frac{\Upsilon_q(\gamma,\sigma,\omega,2)}{[\sigma(1-\omega)|e^{-i\varpi}\cos\varpi| + \Upsilon_q(\gamma,\sigma,\omega,2)]}\sum_{n=1}^{\infty}a_n\tau^n\right\}\\ &= \Re\left\{1+\frac{\Upsilon_q(\gamma,\sigma,\omega,2)}{[\sigma(1-\omega)|e^{-i\varpi}\cos\varpi| + \Upsilon_q(\gamma,\sigma,\omega,2)]}\tau + \frac{\sum_{n=2}^{\infty}\Upsilon_q(\gamma,\sigma,\omega,2)a_n\tau^n}{[\sigma(1-\omega)|e^{-i\varpi}\cos\varpi| + \Upsilon_q(\gamma,\sigma,\omega,2)]}\right\}\\ &\ge 1-\frac{\Upsilon_q(\gamma,\sigma,\omega,2)}{[\sigma(1-\omega)|e^{-i\varpi}\cos\varpi| + \Upsilon_q(\gamma,\sigma,\omega,2)]}r - \frac{\sum_{n=2}^{\infty}|\Upsilon_q(\gamma,\sigma,\omega,n)a_n|\,r^n}{[\sigma(1-\omega)|e^{-i\varpi}cos\varpi| + \Upsilon_q(\gamma,\sigma,\omega,2)]}\\ &\ge 1-\frac{\Upsilon_q(\gamma,\sigma,\omega,2)}{[\sigma(1-\omega)|e^{-i\varpi}\cos\varpi| + \Upsilon_q(\gamma,\sigma,\omega,2)]}r - \frac{\sigma(1-\omega)|e^{-i\varpi}cos\varpi|}{[\sigma(1-\omega)|e^{-i\varpi}\cos\varpi| + \Upsilon_q(\gamma,\sigma,\omega,2)]}r\\ &> 0, \quad |\tau| = r < 1,\end{aligned}$$

by the assertion (3.1) of Theorem 3.2. This obviously demonstrates the inequality (6.6) and thus proved subordination result (6.3).

The inequality (6.4) proceeds from (6.3) through fixing

$$g(\tau) = \frac{\tau}{1-\tau} = \tau + \sum_{n=2}^{\infty} \tau^n \in \mathcal{C}.$$

Next we deliberate the function

$$h(\tau) := \tau - \frac{\sigma(1-\omega)|e^{-i\varpi}\cos\varpi|}{\Upsilon_q(\gamma,\sigma,\omega,2)}\tau^2$$

where $0 \leq \ell < 1,\ 0 \leq \gamma < 1$ and $\Upsilon_q(\gamma,\sigma,\omega,2)$ is assumed by (3.3). Clearly $h \in \mathcal{PT}_q^{\gamma}(\sigma,\varpi,\omega)$. For this function (6.3)becomes

$$\frac{\Upsilon_q(\gamma,\sigma,\omega,2)}{2[\sigma(1-\omega)|e^{-i\varpi}\cos\varpi| + \Upsilon_q(\gamma,\sigma,\omega,2)]} h(\tau) \prec \frac{\tau}{1-\tau}.$$

It is simply confirmed that

$$\min\left\{\Re\left(\frac{\Upsilon_q(\gamma,\sigma,\omega,2)}{2[\sigma(1-\omega)|e^{-i\varpi}\cos\varpi| + \Upsilon_q(\gamma,\sigma,\omega,2)]} h(\tau)\right)\right\} = -\frac{1}{2}, \quad \tau \in \mathcal{U}.$$

This indicates that the constant $\frac{\Upsilon_q(\gamma,\sigma,\omega,2)}{2[\sigma(1-\omega)|e^{-i\varpi}\cos\varpi| + \Upsilon_q(\gamma,\sigma,\omega,2)]}$ cannot be substituted by any larger one. □

7 Concluding Remarks:

By fixing $\gamma = 0$ or $\gamma = 1$ on can effortlessly state the above results for functions $f \in \mathcal{ST}_q(\sigma,\varpi,\omega)$ and $f \in \mathcal{CT}_q(\sigma,\varpi,\omega)$ respectively. Further by appropriately fixing the values of σ,ω,ϖ taking part in Theorem 3.2 to Theorem 6.3, it can express the analogous outcomes for function classes that are somewhat more acquainted. Further by considering the integral operator given in Remark 1.4,

$$I_q^{\ell,\upsilon}\hbar(\tau) = \frac{\Gamma_q(\ell+2)}{\Gamma_q(\ell+\upsilon+2)}\tau + \sum_{n=2}^{\infty} \frac{\Gamma_q(\ell+n+1)}{\Gamma_q(\ell+\upsilon+n+1)}\tau^n$$

and by using definition of convolution, one can find analogues results as in this article.

We have been acquainted with a novel family of spiral-like functions by employing Kober fractional q-calculus operations and the convolution product to get coefficient limits and extreme points. We also present certain integral transform findings, integral means inequalities, and subordination related with these function classes. Therefore, we complete this study by noting that various corollaries and implications may be inferred by appropriately specializing the parameters involved in each of the results reported here.

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Author information

Sunil Dutt Purohit, Department of HEAS (Mathematics), Rajasthan Technical University, Kota-324010, INDIA.
E-mail: `sunil_a_purohit@yahoo.com`

Gangadharan Murugusundaramoorthy, Kaliappan Vijaya, School of Advanced Sciences, Vellore Institute of Technology, Vellore - 632014, INDIA.
E-mail: `gmsmoorthy@yahoo.com; kvijaya@vit.ac.in`

Daya Lal Suthar, Department of Mathematics, Wollo University, P.O. Box: 1145, Dessie, ETHIOPIA.
E-mail: `dlsuthar@gmail.com`

Kamlesh Jangid, Department of Mathematics, Central University of Rajasthan, Ajmer, Rajasthan-305817, INDIA.
E-mail: `jangidkamlesh7@gmail.com`

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