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INDEPENDENT STRONG DOMINATION NUMBER OF INDU-BALA PRODUCT OF GRAPHS

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Abstract A set $D\subset V$ be the strong dominating set of G if every vertex in $V-D$ is strongly dominated by at least one vertex in D. The strong domination number $\gamma_{st}(G)$ of G is the minimum cardinality of a strong dominating set. The independent strong domination number $i_s(G)$ of a graph G is the minimum cardinality of a strong dominating set which is independent. In this paper, by means of the degree sequences (DS) of graphs and some graph theoretical methods, we determine the independent strong domination number of Indu-Bala product of simple connected graphs and null graphs.

1 Introduction

Topological indices are numerical parameters that are very important in Mathematics and have a lot of applications in fields such as Molecular Chemistry,quantitative structure-activity relationships (QSARs), and many other areas. Most of the topological indices are defined in terms of degrees sequence of the vertices. The degree sequence (DS) of a graph is the list of degree of all the vertices. It gives a lot of information about the physico-chemical properties of the graph.

The notion of degree of a graph provides users an area to study various structural properties of graphs and hence attracts the attention of many graph theorists. If the degrees of the vertices v_i of a graph G are d_i for $1\leq i\leq n$, then the degree sequence of G is $\{d_1,d_2,\cdots,d_n\}$. In many research studies, the DS is taken to be a non-decreasing sequence, whenever possible.

If the degree d_i of the vertex v_i is repeated z_i times in the DS of a graph G, then

$$\{\alpha_1={d_1}^{z_1},\alpha_2={d_2}^{z_2},\cdots,\alpha_r={d_r}^{z_r}\}$$

is used instead of $\{d_1,d_2,\cdots,d_n\}$ where $r\leq n$: Here the numbers z_i represent the frequencies of the degrees. In Fig. 1, the DS of $T_{3,2}$ is $\{1,1,2,2,3\}$, or briefly $\{\alpha_1=1^2,\alpha_2=2^2,\alpha_3=3^1\}$.

There are several graph operations used in calculating some chemical invariants of graphs, such as join, cartesian, corona product, union, disjunction and symmetric difference. Several graph operations have been examined in research by means of DS. After discussing the join operation, the DS of Indu-Bala product of graphs will be determined in the present study.

Let G_1 and G_2 be two graphs with n_1 and n_2 vertices and q_1 and q_2 edges, respectively. The join $G_1\vee G_2$ of graphs G_1 and G_2 with disjoint vertex sets $V(G_1)$ and $V(G_2)$ and edge sets $E(G_1)$ and $E(G_2)$ is the graph union $G_1\cup G_2$ together with all the edges joining $V(G_1)$ and $V(G_2)$. Then $|V(G_1\vee G_2)|=n_1+n_2$ and $|E(G_1\vee G_2)|=q_1+q_2+n_1n_2$.

The Indu-Bala product of any two graphs was introduced by G. Indulal and R. Balakrishnan, [9]. This graph product is based on classic loop switching problem analyzed by Graham and Pollack, [10]. The classic loop switching problem is used for wireless communication network theory. The Indu-Bala product $G_1\blacktriangledown G_2$ of graphs G_1 and G_2 is obtained by two disjoint copies of the join $G_1\vee G_2$ where the corresponding vertices of two copies of G_2 are connected by an

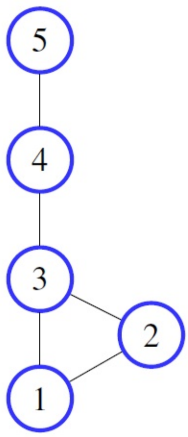

Figure 1: $T_{3,2}$

edge. It is obvious that $|V\left(G_1 \blacktriangledown G_2\right)| = 2|V(G_1 \vee G_2)| = 2\left(\, n_1 + n_2\right)$ and that

$$|E\left(G_1 \blacktriangledown G_2\right)| = 2|E(G_1 \vee G_2)| + n_2 = 2\left(q_1\right) + 2\left(q_2\right) + 2\left(\, n_1 n_2\right) + n_2.$$

Theorem 1.1. Let G_1 and G_2 be two connected graphs with $DS\left(G_1\right) = \left\{{\eta_{11}}^{\lambda 11}, \cdots, {\eta_{1p_1}}^{\lambda 1 p_1}\right\}$ and $DS\left(G_2\right) = \left\{{\eta_{21}}^{\lambda 21}, \cdots, {\eta_{2p_2}}^{\lambda 2 p_2}\right\}$, respectively. Then the DS of the Indu-Bala product of the graphs G_1 and G_2 is given by

$$\left\{\alpha_1 = \left(n_2 + \eta_{11}\right)^{2\lambda 11}, \cdots, \alpha_i = \left(n_2 + \eta_{1p_1}\right)^{2\lambda 1 p_1}, \beta_1 = \left(1 + n_1 + \eta_{21}\right)^{2\lambda 21}, \cdots, \beta_i = \left(1 + n_2 + \eta_{2p_2}\right)^{2\lambda 2 p_2}\right\}.$$

Proof: The DS of the join of the graphs G_1 and G_2 was obtained in [9] as

$$\left\{\alpha_1 = \left(n_2 + \eta_{11}\right)^{\lambda 11}, \cdots, \alpha_i = \left(n_2 + \eta_{1p_1}\right)^{\lambda 1 p_1}, \beta_1 = \left(n_1 + \eta_{21}\right)^{\lambda 21}, \cdots, \beta_i = \left(n_2 + \eta_{2p_2}\right)^{\lambda 2 p_2}\right\}.$$

To obtain $DS(G_1 \vee G_2)$, add the number of vertices n_2 of G_2 to each degree η_{1j} where $1 \leq j \leq p_1$ without changing the powers λ_{1j} and add the number of vertices n_1 of G_1 to each degree η_{2j} where $1 \leq j \leq p_2$ without changing the powers λ_{2j}. The Indu-Bala product of graphs G_1 and G_2 is obtained by two copies of join $G_1 \vee G_2$ where the corresponding vertices of two copies of G_2 are connected by an edge. That is, all the multiplicities of the degrees are multiplied by 2 and all the degrees of β_i are added one. Then the DS of the Indu-Bala product of graphs G_1 and G_2 is

$$\left\{\alpha_1 = \left(n_2 + \eta_{11}\right)^{2\lambda 11}, \cdots, \alpha_i = \left(n_2 + \eta_{1p_1}\right)^{2\lambda 1 p_1}, \beta_1 = \left(1 + n_1 + \eta_{21}\right)^{2\lambda 21}, \cdots, \beta_i = \left(1 + n_2 + \eta_{2p_2}\right)^{2\lambda 2 p_2}\right\}.$$

Theorem 1.2. The null graphs G_1 and G_2 have vertex sets consisting of n_1 and n_2 vertices, respectively. Then, the DS of the Indu-Bala product of the graphs G_1 and G_2 is given by

$$DS\left(G_1 \blacktriangledown G_2\right) = \left\{\alpha_1 = \left(n_1\right)^{2n_2}, \beta_1 = \ \left(1 + n_2\right)^{2n_1}\right\}.$$

Proof: The DS of the join of the graphs G_1 and G_2 was given in [9] as

$$DS(G_1 \vee G_2) = \left\{\alpha_1 = \left(n_1\right)^{n_2},\ \beta_1 = \left(n_2\right)^{n_1}\right\}.$$

To obtain $DS(G_1 \vee G_2)$, add the number of vertices n_2 of G_2 to the degrees of all the vertices of G_1. The Indu-Bala product of the two graphs G_1 and G_2 is obtained by two copies of join $(G_1 \vee G_2)$ and the corresponding vertices of two copies of G_2 are connected by an edge. That is, all the multiplicities of the degrees are multiplied by 2 and all the degrees of β_i are added one. Then the DS of the Indu-Bala product of the graphs G_1 and G_2 is $\{\left(n_1\right)^{2n_2},\ \left(1 + n_2\right)^{2n_1}\}$.

G_1	G_2	$G_1 \blacktriangledown G_2$
P_k	P_n	$\left\{(n+1)^{2(2)},(n+2)^{2(k-2)},(k+2)^{2(2)},(k+3)^{2(n-2)}\right\}$
P_k	C_n	$\left\{(n+1)^{2(2)},(n+2)^{2(k-2)},(k+3)^{2(n)}\right\}$
P_k	$S_{1,n}$	$\left\{(n+2)^{2(2)},(n+3)^{2(k-2)},(k+2)^{2(n)},(k+n+1)^{2}\right\}$
P_k	K_n	$\left\{(n+1)^{2(2)},(n+2)^{2(k-2)},(k+n)^{2(n)}\right\}$
P_k	$W_{1,n}$	$\left\{(n+2)^{2(2)},(n+3)^{2(k-2)},(k+4)^{2},(k+n+1)^{2}\right\}$
C_k	P_n	$\left\{(n+2)^{2(r)},(k+2)^{2(2)},(k+3)^{2(n-2)}\right\}$
C_k	C_n	$\left\{(n+2)^{2(k)},(k+3)^{2(n)}\right\}$
C_k	$S_{1,n}$	$\left\{(n+3)^{2(k)},(k+2)^{2(n)},(k+n+1)^{2}\right\}$
C_k	K_n	$\left\{(n+2)^{2(k)},(k+n-1)^{2(n)}\right\}$
C_k	$W_{1,n}$	$\left\{(n+3)^{2(k)},(k+4)^{2(n)},(r+s+1)^{2}\right\}$
$S_{1,k}$	P_n	$\left\{(n+1)^{2(r)},(n+k)^{2},(k+3)^{2(2)},(k+4)^{2(n-2)}\right\}$
$S_{1,k}$	C_n	$\left\{(n+1)^{2(k)},(n+k)^{2},(k+4)^{2(n)}\right\}$
$S_{1,k}$	$S_{1,n}$	$\left\{(n+2)^{2(k)},(n+k+2)^{2(2)},(k+3)^{2(n)}\right\}$
$S_{1,k}$	K_n	$\left\{(n+2)^{2(k)},(n+k+2)^{2},(k+n+2)^{2(n)}\right\}$
$S_{1,k}$	$W_{1,n}$	$\left\{(n+2)^{2(r)},(n+k+1)^{2},(k+n+2)^{2},(k+5)^{2(n)}\right\}$
K_k	P_n	$\left\{(n+k-1)^{2(k)},(k+2)^{2(2)},(k+3)^{2(n-2)}\right\}$
K_k	C_n	$\left\{(n+k-1)^{2(k)},(k+3)^{2(n)}\right\}$
K_k	$S_{1,n}$	$\left\{(n+k)^{2(k)},(k+2)^{2(n)},(k+n+1)^{2}\right\}$
K_k	K_n	$\left\{(n+k-1)^{2(k)},(k+n)^{2(n)}\right\}$
K_k	$W_{1,n}$	$\left\{(n+k)^{2(k)},(k+4)^{2(n)},(k+n+1)^{2}\right\}$
$W_{1,k}$	P_n	$\left\{(n+k)^{2},(n+3)^{2(k)},(k+3)^{2(2)},(k+4)^{2(n-2)}\right\}$
$W_{1,k}$	C_n	$\left\{(n+k)^{2},(n+3)^{2(k)},(k+4)^{2(n}\right\}$
$W_{1,k}$	$S_{1,n}$	$\left\{(n+k+1)^{2},(n+4)^{2(k)},(k+3)^{2(n))},(k+n+2)^{2}\right\}$
$W_{1,k}$	K_n	$\left\{(n+k)^{2},(n+3)^{2(k)},(k+n+1)^{2(n)}\right\}$
$W_{1,k}$	$W_{1,n}$	$\left\{(n+k+1)^{2},(n+4)^{2(k)},(k+n+2)^{2},(k+5)^{2(n)}\right\}$
$\bar{K}_k$	$\bar{K}_n$	$\left\{(n)^{2(k)},(1+k)^{2(n)}\right\}$

Table 1. Degree sequence of Indu-Bala product of graphs

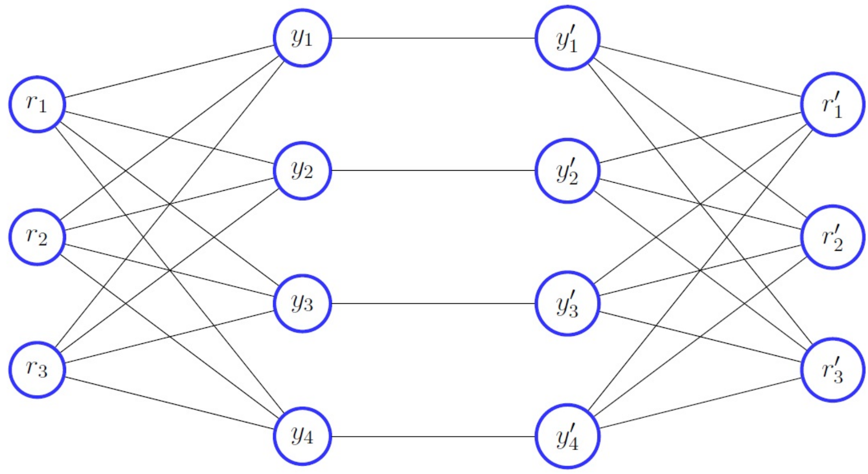

Figure 2: $\bar{K}_3$ and $\bar{K}_4$

2 Independent strong domination number of Indu-Bala product of graphs

Let $G = (V, E)$ be a graph and $u, v \in V$. Then u strongly dominates v if $uv \in E$ and $deg(u) \geq deg(v)$. A set $D \subset V$ be the strong dominating set of G if every vertex in $V - D$ is strongly dominated by at least one vertex in D. The strong domination number $\gamma_{st}(G)$ of G is the minimum cardinality of a strong dominating set. A set $D \subseteq V$ is an independent set if no two vertices of D are adjacent. The independent strong domination number $i_s(G)$ of a graph G is the minimum cardinality of a strong dominating set which is independent. In this section, we determine the independent strong domination (ISD) number of Indu-Bala product of graphs by means of degree sequence.

Theorem 2.1. Let $P_k \blacktriangledown P_n$ be the Indu-Bala product of paths P_k and P_n. Then

$$i_s\left(P_k \blacktriangledown P_n\right) = \begin{cases} \left\lceil \frac{|\alpha_2|}{3} \right\rceil; \; for\, k \equiv 0 (mod\, 3) \\ \left\lceil \frac{|\alpha_1|}{3} \right\rceil + \left\lceil \frac{|\alpha_2|}{3} \right\rceil; \; for\, k \equiv 1, 2 (mod\, 3) \end{cases}, \forall k < n.$$

Proof: Consider the Indu-Bala product of two paths $P_k \blacktriangledown P_n$. Let $(r_1, r_2, r_3, \ldots, r_k)$, $(r'_1, r'_2, r'_3, ..., , r'_k)$ be the vertices of two copies of P_k and $(y_1, y_2, y_3, \ldots, y_n)$, $(y'_1, y'_2, y'_3, \ldots, y'_n)$ be the vertices of two copies of $P_n, \forall k < n$. By the definition of the Indu-Bala product, all the vertices of P_k are adjacent with n vertices of P_n and the corresponding vertices of two copies of P_n are connected by an edge. Since $k < n$, the degree of P_k is always greater than or equal to the degree of P_n. The minimum independent dominating set of P_k is $D = \left\lceil \frac{k}{3} \right\rceil$. The DS of $P_k \blacktriangledown P_n$ is $\left\{\alpha_1 = (n + 1)^{2(2)},\; \alpha_2 = (n + 2)^{2(k-2)},\; \beta_1 = (k + 2)^{2(2)},\; \beta_2 = (k + 3)^{2(n-2)}\right\}$ and the cardinality of $|\alpha_1| = 4$ and $|\alpha_2| = 2(k-2)$. It concludes that,

$$i_s(P_k \blacktriangledown P_n) = \begin{cases} \left\lceil \frac{|\alpha_2|}{3} \right\rceil; \; for\, k \equiv 0 (mod\, 3) \\ \left\lceil \frac{|\alpha_1|}{3} \right\rceil + \left\lceil \frac{|\alpha_2|}{3} \right\rceil; \; for\, k \equiv 1, 2 (mod\, 3) \end{cases}, \forall k < n.$$

Observation 2.1. Let $P_k \blacktriangledown C_n$ be the Indu-Bala product of path P_k and the cycle C_n. Then,

$$i_s\left(P_k \blacktriangledown C_n\right) = \begin{cases} \left\lceil \frac{|\alpha_2|}{3} \right\rceil; \; for\, k \equiv 0 (mod\, 3) \\ \left\lceil \frac{|\alpha_1|}{3} \right\rceil + \left\lceil \frac{|\alpha_2|}{3} \right\rceil; \; for\, k \equiv 1, 2 (mod\, 3) \end{cases}, \forall k < n.$$

Theorem 2.2. The independent strong domination number of the Indu-Bala product of the cycle C_k and the path P_n is given by $i_s\left(C_k \blacktriangledown P_n\right) = \begin{cases} \left\lceil \frac{|\alpha_1|}{3} \right\rceil; \; for\, k \equiv 0, 2 (mod\, 3) \\ \left\lceil \frac{|\alpha_1|}{3} \right\rceil + 1; \; for\, k \equiv 1 (mod\, 3) \end{cases}, \forall 4 \leq k < n.$

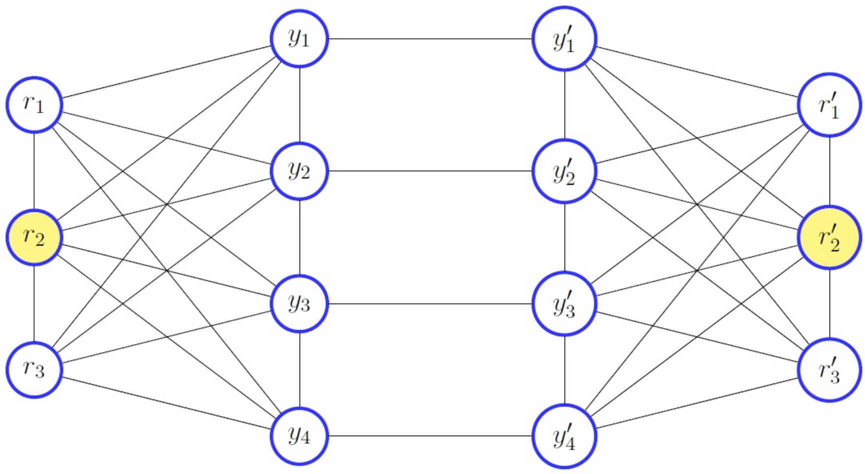

Figure 3: Coloured vertices constitute the ISD set of $P_3\blacktriangledown P_4$

Proof: Consider the Indu-Bala product of two paths $C_k\blacktriangledown P_n$. Let $(r_1, r_2, r_3, \ldots, r_k)$, $(r'_1, r'_2, r'_3, ..., , r'_k)$ be the vertices of two copies of C_k and $(y_1, y_2, y_3, \ldots, y_n)$, $(y'_1, y'_2, y'_3, \ldots, y'_n)$ be the vertices of two copies of $P_n, \forall\, k < n$, respectively. By the definition of Indu-Bala product, all the vertices of C_k are adjacent with n vertices of P_n and the corresponding vertices of two copies of P_n are connected by an edge. Given $k < n$, the degree of C_k is always greater than or equal to the degree of P_n. The minimum independent dominating set of C_k has $\lceil \frac{k}{3} \rceil$ elements. The DS of $C_k\blacktriangledown P_n$ is $\left\{ \alpha_1 = (\text{n} + 2)^{2k},\ \beta_1 = (k + 2)^{2\cdot 2},\ \beta_2 = (k + 3)^{2(n-2)} \right\}$ and $|\alpha_1| = 2k$. It follows that

$$i_s(C_k\blacktriangledown P_n) = \begin{cases} \left\lceil \frac{|\alpha_1|}{3} \right\rceil; \ for\, k \equiv 0, 2 (mod\ 3) \\ \left\lceil \frac{|\alpha_1|}{3} \right\rceil + 1; \ for\, k \equiv 1 (mod\ 3) \end{cases},$$

for $4 \le k < n$.

Observation 2.2. Let the Indu-Bala product of the two cycles C_k and C_n be $C_k\blacktriangledown C_n$. Then for $4 \le k < n$, we have

$$i_s(C_k\blacktriangledown C_n) = \begin{cases} \left\lceil \frac{|\alpha_1|}{3} \right\rceil; \ for\, k \equiv 0, 2 (mod\ 3) \\ \left\lceil \frac{|\alpha_1|}{3} \right\rceil + 1; \ for\, k \equiv 1 (mod\ 3) \end{cases}.$$

Theorem 2.3. The independent strong domination number of the Indu-Bala product of $P_k\blacktriangledown S_{1,n}$ is given by $i_s\left(P_k\blacktriangledown S_{1,n}\right) = 1 + \lceil \frac{k}{3} \rceil$ for $4 \le k < n$.

Proof: Consider the Indu-Bala product of $P_k\blacktriangledown S_{1,n}$. Let $(r_1, r_2, r_3, \ldots, r_k)$, $(r'_1, r'_2, r'_3, \ldots, r'_k)$ be the vertices of two copies of P_k and $(y_0, y_1, y_2, y_3, \ldots, y_n)$, $(y'_0, y'_1, y'_2, y'_3, \ldots, y'_n)$ be the vertices of two copies of $S_{1,n}, \forall\, k < n$. By the definition of Indu-Bala product, all the vertices of P_k are adjacent with all $n+1$ vertices of $S_{1,n}$ and the corresponding vertices of two copies of $S_{1,n}$ are connected by an edge. The DS of $P_k\blacktriangledown S_{1,n}$ is

$$\left\{\alpha_1 = (n+2)^{2\cdot 2},\ \alpha_2 = (n+3)^{2(k-2)},\ \beta_1 = (k+2)^{2n},\ \beta_2 = (k+\text{n+1})^2\right\}.$$

The maximum degree of $P_k\blacktriangledown S_{1,n}$ is β_2 where $|\beta_2| = 2$ dominates all the vertices of the graph $P_k\blacktriangledown S_{1,n}$. If the two vertices of degree β_2 in the dominating set D are considered, then the set D is not an independent set. So one vertex of deg $(\beta_2) \in D$ is included which will dominate all the vertices of $P_k \vee S_{1,n}$ and the minimum independent dominating set of P_k has $\lceil \frac{k}{3} \rceil$ elements. Therefore the independent strong domination number of $P_k\blacktriangledown S_{1,n}$ is $1 + \lceil \frac{k}{3} \rceil$ for $4 \le k < n$.

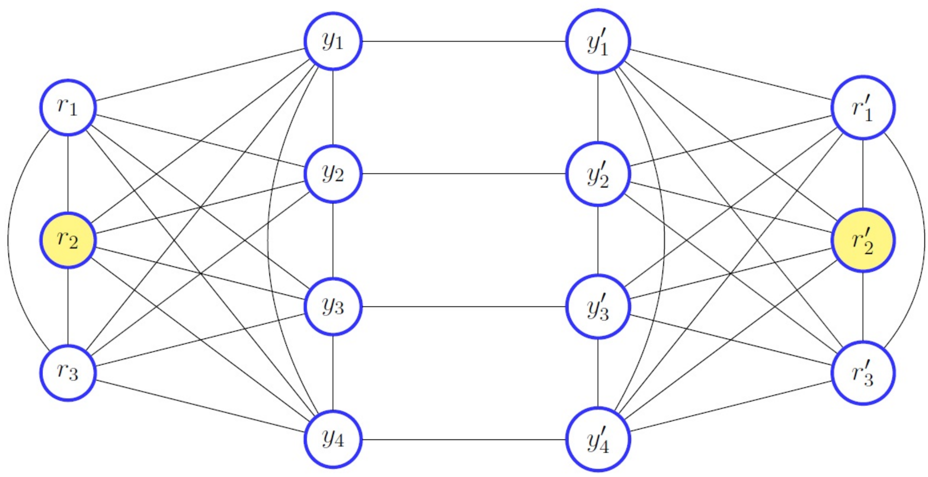

Figure 4: $Coloured\ vertices\ constitute\ the\ ISD\ set\ of\ C_3$ and C_4

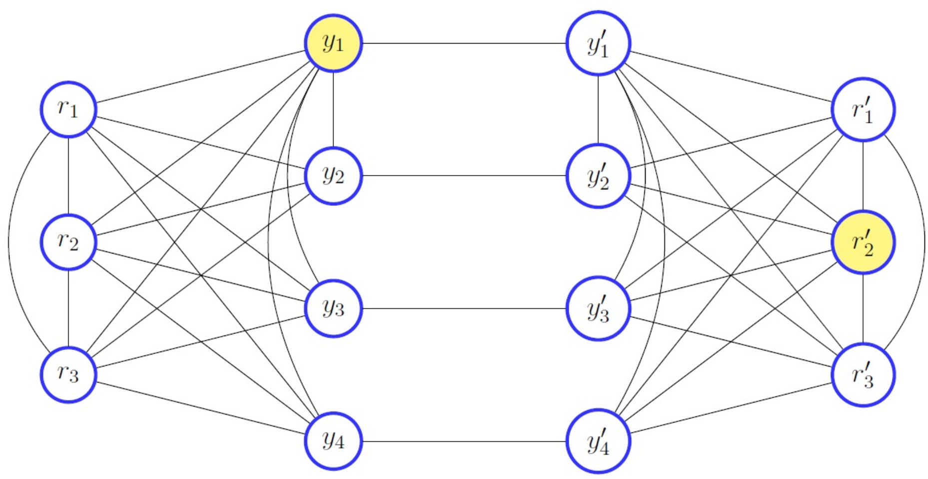

Figure 5: Coloured vertices constitute the ISD set of C_3 and $S_{1,4}$

Observation 2.3. The independent strong domination number of $C_k \blacktriangledown S_{1,n}$ is $i_s (C_k \blacktriangledown S_{1,n}) = 1 + \left\lceil \frac{k}{3} \right\rceil$ for $4 \leq k < n$.

Theorem 2.4. Let $S_{1,n} \blacktriangledown G$ be the Indu-Bala product of star $S_{1,n}$ and a connected graph G. Then $i_s (S_{1,n} \blacktriangledown G) = 2$, $\forall\, 4 \leq n < k$.

Proof: Consider the Indu-Bala product of $S_{1,n} \blacktriangledown P_k$. Let $(y_0, y_1, y_2, \ldots, y_n)$, $(y_0', y_1', y_2', \ldots, y_n')$ be the vertices of two copies of $S_{1,n}$ and $(r_1, r_2, r_3, \ldots, r_k)$, $(r_1', r_2', r_3', \ldots, r_n')$ be the vertices of two copies of P_k for $n < k$. By the definition of Indu-Bala product, all the vertices of $S_{1,n}$ are adjacent with k vertices of P_k and the corresponding vertices of two copies of P_k are connected by an edge. The DS of $S_{1,n} \blacktriangledown P_k$ is given as follows:

$$\left\{ \alpha_1 = (k+1)^{2(n)},\ \alpha_2 = (n+k)^2,\ \beta_1 = (n+3)^{2(2)},\ \beta_2 = (n+4)^{2(k-2)} \right\}.$$

The maximum degree of $S_{1,n} \blacktriangledown P_k$ is α_2, where $|\alpha_2| = 2$ dominates all the vertices of the graph $S_{1,n} \blacktriangledown P_k$. The vertices whose degree is α_2 are non-adjacent in $S_{1,n} \blacktriangledown P_k$. So the vertices of $\deg(\alpha_2)$ are to be included in D and they dominate all the vertices of $S_{1,n} \blacktriangledown P_k$ which implies that the set D is an minimum independent strong dominating set. Therefore $i_s (S_{1,n} \blacktriangledown P_k) = 2$

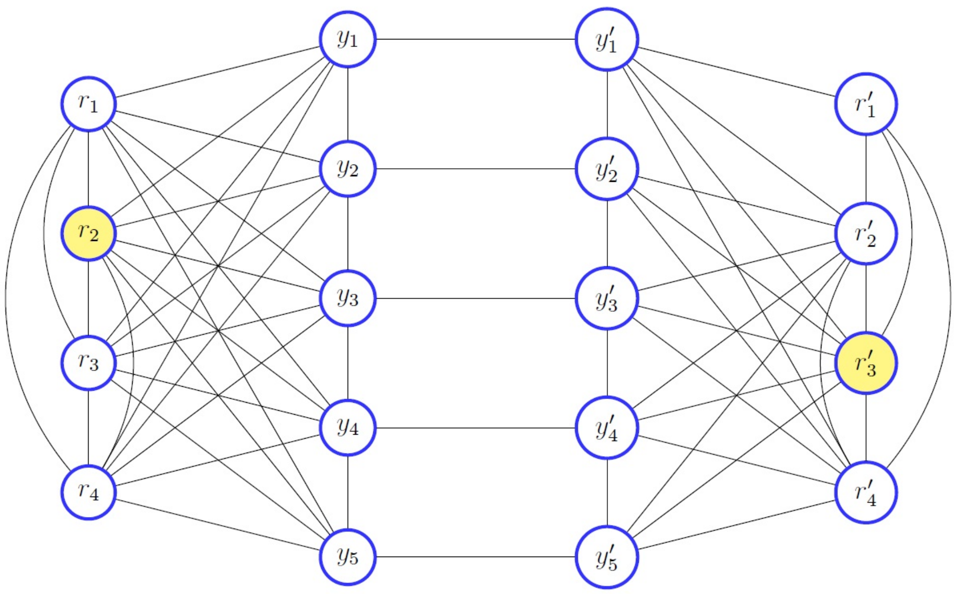

Figure 6: Coloured vertices constitute the ISD set of K_4 and P_5

for $4 \leq n < k$. It follows that the independent strong domination number of $S_{1,n} \blacktriangledown G$ is 2 for $4 \leq n < k$.

Theorem 2.5. The independent strong domination number of the Indu-Bala product of $K_n \blacktriangledown G$ is given by

$$i_s(K_n \blacktriangledown G) = 2, \forall 4 \leq n < k.$$

Proof: Consider the Indu-Bala product of $K_n \blacktriangledown P_k$. Let $(y_1, y_2, y_3, \dots, y_n)$, $(y'_1, y'_2, y'_3 \dots, y'_n)$ be the vertices of two copies of K_n and $(r_1, r_2, r_3, \dots, r_k)$, $(r'_1, r'_2, r'_3 \dots, r'_k)$ be the vertices of two copies of $P_k, \forall n < k$. By the definition of Indu-Bala product, all the vertices of K_n are adjacent with k vertices of P_k and the corresponding vertices of two copies of P_k are connected by an edge. The DS of $K_n \blacktriangledown P_k$ is $\left\{\alpha_1 = (n+k-1)^{2(n)},\ \beta_1 = (n+2)^{2(2)},\ \beta_2 = (n+3)^{2(k-2)}\right\}$. The maximum degree of $K_n \blacktriangledown P_k$ is α_1, where two vertices of degree α_1 are needed to dominate all the vertices of the graph $K_n \blacktriangledown P_k$. That is, one vertex of degree α_1 from each copies of join $K_n \vee P_k$ dominate all the vertices of the graph $K_n \blacktriangledown P_k$. It concludes that two vertices of deg $(\alpha_1) \in D$ will dominate all the vertices of $K_n \blacktriangledown P_k$ and the set D is an minimum independent strong dominating set. Hence, it follows that $i_s(K_n \blacktriangledown P_k) = 2$ for $4 \leq n < k$.

Observation 2.4. The independent strong domination number of the Indu-Bala product of $G \blacktriangledown K_n$ is 2, $\forall 4 \leq n < k$.

Theorem 2.6. Let $W_{1,n} \blacktriangledown G$ be the Indu-Bala product of the wheel $W_{1,n}$ and the connected graph G. Then, $i_s(W_{1,n} \blacktriangledown G) = 2, \forall 5 \leq n < k$.

Proof: Consider the Indu-Bala product of $W_{1,n} \blacktriangledown P_k$. Let $(y_1, y_2, y_3, \dots, y_n)$, $(y'_1, y'_2, y'_3 \dots, y'_n)$ be the vertices of two copies of $W_{1,n}$ and $(r_1, r_2, r_3, \dots, r_k)$, $(r'_1, r'_2, r'_3 \dots, r'_k)$ be the vertices of two copies of $P_k, \forall n < k$. By the definition of Indu-Bala product, all the vertices of $W_{1,n}$ are adjacent with k vertices of P_k and the corresponding vertices of two copies of P_k are connected by an edge. The DS of $W_{1,n} \blacktriangledown P_k$ is

$$\left\{\alpha_1 = (k+n)^2, \alpha_2 = (\text{k+3})^{2(n)}, \beta_1 = (n+3)^{2(2)}, \beta_2 = (n+4)^{2(k-2)}\right\}.$$

The maximum degree of $W_{1,n} \blacktriangledown P_k$ is α_1 and $W_{1,n} \blacktriangledown P_k$ will dominate all the vertices of the graph $W_{1,n} \blacktriangledown P_k$. That is, one vertex of degree α_1 from each copies of join $W_{1,n} \vee P_k$ dominates all the vertices of the graph $W_{1,n} \blacktriangledown P_k$. It concludes that two vertices of deg $(\alpha_1) \in D$ will dominate all the vertices of $W_{1,n}\blacktriangledown P_k$ and the set D is a minimum independent strong dominating set. As a result, the independent strong dominating set of $W_{1,n}\blacktriangledown P_k$ is 2 for $5 \leq n < k$. Similarly, the independent strong domination number of $W_{1,n}\blacktriangledown G$ is 2 for $5 \leq n < k$.

Theorem 2.7. The independent strong domination number of the Indu-Bala product of $P_k\blacktriangledown W_{1,n}$ is given by $i_s\left(C_k\blacktriangledown W_{1,n}\right) = 1 + \left\lceil \frac{k}{3} \right\rceil$ for $5 \leq k < n$.

Proof: Consider the Indu-Bala product of $P_k\blacktriangledown W_{1,n}$. Let $(r_1, r_2, r_3, \ldots, r_k)$, $(r'_1, r'_2, r'_3 \ldots, r'_k)$ be the vertices of two copies of P_k and $(y_0, y_1, y_2, y_3, \ldots, y_n)$, $(y'_0, y'_1, y'_2 \ldots, y'_n)$ be the vertices of two copies of $W_{1,n}, \forall\, k < n$. By the definition of Indu-Bala product, all the vertices of P_k are adjacent with $n+1$ vertices of $W_{1,n}$ and the corresponding vertices of two copies of $W_{1,n}$ are connected by an edge. The DS of $P_k\blacktriangledown W_{1,n}$ is given as follows:

$$\left\{\, \alpha_1 = (n+2)^{2\cdot 2},\;\; \alpha_2 = (n+3)^{2(k-2)},\;\; \beta_1 = (k+2)^{2n},\; \beta_2 = (k+\text{n+1})^2 \right\}.$$

The maximum degree of $P_k\blacktriangledown W_{1,n}$ is β_2, where $|\beta_2| = 2$ dominates all the vertices of the graph $P_k\blacktriangledown W_{1,n}$. If the two vertices of degree β_2 in the dominating set D are considered, then the set D is not an independent set. So, one vertex of degree β_2 is included in D and it dominates all the vertices of $P_k \vee W_{1,n}$; the minimum independent dominating set of P_k is $\left\lceil \frac{k}{3} \right\rceil$. It follows that the independent strong domination number of $P_k\blacktriangledown W_{1,n}$ is $1 + \left\lceil \frac{k}{3} \right\rceil$ for $5 \leq k < n$.

Observe that the independent strong domination number of $C_k\blacktriangledown W_{1,n}$ is $i_s\left(C_k\blacktriangledown W_{1,n}\right) = 1 + \left\lceil \frac{k}{3} \right\rceil$ for $5 \leq k < n$.

3 CONCLUSION

In this work, the degree sequence of the Indu-Bala product $G_1\blacktriangledown G_2$ of two connected graphs is given. The independent strong domination number of the Indu-Bala product of some pairs of families of graphs are determined in terms of degree sequence.

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