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On extended beta type integral operators involving generalized Mittag-Leffler function

Waseem A. Khan[1], M. Ghayasuddin[2] and Moin Ahmad[3]

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Abstract The aim of the present research paper is to establish a new class of extended beta type integral operators involving generalized Mittag-Leffler function, defined by Salim and Faraz [25]. Further, we derive some potentially useful special cases of our main results.

1 Introduction

Recently, a number of authors namely, Choi *et al.* [6, 7, 8], Ahmed and Khan [1], Ali [2], Ali *et al.* [3], Ghayasuddin *et al.*[9], Khan *et al.* [10-14] and Kamarujjama *et al.* [15, 16] have established some interesting integrals operators involving various kind of special functions, which are potentially useful in many diverse field of physics and engineering sciences. In a sequel of such type of works, in this paper, we further establish a new class of beta type integral operators involving generalized Mittag-Leffler function. A number of known and (presumably) new results are also indicated as special cases of our main finding. For purpose our present study, we begin by recalling here the following definitions of some well known functions:

The Swedish mathematician Mittag-Leffler [20] introduced the function $E_\alpha(z)$ as follows:

$$E_\alpha(z)=\sum_{n=0}^{\infty}\frac{z^n}{\Gamma(\alpha n+1)}, \tag{1.1}$$

where $z\in\mathbb{C}$ and $\Gamma(s)$ is the Gamma function.

The Mittag-Leffler function naturally occurs as the solution of fractional order differential equation or fractional order integral equations.

A generalization of $E_\alpha(z)$ was studied by Wiman [29], where he defined the function $E_{\alpha,\beta}(z)$ as

$$E_{\alpha,\beta}(z)=\sum_{n=0}^{\infty}\frac{z^n}{\Gamma(\alpha n+\beta)}, \tag{1.2}$$

where $\alpha,\beta\in\mathbb{C};\Re(\alpha)>0,\Re(\beta)>0$, which is also known as Mittag-Leffler function or Wiman's function.

Afterward, Prabhakar [22] introduced the function $E^{\gamma}_{\alpha,\beta}(z)$ in the form (see also Killbas *et al.* [17]):

$$E^{\gamma}_{\alpha,\beta}(z)=\sum_{n=0}^{\infty}\frac{(\gamma)_n}{\Gamma(\alpha n+\beta)}\frac{z^n}{n!}, \tag{1.3}$$

where $\alpha,\beta,\gamma\in\mathbb{C};\Re(\alpha)>0,\Re(\beta)>0,\Re(\gamma)>0$.

In 2007, Shukla and Prajapati [27] (see also Srivastava and Tomovaski [28]) further introduced and investigated the function $E^{\gamma,q}_{\alpha,\beta}(z)$ as follows:

$$E_{\alpha,\beta}^{\gamma,q}(z)=\sum_{n=0}^{\infty}\frac{(\gamma)_{qn}}{\Gamma(\alpha n+\beta)}\frac{z^n}{n!}, \tag{1.4}$$

where $\alpha,\beta,\gamma\in\mathbb{C};\Re(\alpha)>0,\Re(\beta)>0,\Re(\gamma)>0,q\in(0,1)\bigcup\mathbb{N}$ and $(\gamma)_{qn}=\frac{\Gamma(\gamma+qn)}{\Gamma(\gamma)}$.

Furthermore, Salim [26] gave a new extension of Mittag-Leffler function as follows:

$$E_{\alpha,\beta}^{\gamma,\delta}(z)=\sum_{n=0}^{\infty}\frac{(\gamma)_{n}}{\Gamma(\alpha n+\beta)}\frac{z^n}{(\delta)_n}, \tag{1.5}$$

where $\alpha,\beta,\gamma,\delta\in\mathbb{C};\Re(\alpha)>0,\Re(\beta)>0,\Re(\gamma)>0,\Re(\delta)>0$.

Very recently, Salim and Faraj [25] introduced the following generalized Mittag-Leffler function $E_{\alpha,\beta,p}^{\gamma,\delta,q}(z)$:

$$E_{\alpha,\beta,p}^{\gamma,\delta,q}(z)=\sum_{n=0}^{\infty}\frac{(\gamma)_{qn}}{\Gamma(\alpha n+\beta)}\frac{z^n}{(\delta)_{pn}}, \tag{1.6}$$

where $\alpha,\beta,\gamma,\delta\in\mathbb{C},\min\{\Re(\alpha)>0,\Re(\beta)>0,\Re(\gamma)>0,\Re(\delta)>0\}>0;p,q>0$ and $q<\Re(\alpha)+p$.

The special function (1.6) is a generalization of another type of Mittag-Leffler functions introduced in (1.1)-(1.5).

Now we recall the classical beta function denoted by $B(a,b)$ and is defined by (see [21], see also [18]):

$$B(a,b)=\int_0^1 t^{a-1}(1-t)^{b-1}dt=\frac{\Gamma(a)\Gamma(b)}{\Gamma(a+b)},(\Re(a)>0,\Re(b)>0). \tag{1.7}$$

In 1997, Chaudhary *et al.* [4] presented the following extension of Euler's Beta function:

$$B(a,b;\sigma)=\int_0^1 t^{a-1}(1-t)^{b-1}\exp\left[-\frac{\sigma}{t(1-t)}\right]dt \tag{1.8}$$

For $\sigma=0$, the extended beta function reduces to the classical beta function.

The Gauss hypergeometric function, denoted by $F(a,b,c;z)$ and confluent hypergeometric function of the first kind denoted by $\Phi(b;c;z)$, for $\Re(c)>\Re(b)>0$, are defined as follows (see [24], see also [23]):

$$F(a,b;c;z)=\frac{1}{B(b,c-b)}\int_0^1 t^{b-1}(1-t)^{c-b-1}(1-tz)^{-a}dt, \tag{1.9}$$

$$|arg(1-z)|<\pi$$

and

$$\Phi(b;c;z)=\frac{1}{B(b,c-b)}\int_0^1 t^{b-1}(1-t)^{c-b-1}\exp(zt)dt. \tag{1.10}$$

By using the series expansion of $(1-zt)^{-a}$ and $\exp(zt)$ in (1.9) and (1.10) respectively, we obtain

$$F(a,b;c;z)=\sum_{n=0}^{\infty}\frac{(a)_nB(b+n,c-b)}{B(b,c-b)}\frac{z^n}{n!} \tag{1.11}$$

$$(|z|<1,\Re(c)>\Re(b)>0);$$

$$\Phi(b;c;z) = \sum_{n=0}^{\infty} \frac{B(b+n, c-b)}{B(b, c-b)} \frac{z^n}{n!}, \tag{1.12}$$

$$(\Re(c) > \Re(b) > 0).$$

In 2004, Chaudhary *et al.* [5] (see also [19]) used beta function $B(a, b; \sigma)$ to extended the hypergeometric and confluent hypergeometric function as follows:

$$F(a, b; c; z; \sigma) = \sum_{n=0}^{\infty} \frac{(a)_n B(b+n, c-b; \sigma)}{B(b, c-b)} \frac{z^n}{n!}, \tag{1.13}$$

$$(\sigma \geq 0; |z| < 1, \Re(c) > \Re(b) > 0);$$

$$\Phi(b;c;z;\sigma) = \sum_{n=0}^{\infty} \frac{B(b+n, c-b; \sigma)}{B(b, c-b)} \frac{z^n}{n!}, \tag{1.14}$$

$$(\sigma \geq 0; \Re(c) > \Re(b) > 0);$$

and gave their Euler's type integral representation:

$$F(a, b; c; z; \sigma) = \frac{1}{B(b, c-b)} \int_0^1 t^{b-1}(1-t)^{c-b-1}(1-zt)^{-a} \exp\left[-\frac{\sigma}{t(1-t)}\right] dt, \tag{1.15}$$

$$(\sigma > 0; \sigma = 0; |arg(1-z)| < \pi; \Re(c) > \Re(b) > 0);$$

$$\Phi(b; c; z; \sigma) = \frac{1}{B(b, c-b)} \int_0^1 t^{b-1}(1-t)^{c-b-1} \exp\left[zt - \frac{\sigma}{t(1-t)}\right] dt, \tag{1.16}$$

$$(\sigma > 0; \sigma = 0; \Re(c) > \Re(b) > 0).$$

2 Extended beta type integral operator involving Mittag-Leffler function

Theorem 2.1. If $\alpha, \beta, \gamma, \delta \in \mathbb{C}, \Re(\alpha) > 0, \Re(\beta) > 0, \Re(\gamma) > 0, \Re(\delta) > 0, \Re(A) > 0$, $p, q \geq 0$ and $q < \Re(\alpha) + p$, then

$$\int_0^1 t^{a-1}(1-t)^{b-1} \exp\left(\frac{-A}{t(1-t)}\right) E_{\alpha,\beta,p}^{\gamma,\delta,q}(zt^\alpha) dt = \sum_{n=0}^{\infty} \frac{(\gamma)_{qn} z^n}{\Gamma(\alpha n + \beta)(\delta)_{pn}} B(\alpha n + a, b; A). \tag{2.1}$$

Proof. In order to derive (2.1), we denote L.H.S. of (2.1) by I_1 and then expanding $E_{\alpha,\beta,p}^{\gamma,\delta,q}(zt^\alpha)$ by using (1.6), to get

$$I_1 = \int_0^1 t^{a-1}(1-t)^{b-1} \exp\left(\frac{-A}{t(1-t)}\right) \sum_{n=0}^{\infty} \frac{(\gamma)_{qn} z^n}{\Gamma(\alpha n + \beta)(\delta)_{pn}} dt.$$

Now changing the order of summation and integration (which is guaranteed under the given conditions), to get

$$I_1 = \sum_{n=0}^{\infty} \frac{(\gamma)_{qn} z^n}{\Gamma(\alpha n + \beta)(\delta)_{pn}} \int_0^1 t^{\alpha n + a - 1}(1-t)^{b-1} \exp\left(\frac{-A}{t(1-t)}\right) dt.$$

Now by using (1.8) in the above equation, we get the required result.

Corollary 2.1. For $A = 0$ in Theorem 2.1, we immediately deduce the following result:

$$\frac{1}{\Gamma b}\int_0^1 t^{a-1}(1-t)^{b-1}E^{\gamma,\delta,q}_{\alpha,\beta,p}(zt^{\alpha})dt = E^{\gamma,\delta,q}_{\alpha,a+b,p}(z). \tag{2.2}$$

Theorem 2.2. If $\alpha,\beta,\gamma,\delta,\rho,\mu,\lambda \in \mathbb{C}, \Re(\alpha) > 0, \Re(\beta) > 0, \Re(\gamma) > 0, \Re(\delta) > 0, \Re(\rho), \Re(\mu), \Re(\lambda) > 0, \Re(A) > 0$, $p,q \geq 0$ and $q < \Re(\alpha) + p$; $\mid arg(\frac{b'c+d}{a'c+d}) \mid < \pi$, then

$$\int_a^b (t-a)^{\rho-1}(b-t)^{\mu-1}(ct+d)^{\lambda}\exp\left(\frac{-A}{(t-a)(b-t)}\right)E^{\gamma,\delta,q}_{\alpha,\beta,p}(z(b-t)^f)dt$$

$$= (ac+d)^{\lambda}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{(-A)^m}{m!}\frac{(\gamma)_{qn}z^n}{\Gamma(\alpha n+\beta)(\delta)_{pn}}B(\rho-m,\mu+fn-m)(b-a)^{\rho+\mu+fn-2m-1}$$
$$\times {}_2F_1\left[\rho-m,-\lambda;\rho+\mu+fn-2m;\frac{-(b-a)c}{ca+d}\right]. \tag{2.3}$$

Proof. On the L.H.S. of (2.3), expanding the exponential function and Mittag-Leffler function in their respective series, to get

$$\int_a^b (t-a)^{\rho-1}(b-t)^{\mu-1}(ct+d)^{\lambda}\exp\left(\frac{-A}{(t-a)(b-t)}\right)E^{\gamma,\delta,q}_{\alpha,\beta,p}(z(b-t)^f)dt$$

$$= \int_a^b (t-a)^{\rho-1}(b-t)^{\mu-1}(ct+d)^{\lambda}\sum_{m=0}^{\infty}\frac{(-A)^m}{(t-a)^m(b-t)^m m!}\sum_{n=0}^{\infty}\frac{(\gamma)_{qn}z^n(b-t)^{fn}}{\Gamma(\alpha n+\beta)(\delta)_{pn}}dt.$$

Now changing the order of summation and integration (which is guaranteed under the given conditions),we get

$$\int_a^b (t-a)^{\rho-1}(b-t)^{\mu-1}(ct+d)^{\lambda}\exp\left(\frac{-A}{(t-a)(b-t)}\right)E^{\gamma,\delta,q}_{\alpha,\beta,p}(z(b-t)^f)dt$$

$$= \sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{(-A)^m}{m!}\frac{(\gamma)_{qn}z^n}{\Gamma(\alpha n+\beta)(\delta)_{pn}}\int_a^b (t-a)^{\rho-m-1}(b-t)^{\mu+fn-m-1}(ct+d)^{\lambda}dt,$$

which further on using the integral [1], yields the required result (2.3).

Corollary 2.2. For $A = 0$ in Theorem 2.2, we get

$$\int_a^b (t-a)^{\rho-1}(b-t)^{\mu-1}(ct+d)^{\lambda}E^{\gamma,\delta,q}_{\alpha,\beta,p}(z(b-t)^f)dt$$

$$= (ac+d)^{\lambda}\sum_{n=0}^{\infty}\frac{(\gamma)_{qn}z^n}{\Gamma(\alpha n+\beta)(\delta)_{pn}}B(\rho,\mu+fn)(b-a)^{\rho+\mu+fn-1}$$
$$\times {}_2F_1\left[\rho,-\lambda;\rho+\mu+fn;\frac{-(b-a)c}{ca+d}\right]. \tag{2.4}$$

Corollary 2.3. On setting $a = 0$, $b = 1$ in Theorem 2.2, we get the following result:

$$\int_0^1 t^{\rho-1}(1-t)^{\mu-1}(ct+d)^{\lambda}\exp\left(\frac{-A}{t(1-t)}\right)E^{\gamma,\delta,q}_{\alpha,\beta,p}(z(1-t)^f)dt$$

$$= d^{\lambda} \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \frac{(-A)^m}{m!} \frac{(\gamma)_{qn} z^n}{\Gamma(\alpha n+\beta)(\delta)_{pn}} B(\rho - m, \mu + fn - m)$$

$$\times {}_2F_1 \left[\rho - m, -\lambda; \rho + \mu + fn - 2m; \frac{-c}{d} \right]. \tag{2.5}$$

Theorem 2.3. If $\alpha, \beta, \gamma, \delta, \rho, \mu, \lambda \in \mathbb{C}, \Re(\alpha) > 0, \Re(\beta) > 0, \Re(\gamma) > 0, \Re(\delta) > 0, \Re(\rho) > 0, \Re(\mu) > 0, \Re(\lambda) > 0, \Re(A) > 0, p, q \geq 0$ and $q < \Re(\alpha) + p$, then

$$\int_0^1 t^{\lambda-1}(1-t)^{\mu-\lambda-1}(1-ut^{\rho}(1-t)^{\sigma})^{-a} \exp\left(\frac{-A}{t(1-t)}\right) E_{\alpha,\beta,p}^{\gamma,\delta,q}(zt^{\alpha})dt$$

$$= \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} \frac{(\gamma)_{qn} z^n}{\Gamma(\alpha n+\beta)(\delta)_{pn}} (a)_m B(\lambda + \alpha n + \rho m, \mu - \lambda + \sigma m; A) \frac{u^m}{m!}. \tag{2.6}$$

Proof. On taking L.H.S. of Theorem 2.3, using the definition of generalized Mittag-Leffler function (1.6), and then by changing the order of summation and integration, we get

$$\int_0^1 t^{\lambda-1}(1-t)^{\mu-\lambda-1}(1-ut^{\rho}(1-t)^{\sigma})^{-a} \exp\left(\frac{-A}{t(1-t)}\right) E_{\alpha,\beta,p}^{\gamma,\delta,q}(zt^{\alpha})dt$$

$$= \sum_{n=0}^{\infty} \frac{(\gamma)_{qn} z^n}{\Gamma(\alpha n+\beta)(\delta)_{pn}} \int_0^1 t^{\lambda+\alpha n-1}(1-t)^{\mu-\lambda-1}(1-ut^{\rho}(1-t)^{\sigma})^{-a} \exp\left(\frac{-A}{t(1-t)}\right) dt,$$

which further on using the integral [18], gives the required result (2.6).

Corollary 2.4. On setting $A = 0$ in Theorem 2.3, we get the following result:

$$\int_0^1 t^{\lambda-1}(1-t)^{\mu-\lambda-1}(1-ut^{\rho}(1-t)^{\sigma})^{-a} E_{\alpha,\beta,p}^{\gamma,\delta,q}(zt^{\alpha})dt$$

$$= \sum_{n=0}^{\infty} \frac{(\gamma)_{qn} z^n}{\Gamma(\alpha n+\beta)(\delta)_{pn}} (a)_m B(\lambda + \alpha n + \rho m, \mu - \lambda + \sigma m) \frac{u^m}{m!}. \tag{2.7}$$

Remark. If we consider $p = q = 1$ in (2.1), (2.3) and (2.6), then we get a new class of Beta type integral operators involving the generalized Mittag-Leffler function defined by Salim [26], and the case $\delta = p = 1$ of (2.1), (2.3) and (2.6) is seen to yield the known results of Ahmed and Khan [1].

3 Special Cases

In this section, we establish the following potentially useful integral operators involving generalized Wright hypergeometric functions as special cases of our main results:

1. Setting $\gamma = q = 1$ in Theorem 2.1, we get

$$\int_0^1 t^{a-1}(1-t)^{b-1} \exp\left(\frac{-A}{t(1-t)}\right) {}_2\Psi_2 \left[\begin{array}{ll} (1,1),(1,1); & \\ & zt^{\alpha} \\ (\beta,\alpha),(\delta,p); & \end{array} \right] dt$$

$$= \frac{1}{\Gamma(\delta)} \sum_{n=0}^{\infty} \frac{n! z^n}{\Gamma(\alpha n+\beta)(\delta)_{pn}} B(n\alpha + a, b; A),$$

Here ${}_p\Psi_q$ is the Wright hypergeometric function defined as follows (see [9]).

2. Setting $\alpha=\beta=\gamma=q=1$ in Theorem 2.1, we get

$$\int_0^1 t^{a-1}(1-t)^{b-1}\exp\left(\frac{-A}{t(1-t)}\right){}_1\Psi_1\left[\begin{array}{cc}(1,1); & \\ & zt \\ (\delta,p); & \end{array}\right]dt=\frac{1}{\Gamma(\delta)}\sum_{n=0}^{\infty}\frac{z^n}{(\delta)_{pn}}B(n+a,b;A).$$

3. Setting $\delta=p=1$ in Theorem 2.1, we get

$$\int_0^1 t^{a-1}(1-t)^{b-1}\exp\left(\frac{-A}{t(1-t)}\right){}_1\Psi_1\left[\begin{array}{cc}(\gamma,q); & \\ & zt^{\alpha} \\ (\beta,\alpha); & \end{array}\right]dt$$

$$=\Gamma(\gamma)\sum_{n=0}^{\infty}\frac{(\gamma)_{qn}z^n}{\Gamma(\alpha n+\beta)n!}B(n\alpha+a,b;A).$$

4. Setting $\gamma=q=1$ in Theorem 2.2, we get

$$\int_a^b (t-a)^{\rho-1}(b-t)^{\mu-1}(ct+d)^{\lambda}\exp\left(\frac{-A}{(t-a)(b-t)}\right){}_2\Psi_2\left[\begin{array}{cc}(1,1),(1,1); & \\ & z(b-t)^f \\ (\beta,\alpha),(\delta,p); & \end{array}\right]dt$$

$$=\frac{1}{\Gamma(\delta)}(ac+d)^{\lambda}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{(-A)^m}{m!}\frac{n!z^n}{\Gamma(\alpha n+\beta)(\delta)_{pn}}B(\rho-m,\mu+fn-m)(b-a)^{\rho+\mu+fn-2m-1}$$

$$\times{}_2F_1\left[\rho,-\lambda,\rho+\mu+fn;\frac{(b-a)c}{ca+d}\right].$$

5. Setting $\alpha=\beta=\gamma=q=1$ in Theorem 2.2, we get

$$\int_a^b (t-a)^{\rho-1}(b-t)^{\mu-1}(ct+d)^{\lambda}\exp\left(\frac{-A}{(t-a)(b-t)}\right){}_1\Psi_1\left[\begin{array}{cc}(1,1); & \\ & z(b-t)^f \\ (\delta,p); & \end{array}\right]dt$$

$$=\frac{1}{\Gamma(\delta)}(ac+d)^{\lambda}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{(-A)^m}{m!}\frac{z^n}{(\delta)_{pn}}B(\rho-m,\mu+fn-m)(b-a)^{\rho+\mu+fn-2m-1}$$

$$\times{}_2F_1\left[\rho-m,-\lambda;,\rho+\mu+fn-2m;\frac{(b-a)c}{ca+d}\right].$$

6. Setting $\delta=p=1$ in Theorem 2.2, we get

$$\int_a^b (t-a)^{\rho-1}(b-t)^{\mu-1}(ct+d)^{\lambda}\exp\left(\frac{-A}{(t-a)(b-t)}\right){}_1\Psi_1\left[\begin{array}{cc}(\gamma,q); & \\ & z(b-t)^f \\ (\beta,\alpha); & \end{array}\right]dt$$

$$=\Gamma(\gamma)(ac+d)^{\lambda}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{(-A)^m}{m!}\frac{z^n}{\Gamma(\alpha n+\beta)}B(\rho-m,\mu+fn-m)(b-a)^{\rho+\mu+fn-2m-1}$$

$$\times{}_2F_1\left[\rho,-\lambda,\rho+\mu+fn;\frac{(b-a)c}{ca+d}\right].$$

7. Setting $\gamma=q=1$ in Theorem 2.3, we get

$$\int_0^1 t^{\lambda-1}(1-t)^{\mu-\lambda-1}\{1-ut^{\rho}(1-t)^{\sigma}\}^{-a}\exp\left(\frac{-A}{t(1-t)}\right){}_2\Psi_2\left[\begin{array}{cc}(1,1),(1,1); & \\ & zt^{\alpha} \\ (\beta,\alpha),(\delta,p); & \end{array}\right]dt$$

$$= \frac{1}{\Gamma(\delta)} \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} \frac{n! z^n}{\Gamma(\alpha n + \beta)(\delta)_{pn}} (a)_m B(\lambda + \alpha n + \rho m, \mu - \lambda + \sigma m; A) \frac{u^m}{m!}.$$

8. Setting $\alpha = \beta = \gamma = q = 1$ in Theorem 2.3, we get

$$\int_0^1 t^{\lambda-1}(1-t)^{\mu-\lambda-1}\{1 - ut^{\rho}(1-t)^{\sigma}\}^{-a} \exp\left(\frac{-A}{t(1-t)}\right) {}_1\Psi_1 \left[\begin{array}{cc} (1,1); & \\ & zt \\ (\delta, p); & \end{array} \right] dt$$

$$= \frac{1}{\Gamma(\delta)} \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \frac{z^n}{(\delta)_{pn}} (a)_m B(\lambda + n + \rho m, \mu - \lambda + \sigma m; A) \frac{u^m}{m!}.$$

9. Setting $\delta = p = 1$ in Theorem 2.3, we get

$$\int_0^1 t^{\lambda-1}(1-t)^{\mu-\lambda-1}\{1 - ut^{\rho}(1-t)^{\sigma}\}^{-a} \exp\left(\frac{-A}{t(1-t)}\right) {}_1\Psi_1 \left[\begin{array}{cc} (\gamma, q); & \\ & zt^{\alpha} \\ (\beta, \alpha); & \end{array} \right] dt$$

$$= \Gamma(\gamma) \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} \frac{(\gamma)_{qn} z^n}{\Gamma(\alpha n + \beta) n!} (a)_m B(\lambda + \alpha n + \rho m, \mu - \lambda + \sigma m; A) \frac{u^m}{m!}.$$

4 Concluding remark

In this paper, we have established some beta type integral operators involving generalized Mittag-Leffler function. We have also considered some special cases of our main finding in the form of new beta type integrals involving Wright hypergeometric functions. Moreover, in this section, we briefly consider a variation of the results derived in section 2 by considering the relation between generalized Mittag-Leffler function and generalized Bessel-Maitland function. Since, it can be easily seen that the generalized Mittag-Leffler function $E_{\alpha,\beta,p}^{\gamma,\delta,q}(z)$ is a special case of the generalized Bessel-Maitland function as follows (see [9, p.294]):

$$E_{\alpha,\beta,p}^{\gamma,\delta,q}(z) = J_{\beta-1,\gamma,\delta}^{\alpha,q,p}(-z).$$

Therefore, the results presented in section 2 are easily converted in terms of the generalized Bessel-Maitland function after some suitable parametric replacement.

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Author information

Waseem A. Khan[1], M. Ghayasuddin[2] and Moin Ahmad[3], [1]Department of Mathematics and Natural Sciences, Prince Mohammad Bin Fahd University, P.O Box 1664, Al Khobar 31952, Saudi Arabia, [2]Department of Mathematics, Integral University Campus, Shahjahanpur-242001, India, [3]Department of Mathematics, Shree Krishna Institute, Ahmad Nagar Sitapur-261125,, India.
E-mail: wkhan1@pmu.edu.sa,ghayas.maths@gmail.com, moinah1986@gmail.com

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