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Coefficient bounds for a new class of analytic functions utilizing q-difference operator

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Abstract In this work, we introduce a new class $\mathfrak{D}_q(\varpi,\epsilon)$ of analytic functions utilizing means of q-differential operator in the disk $\mathfrak{U}=\{\xi\in\mathbb{C}:|\xi|<1\}$ defined as follows

$$\mathfrak{D}_q\mathfrak{f}(\xi)=\frac{\mathfrak{f}(\xi)-\mathfrak{f}(q\xi)}{(1-q)\xi}$$

where $q\in(0,1)$. Sufficient conditions involving coefficient inequalities for this class are also obtained. Special cases of our main result are also shown to lead sufficient conditions for the classes $\mathcal{S}_q^*(\epsilon)$ and $\mathcal{K}_q(\epsilon)$, where $\mathcal{S}_q^*(\epsilon)$ and $\mathcal{K}_q(\epsilon)$ denote, respectively, the classes of q- starlike and q- convex functions of order ϵ in $\mathfrak{U}$.

1 Introduction

Let $\mathbb{A}$ denote the class of all functions of the form

$$\mathfrak{f}(\xi)=\xi+\sum_{s=2}^{\infty}a_s\xi^s \tag{1.1}$$

that are analytic in the disk $\mathfrak{U}=\{\xi\in\mathbb{C}:|\xi|<1\}$. Further, let $\mathbb{P}$ denote the class of all analytic functions $\varkappa(\xi)$ in $\mathfrak{U}$ of the form [1]

$$\varkappa(\xi)=1+\sum_{s=1}^{\infty}p_s\xi^s,\qquad \xi\in\mathfrak{U}. \tag{1.2}$$

The utilization of q-calculus operators plays a crucial role in elucidating and addressing diverse challenges in applied science. These challenges encompass ordinary fractional calculus, optimal control, q-difference and q-integral equations, and the geometric function theory of complex analysis. The inception of applying q-calculus can be attributed to Jackson [2]. Specifically, for q belonging to the open interval (0, 1), Jackson's q-derivative (refer to [2]) of a function $\mathfrak{f}\in\mathbb{A}$ is precisely defined.

$$\mathfrak{D}_q\mathfrak{f}(\xi)=\begin{cases}\dfrac{\mathfrak{f}(\xi)-\mathfrak{f}(q\xi)}{(1-q)\xi} & for\quad \xi\neq 0,\\ \mathfrak{f}'(0) & for\quad \xi=0.\end{cases} \tag{1.3}$$

From (1.3), we have

$$\mathfrak{D}_q\mathfrak{f}(\xi)=1+\sum_{s=2}^{\infty}[s]_q a_s\xi^{s-1} \tag{1.4}$$

where

$$[s]_e=\frac{1-e^s}{1-e}=\sum_{i=0}^{s-1}e^i, \tag{1.5}$$

and s called the fundamental number.

For a function $h(\xi) = \xi^s$, we obtain $\mathfrak{D}_q h(\xi) = \mathfrak{D}_q \xi^s = \frac{1-q^s}{1-q}\xi^{s-1} = [s]_q \xi^{s-1}$, and

$$\begin{aligned}\lim_{q\to 1^-\mathfrak{D}q} h(\xi) &= \lim_{q\to 1^-}\left([s]_q \xi^{s-1}\right)\\ &= s\xi^{s-1}\\ &= \mathfrak{f}'(\xi).\end{aligned}$$

For further study on the q-derivative operator $\mathfrak{D}_q$, (see [3]-[17]).

Throughout this paper we will suppose q to be a fixed number and $q \in (0, 1)$.

The operators associated with q-calculus, such as the fractional q-integral and fractional q-derivative operators, play a crucial role in the formation of various subclasses of analytic functions. . A function $\mathfrak{f} \in \mathbb{A}$ given by (1.1) is called q-starlike of order $\epsilon(0 \le \epsilon < 1)$ and denote by $\mathcal{S}_q^*(\epsilon)$ if and only if

$$\text{Re}\left\{\frac{\xi\mathfrak{D}_q\mathfrak{f}(\xi)}{\mathfrak{f}(\xi)}\right\} > \epsilon, \quad (\xi \in \mathfrak{U}). \tag{1.6}$$

Clearly, $\mathcal{S}_q^*(0) = \mathcal{S}_q^*$, the class of q- starlike defined by Ismail et. al. [18].

When $q \to 1^-$, then the class $\mathcal{S}_q^*(\epsilon)$ reduces to the usual class $\mathcal{S}^*(\epsilon)$ of starlike functions of order $\epsilon(0 \le \epsilon < 1)$ in $\mathfrak{U}$.

Also, a function $\mathfrak{f} \in \mathbb{A}$ given by (1.1) is called q-convex of order $\epsilon(0 \le \epsilon < 1)$ and denote by $\mathcal{K}_q(\epsilon)$ if and only if

$$\text{Re}\left\{\frac{\mathfrak{D}_q\mathfrak{f}(\xi) + \varpi\xi\mathfrak{D}'_q\mathfrak{f}(\xi)}{\mathfrak{D}_q\mathfrak{f}(\xi)}\right\} > \epsilon, \quad (\xi \in \mathfrak{U}). \tag{1.7}$$

Clearly, $\mathcal{K}_q(0) = \mathcal{K}_q$, the class of q- convex defined by Ahuja et. al. [3].

When $q \to 1^-$, then the class $\mathcal{K}_q(\epsilon)$ reduces to the usual class $\mathcal{K}(\epsilon)$ of convex functions of order $\epsilon(0 \le \epsilon < 1)$ in $\mathfrak{U}$.

An interesting generalization of the function classes $\mathcal{S}_q^*(\epsilon)$ and $\mathcal{K}_q(\epsilon)$ are provided by the following class:

Definition 1.1. A function $\mathfrak{f} \in \mathbb{A}$ given by (1.1) is in the class $\mathfrak{D}_q(\varpi, \epsilon)$, $\varpi \in [0, 1]$ if

$$\text{Re}\left\{\frac{\xi\mathfrak{D}_q\mathfrak{f}(\xi) + \varpi\xi^2\mathfrak{D}'_q\mathfrak{f}(\xi)}{\varpi\xi\mathfrak{D}_q\mathfrak{f}(\xi) + (1-\varpi)\mathfrak{f}(\xi)}\right\} > \epsilon, \qquad (\xi \in \mathfrak{U}).$$

Clearly, when $q \to 1$, we have $\mathfrak{D}_q(0, \epsilon) = \mathcal{S}_q^*(\epsilon)$ and $\mathfrak{D}_q(1, \epsilon) = \mathcal{K}_q(\epsilon)$.

Building upon the prior research by [19] (refer also to [20]-[27]), we derive conditions, incorporating coefficient inequalities, that are sufficient for functions to belong to $\mathfrak{D}_q(\varpi, \epsilon)$. We also consider various special cases of these coefficients.

The following lemmas are required to our primary result.

Lemma 1.2. *(see [19], [28]) A function $\varkappa(\xi) \in \mathbb{P}$ satisfies Re$\varkappa(\xi) > 0$ $(\xi \in \mathfrak{U})$ if and only if*

$$\varkappa(\xi) \ne \frac{\tau - 1}{\tau + 1} \qquad (\xi \in \mathfrak{U})$$

for all $|\tau| = 1$.

Lemma 1.3. *A function $\mathfrak{f}(\xi) \in \mathbb{A}$ is in $\mathfrak{D}_q(\varpi, \epsilon)$ if and only if*

$$1 + \sum_{s=2}^{\infty} E_s \xi^{s-1} \ne 0$$

where

$$E_s = \frac{((1+\varpi(s-2))[s]_q + \varpi - 1)\tau + (1+\varpi(s-2\epsilon))[s]_q + (1-\varpi)(1-2\epsilon)}{2(1-\epsilon)} a_s. \tag{1.8}$$

Proof. Applying Lemma 1.2, we have

$$\frac{\frac{\xi\mathfrak{D}_q\mathfrak{f}(\xi)+\varpi\xi^2\mathfrak{D}'_q\mathfrak{f}(\xi)}{\varpi\xi\mathfrak{D}_q\mathfrak{f}(\xi)+(1-\varpi)\mathfrak{f}(\xi)}-\epsilon}{1-\epsilon}\neq\frac{\tau-1}{\tau+1}\quad(\xi\in\mathfrak{U};\,\tau\in\mathbb{C};\;|\tau|=1). \tag{1.9}$$

From (1.9), it follows that

$$2(1-\epsilon)\xi+\sum_{s=2}^{\infty}\left[\left(\left(1+\varpi(s-2)\right)[s]_q+\varpi-1\right)\tau+\left(1+\varpi(s-2\epsilon)\right)[s]_q+(1-\varpi)(1-2\epsilon)\right]a_s\xi^s\neq 0$$

$$(\xi\in\mathfrak{U};\tau\in\mathbb{C};|\tau|=1)$$

or, equivalently

$$2(1-\epsilon)\xi \times\left(1+\sum_{s=2}^{\infty}\frac{\left[\left(\left(1+\varpi(s-2)\right)[s]_q+\varpi-1\right)\tau+\left(1+\varpi(s-2\epsilon)\right)[s]_q+(1-\varpi)(1-2\epsilon)\right]}{\left[(1-\varpi)\tau+2(1-\varpi\epsilon)\right]}a_s\xi^{s-1}\right)\neq 0. \tag{1.10}$$

Dividing equation (1.10) by $\left[(1-\varpi)\tau+2(1-\varpi\epsilon)\right]\xi\quad(\xi\neq 0)$, we obtain

$$1+\sum_{s=2}^{\infty}\frac{\left[\left(\left(1+\varpi(s-2)\right)[s]_q+\varpi-1\right)\tau+\varpi(s-2\epsilon)[s]_q+(1-\varpi)(1-2\epsilon)+1\right]}{2(1-\epsilon)}a_s\xi^{s-1}\neq 0$$

where

$$E_s=\frac{\left(\left(1+\varpi(s-2)\right)[s]_q+\varpi-1\right)\tau+\varpi(s-2\epsilon)[s]_q+(1-\varpi)(1-2\epsilon)+1}{2(1-\epsilon)}a_s.$$

□

Remark 1.4. The normalization conditions lead to the following:

$$a_0=0 \text{ and } a_1=1$$

that

$$E_0=\frac{(\varpi-1)\tau+(1-\varpi)(1-2\epsilon)+1}{2(1-\epsilon)}a_0=0,$$

and

$$E_1=\frac{2(1-\epsilon)}{2(1-\epsilon)}a_1=1.$$

2 Coefficient conditions for the class $\mathfrak{D}_q(\varpi,\epsilon)$

In this section, using Lemma 1.3, we have the following result.

Theorem 2.1. *If $\mathfrak{f}(\xi)\in\mathbb{A}$ satisfies the condition:*

$$\sum_{s=2}^{\infty}\left(\left|\sum_{\sigma=1}^{s}\left\{\sum_{u=1}^{\sigma}\left[\left(1+\varpi(u-2\epsilon)\right)[u]_q+(1-\varpi)(1-2\epsilon)\right](-1)^{\sigma-u}\binom{\varpi}{\sigma-u}a_u\right\}\binom{\delta}{s-\sigma}\right|\right.$$
$$\left.+\left|\sum_{\sigma=1}^{s}\left\{\sum_{u=1}^{\sigma}\left(\left(1+\varpi(u-2)\right)[u]_q+\varpi-1\right)(-1)^{\sigma-u}\binom{\varpi}{\sigma-u}a_u\right\}\binom{\delta}{s-\sigma}\right|\right)$$
$$\leq 2(1-\epsilon),$$

then $\mathfrak{f}(\xi)\in\mathfrak{D}_q(\varpi,\epsilon)$.

Proof. To prove that $1+\sum\limits_{s=2}^{\infty} E_s\xi^{s-1}\neq 0$, it is sufficient that

$$\left(1+\sum_{s=2}^{\infty} E_s\xi^{s-1}\right)(1-\xi)^{\varpi}(1+\xi)^{\delta}$$
$$=1+\sum_{s=2}^{\infty}\left[\sum_{\sigma=1}^{s}\left\{\sum_{u=1}^{\sigma} E_u(-1)^{\sigma-u}\binom{\varpi}{\sigma-u}\right\}\binom{\delta}{s-\sigma}\right]\xi^{s-1}\neq 0.$$

Thus, if $\mathfrak{f}(\xi)$ satisfies

$$\sum_{s=2}^{\infty}\left|\sum_{\sigma=1}^{s}\left\{\sum_{u=1}^{\sigma} E_u(-1)^{\sigma-u}\binom{\varpi}{\sigma-u}\right\}\binom{\delta}{s-\sigma}\right|\leq 1,$$

that is, if

$$\frac{1}{2(1-\epsilon)}\sum_{s=2}^{\infty}\left|\sum_{\sigma=1}^{s}\left\{\sum_{u=1}^{\sigma}\Psi(u,q,\epsilon)(-1)^{\sigma-u}\binom{\varpi}{\sigma-u}a_u\right\}\binom{\delta}{s-\sigma}\right|\leq 1.$$

where

$$\Psi(u,q,\epsilon)=((1+\varpi(u-2))[u]_q+\varpi-1)\tau+\varpi(u-2\epsilon)[u]_q+(1-\varpi)(1-2\epsilon)+1.$$

Now

$$\frac{1}{2(1-\epsilon)}\sum_{s=2}^{\infty}\left|\sum_{\sigma=1}^{s}\left\{\sum_{u=1}^{\sigma}\Psi(u,q,\epsilon)(-1)^{\sigma-u}\binom{\varpi}{\sigma-u}a_u\right\}\binom{\delta}{s-\sigma}\right|$$
$$\leq\frac{1}{2(1-\epsilon)}\sum_{s=2}^{\infty}\left(\left|\sum_{\sigma=1}^{s}\left\{\sum_{u=1}^{\sigma}(1+\varpi(u-2\epsilon)[u]_q+(1-\varpi)(1-2\epsilon))(-1)^{\sigma-u}\binom{\varpi}{\sigma-u}a_u\right\}\binom{\delta}{s-\sigma}\right|\right.$$
$$\left.+|\tau|\left|\sum_{\sigma=1}^{s}\left\{\sum_{u=1}^{\sigma}((1+\varpi(u-2))[u]_q+\varpi-1)(-1)^{\sigma-u}\binom{\varpi}{\sigma-u}a_u\right\}\binom{\delta}{s-\sigma}\right|\right)$$
$$\leq 1\qquad(\tau\in\mathbb{C};\ |\tau|=1),$$

then $\mathfrak{f}(\xi)\in\mathfrak{D}_q(\varpi,\epsilon)$ and so the proof is complete. □

If we let $\varpi=0$, then Theorem 2.1 gives the following corollary.

Corollary 2.2. *If* $\mathfrak{f}(\xi)\in\mathbb{A}$ *satisfies the condition:*

$$\sum_{s=2}^{\infty}\left(\left|\sum_{\sigma=1}^{s}\left\{\sum_{u=1}^{\sigma}([u]_q+1-2\epsilon)(-1)^{\sigma-u}\binom{\varpi}{\sigma-u}a_u\right\}\binom{\delta}{s-\sigma}\right|\right.$$
$$\left.+\left|\sum_{\sigma=1}^{s}\left\{\sum_{u=1}^{\sigma}([u]_q-1)(-1)^{\sigma-u}\binom{\varpi}{\sigma-u}a_u\right\}\binom{\delta}{s-\sigma}\right|\right)$$
$$\leq 2(1-\epsilon),$$

then $\mathfrak{f}(\xi)\in\mathcal{S}_q^*(\epsilon)$. *In particular,* for $\epsilon=0$, *if* $\mathfrak{f}(\xi)\in A$ *satisfies the condition:*

$$\sum_{s=2}^{\infty}\left(\left|\sum_{\sigma=1}^{s}\left\{\sum_{u=1}^{\sigma}([u]_q+1)(-1)^{\sigma-u}\binom{\varpi}{\sigma-u}a_u\right\}\binom{\delta}{s-\sigma}\right|\right.$$
$$\left.+\left|\sum_{\sigma=1}^{s}\left\{\sum_{u=1}^{\sigma}([u]_q-1)(-1)^{\sigma-u}\binom{\varpi}{\sigma-u}a_u\right\}\binom{\delta}{s-\sigma}\right|\right)$$
$$\leq 2,$$

then $\mathfrak{f}(\xi)$ *is* q*-starlike in* $\mathfrak{U}$.

If we let $\varpi=1$, then Theorem 2.1 gives the following corollary

Corollary 2.3. *If* $\mathfrak{f}(\xi) \in \mathbb{A}$ *satisfies the condition:*

$$\sum_{s=2}^{\infty}\left(\left|\sum_{\sigma=1}^{s}\left\{\sum_{u=1}^{\sigma}(u+1-2\epsilon)\,[u]_q(-1)^{\sigma-u}\binom{\varpi}{\sigma-u}a_u\right\}\binom{\delta}{s-\sigma}\right| + \left|\sum_{\sigma=1}^{s}\left\{\sum_{u=1}^{\sigma}(u-1)[u]_q(-1)^{\sigma-u}\binom{\varpi}{\sigma-u}a_u\right\}\binom{\delta}{s-\sigma}\right|\right) \le 2(1-\epsilon),$$

then $\mathfrak{f}(\xi) \in \mathcal{K}_q(\epsilon)$. *In particular,* for $\epsilon = 0$, if $\mathfrak{f}(\xi) \in \mathbb{A}$ *satisfies the following condition:*

$$\sum_{s=2}^{\infty}\left(\left|\sum_{\sigma=1}^{s}\left\{\sum_{u=1}^{\sigma}(u+1)[u]_q(-1)^{\sigma-u}\binom{\varpi}{\sigma-u}a_u\right\}\binom{\delta}{s-\sigma}\right| + \left|\sum_{\sigma=1}^{s}\left\{\sum_{u=1}^{\sigma}(u-1)[u]_q(-1)^{\sigma-u}\binom{\varpi}{\sigma-u}a_u\right\}\binom{\delta}{s-\sigma}\right|\right) \le 2,$$

then $\mathfrak{f}(\xi)$ *is* q*-convex in* $\mathfrak{U}$.

If we put $\varpi = \delta = 0$ in Theorem 2.1, we have the following corollary

Corollary 2.4. *If* $\mathfrak{f}(\xi) \in \mathbb{A}$ *satisfies the condition:*

$$\sum_{s=2}^{\infty}[1+\varpi(s-\epsilon-1)[s]_q - \epsilon(1-\varpi)]\,|a_s| \le 1-\epsilon,$$

then $\mathfrak{f}(\xi) \in \mathfrak{D}_q(\varpi, \epsilon)$.

Remark 2.5. (i) Letting $q \to 1^-$in Corollary 2.2 and Corollary 2.3, we get the sufficient conditions for the classes $\mathcal{S}^*(\epsilon)$ and $\mathcal{K}(\epsilon)$ obtained by Hayami et al.*[19].*

(ii) *Letting* $q \to 1^-$*and* $\varpi = 0$*,* $\varpi = 1$ *in Corollary 2.4, we get the following well-known coefficient conditions for the familiar classes* $\mathcal{S}^*$ *and* $\mathcal{K}$*, respectively.*

3 Conclusions

In this paper, we investigate a new class $\mathfrak{D}_q(\varpi, \epsilon)$ of analytic functions defined in the disk $\mathfrak{U} = \{\xi \in \mathbb{C} : |\xi| < 1\}$, which are associated with q-calculus operators. Further, we gave sufficient conditions involving coefficient inequalities for functions in this class. If we let $q \to 1^-$, we observe that the inequalities in Theorem 2.1 provide the sufficient conditions for the classes $\mathcal{S}^*(\epsilon)$ and $\mathcal{K}(\epsilon)$ due to Hayami et al.*[19].*

We also conclude by remarking that the class $\mathfrak{D}_q(\varpi, \epsilon)$ defined in Section 1 can be used to investigate some properties for functions in this class like, coefficient estimates, distortion theorems, etc.

References

[1] P. L. Duren, *Univalent Functions*, Grundlehren der Mathematischen Wissenschaften, 259, Springer-Verlag, New York (1983).

[2] H. Aldweby and M. Darus, Coefficient estimates of classes of q-starlike and q-convex functions, *Advanced Studies in Contemporary Mathematics*, vol. **26**(1) (2016) 21–26.

[3] O. P. Ahuja, A. Cetinkaya and Y. Polatoglu, Bieberbach-de Branges and Fekete-Szego inequalities for certain families of q-convex and q-close to convex functions, *J.Comput. Anal. Appl.*, **26**(4) (2019) 639–649.

[4] A. Amourah, B. Aref Frasin, T. Al-Hawary, Coefficient Estimates for a Subclass of Bi-univalent Functions Associated with Symmetric q-derivative Operator by Means of the Gegenbauer Polynomials, *Kyungpook Mathematical Journal,* 2022, 62 (2), 257-269.

[5] B. A. Frasin, M. Darus, Subclass of analytic functions defined by q-derivative operator associated with Pascal distribution series, *AIMS Mathematics*, 2021, 6(5): 5008-5019. doi: 10.3934/math.2021295.

[6] B. A. Frasin, N. Ravikumar and S. Latha, A subordination result and integral mean for a class of analytic functions defined by q-differintegral operator, *Ital. J. Pure Appl. Math.*, N. **45** (2021) 268–277.

[7] M. Govindaraj and S. Sivasubramanian, On a class of analytic functions related to conic domains involving q-calculus, *Analysis Mathematica*, **43**(3) (2017) 475–487.

[8] S. Kanas and D. Răducanu, Some subclass of analytic functions related to conic domains,*Math. Slovaca* **64**(5) (2014) 1183–1196.

[9] F. Müge Sakar, Muhammad Naeem, Shahid Khan and Saqib Hussain, Hankel determinant for class of analytic functions involving q-derivative operator, *J. Adv. Math. Stud.* **14**(2) (2021) 265-278.

[10] C. Ramachandran, S. Annamalai and B.A. Frasin, The q-difference operator associated with the multivalent function bounded by conical sections, *Bol. Soc. Paran. Mat.*, **39**(1) (2021) 133–146.

[11] T. M. Seoudy and M. K. Aouf, Coefficient estimates of new classes of q-starlike and q-convex functions of complex order, *Journal of Mathematical Inequalities*, **10**(1) (2007) 135–145.

[12] T. M. Seoudy and M. K. Aouf, Convolution properties for certain classes of analytic functions defined by q-derivative operator, *Abstract and Applied Analysis*, vol. 2014, Article ID 846719, 7 pages, 2014.

[13] H. Shamsan, S. Latha and B.A. Frasin, Convolution conditions for q-Sakaguchi-Janowski type functions, *Ital. J. Pure Appl. Math.*, N. 44-2020, pp. 521–529.

[14] H. M. Srivastava, Operators of basic (or q-) calculus and fractional q-calculus and their applications in geometric function theory of complex analysis, *Iran. J. Sci. Technol. Trans. A: Sci.*, **44** (2020) 327–344.

[15] H. M. Srivastava, Some generalizations and basic (or q-) extensions of the Bernoulli, Euler and Genocchi polynomials,*Appl. Math. Inf. Sci.* **5**(34) (2011) 390-444.

[16] A. Alsoboh, A. Amourah, M. Darus and C. A. Rudder, Investigating New Subclasses of Bi-Univalent Functions Associated with q-Pascal Distribution Series Using the Subordination Principle, *Symmetry*, **15. 5**(2023) 1109.

[17] A. Amourah, A. Alsoboh, O. Ogilat, G. M. Gharib, R. Saadeh and M. Al Soudi, A generalization of Gegenbauer polynomials and bi-univalent functions, *Axioms.* **12**(2) (2023) 128.

[18] M. E. H. Ismail, E. Merkes, D. Steyr, A generalization of starlike functions, *Complex Variables Theory Appl.* **14** (1990) 77-84.

[19] T. Hayami, S. Owa and H.M. Srivastava, Coefficient inequalities for certain classes of analytic and univalent functions, *J. Ineq. Pure Appl. Math.*Vol. **8** (2007), Issue 4, Article 95, 1-10.

[20] B. Frasin, M. Darus, T. Al-Hawary, Coefficient inequalities for certain classes of analytic functions associated with the Wright generalized hypergeometric function, *Journal of Mathematics and Applications*, 2011, 34, 27-34.

[21] B.A. Frasin, Coefficient inequalities for certain classes of Sakaguchi type functions, *International Journal of Nonlinear Science*, 1749-3889, **10**(2) (2010) 206-211.

[22] B.A. Frasin, Coefficient inequalities for certain classes of analytic functions of complex order,*Math. Vesnik* **63** (1) (2011) 73–78.

[23] T. Hayami and S. Owa, Coefficient conditions for certain univalent functions, Int. {*J. Open Problems Comput. Sci. Math.*, **1**(1) (2008).

[24] S. Latha, Certain class of analytic and univalent functions involving the Ruscheweyh derivative operator, *Journal of Math. Analysis*, Vol. **3**(33) 2009 1633 - 1644.

[25] S. Latha, Coefficient inequalities and convolution conditions, *Int. J. Contemp. Math. Sciences*, **3**(30) (2008) 1461 - 1467.

[26] F. Müge Sakar and H. Özlem Güney, Coefficient Bounds for Certain Subclasses of m-fold Symmetric Bi-univalent Functions Defined by the Q-derivative Operator, *Konuralp Journal of Mathematics*, **6**(2) (2018) 279-285.

[27] F.M. Sakar, S.M. Aydogan, Coefficient Bounds for Certain Subclasses of m-fold Symmetric Bi-univalent Functions Defined by Convolution, *Acta Universitatis Apulensis*, 55, 11-21., Doi: doi: 10.17114/j.aua.2018.55.02, 2018).

[28] H. Silverman, E. M. Silvia, and D. Telage, Convolution conditions for convexity, starlikeness and spirallikeness, *Math. Z.*, **162** (1978) 125-130.

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