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A QUALITATIVE RESULT FOR A NONLINEAR FRACTIONAL DIFFERENTIAL SYSTEM WITH VARIABLE DELAY

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Abstract This article deals with the problem of asymptotic stability of a fractional mathematical model as a nonlinear fractional differential system with a variable delay. Sufficient conditions of the asymptotic stability are obtained in relation to the system with a variable time delay using a suitable Lyapunov-Krasovskiĭ functional (LKF). The result of this article has new contributions to the existing ones in the literature. We also present an example as the numerical application of the result.

1 Introduction

In the past years in mathematics and engineering sciences, stability, asymptotic stability, exponential stability etc. of solutions of fractional differential equations, fractional integro-differential equations, etc., have been studied. Many types of qualitative behaviors have been studied by many researchers and many important and remarkable results have been obtained on the subject, see, Abbas et al. [1], Alkhazzan et al. [2], Bachir et al. [3], Bohner et al. [4], Brandibur and Kaslik [5], Duarte-Mermoud et al. [6], Liu et al. [10], Liu et al. [11], Podlubny [12], Raza et al. [13], Tan [14], Tunç [15], Tunç [16], Tunç and Tunç [17], Tunç et al. [18], Zada et al. [19], Zhou et al. [20] and the references included in these sources.

In this research paper, the asymptotic stability of the zero solution of a certain nonlinear Riemann-Liouville fractional differential system (RLFrDS) will be investigated with the help of the LKF method.

As for the main motivation of this work, Liu et al. [10] investigated the asymptotic stability of zero solution of the following nonlinear RLFrDS:

$$ {}_{t_0}D_t^q x(t) = Ax(t) + Bx(t-\tau(t)) + F_1(x(t)) + F_2(x(t-\tau(t))), \tag{1.1}$$

where $q \in (0,1)$, $x \in \mathbb{R}^n$ is the state vector, $A,\ B \in \mathbb{R}^{n\times n}$, $n \in \mathbb{N}$, $\tau \in C^1[\mathbb{R}^+,\ (0,\infty)]$ such that $\tau'(t) \le d < 1$. Additionally, $F_i \in \mathbb{R}^n$ are continuous and $F_i(0) = 0$, $i = 1,\ 2$, such that

$$\lim_{\|x\|\to 0} \frac{\|F_i(x)\|}{\|x\|} = 0.$$

Liu et al. [10, Theorem 4.1] proved the following theorem.

Theorem 1.1. *The trivial solution of the RLFrDS (1.1) is asymptotically stable if there exist two symmetric and positive definite matrices P and Q such that the following estimates hold, simultaneously:*

$$PA + A^TP + 2Q = 0,$$
$$\|PB\| \le \lambda_{min}(Q)\sqrt{1-d}.$$

In this paper, taking into account the RLFrDS (1.1) and the result of Liu et al. [10, Theorem 4.1], we will take into account the following RLFrDS with a variable delay

$$\begin{aligned} {}_{t_0}D_t^q x(t) =& Ax(t) + BF_0(x) + F(t,x(t)) + M\Phi(x(t-\tau(t))) \\ &+ G(x(t-\tau(t))) + H(t,x(t),x(t-\tau(t))), \end{aligned} \tag{1.2}$$

where $q \in (0,1)$, $t \geq t_0$, $x \in \mathbb{R}^n$ *is the state vector,* $A,\ B,\ M \in \mathbb{R}^{nxn}$ *and* $\tau \in C^1[\mathbb{R}^+,\ (0,\infty)]$ *is the variable delay.*

In the entire article, we need the conditions below:

(A1)

$$F_0,\ \Phi,\ G \in C[\mathbb{R}^n, \mathbb{R}^n],$$

$$F \in C[[0,\infty) \times \mathbb{R}^n, \mathbb{R}^n], H \in C[[0,\infty) \times \mathbb{R}^n \times \mathbb{R}^n, \mathbb{R}^n],$$

$$F_0(0) = 0, F(t,0) = 0, \Phi(0) = 0, G(0) = 0, H(t,0,0) = 0,$$

$$\tau'(t) \leq d < 1,\ d \in \mathbb{R},\ d > 0.$$

(A2) The functions F_0,F,Φ, G *and* H *satisfy the following relations:*

$$\lim_{\|x\|\to 0} \frac{\|F_0\,(x)\|}{\|x\|} = 0,\ \lim_{\|x\|\to 0} \frac{\|F\,(t,x)\|}{\|x\|} = 0,\ \lim_{\|x\|\to 0} \frac{\|\Phi\,(x)\|}{\|x\|} = 0,$$

$$\lim_{\|x\|\to 0} \frac{\|G\,(x)\|}{\|x\|} = 0,\ \lim_{\|x\|\to 0} \frac{\|H\,(t,x)\|}{\|x\|} = 0.$$

In this paper, we benefit from the below notations: $\mathbb{R}^n$ *represents n-dimensional Euclidean space,* $\mathbb{R}^{n\times n}$ *is the set of all* $n\times n$ *-dimensional real matrices,* K *is a symmetric matrix and* K^T *denotes transpose of this matrix. If the matrix* K *is positive definite, then* $\langle Kx, x\rangle > 0$,$x \neq 0$. $\|.\|$ *represents the Euclidean norm,* $\|K\| = \sqrt{\lambda_{\max}(K^T K)}$ *denotes the spectral norm of this matrix,* $\lambda_{\max}(K\ )$ *and* $\lambda_{\min}(K)$ *denote the maximum and minimum values of the eigenvalues of the matrix* K*, respectively.*

2 Basic results

We will give some lemmas as basic results which are used advances.

Lemma 2.1. *(Kilbas et al. [8]). If* $p > q > 0$*, then*

$${}_{t_0}D_t^q({}_{t_0}D_t^{-p}x(t)) = {}_{t_0}D_t^{q-p}x(t)$$

holds for "sufficiently good" functions $x(t)$*. In particular, this relation holds if* $x(t)$ *is integrable.*

Lemma 2.2. *(Duarte-Mermoud et al. [6]). Let* $x \in \mathbb{R}^n$ *be a vector of differentiable functions. Then, for any time instant* $t \geq t_0$*, the following inequality holds:*

$$\frac{1}{2}{}_{t_0}D_t^q(x^T(t)Px(t)) \leq x^T(t)P{}_{t_0}D_t^q x(t),\ \ \forall q \in (0,1),$$

where $P \in \mathbb{R}^{n\times n}$ *is a symmetric and positive-definite matrix.*

Lemma 2.3. *(Liu et al. [9]). For any* $x,\ y \in \mathbb{R}^n$ *and* $\varepsilon > 0$*, the following inequality holds:*

$$2x^T y \leq \varepsilon x^T x + \frac{1}{\varepsilon} y^T y\ .$$

Lemma 2.4. *(Tan [14]). Let* $U > 0$ *and* $V \geq 0$ *be real symmetric matrices and* η *be a positive number. Then*

$$\eta U > V \Leftrightarrow \lambda_{max}(VU^{-1}) < \eta \Leftrightarrow \lambda_{max}(U^{-\frac{1}{2}}VU^{-\frac{1}{2}}) < \eta.$$

3 Stability result

The main stability result of this paper is presented in the bellow theorem.

Theorem 3.1. *The trivial solution of the RLFrDS (1.2) with a variable delay is asymptotically stable if there exist symmetric and positive definite matrices P and Q such that the bellow relations fulfill:*

$$PA + A^T P + 3Q = 0, \tag{3.1}$$

$$\frac{1}{\lambda_{min}(Q)} \frac{1}{\sqrt{1-d}} \|PM\| < 1 \ \text{ and } \frac{1}{\lambda_{min}(Q)} \frac{1}{\sqrt{1-d}} \|PB\| < 1. \tag{3.2}$$

Proof. To prove this theorem, we define an LKF by

$$V(t) = {}_{t_0}D_t^{q-1}(x^T(t)Px(t)) + \int\limits_{t-\tau(t)}^{t} x^T(s)Qx(s)ds.$$

It is obvious that this LKF is positive definite since the matrices P and Q are positive definite. Next, fulfilling the time derivative of the LKF $V(t)$ along the solutions of the RLFrDS (1.2) and using Lemma 2.2, we find

$$\begin{aligned}
\frac{d}{dt}V(t) =& {}_{t_0}D_t^q(x^T(t)Px(t)) + x^T(t)Qx(t) \\
& - (1-\tau'(t))x^T(t-\tau(t))Qx(t-\tau(t)) \\
\le& 2x^T(t)P\,{}_{t_0}D_t^q x(t) + x^T(t)Qx(t) \\
& - (1-d)x^T(t-\tau(t))Qx(t-\tau(t)) \\
=& 2x^T(t)P\left[Ax(t) + BF_0(x) + F(t,x(t)) + M\Phi(x(t-\tau(t)))\right. \\
& \left. + G(x(t-\tau(t))) + H(t,x(t),x(t-\tau(t)))\right] \\
& + x^T(t)Qx(t) \ - (1-d)x^T(t-\tau(t))Qx(t-\tau(t)) \\
=& x^T(t)\left[PA + A^TP + 3Q\right]x(t) - x^T(t)\,(2Q)\,x(t) \\
& + 2x^T(t)PBF_0(x(t)) + 2x^T(t)PF(t,x(t)) \\
& + 2x^T(t)PM\Phi(x(t-\tau(t)) + 2x^T(t)PG(x(t-\tau(t))) \\
& + 2x^T(t)PH(t,x(t),x(t-\tau(t))) \\
& - (1-d)x^T(t-\tau(t))Qx(t-\tau(t)).
\end{aligned} \tag{3.3}$$

According to the inequality $2x^T y \le \varepsilon x^T x + \frac{1}{\varepsilon} y^T y$, (see, Lemma 2.3), we get the following inequalities, respectively:

$$\begin{aligned}
2x^T(t)PBF_0(x(t)) =& 2x^T(t)PBQ^{-\frac{1}{2}}Q^{\frac{1}{2}}F_0(x(t)) \\
\le& \frac{1}{m(1-d)}x^T(t)\left(PBQ^{-1}B^TP^T\right)x(t) \\
& + m(1-d)\left({F_0}^T(x(t))QF_0(x(t))\right); \\
2x^T(t)PF(t,x(t)) \le& \frac{1}{\beta}x^T(t)P^2x(t) + \beta F^T(t,x(t))F(t,x(t)); \\
2x^T(t)PM\Phi(x(t-\tau(t)) =& 2x^T(t)PMQ^{-\frac{1}{2}}Q^{\frac{1}{2}}\Phi(x(t-\tau(t))) \\
\le& \frac{1}{n(1-d)}x^T(t)PMQ^{-1}M^TP^Tx(t) \\
& + n(1-d)\Phi^T(x(t-\tau(t)))Q\Phi(x(t-\tau(t))); \\
2x^T(t)PG(x(t-\tau(t))) \le& \frac{1}{g}x^T(t)P^2x(t) + gG^T(x(t-\tau(t)))G(x(t-\tau(t)));
\end{aligned}$$

$$
\begin{aligned}
2x^T(t)PH(t,x(t),x(t-\tau(t))) \le & \frac{1}{h}x^T(t)P^2x(t) \\
& + hH^T(t,x(t),x(t-\tau(t)))H(t,x(t),x(t-\tau(t))),
\end{aligned}
$$

where $\beta,\ g,\ h,\ m,\ n$ are some positive constants.

Hence, substituting the inequalities above into (3.3), we obtain

$$
\begin{aligned}
\dot{V}(t) \le & x^T(t)\left[\frac{1}{m\,(1-d)}PBQ^{-1}B^TP^T\right. \\
& \left. + \frac{1}{n\,(1-d)}PMQ^{-1}M^TP^T - 2Q + \left(\frac{1}{\beta}+\frac{1}{g}+\frac{1}{h}\right)P^2\right]x(t) \\
& + m(1-d){F_0}^T(x(t))QF_0(x(t)) \ + \beta F^T(t,x(t))F(t,x(t)) \\
& + n(1-d)\Phi^T(x(t-\tau(t)))Q\Phi(x(t-\tau(t))) \\
& + gG^T(x(t-\tau(t)))G(x(t-\tau(t))) \\
& + hH^T(t,x(t),x(t-\tau(t)))H(t,x(t),x(t-\tau(t))) \\
& - \ (1-d)x^T(t-\tau(t))Qx(t-\tau(t)).
\end{aligned}
\tag{3.4}
$$

According to the description of the spectral norm, we get

$$
\begin{aligned}
& \left[\lambda_{max}\left(Q^{-\frac{1}{2}}P\left(\frac{1}{1-d}BQ^{-1}B^T\right)P^TQ^{-\frac{1}{2}}\right)\right]^{\frac{1}{2}} \\
= & \left[\frac{1}{1-d}\lambda_{max}\left(Q^{-\frac{1}{2}}PBQ^{-1}B^TP^TQ^{-\frac{1}{2}}\right)\right]^{\frac{1}{2}} \\
= & \left[\frac{1}{1-d}\lambda_{max}\left\|Q^{-\frac{1}{2}}PBQ^{-\ \frac{1}{2}}\right\|^2\right]^{\frac{1}{2}} \\
\le & \frac{1}{\sqrt{1-d}}\left\|Q^{-\frac{1}{2}}\right\|^2\|PB\| \\
\le & \frac{1}{\sqrt{1-d}}\left\|Q^{-\frac{1}{2}}\right\|^2\|PB\| \\
= & \frac{1}{\lambda_{min}\,(Q)}\frac{1}{\sqrt{1-d}}\|PB\|\,.
\end{aligned}
$$

By the same way, doing similar calculations as the above, we derive

$$
\begin{aligned}
& \left[\lambda_{max}\left(Q^{-\ \frac{1}{2}}P\left(\frac{1}{1-d}MQ^{-1}M^T\right)P^TQ^{-\ \frac{1}{2}}\right)\right]^{\frac{1}{2}} \\
= & \left[\frac{1}{1-d}\lambda_{max}\left(Q^{-\frac{1}{2}}PMQ^{-1}M^TP^TQ^{-\frac{1}{2}}\right)\right]^{\frac{1}{2}} \\
= & \left[\frac{1}{1-d}\left\|Q^{-\frac{1}{2}}PMQ^{-\frac{1}{2}}\right\|^2\right]^{\frac{1}{2}} \\
\le & \frac{1}{\sqrt{(1-d)}}\left\|Q^{-\frac{1}{2}}\right\|^2\|PM\| \\
= & \frac{1}{\lambda_{min}\,(Q)}\frac{1}{\sqrt{(1-d)}}\|PM\|\,.
\end{aligned}
$$

Hence, using the estimate (3.2), we have

$$
\left[\lambda_{max}\left(Q^{-\frac{1}{2}}P\left(\frac{1}{1-d}BQ^{-1}B^T\right)P^TQ^{-\frac{1}{2}}\right)\right]^{\frac{1}{2}} < 1. \tag{3.5}
$$

Next, it follows from (3.5) that there exists a $\mu > 0$, $\mu \in \mathbb{R}$, such that

$$\left[\lambda_{max}\left(Q^{-\frac{1}{2}}P\left(\frac{1}{1-d}BQ^{-1}B^T\right)P^TQ^{-\frac{1}{2}}\right)\right]^{\frac{1}{2}} < \mu < 1.$$

Since $P > 0$ and$BQ^{-1}B^T \geq 0,$ then it follows from Lemma 2.4 that

$$P\left(\frac{1}{1-d}BQ^{-1}B^T\right)P^T < \mu Q\ .$$

Choose a $\mu \in \mathbb{R}$, $0 < m < 1$, such that $0 < \frac{\mu}{m} < 1$. Then, we have,

$$P\left(\frac{1}{m(1-d)}BQ^{-1}B^T\right)P^T - \frac{\mu}{m}Q < 0$$

and

$$P\left(\frac{1}{m(1-d)}BQ^{-1}B^T\right)P^T - Q < \left(\frac{\mu}{m} - 1\right)Q < 0.$$

By a similar way, we get

$$\left[\lambda_{max}\left(Q^{-\frac{1}{2}}P\left(\frac{1}{1-d}MQ^{-1}M^T\right)P^TQ^{-\frac{1}{2}}\right)\right]^{\frac{1}{2}} < 1. \tag{3.6}$$

From (3.6), we arrive that there is a $\mu > 0$, $\mu \in \mathbb{R}$,such that

$$\lambda_{max}\left(Q^{-\frac{1}{2}}P\left(\frac{1}{1-d}MQ^{-1}M^T\right)P^TQ^{-\frac{1}{2}}\right) < \mu < 1.$$

Since $P > 0$ and $MQ^{-1}M^T \geq 0,$ then it follows from Lemma 2.4 that

$$P\frac{1}{1-d}MQ^{-1}M^TP^T < \mu Q.$$

Choose $n < 1$ such that $0 < \frac{\mu}{n} < 1$. Then, we have

$$\frac{1}{n(1-d)}PMQ^{-1}M^TP^T - \frac{\mu}{n}Q < 0$$

and

$$\frac{1}{n(1-d)}PMQ^{-1}M^TP^T - Q < \left(\frac{\mu}{n} - 1\right)Q < 0. \tag{3.7}$$

We note that $m,\ n < 1$. Next, appropriate positive numbers $\beta,\ g,\ h$ can be chosen so that the following relation fulfills:

$$\begin{aligned} K_0 = \Bigg[&\frac{1}{m(1-d)}PBQ^{-1}B^TP^T \\ &+\frac{1}{n(1-d)}PMQ^{-1}M^TP^T - 2Q + \left(\frac{1}{\beta} + \frac{1}{g} + \frac{1}{h}\right)P^2\Bigg] < 0. \end{aligned}$$

Let $g + h + n < 1$. Hence, since $g + h + n < 1,\ 0 < d < 1$ and the matrix Q is positive, then it is clear that

$$K_1 = (g + h + n - 1)(1 - d)Q < 0.$$

Thus, by virtue of the discussion above, we arrive

$$\begin{aligned} \dot{V}(t) \leq& x^T(t)K_0x(t) + m(1-d){F_0}^T(x(t))QF_0(x(t)) + \beta F^T(t, x(t))F(t, x(t)) \\ &+ n(1-d)\boldsymbol{\Phi}^T(x(t-\tau(t)))Q\boldsymbol{\Phi}(x(t-\tau(t))) \\ &+ gG^T(x(t-\tau(t)))G(x(t-\tau(t))) \\ &+ hH^T(t, x(t), x(t-\tau(t)))H(t, x(t), x(t-\tau(t))) \\ &- \ (1-d)x^T(t-\tau(t))Qx(t-\tau(t)) \\ \leq& x^T(t)K_0x(t) + \beta\|F(t, x(t))\|^2 + m(1-d)\left\|{F_0}^T(x(t))QF_0(x(t))\right\|^2 \\ &+ n(1-d)\left\|\boldsymbol{\Phi}^T(x(t-\tau(t)))Q\boldsymbol{\Phi}(x(t-\tau(t)))\right\|^2 + g\|G(x(t-\tau(t)))\|^2 \end{aligned}$$

$$
\begin{aligned}
&+ h\|H(t,x(t),x(t-\tau(t)))\|^2 - \ (1-d)\left\|x^T(t-\tau(t))Qx(t-\tau(t))\right\|^2 \\
\leq& x^T(t)K_0x(t) + \beta\|F(t,x(t))\|^2 + m(1-d)\left\|{F_0}^T(x(t))QF_0(x(t))\right\|^2 \\
&+ (n+g+h-1)(1-d)\left\|x^T(t-\tau(t))Qx(t-\tau(t))\right\|^2. \qquad (3.8)
\end{aligned}
$$

According to (A2), for the positive constants $\beta,\ g,\ h,\ \ m,\ \ n,$ there is an available constant $\theta > 0$ such that when $\|x(t)\| < \theta,\ t \geq t_0$,and $K_0 + \theta I < 0$, where I is a suitable identity matrix, the following inequalities can be fulfilled:

$$\|F(t,x(t))\|^2 \leq \frac{\theta}{\beta}\|x(t)\|^2;$$

$$\left\|{F_0}^T(x(t))QF_0(x(t))\right\|^2 \leq \frac{\theta}{m\,(1-d)}\|x(t)\|^2;$$

$$\|G(x(t-\tau(t)))\|^2 \leq \frac{\theta}{g}\|x(t-\tau(t))\|^2;$$

$$\|H(t,x(t),x(t-\tau(t)))\|^2 \leq \frac{\theta}{h}\|x(t-\tau(t))\|^2;$$

$$\left\|\Phi^T(x(t-\tau(t)))Q\Phi(x(t-\tau(t)))\right\|^2 \leq \frac{\theta}{n\,(1-d)}\|x(t-\tau(t))\|^2.$$

By virtue of the inequalities above and (3.8), we obtain

$$\dot{V}(t) \leq x^T(t)(K_0+\theta I)x(t) + x^T(t-\tau(t))K_1x(t-\tau(t)).$$

In view of $K_1 = (g+h+n-1)(1-d)Q < 0$ and $K_0 + \theta I < 0$, we can derive that $\dot{V}(t)$ is negative definite. Therefore, the zero solution of the nonlinear RLFrDS (1.2) with a variable delay is asymptotically stable. This result completes the proof of Theorem 3.1. □

4 Numerical result

In Section "4. Numerical result", we provide Example 4.1, which is included by RLFrDS (1.2), for an application of Theorem 3.1.

Example 4.1. In a specific subcase of the RLFrDS (1.2) with a variable delay, we consider the following nonlinear RLFrDS with a variable retardation:

$$
\begin{aligned}
{}_{t_0}D_t^q x(t) =& Ax(t) + BF_0(x) + F(t,x(t)) + M\Phi(x(t-\tau(t))) \\
&+ G(x(t-\tau(t))) + H(t,x(t),x(t-\tau(t))), \qquad (4.1)
\end{aligned}
$$

where

$$q \in (0,1)\ ,\ x(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix},$$

$$\tau(t) = 0.3t,\ d = 0.3,$$

$$F_0(x(t)) = \begin{bmatrix} x_1(t)\exp(-{x_1}^2(t)) \\ x_2(t)\exp(-{x_2}^2(t)) \end{bmatrix},$$

$$F(t,x(t)) = \begin{bmatrix} x_1(t)\exp(-{x_1}^2(t)) \\ x_2(t)\exp(-{x_2}^2(t)) \end{bmatrix},$$

$$G(x(t-\tau(t))) = \begin{bmatrix} x(t-\tau(t))\exp(-{x_1}^2(t-\tau(t))) \\ x(t-\tau(t))\exp(-{x_2}^2(t-\tau(t))) \end{bmatrix},$$

$$\Phi(x(t-\tau(t))) = 0,4\begin{bmatrix} \sin(x_1(t-\tau(t)))\exp(-{x_2}^2(t-\tau(t))) \\ \sin(x_2(t-\tau(t)))\exp(-{x_1}^2(t-\tau(t))) \end{bmatrix},$$

$$H(t, x(t), x(t-\tau(t))) = \begin{bmatrix} \sin(x_1(t-\tau(t)))\exp(-{x_1}^2(t-\tau(t))) \\ \sin(x_2(t-\tau(t)))\exp(-{x_2}^2(t-\tau(t))) \end{bmatrix},$$

$$A = \begin{pmatrix} -4 & 0 \\ 0 & -2 \end{pmatrix}, B = \begin{pmatrix} 0.001 & 0.1 \\ 0 & -0,1 \end{pmatrix}, M = \begin{pmatrix} 0,008 & 0,002 \\ 0,01 & 0,04 \end{pmatrix}.$$

Let

$$P = \begin{pmatrix} 6 & 0 \\ 0 & 4 \end{pmatrix}, \; Q = \begin{pmatrix} 16 & 0 \\ 0 & 16/3 \end{pmatrix}.$$

Hence, we can obtain the following data:

$$PA + A^T P + 3Q = 0,$$

$$\frac{\|PM\|}{\sqrt{1-d}} = 0.1920, \lambda_{\min}(Q) = 16/3,$$

$$\lambda_{min}(Q) - \frac{1}{\sqrt{1-d}}\|PM\| = 5.1413 > 0,$$

$$\frac{\|PB\|}{\sqrt{1-d}} = 0.8619, \lambda_{\min}(Q) = 16/3,$$

$$\lambda_{min}(Q) - \frac{1}{\sqrt{1-d}}\|PB\| = 4.4714 > 0.$$

Thus, the conditions (3.1), (3.2) and the others are satisfied. Hence, the trivial solution of nonlinear RLFrDS (4.1) with a variable delay is asymptotically stable.

5 Conclusion

In this article, we take into consideration an RLFrDS. The considered RLFrDS is a different Riemann-Liouville fractional differential mathematical model than those in the present literature. Here, a new theorem, which includes sufficient conditions, is established on asymptotic stability of this RLFrDS. The technique called LKF method is utilized to prove that stability result. We aim to have new contributions to the qualitative topic of the RLFrDS.

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