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On a faster iterative method for solving nonlinear fractional integro-differential equations with impulsive and integral conditions

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Abstract In this article, we find the solution of a nonlinear fractional integro-differential equations with impulsive and integral conditions via an efficient method. The method of solution in this article involves the concept of fixed point theory in Banach spaces.

1 Introduction

A mapping g on a nonempty subset $\mathcal{K}$ of a Banach $\mathscr{G}$ is called contraction if there exists a constant $\delta \in [0, 1)$ such that

$$\|g(x) - g(y)\| \leq \delta\|x - y\|, \text{ for all } x, y \in \mathcal{K}. \tag{1.1}$$

A point x in $\mathcal{K}$ is said to be a fixed point of g if $g(x) = x$. We denote the set of all fixed points of g by $\mathcal{F}(g) = \{x \in \mathcal{K} : x = g(x)\}$. Let $\Re$ denote the set of all real numbers and $\mathbb{N}$ be the set of all natural numbers.

In this paper, we will study the the approximation of solution of the following fractional integral integro-differential equations with impulsive and integral conditions given by

$$\begin{aligned} {}^{\mathrm{C}}\mathcal{D}^{\nu}x(t) &= f(t, x(t), \int_0^t k(t, s, x(s))ds), \, t \in \mho' = \mho\backslash\{t_1, \cdots, t_m\}, & (1.2)\\ & \quad \mho = [0, 1], & \\ x(t_k^+) &= x(t_k^-) + x_k, \;\; x_k \in \Re, & (1.3)\\ x(0) &= \int_0^1 \Bbbk(s)x(s)ds, & (1.4) \end{aligned}$$

where $k = 1, \cdots, m$, $0 < \nu \leq 1$, ${}^{\mathrm{C}}\mathcal{D}^{\nu}$ is the Caputo fractional derivative, $\mathscr{G}$ denotes a Banach space, $f : \mho \times \mathscr{G} \times \mathscr{G} \to \mathscr{G}$ is a given function, $k : J \times \mathscr{G} \to \mathscr{G}$, $\Bbbk \in L^1([0, 1], \Re_+)$, $\Bbbk(t) \in [0, 1)$ and $J = \{(t, s) : 0 \leq s \leq t \leq 1\}$, $0 = t_0 < t_1 < \cdots < t_m < t_{m+1} = 1$, $Jx|_{t=t_k} = x(t_k^+) - x(t_k^-)$, $x(t_k^+) = \lim_{h \to 0^+} x(t_k + h)$ and $x(t_k^-) = \lim_{h \to 0^-} x(t_k + h)$ represent the right and left limits of $x(t)$ at $t = t_k$. We set

$$\psi x(t) = \int_0^t k(t, s, x(s))ds.$$

Fractional differential equations arise in many engineering and scientific disciplines as the mathematical modeling of systems and processes in the fields of physics, chemistry, aerodynamics, viscoelasticity, electro-chemistry, signal processing, control theory, porous media, electromagnetic and so forth (see [1, 2, 3, 7, 8, 14, 9, 10, 36] and the references therein). There has been a significant theoretical development in fractional differential equations in recent years (see [5, 13, 15, 28, 30, 31, 32, 33] and the references therein).

The dynamics of evolving processes is often subjected to abrupt changes such as shocks, harvesting, or natural disasters, and so on. Often these short-term perturbations are treated as having acted instantaneously or in the form of impulses. The conditions (1.3) include such a kind of dynamics. The classical impulsive differential equations have become important in recent years as mathematical models of phenomena in physical, engineering and biomedical sciences (see [4] and the references therein).

The following definition and lemmas will be important in this study.

Definition 1.1. The Caputo derivative of fractional order ν for a function $f(t)$ is defined by

$$({}^{\mathrm{C}}\mathfrak{D}^{\nu} f)(t) = \frac{1}{\Gamma(n-\nu)} \int_0^t \frac{f^{(n)}(s)}{(t-s)^{\nu-n+1}} ds, \tag{1.5}$$

where $n = [\nu] + 1$ and $[\nu]$ denotes the integer part of ν.

Lemma 1.2. *[4] If $\Omega(\tau) = \int_\tau^1 \mathbb{k}(s)(s-\tau)^{\nu-1} ds$, for $\tau \in [0,1]$, and if $\mathbb{k} \in L^1([0,1], \Re_+)$ satisfies $0 \le \mathbb{k}(s) \le 1$ $(0 \le s \le 1)$, then*

$$\frac{\Omega(\tau)}{\Gamma(\nu)} < \eta \quad \textit{and} \quad \frac{\int_0^t (t-s)^{\nu-1}}{\Gamma(\nu)} < \eta.$$

Lemma 1.3. *[35] Let $\{\theta_n\}$ be a nonnegative real sequence satisfying the following inequality:*

$$\theta_{n+1} \le (1-\sigma_n)\theta_n,$$

where $\sigma_n \in (0,1)$ for all $n \in \mathbb{N}$ and $\sum_{n=0}^{\infty} \sigma_n = \infty$, then $\lim_{n\to\infty} \theta_n = 0$.

Consider the set of functions

$$\begin{aligned} PC(\mho, \mathscr{G}) \;=\; & \{y : \mho \to \mathscr{G} : y \in C((t_k, t_{k+1}], \mathscr{G}), k = 0, \cdots, m \text{ and there exist} \\ & y(t_k^-) \text{ and } y(t_k^+),\ k = 1, \cdots, m \text{ with } y(t_k^-) = y(t_k)\}. \end{aligned}$$

The set defined above is a Banach when endowed with the norm

$$\|y\|_{PC} = \sup_{t\in I} |y(t)|.$$

A function $x \in PC(\mho, Y)$ whose ν-derivative exits on $\mho'$ is said to be solution of the problem (1.2)–(1.4) if x satisfies the equation ${}^{\mathrm{C}}\mathfrak{D}^{\nu} x(t) = f(t, x(t), \psi x(t))$ a.e. on $\mho'$, and satisfies the conditions (1.3)–(1.4).

The problem (1.2)–(1.4) can be transform into the following integral equation [4]:

$$\begin{aligned} x(t) \;=\; & \frac{1}{(1-\mu)\Gamma(\nu)} \int_0^1 \Omega(\tau) f(\tau, x(\tau), \psi x(\tau), \psi x(\tau)) d\tau \\ & + \frac{1}{\Gamma(\nu)} \int_0^t (t-s)^{\nu-1} f(s, x(s), \psi x(s)) ds + \frac{\nu}{(1-\mu)} \sum_{i=1}^{k} x_i, \end{aligned}$$

where $\mu = \int_0^1 \mathbb{k}(s) ds$. Suppose the following conditions are verified:

(V_1) The function $f : \mho \times \mathscr{G} \times \mathscr{G} \to \mathscr{G}$ is jointly continuous.

(V_2) There exists a constant $e > 0$ such that $|f(t, \xi, \omega) - f(t, \overline{\xi}, \overline{\omega})| \le e[|\xi - \overline{\xi}| + |\omega - \overline{\omega}|]$, for each $t \in \mho$, and each $\xi, \omega, \overline{\xi}, \overline{\omega} \in \mathscr{G}$.

(V_3) $k : J \times \mathscr{G} \to \mathscr{G}$ is continuous and there exists a constant $e_1 > 0$, such that $|k(t,s,\xi) - k(t,s,\omega)| \le e_1|\xi - \omega|, \forall \xi, \omega \in \mathscr{G}, t, s \in J$.

(V_4) If $\mu = \int_0^1 g(s) ds$ as

$$\left[\frac{(e + ee_1)}{\Gamma(\nu+1)} + \frac{\eta(e + ee_1)}{(1-\mu)}\right] < 1.$$

Anguraj et al. [4] proved the following existence result for the problem (1.2)–(1.4).

Theorem 1.4. *Under the assumptions (V_1)–(V_4), the problem* (1.2)–(1.4) *has a unique solution on* $PC(\mho, Y)$.

Existence theorem for fixed points of an operator is concerned with establishing sufficient conditions in which the operator will have solution, but does not necessarily show how to find it. On the other hand, iteration method of fixed points is concerned with approximation or computation of sequences which converge to the solution of such operator. There are several results in the literature concerning approximation of various nonlinear integral and differential equations, see for example, [16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 27]. In this article, we will approximate the solution of nonlinear fractional integro-differential equations with impulsive and integral conditions by utilizing the following iterative algorithm recently introduced by Ofem et al. [16]:

$$\begin{cases} x_1 \in \mathcal{K}, \\ s_n = (1 - v_n)x_n + v_n T x_n, \\ z_n = (1 - u_n)T x_n + u_n T s_n, \qquad \forall n \geq 1, \\ y_n = T z_n, \\ x_{n+1} = T y_n, \end{cases} \tag{1.6}$$

where $\{u_n\}$ and $\{v_n\}$ are sequences in (0,1).

Remark 1.5. It is shown in [16] that the iterative algorithm (1.6) has a better speed of convergence than S [6], Picard-S [11], Thakur [34] and M [37] iteration processes for single-valued generalized α-nonexpansive mappings.

2 Main result

Now we approximate the solution of the problem (1.2)–(1.4) using the iterative process (1.6).

Theorem 2.1. *Let* $\{x_n\}$ *be the iterative procedure* (1.6) *with sequences* $\{u_n\}, \{v_n\} \in (0, 1)$ *such that* $\sum_{n=0}^{\infty} u_n v_n = \infty$. *Then the problem* (1.2)–(1.4) *has a unique solution, say,* $q \in PC(\mho, Y)$ *and* $\{x_n\}$ *converges to* q.

Proof. Let $\{x_n\}$ be an iterative sequence generated by the iteration process (1.6) for the operator $T : PC(\mho, \mathscr{G}) \to PC(\mho, \mathscr{G})$ define by

$$\begin{aligned} T(x)(t) &= \frac{1}{(1-\mu)\Gamma(\nu)} \int_0^1 \Omega(\tau) f(\tau, x(\tau), \psi x(\tau)) d\tau \\ &\quad + \frac{1}{\Gamma(\nu)} \int_0^t (t-s)^{\nu-1} f(s, x(s), \psi x(s)) ds + \frac{\mu}{(1-\mu)} \sum_{i=1}^{k} x_i. \end{aligned}$$

Let $x, y \in PC(\mho, \mathscr{G})$. Then, for each $t \in \mho$, from (1.6) we have

$$\begin{aligned}
\|s_n - q\| &= \|(1-v_n)x_n + v_n T x_n - Tq\| \\
&\leq (1-v_n)\|x_n - q\| + v_n\|Tx_n - Tq\| \\
&= (1-v_n)|x_n(t) - q(t)| + v_n|T(x_n)(t) - T(q)(t)| \\
&= (1-v_n)|x_n(t) - q(t)| \\
&\quad + v_n \Bigg| \frac{1}{(1-\mu)\Gamma(\nu)} \int_0^1 \Omega(\tau) f(\tau, x_n(\tau), \psi x_n(\tau)) d\tau \\
&\quad + \frac{1}{\Gamma(\nu)} \int_0^t (t-s)^{\nu-1} f(s, x_n(s), \psi x_n(s)) ds + \frac{\mu}{(1-\mu)} \sum_{i=1}^k x_i \\
&\quad - \frac{1}{(1-\mu)\Gamma(\nu)} \int_0^1 \Omega(\tau) f(\tau, q(\tau), \psi q(\tau)) d\tau \\
&\quad - \frac{1}{\Gamma(\nu)} \int_0^t (t-s)^{\nu-1} f(s, x(s), \psi q(s)) ds - \frac{\mu}{(1-\mu)} \sum_{i=1}^k x_i. \Bigg| \\
&\leq (1-v_n)|x_n(t) - q(t)| \\
&\quad + v_n \Bigg\{ \frac{1}{(1-\mu)\Gamma(\nu)} \int_0^1 \Omega(\tau) |f(\tau, x_n(\tau), \psi x_n(\tau)) - f(\tau, q(\tau), \psi q(\tau))| d\tau \\
&\quad + \frac{1}{\Gamma(\nu)} \int_0^t (t-s)^{\nu-1} |f(s, x_n(s), \psi x_n(s)) - f(s, q(s), \psi q(s))| ds \Bigg\} \\
&\leq (1-v_n)|x_n(t) - q(t)| + \\
&\quad + v_n \Bigg\{ \frac{\eta e}{(1-\mu)} \int_0^1 [|x_n(\tau) - q(\tau)| + |\psi x_n(\tau) - \psi q(\tau)|] d\tau \\
&\quad + \frac{1}{\Gamma(\nu)} \int_0^t (t-s)^{\nu-1} [|x_n(s) - q(s)| + |\psi x_n(s)) - \psi q(s))|] ds \Bigg\} \\
&\leq (1-v_n)\|x_n - q\| + v_n \left\{ \frac{\eta(e + ee_1)}{(1-\mu)} \|x_n - q\| + \frac{(e + ee_1)}{\Gamma(\nu+1)} \|x_n - q\| \right\} \\
&= \left(1 - v_n \left(1 - \left[\frac{\eta(e + ee_1)}{(1-\mu)} + \frac{(e + ee_1)}{\Gamma(\nu+1)}\right]\right)\right) \|x_n - q\|. \qquad (2.1)
\end{aligned}$$

$$\begin{aligned}
\|z_n - q\| &\leq (1-u_n)\|Tx_n - Tq\| + u_n\|Ts_n - Tq\| \\
&\leq (1-u_n) \Bigg\{ \frac{1}{(1-\mu)\Gamma(\nu)} \int_0^1 \Omega(\tau) |f(\tau, x_n(\tau), \psi x_n(\tau)) - f(\tau, q(\tau), \psi q(\tau))| d\tau \\
&\quad + \frac{1}{\Gamma(\nu)} \int_0^t (t-s)^{\nu-1} |f(s, x_n(s), \psi x_n(s)) - f(s, q(s), \psi q(s))| ds \Bigg\} \\
&\quad + u_n \Bigg\{ \frac{1}{(1-\mu)\Gamma(\nu)} \int_0^1 \Omega(\tau) |f(\tau, s_n(\tau), \psi s_n(\tau)) - f(\tau, q(\tau), \psi q(\tau))| d\tau \\
&\quad + \frac{1}{\Gamma(\nu)} \int_0^t (t-s)^{\nu-1} |f(s, s_n(s), \psi s_n(s)) - f(s, q(s), \psi q(s))| ds \Bigg\}
\end{aligned}$$

$$
\begin{aligned}
&\leq (1-u_n)\left\{\frac{\eta e}{(1-\mu)}\int_0^1[|x_n(\tau)-q(\tau)|+|\psi x_n(\tau)-\psi q(\tau)|]d\tau\right.\\
&\left.+\frac{1}{\Gamma(\nu)}\int_0^t(t-s)^{\nu-1}[|x_n(s)-q(s)|+|\psi x_n(s))-\psi q(s))|]ds\right\}\\
&u_n\left\{\frac{\eta e}{(1-\mu)}\int_0^1[|s_n(\tau)-q(\tau)|+|\psi s_n(\tau)-\psi q(\tau)|]d\tau\right.\\
&\left.+\frac{1}{\Gamma(\nu)}\int_0^t(t-s)^{\nu-1}[|s_n(s)-q(s)|+|\psi s_n(s))-\psi q(s))|]ds\right\}\\
&\leq (1-u_n)\left\{\frac{\eta(e+ee_1)}{(1-\mu)}\|x_n-q\|+\frac{(e+ee_1)}{\Gamma(\nu+1)}\|x_n-q\|\right\}\\
&+u_n\left\{\frac{\eta(e+ee_1)}{(1-\mu)}\|s_n-q\|+\frac{(e+ee_1)}{\Gamma(\nu+1)}\|s_n-q\|\right\}\\
&= (1-u_n)\left[\frac{\eta(e+ee_1)}{(1-\mu)}+\frac{(e+ee_1)}{\Gamma(\nu+1)}\right]\|x_n-q\|\\
&u_n\left[\frac{\eta(e+ee_1)}{(1-\mu)}+\frac{(e+ee_1)}{\Gamma(\nu+1)}\right]\|s_n-q\|. \qquad (2.2)
\end{aligned}
$$

$$
\begin{aligned}
\|y_n-q\| &= \|Tz_n-Tq\|\\
&= |T(z_n)(t)-T(q)(t)|\\
&\leq \frac{1}{(1-\mu)\Gamma(\nu)}\int_0^1\Omega(\tau)|f(\tau,z_n(\tau),\psi z_n(\tau))-f(\tau,q(\tau),\psi q(\tau))|d\tau\\
&+\frac{1}{\Gamma(\nu)}\int_0^t(t-s)^{\nu-1}|f(s,z_n(s),\psi z_n(s))-f(s,q(s),\psi q(s))|ds\\
&\leq \frac{\eta e}{(1-\mu)}\int_0^1[|z_n(\tau)-q(\tau)|+|\psi z_n(\tau)-\psi q(\tau)|]d\tau\\
&+\frac{1}{\Gamma(\nu)}\int_0^t(t-s)^{\nu-1}[|z_n(s)-q(s)|+|\psi z_n(s))-\psi q(s))|]ds\\
&\leq \frac{\eta(e+ee_1)}{(1-\mu)}\|z_n-q\|+\frac{(e+ee_1)}{\Gamma(\nu+1)}\|z_n-q\|\\
&= \left[\frac{\eta(e+ee_1)}{(1-\mu)}+\frac{(e+ee_1)}{\Gamma(\nu+1)}\right]\|z_n-q\|. \qquad (2.3)
\end{aligned}
$$

$$
\begin{aligned}
\|x_{n+1}-q\| &= \|Ty_n-Tq\|\\
&= |T(y_n)(t)-T(q)(t)|\\
&\leq \frac{1}{(1-\mu)\Gamma(\nu)}\int_0^1\Omega(\tau)|f(\tau,y_n(\tau),\psi y_n(\tau))-f(\tau,q(\tau),\psi q(\tau))|d\tau\\
&+\frac{1}{\Gamma(\nu)}\int_0^t(t-s)^{\nu-1}|f(s,y_n(s),\psi y_n(s))-f(s,q(s),\psi q(s))|ds
\end{aligned}
$$

$$
\begin{aligned}
&\le \frac{\eta e}{(1-\mu)}\int_0^1 [|y_n(\tau)-q(\tau)|+|\psi z_n(\tau)-\psi q(\tau)|]d\tau \\
&\quad +\frac{1}{\Gamma(\nu)}\int_0^t (t-s)^{\nu-1}[|y_n(s)-q(s)|+|\psi y_n(s))-\psi q(s))|]ds \\
&\le \frac{\eta(e+ee_1)}{(1-\mu)}\|y_n-q\|+\frac{(e+ee_1)}{\Gamma(\nu+1)}\|y_n-q\| \\
&= \left[\frac{\eta(e+ee_1)}{(1-\mu)}+\frac{(e+ee_1)}{\Gamma(\nu+1)}\right]\|y_n-q\|. \qquad (2.4)
\end{aligned}
$$

Using (2.1), (2.2), (2.3) and (2.4), we obtain

$$
\begin{aligned}
\|x_{n+1}-q\| &= \left[\frac{\eta(e+ee_1)}{(1-\mu)}+\frac{(e+ee_1)}{\Gamma(\nu+1)}\right]^3 \times \\
&\quad \left(1-u_nv_n\left(1-\left[\frac{\eta(e+ee_1)}{(1-\mu)}+\frac{(e+ee_1)}{\Gamma(\nu+1)}\right]\right)\right)\|x_n-q\|. \qquad (2.5)
\end{aligned}
$$

Since by assumption (V_4) we have $\left[\frac{\eta(e+ee_1)}{(1-\mu)}+\frac{(e+ee_1)}{\Gamma(\nu+1)}\right]<1$, then it follows that $\left[\frac{\eta(e+ee_1)}{(1-\mu)}+\frac{(e+ee_1)}{\Gamma(\nu+1)}\right]^3<1$. Thus, (2.5) becomes

$$
\|x_{n+1}-q\| \le \left(1-u_nv_n\left(1-\left[\frac{\eta(e+ee_1)}{(1-\mu)}+\frac{(e+ee_1)}{\Gamma(\nu+1)}\right]\right)\right)\|x_n-q\|. \qquad (2.6)
$$

Now define $\sigma_n=u_nv_n\left(1-\left[\frac{\eta(e+ee_1)}{(1-\mu)}+\frac{(e+ee_1)}{\Gamma(\nu+1)}\right]\right)$, then $\sigma_n\in(0,1)$ such that $\sum_{n=1}^{\infty}\sigma_n=\infty$ and set $\theta_n=\|x_n-q\|$. Then (2.6) can be rewritten as

$$
\theta_{n+1}=(1-\sigma_n)\theta.
$$

Therefore, all the conditions of Lemma 1.3 are satisfied. Hence, $\lim_{n\to\infty}\|x_n-q\|=0$. □

3 conclusion

In this paper, using the iterative method (1.6) which has been shown by Ofem et al. [16] to be faster than several existing iterative methods, we solved a fractional integro-differential equations with impulsive and integral conditions.

References

[1] T. Abdeljawad, F Jarad, A Atangana and PO Mohammed, On a new type of fractional difference operators on h-step isolated timescales: Weighted fractional difference operators, Journal of Fractional Calculus and Nonlinear Systems 1 (1)(2020), 46-74.

[2] M. S. Abdo, Further results on the existence of solutions for generalized fractional quadratic functional integral equations: Generalized fractional quadratic functional integral equations, Journal of Mathematical Analysis and Modeling 1 (1) (2020), 33-46.

[3] A. Almalahi and S. K. Panchal, Some existence and stability results for ϕ-Hilfer fractional implicit differentialequation with periodic conditions: ϕ-Hilfer Fractional Implicit Differential Equation, Journal of Mathematical Analysis and Modeling 1 (1) (2020), 1-19

[4] A. Anguraj, P. Karthikeyan, M. Rivero, J.J. Trujillo, On new existence results for fractional integro-differential equations with impulsive and integral conditions, *Computers and Mathematics with Applications*, 2013, available from doi:10.1016/j.camwa.2013.01.034.

[5] R. P. Argawal, M. Benchohra, Hamani, Boundary value problems for fractional differential equations, *Adv. Stud. Contemp. Math.*, **16**:181–196, 2008.

[6] R. P. Argawal, D. O. Regan, D. R. Sahu, Iterative construction of fixed points of nearly asymptotically nonexpansive mappings, *J Nonlinear Convex Anal.*, **8**:61–79, 2007.

[7] B Daşbaşi, Stability analysis of an incommensurate fractional-order SIR model Mathematical, Modelling and Numerical Simulation with Applications 1 (1), 44-55

[8] I. M. Esuabana, U. A. Abasiekwere, J. A. Ugboh, Z. Lipcsey, *Equivalent construction of ordinary differential equations from impulsive system*, Acad. J. Appl. Math. Sci. **4** (2018), no. 8, 77–89.

[9] I. M. Esuabana, J. A. Ugboh, *Marching method: A new numerical method for finding roots of algebraic and transcendental equations*, Am. J. Comput. Appl. Math. **9** (2019), no. 1, 6–11.

[10] I. M. Esuabana, J. A., Ugboh, *Survey of impulsive differential equations with continuous delay*, Int. J. Math. Trends Tech. **60** (2018), no. 1, 22–28.

[11] F. Gursoy, V Karakaya, A Picard–S hybrid type iteration method for solving a differential equation with retarded argument, 2014, available from arXiv:1403.2546v2.

[12] Z Hammouch, M Yavuz and N Özdemir, Numerical solutions and synchronization of a variable-order fractional chaotic system, Mathematical Modelling and Numerical Simulation with Applications 1 (1)(2021), 11-23.

[13] A. A. Kilbas, S. A. Marzan, Nonlinear differential equations with the Caputo fractional derivative in the space of continuously differentiable functions, *Differ. Equ.*, **41**(1):84–89, 2005.

[14] Z. Lipcsey, I. M. Esuabana, J. A. Ugboh, I. O. Isaac, *Integral representation of functions of bounded variation*, J. Math. **2019** (2019), Article ID 1065946. https://doi.org/10.1155/2019/1065946

[15] Z. Liu, R. Wang, A note on fractional Equations of Volterra Type with nonlocal boundary condition, *Abstract and Applied Analysis*, 2013, Article ID 432941, 8 pages, available from http://dx.doi.org/10.1155/2013/432941.

[16] A. E. Ofem, U. E. Udofia and D. I. Igbokwe, Solutions of constrained convex minimization problem and split feasibility problem via a new faster iterative algorithm, European Journal of Mathematical Analysis, 1 (2021), 106–132.

[17] A. E. Ofem, U.E. Udofia, Iterative solutions for common fixed points of nonexpansive mappings and strongly pseudocontractive mappings with applications, *Canad. J. Appl. Math.*, **3**(1):18–36, 2021.

[18] A. E. Ofem, D. I. Igbokwe, *A new faster four step iterative algorithm for Suzuki generalized nonexpansive mappings with an application*, Adv. Theory Nonlinear Anal. Appl. **5** (2021), no. 3, 482–506. https://doi.org/10.31197/atnaa.869046.a

[19] A. E. Ofem, H. Işik, F. Ali, J. Ahmad, *A new iterative approximation scheme for Reich-Suzuki type nonexpansive operators with an application*, J. Inequal. Appl. **2022** (2022), Paper No. 28. https://doi.org/101186/s13660-022-02762-8

[20] A. E. Ofem, U. E. Udofia, D. I. Igbokwe, *A robust iterative approach for solving nonlinear Volterra delay integro-differential equations*, Ural Math. J. **7** (2021), no. 2, 59–85.

[21] A. E. Ofem, H. Isik, G.C. Ugwunnadi, R. George, O. K. Narain, *Approximating the solution of a nonlinear delay integral equation by an efficient iterative algorithm in hyperbolic spaces*, AIMS Math. **8** (2023) no. 7, 14919–14950.

[22] A.E. Ofem, J. A. Abuchu, R. George, G. C. Ugwunnadi, O. K. Narain, *Some new results on convergence, weak w2-stability and data dependence of two multivalued almost contractive mappings in hyperbolic spaces*, Math. **10** (2022), no. 20, Paper No. 3720.

[23] A. E. Ofem, J. A. Abuchu, G. C. Ugwunnadi, H. Işik, O. K. Narain, *On a four-step iterative algorithm and its application to delay integral equations in hyperbolic spaces*, Rend. Circ. Mat. Palermo Ser. 2 (in press). https://doi.org/10.1007/s12215-023-00908-1

[24] A. E. Ofem, G. C. Ugwunnadi, O. K. Narain, J. K. Kim, *Approximating common fixed point of three multivalued mappings satisfying condition (E) in hyperbolic spaces*, Nonlinear Funct. Anal. Appl. **28** (20233), no. 3, 623–646.

[25] G. A. Okeke and A. E. Ofem, A novel iterative scheme for solving delay differential equations and nonlinear integral equations in Banach spaces, Mathematical Methods in the Applied Sciences,

[26] A. E. Ofem, M. O. Udo, O. Joseph, R. George and C. F. Chikwe, Convergence Analysis of a New Implicit Iterative Scheme and Its Application to Delay Caputo Fractional Differential Equations, Fractal and Fractional, 7 (3) (2023), 212.

[27] A.E. Ofem, A. Hussain, O. Joseph, M. O. Udo, U. Ishtiaq, H. Al Sulami, Solving Fractional Volterra–Fredholm Integro–Differential Equations via A^{**} Iteration Method, Axioms, 11 (9) (2022), 470.

[28] S. G. Samko, A. A. Kilbas, O. I. Marichev, *Fractional Integrals and Derivatives*, Theory and Applications, Gordon and Breach, Yverdon, 1993.

[29] N. Sene, Fractional diffusion equation described by the Atangana-Baleanu fractional derivative and its approximate solution, Journal of Fractional Calculus and Nonlinear Systems, 2 (1) (2021), 60-75.

[30] F Jarad, E Uğurlu, T Abdeljawad, D Baleanu, On a new class of fractional operators, Advances in Difference Equations 2017 (1), 1-16.

[31] F Jarad, T Abdeljawad and D Baleanu, Caputo-type modification of the Hadamard derivatives, Advances in Difference Equations 2012 (1), 1-8.

[32] T Abdeljawad, On comformable fractional calculus, Journal of computational and Applied Mathematics 279, 57-66.

[33] T Abdeljawad, D Baleanu, Discrete fractional differences with non-singular Mittag-Leffler Kernels, Advances in Difference Equations 2016 (1), 1-18

[34] B. S. Thakur, D. Thakur and M. Postolache, A new iteration scheme for approximating fixed points of nonexpansive mappings, Filomat, 30(2016), 2711–2720.

[35] X. Weng, Fixed point iteration for local strictly pseudocontractive mapping, *Proc. Am. Math. Soc.*, **113**:727–731, 1991.

[36] J. A. Ugboh and I. M. Esuabana, Existence and Uniqueness Result for a Class of Impulsive Delay Differential Equations, 2(4) (2018), https://dx.doi.org/10.22161/ijcmp.2.4.1

[37] K. Ullah, M. Arshad, Numerical Reckoning Fixed Points for Suzuki's Generalized Nonexpansive Mappings via New Iteration Process, *Filomat*, **32**:187–196, 2018.

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