

A Knowledge discovery framework using fuzzy and wavelet methods for multi-criteria ranking

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Abstract The sentiment analysis approach or opinion mining for product ranking deals with computing people's opinions using structured and unstructured data from blogs, review sites/articles and social media. The fast pace of changing user preferences in different age groups and geographical regions has made product ranking a valuable research area. The product ranking based on user's opinion for multi-criteria decision-making has gained prominence with the rise in e-commerce and online selling of goods and services. The reviews on online products display significant impacts on decisions made by consumers purchase. Ranking of the products through online reviews influences consumers' purchase decisions and a source for sellers for evaluating market response to their product. The increasing number of competitive business models has made the right product selection by the end user based on other users opinion and feedback on different forums a challenging task. Aggregation of product features by the e-commerce portals based on data collected from various sources is not enough for the buyer to make decision on appropriate product choice. The present research introduces a novel approach for ranking the alternatives based on machine learning techniques, fuzzy analytical hierarchy process, the Technique for Order Preference by Similarity to Ideal Solution (TOPSIS), and wavelet transformations. Experiments are conducted over the real data sets, and efficacy of the proposed method is assessed and compared the results with the rank given by the domain experts.

1 Introduction

Sentiment analysis (Opinion mining) of structured and unstructured data is required for better purchasing decision, customer centric product placing, services and better business management. Opinion mining/classification for multi criteria decision making is a relatively new research area which has gained importance due to digital mode of business. The most often used supervised machine learning approaches for sentiment classification are Naive Bayes (NB) and Support Vector Machines (SVM). The sentiment classification examines the sentiment polarity of the filtered text categorized as neutral, negative or positive. Internet has given rise to e-commerce portals, digital social media, blogging site and news web sites, making a strong influence on various sectors including manufacturing, trade and supply chain, service-oriented industries like, health care, tourism, hotel management, etc. The prospective buyer makes opinion by accessing the information and sharing their views with other buyers. The advent of easily accessible Business to-Consumer (B2C) web sites had transformed the traditional interactive shopping by customers to online shopping platforms, saving time and resources and identify the best sources to get the most suitable product. The B2C web sites features the direct purchase of goods or services online by the customers from manufacturers restricting the need of middlemen and other third-party sellers. This results in reduction of overall costing of products making it more affordable. These B2C sites for online shopping facilitate consumers access the various product specifications and compare them. But merely knowing the product specification by the sites are not supportive enough for customers to make decision while purchasing. They are more interested in the combination of product reviews made by existing users and the competitive pricing of the product. These sites keep record of customer's product review and are reliable source for new customers to take inference while making a product purchase. These reviews are also reviewed by manufac-

turers to inspect the user sentiments regarding the product, enabling them to utilize the positive features in marketing and negative ones to be improved to maximize the customer satisfaction. The variance in user's opinions of same product across various shopping platforms makes analysis practically difficult task for the analyst due to unstructured nature of data. Though some web sites provide a comparison platform for the products taking basic features and price into consideration, still none of them can efficiently perform a comprehensive ranking of them. Moreover, the comparisons made based on consumer reviews for the product are not available. A curative testimonial obtained from these different B2C web sites in a common format is crucial for making an analysis while implementing different meta data and content-based features that deduce the rank scores for different product alternatives. This has proven to be beneficial for both new customers and manufacturers [4, 5, 13, 19, 20, 23, 26, 36, 45, 47].

The Multi Criteria Decision Making (MCDM) method is commonly used to identify the best options by weighing numerous comparable and competing criteria. In the current study, a set of eight characteristics, such as rating, user verification status, title, content, and usefulness based on meta data and review document contents, were used to rank distinct online product alternatives. The features identified from review documents are ranked using Analytic Hierarchy Process (AHP), Saaty nine point scale of numeric value and linguistic meaning are used in relative score matrix generation where each pair convert this matrix into the fuzzy matrix using triangular membership function; and compute fuzzy weights and rank. The fuzzy weight is multiplied by each column of the decision matrix and translated into the wavelet domain, and the decision matrix is constructed from review papers in order to rank distinct options using TOPSIS. Section 2 discusses related works in the same field, Section 3 discusses preliminary work, Section 4 discusses recommended approach, Section 5 discusses experimental findings, and Section 6 discusses conclusion and future work.

2 Related Works

Rekik et al. [2] presented a database of customer satisfaction measurements for a feedback-based diagnostic system. These measurements were connected to the many services that e-commerce provides to its clients. A text mining approach was proposed by Kamal et al. [3], Hyperlink-Induced Topic Search (HITS) algorithm has been used in order to generate reliability scores, opinions and product features. Chen et al. [10] developed a prototype system that combines the advantages of the laddering technique with the radial basis characteristic (RBF) neural network for customer needs gathering and multicultural factor analyses. Hwang and Yoon [11] proposed stock ranking method, which includes overall performance under a stochastic environment and proved it's significance, better understanding of the comparison within the same sector of companies. Cano and Morisio [15] published a review on hybrid recommender systems, which are software programmes that are used to develop and deliver recommendations for products and other entities to consumers using a variety of methodologies. Two or more recommendation algorithms are merged in different ways in hybrid recommender systems to profit from their complementing benefits. Kujawski [16] conducted study on the difficulties of Multi-Criteria Decision Analysis (MCDA), which are illustrated through a case study of 2002 Olympics. The study compares various MCDA models and what they have to offer. Several challenging tasks faced by analysts working in MCDAI siklar and Buyukozkan [17] used an AHP and a TOPSIS to evaluate a Multi Criteria Decision Making approach. Hsieh and Wu [21] presented a support vector regression model to assist users in their analysis and ranking of reviews based on a set of linguistic variables. Using deep learning models, review ranking has been offered as a strategy to deal with it. Yu et al. [18] evaluated four different 3G licensing mechanisms in Taiwan using a fuzzy multicriteria decision-making approach auction, beauty contest, tender, and beauty contest with a fixed license fee. The findings provide meaningful insights to the policy-makers to select the most appropriate licensing policy for the country that can enhance national income, improve telecommunication technologies and services, along with significant ROI for 3G licensees. Salmeron Herrero [25] presented a model based on AHP to rate important success elements of executive information systems. Yang et al. [28] developed a domain sentiment dictionary using external textual data that facilitate extracting all the sentiments expressed in a document with a multitude of expressions using a single dictionary for NLP applications. A hybrid

model was developed by integrating different single models that is shown to be more effective than single baseline models. Zhang et al. [29] developed feature-based product ranking technique that ranks different features of a product based on the mining of thousands of customer reviews to aid in the purchase decision-making of online shoppers. Lakiotaki et al. [30] enhanced the performance of Multi-rating Recommender Systems by combining approaches from Multiple Criteria Decision Analysis with the Collaborative Filtering methodology. Mookiah [33] developed a graph-based strategy for tailored news recommendation. Zhang et al. [14] describe a hesitant fuzzy set and emotion word architecture for rating products based on internet reviews. Chen [41] suggested an algorithm for rating E-Commerce brands based on user assessment and sentiment analysis. Liu et al. [49] created a system for rating products based on internet reviews using sentiment analysis and intuitionist fuzzy set theory. Jurgitasinskas and Kabasinskas [27] examined explainable artificial intelligence-based multi-criteria decision-making strategies in banking. Mohammed et al. [38] created a control sequence rating for critical systems based on equipment health and Elahi et al. [37] developed and evaluated a university recommender system using SVD and KNN algorithm.

3 Preliminaries

This section illustrates technical details about fuzzy, multicriteria decision-making techniques fuzzy AHP, TOPSIS and wavelet transform that focused on features and product ranking in our proposed methodology.

3.1 Fuzzy Logic

Fuzzy logic is a mathematical language used to represent something with a fuzzy set. L.A. Zadeh created fuzzy logic in 1965; the definition of fuzzy is unclear. The synonym of the fuzzy can be, noisy, unclear, blurry etc. Antonym of the fuzzy is crisp, crisp can be yes or no, true or false, but the fuzzy answer means, may be, may not be, absolutely, partially etc. Fuzzy system contains fuzzy elements, fuzzy sets, fuzzy rule, fuzzy implications. The fuzzy set is given by [1, 8, 34] $F = \{(s, \mu) : s \in X\}$ and $\mu(s)$ is the degree of s .

It is a collection of ordered pairs, inclusion of an element $x \in X$ into F is fuzzy.

The membership function thoroughly defines a fuzzy set. As a result, it is critical to understand how a membership function might be represented (mathematically or otherwise). A membership function might be either a discrete or continuous universe of speech. Of course, a membership function on a discrete universe is trivial. A membership function on a continuous universe of speech requires particular consideration. Following functions are typical examples of membership functions.

- (i.) Triangular membership function,
- (ii.) Trapezoidal membership function,
- (iii.) Gaussian membership function,
- (iv.) Generalized bell membership function and
- (v.) Sigmoid membership function.

In this article, we will apply the triangle membership function to transform the relative criterion matrix into the fuzzy relative criteria matrix, as provided by

$$\Delta(x, a, b, c) = \begin{cases} 0, & \text{if } x \leq a, \\ \frac{x-a}{b-a}, & \text{if } a \leq x \leq b, \\ \frac{c-x}{c-b}, & \text{if } b \leq x \leq c, \\ 0, & \text{if } c \leq x, \end{cases} \quad (3.1)$$

the graph of the triangular membership function is given in figure 1. Further, addition of two fuzzy numbers A_1 and A_2 can be given by,

$$A_1 \oplus A_2 = (l_1, m_1, u_1) \oplus (l_2, m_2, u_2) = (l_1 + l_2, m_1 + m_2, u_1 + u_2).$$

Similarly multiplication of two fuzzy numbers A_1 and A_2 can be given by,

$$A_1 \otimes A_2 = (l_1, m_1, u_1) \otimes (l_2, m_2, u_2) = (l_1 \times l_2, m_1 \times m_2, u_1 \times u_2).$$

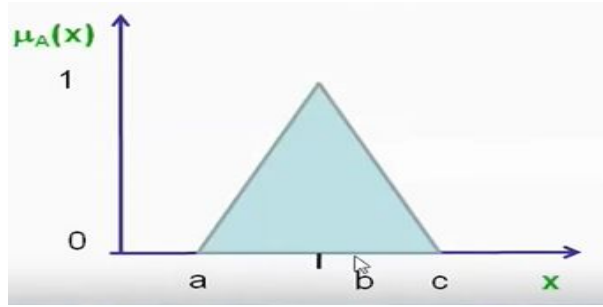


Figure 1. triangular membership function

3.2 Fuzzy Analytic Hierarchy Process

In the year 1980, Thomas L. Saaty developed the AHP that is the most often used approach for ranking various characteristics. It separates criteria, sub-criteria, and alternatives into a graded framework by fractionating a typical, unstructured complicated condition into several constituent fractions. To compare different alternatives, AHP performs the pairwise comparison of criteria assigning some numeric weights. The experts implement their knowledge and expertise to suggest weights on a priority scale and the AHP a non-linear approach, estimates whether these pairwise criteria by the experts are consistent or not. The authors stated that the generalized AHP faces a rank reversal problem in which the ranking of alternatives may change on adding some new ones while implementing AHP over them. Despite several cons, AHP is still the widely used MCDM models employed for solving decision making problems [16, 35, 42, 43, 44]. The paper provides a detailed description of multi-criteria decision analysis techniques highlighting its pitfalls, limitations, and practical problems encountered while implementation. The following paragraphs provide concise explanations of the procedures utilized in fuzzy AHP to rank a given list of options.

Step-1 : Create a relative criteria matrix

$$M = \begin{pmatrix} a_{1,1} & a_{1,2} & \dots & a_{1,m} \\ a_{2,1} & a_{2,2} & \dots & a_{2,m} \\ a_{3,1} & a_{3,2} & \dots & a_{3,m} \\ \cdot & \cdot & \cdot & \dots \\ \cdot & \cdot & \cdot & \dots \\ a_{n,1} & a_{n,2} & \dots & a_{n,m} \end{pmatrix}$$

$M = [a_{i,j}]$, $1 \leq i \leq n$ and $1 \leq j \leq m$, here $a_{i,j}$ are given by Saaty nine point scale of numeric value and linguistic meaning.

1	a_1 and a_2 have equal importance
3	a_1 is slightly more important than a_2
5	a_1 is more important than a_2
7	a_1 is strongly more important than a_2
9	a_1 is extremely more important than a_2
2,4,6,8	intermediate value of importance

Step-2 : Fuzzyfication of each entry of the matrix $M = [a_{i,j}]$ can be done using following table.

1=(1,1,1)	6=(5,6,7)
2=(1,2,3)	7=(6,7,8)
3=(2,3,4)	8=(7,8,8)
4=(3,4,5)	9=(9,9,9)
5=(4,5,6)	

Also, the inverse of an element $A = (l, m, u)$ can be given by $A^{-1} = (\frac{1}{u}, \frac{1}{m}, \frac{1}{l})$. For example inverse of $3 = (2, 3, 4)$ can be written as $\frac{1}{3} = (\frac{1}{4}, \frac{1}{3}, \frac{1}{2})$. from here we can obtained the fuzzyfied matrix Table 1 $F(M) = [l_{i,j}, m_{i,j}, u_{i,j}]$, $1 \leq i \leq n$ and $1 \leq j \leq m$ of the relative criteria matrix $M = [a_{i,j}]$, $1 \leq i \leq n$ and $1 \leq j \leq m$ see Table 1.

Table 1. Fuzzified matrix F(M)

$(l_{1,1}, m_{1,1}, u_{1,1})$	$(l_{1,2}, m_{1,2}, u_{1,2})$...	$(l_{1,m}, m_{1,m}, u_{1,m})$
$(l_{2,1}, m_{2,1}, u_{2,1})$	$(l_{2,2}, m_{2,2}, u_{2,2})$...	$(l_{2,m}, m_{2,m}, u_{2,m})$
$l_{3,1}, m_{3,1}, u_{3,1})$	$(l_{3,2}, m_{3,2}, u_{3,2})$...	$(l_{3,m}, m_{3,m}, u_{3,m})$
.
.
$l_{n,1}, m_{n,1}, u_{n,1})$	$(l_{n,2}, m_{n,2}, u_{n,2})$ $(l_{n,m}, m_{n,m}, u_{n,m})$

Step-3 : Now Calculate the fuzzy geometric mean R_i from the fuzzified matrix $F(M)$ by,

$$R_i = [(l_1, m_1, u_1) \otimes (l_2, m_2, u_2) \dots (l_n, m_n, u_n)]^{\frac{1}{n}}$$

$$R_i = [(l_1 \times l_2 \times \dots \times l_n)^{\frac{1}{n}}, (m_1 \times m_2 \times \dots \times m_n)^{\frac{1}{n}}, (u_1 \times u_2 \times \dots \times u_n)^{\frac{1}{n}}]$$

Step-4 : Calculate fuzzy weight $\tilde{w} = R_i \otimes (R_1 \oplus R_2 \oplus \dots \oplus R_n)^{-1}$ and then find the weight $w_i = \frac{l_i + m_i + u_i}{3}$.

Step-5: Let $S = w_1 + w_2 + \dots + w_n$, now find normalize weight \hat{w} by dividing each element of w_i by S , i.e.

$$\hat{w} = \left[\frac{w_i}{S} \right]_{1 \leq i \leq n}$$

3.3 Technique for Order Preference by Similarity to Ideal Solution

Hwang and Yoon developed a technique for order preference by similarity to ideal solution(TOPSIS) in 1980. It is the most widely implemented ranking approach that labels the criteria into two different classes where onedenotes the criteria with positive outcome of target and the other signifies negative ones. As a result, it computes two opposing best ideal situations, the best and worst solutions. The greatest solution has the most positive criteria values and the fewest negative criteria values, whereas the worst ideal option has the most negative criteria values and fewest positive criteria values. Finally, the TOPSIS technique employs the Euclidean distance to compute the relative proximity of the alternatives to the ideal criterion answers while ranking them. The pros of TOPSIS includes its easy to usage, simple and programmability. The major con of TOPSIS lies within the usage of Euclidean distance that considers criteria correlation. The following section of the paper provides the necessary steps for TOPSIS to rank the alternatives [11, 35]. The following paragraphs provide a quick overview of the TOPSIS technique for ranking options.

Step-1 : Create a decision matrix $D_{m,n}$, m and n are alternatives and criteria respectively.

$$D = \begin{pmatrix} d_{1,1} & d_{1,2} & \dots & \dots & d_{1,m} \\ d_{2,1} & d_{2,2} & \dots & \dots & d_{2,m} \\ d_{3,1} & d_{3,2} & \dots & \dots & d_{3,m} \\ \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots \\ d_{n,1} & d_{n,2} & \dots & \dots & d_{n,m} \end{pmatrix}$$

Step-2: Normalize the decision matrix D , let $\hat{D} = [\hat{d}_{i,j}]$ is normalize decision matrix, whose

element are given by $\hat{d}_{i,j} = \frac{d_{i,j}}{\sqrt{\sum_i^m d_{i,j}^2}}$, $i = 1, 2, \dots, m$ and $j = 1, 2, \dots, n$.

$$\hat{D} = \begin{pmatrix} \hat{d}_{1,1} & \hat{d}_{1,2} & \cdot & \cdot & \cdot & \hat{d}_{1,m} \\ \hat{d}_{2,1} & \hat{d}_{2,2} & \cdot & \cdot & \cdot & \hat{d}_{2,m} \\ \hat{d}_{3,1} & \hat{d}_{3,2} & \cdot & \cdot & \cdot & \hat{d}_{3,m} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \hat{d}_{n,1} & \hat{d}_{n,2} & \cdot & \cdot & \cdot & \hat{d}_{n,m} \end{pmatrix}$$

Step-3: After calculating the normalize decision matrix \hat{D} , we calculate the weighted normalize decision matrix by multiplying each column of \hat{D} with the corresponding criteria rank score, which is obtained from fuzzy AHP. Let $\Gamma = [\tau_{i,j}]$, whose element are given by $\tau_{i,j} = \hat{d}_{i,j} \times s_j$, s_j are criteria rank score of the j^{th} criteria.

$$\Gamma = \begin{pmatrix} \hat{\tau}_{1,1} & \hat{\tau}_{1,2} & \cdot & \cdot & \cdot & \hat{\tau}_{1,m} \\ \hat{\tau}_{2,1} & \hat{\tau}_{2,2} & \cdot & \cdot & \cdot & \hat{\tau}_{2,m} \\ \hat{\tau}_{3,1} & \hat{\tau}_{3,2} & \cdot & \cdot & \cdot & \hat{\tau}_{3,m} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \hat{\tau}_{n,1} & \hat{\tau}_{n,2} & \cdot & \cdot & \cdot & \hat{\tau}_{n,m} \end{pmatrix}$$

Step-4: Let F be the criteria set and partition of F are F^+ and F^- which are positive and negative impact on the goal. The best and worst ideal solutions are $B = (b_1, b_2, b_3, \dots, b_n)$ and $W = (w_1, w_2, w_3, \dots, w_n)$, which are given by

$$b_j = \begin{cases} \max_{i=1}^m \{\tau_{i,j}\}, & \text{if } F[j] \in F^+ \\ \min_{i=1}^m \{\tau_{i,j}\}, & \text{if } F[j] \in F^- \end{cases}$$

and

$$w_j = \begin{cases} \min_{i=1}^m \{\tau_{i,j}\}, & \text{if } F[j] \in F^+ \\ \max_{i=1}^m \{\tau_{i,j}\}, & \text{if } F[j] \in F^- \end{cases}$$

Step-5: Calculate the Euclidean distance from each alternatives with ideal and worst solution. Let $d_b[i]$ and $d_w[i]$ are Euclidean distances from each alternatives with ideal and worst solution, then it is given by

$$d_b[i] = \sqrt{\sum_{j=1}^n (\tau_{i,j} - b_j)^2}$$

and

$$d_w[i] = \sqrt{\sum_{j=1}^n (\tau_{i,j} - w_j)^2}$$

Step-5: The final step is to calculate the rank score of each alternative by

$$R[i] = \frac{d_w[i]}{d_b[i] + d_w[i]}$$

3.4 Wavelets

In the last two decades, wavelets have come up as a prominent tool in digital signal processing and mathematics. These exhibits orthogonality, symmetry, completeness making it a good area of research. Wavelets find its applicability as a solution for problems dealing with differential equations, integral equation, image processing, signal processing, cryptography, coding theory, data analytics, cloud computing, weather forecasting, seismology, time series analysis,

turbulence, artificial and convolution neural network, finance, geophysics, biotechnology, and bio-informatics etc [6, 7, 9, 12, 22, 24, 31, 39, 40]. The wavelets are square, integrable signals or functions that keeps check on admissibility condition. Also, the wavelet transform is an integral transform which exhibits the features of dilation and translation while implementing wavelet function.

The Haar function is given by

$$\phi(x) = \begin{cases} 1, & 0 \leq x < 1 \\ 0, & \text{otherwise} \end{cases}$$

The space $V_0 = \text{Span}\{\dots\phi(x+1), \phi(x), \phi(x-1), \phi(x-2)\dots\} \cap L^2(\mathbb{R})$
 $\langle \phi(x-i), \phi(x-j) \rangle = 0$ and $\int_{\mathbb{R}} |\phi(x-k)|^2 dx = 1$.

Hence, the set $\{\phi(x-k)\}_{k \in \mathbb{Z}}$ forms an orthonormal basis for the space V_0 . Further, the space V_j is given by

$$V_j = \text{Span}\{\dots\phi(2^j x + 1), \phi(2^j x), \phi(2^j x - 1), \phi(2^j x - 2)\dots\} \cap L^2(\mathbb{R}).$$

$$V_j = \text{Span}\{\phi(2^j x - k)\}_{j, k \in \mathbb{Z}} \cap L^2(\mathbb{R}).$$

The space V_j are called Haar space as it is generated from Haar function.

The set $\{\phi_{j,k} = 2^{j/2} \phi(2^j x - k)\}_{j, k \in \mathbb{Z}}$ is an orthonormal basis for V_j , support of the $\phi_{j,k}$ is $[\frac{k}{2^j}, \frac{k+1}{2^j}]$. Also, the sequence of space V_j form multiresolution analysis which satisfies the following properties.

(i.) Nestedness property $\dots V_{-2} \subseteq V_{-1} \subseteq V_0 \subseteq V_1 \subseteq V_2 \dots$,

(ii.) If $f(x) \in V_j$ then, $f(2x) \in V_{j+1}$,

(iii.) $\bigcap_{j \in \mathbb{Z}} V_j = \dots \bigcap V_{-1} \cap V_0 \cap V_1 \dots = \{0\}$,

(iv.) $\overline{\bigcup_{j \in \mathbb{Z}} V_j} = \dots \bigcup V_{-1} \cup V_0 \cup V_1 \dots = L^2(\mathbb{R})$.

The Haar wavelet is given by

$$\psi(x) = \begin{cases} 1, & 0 \leq x < \frac{1}{2} \\ -1, & \frac{1}{2} \leq x < 1 \\ 0, & \text{otherwise} \end{cases}$$

The space $W_0 = \text{Span}\{\dots\psi(x+1), \psi(x), \psi(x-1), \psi(x-2)\dots\} \cap L^2(\mathbb{R})$

$$\langle \psi(x-k), \psi(x-m) \rangle = \begin{cases} 1, & k = m \\ 0, & k \neq m \end{cases}$$

The set $\{\psi(x-k)_{k \in \mathbb{Z}}\}$ forms an orthonormal basis for W_0 .

The general Haar wavelet space is given by $W_j = \text{Span}\{\psi(2^j x - k)\}_{j, k \in \mathbb{Z}} \cap L^2(\mathbb{R})$

$\{\psi_{j,k} = 2^{j/2} \psi(2^j x - k)\}_{j, k \in \mathbb{Z}}$ also $\|\psi_{j,k}(x)\| = 1$, $W_j = \text{Span}\{\dots\psi(2^j x + 1), \psi(2^j x), \psi(2^j x - 1), \psi(2^j x - 2)\dots\} \cap L^2(\mathbb{R})$ and $V_{j+1} = V_j \oplus W_j = V_0 \oplus W_0 \oplus W_1 \dots \oplus W_j$.

The haar function $\phi_{j,k}$ satisfy the dilation equation $\phi_{j,k}(x) = \frac{1}{\sqrt{2}} \phi_{j+1,2k}(x) + \frac{1}{\sqrt{2}} \phi_{j+1,2k}(x)$, $\forall j, k \in \mathbb{Z}$ and Haar wavelet function satisfy $\psi_{j,k}(x) = \frac{1}{\sqrt{2}} \psi_{j+1,2k}(x) - \frac{1}{\sqrt{2}} \psi_{j+1,2k}(x)$, $\forall j, k \in \mathbb{Z}$.

Further, the discrete wavelet transform of a vector v of size N is given by $W_N \times v$, $W_N = [\frac{H_{N/2}}{G_{N/2}}]$, here H and G are called average and detail matrix. The Haar wavelet matrix is given by

$$W = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 & 0 & 0 & \dots & 0 & 0 \\ 0 & 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 & \dots & 0 & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & 0 & 0 & 0 & 0 & \cdot & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 & 0 & 0 & \dots & 0 & 0 \\ 0 & 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 & \dots & 0 & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & 0 & 0 & 0 & 0 & \cdot & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix}$$

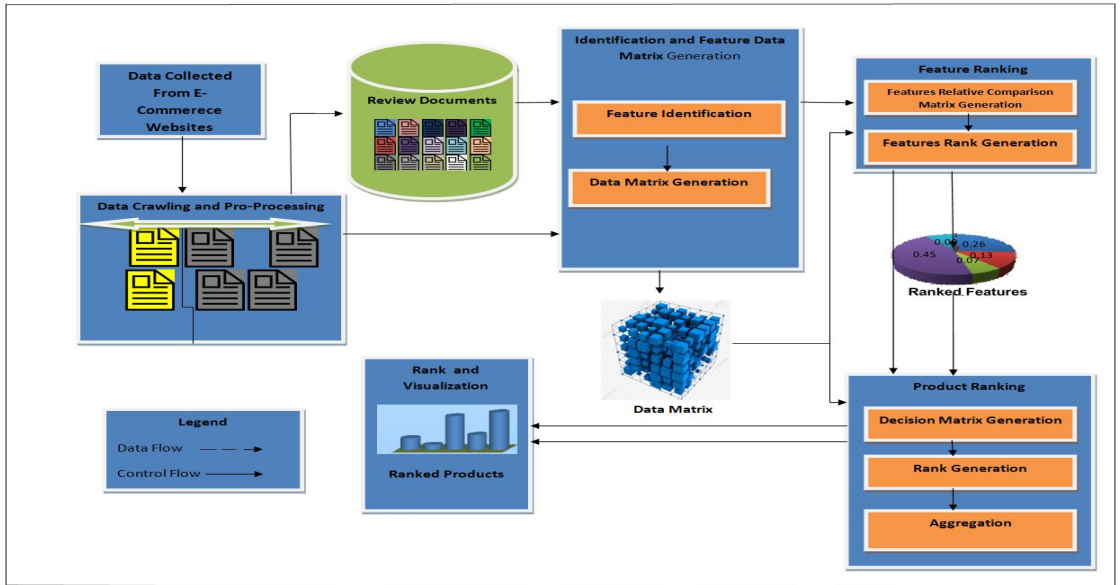


Figure 2. Flowchart of the Proposed Framework

4 Proposed Methodology

The main objective of this section is to discuss the functioning details of the proposed framework. The first task is to identify the features and weighting, based on the comments of the user, such as camera, price, battery and processor etc. The next task is to rank these features. The feature ranking task is performed using the fuzzy AHP. Using Saaty nine point scale of numeric value and linguistic meaning the relative score matrix is generated according to the expert suggestion for each pairs. Now convert this matrix into the fuzzy matrix using triangular membership function and compute fuzzy weights and rank the features. For stability and consistency of the system we refer [32, 46, 48, 50]. The next task of the algorithm is to rank the alternatives. The decision matrix has been generated using customer reviews and comments for each product/alternative with respect to the features, then fuzzy weight is multiplied with the each column of the decision matrix and transform into the wavelet domain. Apply the TOPSIS discussed in the previous section of the manuscript and rank the alternatives. The graphical representation of the proposed framework is given in figure 2.

Algorithm Input

Step 1: Generate decision matrix for weighting criteria,

Step 2: Convert the decision matrix into fuzzy matrix F ,

Step 3: Calculate the fuzzy geometric mean and then find fuzzy weight,

Step 4: Normalize the fuzzy weight \hat{W} ,

Step 5: Check the consistency of the system,

Step 6: If the system is consistent then proceed otherwise go to step 1 and then generate different decision matrix for weighting criteria ,

Step 7: Generate the decision matrix from the data set and multiply each column with their corresponding weight obtained in step 4,

Step 8: Transform the column of the matrix into the wavelet domain by multiplying each column with wavelet matrix and normalize the transform matrix,

Step 9: Find the ideal best and worst solution by minimizing and maximizing the element row wise,

Step 10: Calculate the Euclidean distance from each column with ideal best and worst solution and then rank the alternatives.

% Generate feature relative score matrix
for $i \leftarrow 1$ to n **do**


```

for  $j \leftarrow 1$  to  $n$  do
  if  $i < j$  then
     $a[i, j] \leftarrow a[i][j] \leftarrow$  read expert input
  end if
  if  $(i = j)$  then
     $a[i][j] \leftarrow 1$ 
  else
     $a[i][j] \leftarrow \frac{1}{a[j][i]}$ 
  end if
end for
end for
% fuzzification of feature relative score matrix
for  $i \leftarrow 1$  to  $n$  do
  for  $j \leftarrow 1$  to  $n$  do
     $M1[i, j] \leftarrow$  read expert input of fuzzy
     $M2[i, j] \leftarrow$  read expert input of fuzzy
     $M3[i, j] \leftarrow$  read expert input of fuzzy
  end for
end for
for  $i \leftarrow 1$  to  $n$  do
   $R1[i] \leftarrow \text{geomean}(M1[i, :])$ 
   $R2[i] \leftarrow \text{geomean}(M2[i, :])$ 
   $R3[i] \leftarrow \text{geomean}(M3[i, :])$ 
end for
 $r \leftarrow [R1; R2; R3]$ 
 $Com1 = 1/\text{sum}(R3)$ 
 $Com2 = 1/\text{sum}(R2)$ 
 $Com3 \leftarrow 1/\text{sum}(R1)$ 
 $w1 \leftarrow R1 \times com1$ 
 $w2 \leftarrow R2 \times com2$ 
 $w3 \leftarrow R3 \times com3$ 
 $wgt \leftarrow [w1; w2; w3]$ 
for  $i \leftarrow 1$  to  $n$  do
   $w[i] \leftarrow \text{geomean}(wgt[:, i])$ 
end for
 $nW = w/\text{sum}(w)$ 
%Generate decision matrix from the data set
%Transform each column into the wavelet domain
%D is the decision matrix and H is the Haar wavelet matrix
for  $i \leftarrow 1$  to  $n$  do
   $W2[i, :] \leftarrow H \times D[i, :]$ 
end for
for  $i \leftarrow 1$  to  $n$  do
  for  $j \leftarrow 1$  to  $n$  do
     $nD[i][j] \leftarrow \frac{W2[i][j]}{\text{sqrt}(\text{sum}(D[i][j])^2)}$ 
  end for
end for
for  $i \leftarrow 1$  to  $n$  do
  for  $j \leftarrow 1$  to  $n$  do
     $wnD \leftarrow nD[i][j] \times nW[j]$ 
  end for
end for
for  $i \leftarrow 1$  to  $n$  do
  if  $(F[i] \in F^{(+)})$  then
     $Best[i] \leftarrow \text{max}(wnD[i][j])$ 
     $Worst[i] \leftarrow \text{min}(wnD[i][j])$ 
  end if
end for

```

```

if ( $F[i] \in F^{(-)}$ ) then
     $Best[i] \leftarrow \min(wnD[i][j])$ 
     $Worst[i] \leftarrow \max(wnD[i][j])$ 
end if
end if
end for
for  $i \leftarrow 1$  to  $n$  do
     $bdist \leftarrow \sqrt{\sum_{1 \leq j \leq n} (wnD[i][j] - Best[j])^2}$ 
     $wdist \leftarrow \sqrt{\sum_{1 \leq j \leq n} (wnD[i][j] - Worst[j])^2}$ 
     $Rank \leftarrow \frac{wdist}{bdist + wdist}$ 
end for
    
```

5 Experimental Results

In this section we we presents experimental results by applying our method for ranking of mobile phones. We took the review document of each mobile phone, which have been taken from the e-commerce web site Flipkart, after preprocessing of the data we extracted the features and data matrix were generated. We took features as camera, battery, display, price, customer review, processor, user interface and warranty. In the first stage we generated the feature relative score criteria matrix using experts input which is given in matrix M Table 2 as given in step-1 of the section 3.2.

Table 2. Relative Criteria Score Matrix M

1	3	5	7	3	7	9	5
$\frac{1}{3}$	1	3	3	5	5	7	9
$\frac{1}{5}$	$\frac{1}{3}$	1	5	3	3	3	4
$\frac{1}{7}$	$\frac{1}{3}$	$\frac{1}{5}$	1	$\frac{1}{5}$	$\frac{1}{3}$	2	3
$\frac{1}{3}$	$\frac{1}{5}$	$\frac{1}{3}$	5	1	3	5	5
$\frac{1}{7}$	$\frac{1}{5}$	$\frac{1}{3}$	3	$\frac{1}{3}$	1	3	5
$\frac{1}{9}$	$\frac{1}{7}$	$\frac{1}{3}$	$\frac{1}{2}$	$\frac{1}{5}$	$\frac{1}{3}$	1	$\frac{1}{3}$
$\frac{1}{5}$	$\frac{1}{9}$	$\frac{1}{4}$	$\frac{1}{3}$	$\frac{1}{5}$	$\frac{1}{5}$	3	1

After generating feature relative score criteria matrix, we convert it into fuzzy matrix $F(M)$ as discussed in section 3.2 step-2 which is given in Table 3 and by applying the rest part of the proposed algorithm we obtained the fuzzy geometric mean, fuzzy weight and normalized fuzzy weight as given in step-3, 4 and step-5 of section 3.2. Also fuzzy geometric mean, fuzzy weight and normalized fuzzy weight have been calculated accordingly and given as R, W and \hat{W} respectively in the following part of this section, see Table 4 for fuzzy geometric mean. Features and their corresponding weights generated using fuzzy AHP is given in Table 5.

Table 3. Fuzzified Relative Criteria Score Matrix $F(M)$

(1,1,1)	(2,3,4)	(4,5,6)	(6,7,8)	(2,3,4)	(6,7,8)	(9,9,9)	(4,5,6)
$(\frac{1}{4}, \frac{1}{3}, \frac{1}{2})$	(1,1,1)	(2,3,4)	(2,3,4)	(4,5,6)	(4,5,6)	(6,7,8)	(9,9,9)
$(\frac{1}{6}, \frac{1}{5}, \frac{1}{4})$	$(\frac{1}{4}, \frac{1}{3}, \frac{1}{2})$	(1,1,1)	(4,5,6)	(2,3,4)	(2,3,4)	(2,3,4)	(3,4,5)
$(\frac{1}{8}, \frac{1}{7}, \frac{1}{6})$	$(\frac{1}{4}, \frac{1}{3}, \frac{1}{2})$	$(\frac{1}{6}, \frac{1}{5}, \frac{1}{4})$	(1,1,1)	$(\frac{1}{6}, \frac{1}{5}, \frac{1}{4})$	$(\frac{1}{4}, \frac{1}{3}, \frac{1}{2})$	(1,2,3)	(2,3,4)
$(\frac{1}{4}, \frac{1}{3}, \frac{1}{2})$	$(\frac{1}{6}, \frac{1}{5}, \frac{1}{4})$	$(\frac{1}{4}, \frac{1}{3}, \frac{1}{2})$	(4,5,6)	(1,1,1)	(2,3,4)	(4,5,6)	(4,5,6)
$(\frac{1}{8}, \frac{1}{7}, \frac{1}{6})$	$(\frac{1}{6}, \frac{1}{5}, \frac{1}{4})$	$(\frac{1}{4}, \frac{1}{3}, \frac{1}{2})$	(2,3,4)	$((\frac{1}{4}, \frac{1}{3}, \frac{1}{2}))$	(1,1,1)	(2,3,4)	(4,5,6)
$(\frac{1}{9}, \frac{1}{9}, \frac{1}{9})$	$(\frac{1}{8}, \frac{1}{7}, \frac{1}{6})$	$(\frac{1}{4}, \frac{1}{3}, \frac{1}{2})$	$(\frac{1}{3}, \frac{1}{2}, \frac{1}{1})$	$(\frac{1}{6}, \frac{1}{5}, \frac{1}{4})$	$(\frac{1}{4}, \frac{1}{3}, \frac{1}{2})$	(1,1,1)	$((\frac{1}{4}, \frac{1}{3}, \frac{1}{2}))$
$(\frac{1}{6}, \frac{1}{5}, \frac{1}{4})$	$(\frac{1}{9}, \frac{1}{9}, \frac{1}{9})$	$(\frac{1}{5}, \frac{1}{4}, \frac{1}{3})$	$(\frac{1}{4}, \frac{1}{3}, \frac{1}{2})$	$(\frac{1}{6}, \frac{1}{5}, \frac{1}{4})$	$(\frac{1}{6}, \frac{1}{5}, \frac{1}{4})$	(2,3,4)	(1,1,1)

Table 4. Fuzzy geometric mean R

1.915828444	2.834879198	4.104540197
1.287746006	1.937568857	2.902348207
0.657693414	1.05316091	1.66220773
0.210114336	0.335402083	0.543269277
0.57330184	0.877120503	1.379456825
0.340887314	0.527618073	0.837835759
0.134251106	0.199121832	0.329963518
0.160945546	0.235128544	0.359827768

$$\text{Fuzzy weight } W \begin{pmatrix} 2.95174928 \\ 2.042554357 \\ 1.124354018 \\ 0.362928565 \\ 0.943293056 \\ 0.568780382 \\ 0.221112152 \\ 0.251967286 \end{pmatrix} \text{ Normalized weight } \hat{W} \begin{pmatrix} 0.348628822 \\ 0.241244514 \\ 0.132796583 \\ 0.042865212 \\ 0.111411613 \\ 0.067178211 \\ 0.026115385 \\ 0.029759661 \end{pmatrix}$$

Table 5. Features and their corresponding weights generated using fuzzy AHP.

Features	Rank	Score
Price	1	0.348628822
Battery	2	0.241244514
Camera	3	0.132796583
Processor	6	0.042865212
Screen	4	0.111411613
User Interface	5	0.067178211
Customer Review	8	0.026115385
Warranty details	7	0.029759661

After that we applied second half part of the algorithm to rank the alternatives. In this example we have taken eight mobile phones naming Samsung galaxy M 12 as alternate 1, iphone 12 mini as as alternate 2, Oppo A53 as alternate 3, Vivo Y20G as alternate 4, realme 8 as alternate 5, Mi 11i as alternate 6, Moto G40 as alternate 7 and Infinix hot 115 as alternate 8. And the decision data matrix has been generated on the basis of 8567 ratings and 606 comments for the alternative 1, 159508 ratings and 10944 comments for the alternative 2, 13157 ratings and 1065 comments for the alternative 3, 4791 ratings and 327 comments for the alternative 4, 75193 ratings and 7285 comments for the alternative 5, 5913 ratings and 795 comments for the alternative 6, 37433 ratings and 4059 comments for the alternative 7 and 19646 ratings and 2134 comments for the alternative 8 respectively. Based on the comments and ratings we obtained decision matrix and we calculated the weighted decision matrix Table 6 generated from the data-set by multiplying decision matrix with the weight given in Table 5. Further, we transform each column of Table 7 in wavelet domain by multiplying with Haar wavelet matrix and then we applied the rest part of algorithm 2, which gives the rank score of different alternatives of the smartphone. Further, consider the Table 7 we can observe that Alternate 4 receive the highest rank 1, Alternate 3 secured rank 2, Alternate 2 secured rank 3, Alternate 1 secured rank 4, Alternate 8 secured rank 5, Alternate 6 secured rank 6, Alternate 7 secured rank 7 and Alternate 5 received the lowest rank 8. Also the rank of different alternatives of the smartphone given by the expert and proposed method has been given in Table 8.

Table 6. weighted decision matrix generated from the dataset.

Feat./ Alter.	Camera	Battery	Display	Price	Cust. Rev.	Processor	User Int.	Warranty
Alt 1	0.4913	0.9891	0.4345	1.3248	0.1097	0.0857	0.4367	0.0298
Alt 2	0.5976	0.9409	0.5014	1.4991	0.1175	0.0772	0.3628	0.0298
Alt 3	0.5046	0.9891	0.4568	1.3945	0.1123	0.0943	0.4380	0.0476
Alt 4	0.5046	0.9650	0.4456	1.3597	0.1123	0.0986	0.4367	0.0476
Alt 5	0.4913	1.0132	0.4679	1.3945	0.1123	0.0857	0.4299	0.0476
Alt 6	0.4648	1.0374	0.4902	1.3248	0.1097	0.1072	0.4481	0.0476
Alt 7	0.5046	1.0374	0.4568	1.4294	0.1123	0.0986	0.4555	0.0476
Alt 8	0.5179	0.9891	0.4679	1.4642	0.1123	0.0857	0.4555	0.0476

Table 7. Rank Score of different alternatives of the smartphone.

Features	Score	Rank
Alt 1	0.9080	4
Alt 2	0.96709	3
Alt 3	0.9710	2
Alt 4	0.9803	1
Alt 5	0.0345	8
Alt 6	0.0501	6
Alt 7	0.0358	7
Alt 8	0.0548	5

6 Conclusion

Fuzzy and wavelet based multi-criteria ranking framework has been introduced and applied successfully on real data set of smart phones collected from the e-commerce web site and results for ranking have been tabulated. We presents a novel method for decision making, which include fuzzy AHP, TOPSIS and wavelet theory to solve the problem of product ranking. This study proposes a novel challenge for rating products based on internet reviews. In the problem, alternative items, product features, and product feature weights are calculated based on the customer's suited choice. The issue has a lot of practical implications and deserves further attention. There are several decision-making models and validation approaches available. It is impossible to say whether one technique is better than another, however the presented method has a clear logic and a straightforward calculation procedure. Furthermore, based on the abrupt changes in the data set, the outcomes can be adjusted by providing precise weighting in the decision matrix

Table 8. Comparison of rank of different alternatives of the smartphone given by the expert and proposed method.

Alternatives	Expert 1	Expert 2	Expert 3	Proposed Method
Alt 1	3	3	4	4
Alt 2	4	4	1	3
Alt 3	2	2	2	2
Alt 4	1	1	3	1
Alt 5	7	8	8	6
Alt 6	8	6	7	8
Alt 7	6	7	6	7
Alt 8	5	5	5	5

and employing a different wavelet matrix. Although the accuracy of the proposed method can be modified by choosing different membership functions in fuzzy to change the relative criteria matrix given in Table 1. The different wavelet like Daubechies, Symlet, coifletetc can be used to transform the column of the decision matrix into the wavelet domain.

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