

## MATHEMATICAL MUSINGS II

### Cassius J. Keyser: A Doctrine of Ideals

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**Abstract** American intellectual Cassius Jackson Keyser (1862–1947) set down a personal philosophy of ideals that underpins the essential elements of aspiration and motivation all of us need for a fulfilled existence—as mathematician, as citizen, *sapientia ianua vitae* never ceases to apply. Here we have an intriguing dose of percipience from a notable mathematical writer and once regular commentator on the benefits available from mathematics to the human condition.

Cassius J. Keyser was an American mathematician, educator and expositor with a great interest in philosophical matters as they related to the mathematics and the mathematical community of his era—some of his thoughts still have much relevance today. A firm advocate of the humanistic side to mathematical familiarity, appreciation and study (including research, for specialists), he believed the discipline had much to offer in supporting and maintaining the wellbeing of mankind; the following excerpt from the opening piece of a 1927 text directs and emboldens everyone to set their sights on lofty achievements based on his notion of the ‘unattainable ideal’:

“... Ideals are of two sorts—those that can be realized and those that can not. The former are spurious; the latter, genuine. Genuine ideals are unattainable—they are ever-flying perfects, as Emerson said.<sup>1</sup>

There is a flourishing school of philosophy in which it is held that such ideals, because they are unattainable, tend to beget despair, to depress, to dishearten, to devitalize, to mock legitimate aspiration and defeat the ends of life. The opposite is true. Genuine ideals are beautiful—the supreme ones ineffably beautiful—and beauty is life’s elixir, its chief stimulant. Beauty is the generative principle of what is good in the world.

Unattainable ideals have made possible the great triumphs of the human soul. But for the unattainable ideal of perfect justice, we should not have the Republic of Plato. But for the unattainable ideal of logical perfection, we should be without the miracles of modern Mathematics. But for the unattainable ideal of perfect conquest of Nature, we should not have the marvels of modern Invention and Empirical Science. But for the unattainable ideal of perfect drama, we should not have the works of Shakespere or those of the Greek masters. And so on and on.

Beautiful indeed are the sun and moon and stars, and they give precious light, but the light they give is external. The inner light—the radiance that lures to excellence and illuminates the endless way to perfection—is the light of ideals—of ideals that are unattainable.

There are interesting creatures known as moles. Their eyes are very small and they have but little vision. One variety of these humble animals has a distinguished name—it is the star-nosed mole. In the minute anatomy of its nose there have been found as many as 30,000 receptors. These no doubt serve it well in its hard struggle for existence, unvisited by utopian dreams but dealing always and only, in practician manner, with such concrete realities as are present to its sense. Its life is, however, a mole’s life, dark, subterranean—no “angel of progress, leaning from the far horizon, beckons” it onward to ever better and better things.

<sup>1</sup>Ralph Waldo Emerson (1803–1882)—American philosopher, essayist and poet.

Human beings, too, have receptors and they have eyes, too, better than a mole's. But above all they have the power of *Idealization* giving them dreams and visions of goods beyond the attainable. It is this power that makes humans human; it is this that quickens them to higher life and sustains the toil that creates civilization. Pursuit of the unattainable is the proper vocation of man.

So the philosophy that counsels us to eschew genuine ideals on the alleged ground that, being unattainable, they weaken the springs of life and produce despair, is profoundly ignorant of human nature. As a philosophy of human conduct it is mean. It is at best the philosophy of the star-nosed mole. Its ethical principle resembles that of a beetle—buzz, fight and hit where you can—only, a beetle has wings and can fly a little but that philosophy is wingless—it crawls.” [1, pp. 2–5].



A Photograph of Cassius J. Keyser (Wikimedia Commons)

We could do much worse than to adopt Keyser's strategy in our own mathematical endeavours, at whatever level they occur. In fact it could be argued that authentic research mathematicians adhere to his guiding doctrine as natural instinct—taking the form of a mindset, if you will, which fosters activity in mathematics and crystallises into an inbuilt disposition that safeguards mental stimulation, sustains enjoyment of the subject, and frames the close bond we have with it—lest we place ourselves (consciously or otherwise) in the enervating confusion of *terra nullius* and lose sight of our professional purpose in passing on the baton of knowledge, gaining fresh insights, and clearing new intellectual pathways for others to tread;<sup>2</sup> after all, as Keyser concluded, “It has been truly said that the style is the man. In this connection it is significant

<sup>2</sup>We see echoes of Keyser's mantra—in both tone and substance essentially—within the words of British mathematician, philosopher, logician, and public intellectual Bertrand A.W. Russell (1872–1970); in his oft cited article ‘The Study of Mathematics’ (*The New Quart.: A Rev. Sci. Lit.* No. 1, 31–44 (1907)), he penned the following:

“What is best in mathematics deserves not merely to be learnt as a task, but to be assimilated as a part of daily thought, and brought again and again before the mind with ever renewed encouragement. Real life is, to most men, a long second-best, a perpetual compromise between the ideal and the possible; but the world of pure reason knows no compromise, no practical limitations, no barrier to the creative activity embodying in splendid edifices the passionate aspiration after the perfect from which all great work springs. Remote from human passions, remote even from the pitiful facts of nature, the generations have gradually created an ordered cosmos, where pure thought can dwell as in its natural home, and where one, at least, of our nobler impulses can escape from the dreary exile of the actual world!” (pp. 33–34).

Concession to the reality of, and limitations in, our abilities is inevitable and necessary, of course, but the importance of aiming high, as it were, is locked into the psyche of many mathematicians as a default *modus operandi*—we may struggle with our work, and wrestle with ourselves at times, but our creative gaze is always fixed upwards.

that the philosophy in question is formless. It is so inevitably. For when genuine ideals are condemned as vicious, art is impossible.” (p. 5).

As an aside, and to finish, we note that Keyser discussed the topic at other times:

“... No matter whether an ideal be one of justice or of injustice, of freedom or of tyranny, of beauty or of ugliness, of happiness or of misery, of wisdom or of un-wisdom, of moral good or of moral evil, of power or of impotence, of clarity or of obscurity, of skill or of unskill, of piety or of impiety, or of any other distinction, it will be a *genuine* ideal, if and only if, like a mathematical limit, it admits of being approached through an endless sequence of closer and closer approximations, and is, again like a mathematical limit, incapable of being actually attained. Many “ideals,” so-called in current usage, are attainable. I call them *spurious* ideals, not to imply that they are ignoble, for often they are not, but to distinguish them from genuine ideals, from which they are separated by an unbridgeable chasm of difference. Spurious ideals differ from genuine ones as a high degree of rational plausibility differs from flawless reason or as any attainable form of excellence differs from perfection.

And so it is clear that, as I have elsewhere [Ref.<sup>3</sup>] said, genuine ideals “are not things to gush over or to sigh and sentimentalize about; they are not what would be left if that which is hard in reality were taken away; they are themselves the very flint of reality, ... , hard, cold, intellectual, logical, austere.” ” [2, p. 27].

Keyser, and his ilk, never failed to project and promote mathematics as something that could most assuredly enrich the lives of everyone. This, as has been discovered by thousands of people to their great benefit, remains as true today as it did centuries ago—the discipline may evolve, but human nature has basic wants, needs and traits baked into it which the study of mathematics can touch in meaningful and purposeful ways. Its reach and commonalities of value have aspects that are independent of specifics such as race, gender, religion, wealth, circumstance, society, and so on, giving mathematics an invariant worth through some profound and transcending shared experiences—anyone in doubt is advised to read Francis Edward Su’s award winning 2020 text *Mathematics for Human Flourishing* (Yale University Press, New Haven) which details inspiring testimonies as evidence.

I often find myself wondering if, had Keyser raised a family, any of his descendants would have brought their own philosophies to bear in analyses of modern day life and modern day mathematics—and in doing so continue an academic lineage in re-capturing his level of commentary and observation with trademark poignancy, sagacity, eloquence and wisdom.

## References

- [1] C. J. Keyser, Mole philosophy, *Mole Philosophy and Other Essays*, E.P. Dutton and Co., New York, 1–5 (1927).
- [2] C. J. Keyser, The bearings of mathematics, *Mathematics As a Culture Clue, And Other Essays* (Vol. 1 of *The Collected Works of Cassius Jackson Keyser*), Scripta Mathematica (Yeshiva University), New York, 20–44 (1947).

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<sup>3</sup>Keyser here cites p. 294 of his 1922 text (comprising a series of 21 ‘Lectures for Educated Laymen’) *Mathematical Philosophy: A Study of Fate and Freedom* (E.P. Dutton & Co., New York), where he seems to have first used the metaphor of the limit to make this point under a section titled ‘Mathematical Limit Processes Viewed as Species of Idealization’ as part of Lecture XIV therein (pp. 265–296).