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GENERALIZATION AND APPLICATION OF LOCALLY SEMI-COMPACT SPACES

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Abstract The definition of locally semi-compact spaces originates from the study of semi-compact spaces. In this paper, we obtain some new properties of locally semi-compact spaces and define semi-k spaces as a generalization of locally semi-compact spaces. The relation between s-topological groups and locally semi-compact spaces is also established.

1 Introduction

Generalized open sets play a significant role in General Topology, and they are now the research topics of many topologists worldwide. Levine [1] introduced the concept of semi-open sets and semi-continuity in general topological spaces. A subset A in a topological space X is said to be a *semi-open* set if and only if $A \subset A^{0-}$, or equivalently if there exists an open subset U of X such that $U \subset A \subset U^{-}$. The complement of a semi-open set is said to be a *semi-closed* set ($A^{-0} \subset A$). Every open (respectively closed) set is semi-open (respectively semi-closed), but the converse may not be true. For example, suppose $X = \{a, b, c\}$ and $T = \{\varnothing, X, \{a\}, \{b\}, \{a, b\}\}$. Then (X, T) is a topological space. The sets $\{b, c\}$ and $\{a, c\}$ are semi-open but not open sets. Since the semi-open sets played a significant role in the study of topological spaces, many mathematicians introduced and investigated generalized open sets in topological spaces, such as α-open sets ($A \subset A^{0-0}$) [2], α-closed sets ($A^{0-0} \subset A$) [2], regular-open sets ($A = A^{-0}$) [3], b-open sets [4], pre-regular p-open sets [5].

As applications, the authors [6] used fuzzy semi-open sets and fuzzy semi-continuous mappings to generalize the fuzzy bitopological space and introduced fuzzy soft semi-pre-interior and fuzzy soft semi-pre-closure [7] in fuzzy soft topological space. To minimize the topology conditions for different reasons or preserve some properties under fewer conditions than those on topology, the authors [8] introduced a concept of supra semi-limit points of a set and studied main properties. In the paper [9], the authors examined almost semi-correspondence between known relations of similarity and semi-correspondence and studied this relationship in the family of density-type topologies. The properties of regular closed sets, semi-open sets, regular semi-open sets, pre-open sets, and β-open sets in generalized topological spaces analogous to their properties in topological spaces are also studied in [10].

In this direction, in 1978, Maheshwari and Prasad [11] used semi-open sets to define s-regular spaces. A space X is said to be *s-regular* if, for any closed set F and $x \notin F$, there exist disjoint semi-open sets U and V such that $x \in U$ and $F \subset V$. A space X is said to be *semi-*T_3 if X is s-regular and T_1. A topological space X is said to be *s-normal* [11] if, for every pair of disjoint closed sets A and B of X, there exist disjoint semi-open sets U and V such that $A \subset U$ and $B \subset V$. It is evident that every regular space is s-regular, and every normal space is s-normal. However, the converse need not be true.

One of the critical applications of semi-open sets is compactness. A space X is said to be a *semi-compact* space [12] if every cover of X by semi-open sets has a finite subcover. A space X is said to be *s-paracompact* [13] if every open cover of X has a locally finite semi-open refinement. The authors in [13] also investigated the relationship between s-paracompact spaces and semi-compact spaces.

In 1984, the definition of locally semi-compact spaces [14], which are closely related to semi-compact spaces, was introduced. A space X is said to be *locally semi-compact* if every point of X has an open neighborhood which is a semi-compact subspace of X. Obviously, every semi-compact space is a locally semi-compact space, but the converse may not be true. For example, if $X = \mathbb{N}$ and $T = \{A : A \subset X\}$, then (X, T) is an infinite discrete topological space. Hence, $\{\{x_n\} : x_n \in X\}$ is a semi-open cover of X, but it does not have a finite subcover. Thus, X is locally semi-compact but not semi-compact. Furthermore, the study of locally semi-compact spaces is wide open. In 2014, the authors in [15] studied the s-topological groups, and a wider class of S-topological groups, which are defined using semi-open sets and semi-continuity. In 2015, two types of topological groups, which are irresolute-topological and Irr-topological [16], were introduced and studied. At the same time, many authors use generalized open sets to replace open sets when they study topological groups, such as [17] and [18].

Our aim in this paper is to continue the investigation of locally semi-compact spaces and introduce the definition of semi-k spaces as a generalization of locally semi-compact spaces. One of the cores of investigating spaces using mappings is establishing extensive connections between spaces with specific topological properties. Therefore, we improve upon some results concerning locally semi-compact spaces and show what mappings preserve locally semi-compactness.

Throughout this paper, X and Y are always topological spaces on which no separation axioms are assumed. The set of positive integers is denoted as $\mathbb{N}$. The real line is denoted as $\mathbb{R}$. The interior and semi-interior of A in X are denoted as A^0 and $sop(A)$. The closure and semi-closure of A in X are denoted as $\overline{A}$ and $sclA$. Suppose X and Y are topological spaces, and $f : X \to Y$ is a mapping between X and Y. If $A \subseteq X$, the restriction of f to A is the function $f|_A : A \to f(A)$ defined by $f|_A(x) = f(x)$ for each x in A. Throughout this paper, G denotes a group $(G, *)$ endowed with a topology. If G is a group, then e denotes its identity element. If H is an invariant subgroup of G, then G/H denotes the quotient group. For definitions not defined here, we refer the reader to [19].

2 Locally semi-compact space

Locally semi-compact spaces are one of the essential classes of topological spaces, and many results have been obtained. However, some topological properties, such as hereditary property and preservation under cartesian product, have not been obtained. The connection between locally semi-compact spaces and other spaces is also not obtained. Thus, this section is mainly devoted to studying locally semi-compact spaces and addresses the questions mentioned above.

Definition 2.1. [20] A mapping $f : X \to Y$ is said to be *irresolute* if, for each semi-open set $V \subset Y$, $f^{-1}(V)$ is a semi-open set in X.

Definition 2.2. [19] A mapping $f : X \to Y$ is said to be *almost open* if, for each $y \in Y$, there is $x \in f^{-1}(y)$ such that $f(U)$ is a neighborhood of y in Y whenever U is a neighborhood of x.

Theorem 2.3. *[14] If $f : X \to Y$ is an open irresolute surjection and X is locally semi-compact, then Y is locally semi-compact.*

Since f is an open mapping, it follows that f is an almost open mapping [19], but the converse may not be true. The following is a generalization of theorem 2.3 to locally semi-compact spaces.

Theorem 2.4. *If $f : X \to Y$ is an almost open irresolute surjection and X is locally semi-compact, then Y is locally semi-compact.*

Proof. Suppose y in Y. Since f is an almost open mapping and X is locally semi-compact, it follows that there is x in $f^{-1}(y)$ and a semi-compact neighborhood V containing x such that $f(V)$ is a neighborhood of y. Suppose $\{W_\alpha\}_{\alpha \in I}$ is a semi-open cover of $f(V)$. Since f is an irresolute surjection, it follows that $\{f^{-1}(W_\alpha)\}_{\alpha\in I}$ is a semi-open cover of V. Then there exists a finite semi-open cover $\{f^{-1}(W_1), f^{-1}(W_2), \cdots, f^{-1}(W_r)\}$ of V. Hence, $V \subset \cup_{i=1}^{r} f^{-1}(W_i)$ and $f(V) \subset \cup_{i=1}^{r} W_i$. Therefore, $f(V)$ is a semi-compact neighborhood of y, and Y is locally semi-compact. □

The following example shows that Y need not be locally semi-compact when $f : X \to Y$ is an irresolute surjection and X is a locally semi-compact space. Hence, the hypothesis that f is almost open is essential in Theorem 2.4.

Example 2.5. Suppose $X = \{-1\} \cup (0, 1]$ is a subspace of the Euclidean topology space $\mathbb{R}$. Then X is a locally semi-compact space. Let $Y = \{(x, \sin(1/x)) : 0 < x \le 1\} \cup \{(0, 0)\}$ and Y be a subspace of Euclidean topology space $\mathbb{R}^2$. Suppose V is an open neighborhood of $(0, 0)$. Then there exists an open ball E_r centered at $(0, 0)$, where r is the radius of E_r, such that $E_r \cap Y \subset V$. Then $\{(x, r/3) : x \in \mathbb{R}\} \cap E_r \cap Y$ is an infinite set of V and does not have semi-cluster point in Y. Then V is not semi-compact. Thus, Y is not locally semi-compact. Let $f : X \to Y$ be a following mapping.

$$f(x) = \begin{cases} (0,0), & if\ x = -1 \\ (x, \sin(1/x)), & if\ 0 < x \le 1 \end{cases}$$

Then $f : X \to Y$ is an irresolute surjection, and X is a locally semi-compact space, but the locally semi-compact is not preserved under f.

Definition 2.6. [20] A mapping $f : X \to Y$ is said to be *semi-open* (respectively *pre-semi-open*) if, for each semi-open set $V \subset X$, $f(V)$ is an open (respectively semi-open)set.

Theorem 2.7. *[14] Each locally semi-compact space is a locally compact space.*

Lemma 2.8. *If $f : X \to Y$ is a semi-open continuous bijection and Y is locally compact, then X is locally semi-compact.*

Proof. Suppose $x \in X$ and $y = f(x)$. Then there exists a compact neighborhood U such that $y \in U$ and $x \in f^{-1}(U)$. Then $f^{-1}(U)$ is a neighborhood of x. Suppose $\{V_\alpha : \alpha \in I\}$ is a semi-open cover of $f^{-1}(U)$. Since f is a semi-open surjection, it follows that $\{f(V_\alpha) : \alpha \in I\}$ is an open cover of U. Thus, there is a finite subcover $\{f(V_1), f(V_2), \cdots, f(V_r)\}$ of U. Then there is a finite subcover $\{V_1, V_2, \cdots, V_r\}$ of $f^{-1}(U)$. Therefore, $f^{-1}(U)$ is a compact neighborhood of x, and X is locally semi-compact. □

According to Lemma 2.8 and Theorem 2.7, we obtain the following corollary direectly.

Corollary 2.9. *If $f : X \to Y$ is a semi-open continuous bijection and Y is locally compact, then X is locally compact.*

Lemma 2.10. *If $f : X \to Y$ is a pre-semi-open continuous bijection and Y is locally semi-compact, then X is locally semi-compact.*

Proof. The proof is similar to Lemma 2.8, and it is omitted. □

Theorem 2.11. *[19] Each T_2 locally compact space is a Tychonoff space.*

Since each regular second-countable space is metrizable and each Tychonoff space is regular, by Theorem 2.4, Theorem 2.7 and Theorem 2.11, we arrive at the following theorem.

Theorem 2.12. *Suppose X is a locally semi-compact space and $f : X \to Y$ is an almost open irresolute surjection. Then Y is metrizable when Y is a T_2 second-countable space.*

Corollary 2.13. *Suppose X is a semi-T_2 locally semi-compact space and $f : X \to Y$ is a semi-open irresolute bijection. Then the following properties are equivalent.*
(1) *Y is a semi-T_3 space.*
(2) *Y is an s-regular space.*
(3) *Y is a semi-T_2 space.*
(4) *Y is a semi-T_1 space.*
(5) *Y is a semi-T_0 space.*

(6) *Y is a Tychonoff space.*
(7) *Y is a T_3 space.*
(8) *Y is a regular space*
(9) *Y is a T_2 space.*
(10) *Y is a T_1 space.*
(11) *Y is a T_0 space.*

Proof. It is clear that $(7) \Rightarrow (1) \Rightarrow (2) \Rightarrow (3) \Rightarrow (4) \Rightarrow (5)$. For every distinct y_1 and y_2, there exists distinct x_1 and x_2, such that $y_1 = f(x_1)$ and $y_2 = f(x_2)$. Then there exist disjoint semi-open sets U and V such that $x_1 \in U$ and $x_2 \in V$. Then $f(U)$ and $f(V)$ are open sets and $f(U) \cap f(V) = \varnothing$. Thus, Y is T_2. Since f is a semi-open, it follows that f is almost open. By Theorem 2.4 and Theorem 2.7, Y is locally compact. By Theorem 2.11, Y is a Tychonoff space. Then $(6) \iff (7) \iff (8) \iff (9) \iff (10) \iff (11)$ and $(5) \Rightarrow (6)$. □

Theorem 2.14. *[14] Suppose $A \subset B \subset X$ and B is an α-open set. Then A is semi-compact relative to B if and only if A is semi-compact relative to X.*

Corollary 2.15. *If A and B are locally semi-compact α-open in X, then $A \cup B$ is locally semi-compact.*

Proof. Suppose x in $A \cup B$. We may assume x in A. Then there exists a semi-compact open neighborhood $U \subset A$ such that x in U and $U \subset A \subset A \cup B$. Since each open set is semi-open, by Theorem 2.14, U is semi-compact relative to $A \cup B$. Therefore, $A \cup B$ is locally semi-compact. □

The following example shows that $X \cup Y$ need not be locally semi-compact when X and Y are locally semi-compact spaces.

Example 2.16. Suppose $\mathbb{R}$ is the Euclidean topology space, $X = \{(x, y) \in \mathbb{R}^2 : x > 0\}$ and $Y = \{(0, 0)\}$. Then X and Y are locally semi-compact spaces of $\mathbb{R}^2$. Since $(0, 0) \in X \cup Y$, and $(0, 0)$ does not have a semi-compact open neighborhood in $X \cup Y$, it follows that $X \cup Y$ is not locally semi-compact.

It is well known that the intersection of two semi-open sets need not be semi-open. To investigate this question, we introduce the following property of semi-open sets.

Definition 2.17. A topological space X is said to have the *SOP* property (semi-openness preservation) if, for all open subsets A and B, we have $\overline{A} \cap \overline{B} = \overline{A \cap B}$.

Example 2.18. Every finite T_1 space has the SOP property.

If X has the SOP property, U and V are semi-open sets, then there exist two open sets, A and B, such that $A \subset U \subset \overline{A}$, $B \subset V \subset \overline{B}$. Thus, $A \cap B \subset U \cap V \subset \overline{A} \cap \overline{B} = \overline{A \cap B}$. Then we arrive at the following result directly. If X has the SOP property, then the finite intersection of semi-open sets is a semi-open set of X, and the family of all semi-open sets forms a topology on X.

Proposition 2.19. *If X and Y have the SOP property, then $X \times Y$ has the SOP property.*

Proof. Suppose A and B are open sets in $X \times Y$. Let $A = A_1 \times A_2$ and $B = B_1 \times B_2$. Then $\overline{A \cap B} = \overline{(A_1 \times A_2) \cap (B_1 \times B_2)} = \overline{(A_1 \cap B_1) \times (A_2 \cap B_2)} = \overline{(A_1 \cap B_1)} \times \overline{(A_2 \cap B_2)}$. Since X and Y have the SOP property, it follows that $\overline{(A_1 \cap B_1)} \times \overline{(A_2 \cap B_2)} = (\overline{A_1} \cap \overline{B_1}) \times (\overline{A_2} \cap \overline{B_2})$. Since $(\overline{A_1} \cap \overline{B_1}) \times (\overline{A_2} \cap \overline{B_2}) = (\overline{A_1} \times \overline{A_2}) \cap (\overline{B_1} \times \overline{B_2}) = \overline{A_1 \times A_2} \cap \overline{B_1 \times B_2} = \overline{A} \cap \overline{B}$, it follows that $\overline{A \cap B} = \overline{A} \cap \overline{B}$. Then $X \times Y$ has the SOP property. □

Since each T_2 second-countable locally compact space is paracompact, [19] and locally semi-compact space is locally compact, by Theorem 2.4, we arrive at the following corollary.

Corollary 2.20. *If $f : X \to Y$ is an almost open irresolute surjection, X is a locally semi-compact space, and Y is a semi-T_2 second-countable space with the SOP property, then Y is a paracompact space.*

Lemma 2.21. *If X is a semi-T_2 space with the SOP property, then each semi-compact set is semi-closed in X.*

Proof. Suppose A is a semi-compact set in X and $x \in A$. Fix $y \in X - A$. Since X is a semi-T_2 space, it follows that there exist two semi-open sets, U_x and V_x, such that $x \in U_x$, $y \in V_x$, and $U_x \cap V_x = \varnothing$. Then $\{U_x : x \in A\}$ is a semi-open cover of A. Thus, there exists a finite subcover $\{U_{x_1}, U_{x_2}, \cdots, U_{x_p}\}$ of A and $A \subset \cup_{i=1}^{p} U_{x_i}$. Let $U = \cup_{i=1}^{p} U_{x_i}$ and $V = \cap_{i=1}^{p} V_{x_i}$. Since X has the SOP property, it follows that V is a semi-open set and $U \cap V = \varnothing$. Then $A \cap V = \varnothing$. Therefore, A is semi-closed in X. □

Lemma 2.22. *If X is a semi-compact space and Y is semi-closed in X, then Y is semi-compact.*

Proof. Suppose $\{A_\alpha : \alpha \in I\}$ is a semi-open cover of Y. For each A_α, there exists an open set $B_\alpha \subset Y$ such that $B_\alpha \subset A_\alpha \subset \overline{B_\alpha}$. Then there exists an open set C_α in X such that $B_\alpha = C_\alpha \cap Y$. Thus, $\overline{A_\alpha} \subset \overline{B_\alpha} \subset \overline{C_\alpha}$. Let $D_\alpha = C_\alpha \cup A_\alpha$. Then $D_\alpha \cap Y = A_\alpha$. Since $C_\alpha \subset D_\alpha \subset \overline{D_\alpha}$ and $\overline{D_\alpha} = \overline{C_\alpha \cup A_\alpha} = \overline{C_\alpha}$, it follows that D_α is semi-open in X. Then $\{D_\alpha : \alpha \in I\} \cup \{X - Y\}$ is a semi-open cover of X. There exists a semi-open cover $\{D_1, D_2, \cdots, D_r\} \cup \{X - Y\}$ in X. Therefore, there exists a semi-open cover $\{A_1, A_2, \cdots, A_r\}$ of Y, and Y is semi-compact. □

The following result is an immediate consequence of Lemma 2.21 and Lemma 2.22.

Theorem 2.23. *Suppose X is a semi-compact semi-T_2 space with the SOP property. Then $A \subset X$ is semi-closed if and only if A is semi-compact.*

Lemma 2.24. *If X is a semi-compact semi-T_2 space with the SOP property, then X is s-regular.*

Proof. The proof is similar to the proof of Lemma 2.21 and thus omitted. □

The following theorem will be helpful to show the next result.

Theorem 2.25. [10] *Suppose $A \subset B \subset X$, where X is a topological space. Then*
(1) *If A is semi-open in X, then A is semi-open in B.*
(2) *If A is semi-open in B and B is semi-open in X, then A is semi-open in X.*

Lemma 2.26. *If $A \subset B \subset X$, A is semi-closed in B, and B is semi-closed in X, then A is semi-closed in X.*

Proof. If x is not in A, then x in $X - B$ or x in $B - A$. Suppose x in $X - B$. Since B is semi-closed in X, it follows that there is a semi-open set U in X such that x in U and $U \cap B = \varnothing$, and hence A is semi-closed in X. Suppose $x \in B - A$. Since A is semi-closed in B, it follows that there is a semi-open set V in B such that x in V and $V \cap A = \varnothing$. Then $V \subset B \subset X$ and $V \subset sop(B) \subset X$. By Theorem 2.25, V is a semi-open set of X. Therefore, A is semi-closed in X. □

Corollary 2.27. *Suppose X is locally semi-compact semi-T_2 with the SOP property and Y is α-open in X. Then Y is locally semi-compact when Y is semi-closed in X.*

Proof. For each $x \in Y$, there exists a semi-compact open set U in X such that $x \in U$. By Lemma 2.21, U is semi-closed in X. Since Y is semi-closed in X, it follows that $U \cap Y$ is semi-closed in X. Then $U \cap Y$ is semi-closed in U. By Lemma 2.22, $U \cap Y$ is semi-compact in U. Since U is open in X, it follows that U is α-open. By Theorem 2.14, $U \cap Y$ is semi-compact in X. Then $U \cap Y$ is semi-compact in Y. Therefore, Y is locally semi-compact. □

Lemma 2.28. *If Y is semi-open in X and X is a semi-T_2 space with the SOP property, then Y is a semi-T_2 space.*

Proof. Suppose $x \neq y$, x and y in Y. Then there exists a semi-open set $U \subset X$ containing x and a semi-open set $V \subset X$ containing y such that $U \cap V = \varnothing$. Since X has the SOP property and Y is semi-open, it follows that $Y \cap U$ and $Y \cap V$ are semi-open in X. According to Theorem 2.25, $Y \cap U$ and $Y \cap V$ are semi-open in Y and $(Y \cap U) \cap (Y \cap V) = \varnothing$. Therefore, Y is a semi-T_2 space. □

Theorem 2.29. *Suppose X is a locally semi-compact space with the SOP property. Then X is s-regular when X is semi-T_2.*

Proof. Suppose $A \subset X$ is closed and $x \notin A$. Then there exists an open set U containing x such that $U \cap A = \emptyset$. Since X is locally semi-compact, it follows that there exists a semi-compact open neighborhood V in X containing x. According to Lemma 2.28, V is semi-T_2. Let $W = U \cap V$. Then W is semi-open in X. By Theorem 2.25, W is semi-open in V. Then $F = V - W$ is semi-closed in V. By Lemma 2.21, V is semi-closed in X. By Lemma 2.26, F is semi-closed in X. By Lemma 2.24, V is s-regular. Since $x \notin F$, it follows that there exists a semi-open set G in V containing x and a semi-open set H in V containing F such that $G \cap H = \emptyset$. By Theorem 2.25, G is semi-open in X. Let $C = V - H$. Then C is semi-closed in V. By Lemma 2.26, C is semi-closed in X and $X - C$ is semi-open. Then $x \in G \subset C \subset W \subset U$ and $A \subset (X - C)$. Therefore, $(X - C) \cap G = \emptyset$ and X is s-regular. □

Since each T_1 s-regular space is a semi-T_3 space and according to Theorem 2.29, we obtain the following corollary directly.

Corollary 2.30. *Suppose X is a T_1 locally semi-compact space with the SOP property. Then the following properties are equivalent.*
(1) *X is a semi-T_3 space.*
(2) *X is an s-regular space.*
(3) *X is a semi-T_2 space.*

Theorem 2.31. *If X and Y are locally semi-compact spaces with the SOP property, then $X \times Y$ is locally semi-compact.*

Proof. Suppose $x_0 \in X$ and $y_0 \in Y$. Since X and Y are locally semi-compact spaces, it follows that there is a semi-compact neighborhood U in X such that $x_0 \in U$ and a semi-compact neighborhood V in Y such that $y_0 \in V$. Suppose $\mathscr{A}$ is a semi-open cover of $U \times V$. For each $x \in U$, $\{x\} \times V$ is semi-compact in $U \times V$. Then $\mathscr{A}$ is a semi-open cover of $\{x\} \times V$ and there exists a finite cover $\mathscr{A}_x \subset \mathscr{A}$ of $\{x\} \times V$. Let $\mathscr{A}_x = \{U_{x_1} \times V_{x_1}, U_{x_2} \times V_{x_2}, \cdots, U_{x_r} \times V_{x_r}\}$. Since X has the SOP property, it follows that $W_x = \cap_{i=1}^{r} U_{x_i}$ containing x is semi-open. Then $W_x \times V = W_x \times (V_{x_1} \cup V_{x_2} \cup \cdots \cup V_{x_r}) \subset U_{x_1} \times V_{x_1} \cup U_{x_2} \times V_{x_2} \cup \cdots U_{x_r} \times V_{x_r}$ and $\mathscr{A}_x$ is a semi-open cover of $W_x \times V$. Since $\{W_x : x \in U\}$ is a semi-open cover of U and U is semi-compact, it follows that there exists a semi-open cover $\mathscr{W} = \{W_{x_1}, W_{x_2}, \cdots, W_{x_s}\}$ of U. Then there exists a finite semi-open cover $\mathscr{A}_{x_i}$ of $W_{x_i} \times V$ for each $W_{x_i} \in \mathscr{W}$ and $i = 1, 2, \cdots, s$. Let $\mathscr{A}_0 = \mathscr{A}_{x_1} \cup \mathscr{A}_{x_2} \cup \cdots \cup \mathscr{A}_{x_s}$. Then $U \times V = (W_{x_1} \cup W_{x_2} \cup \cdots \cup W_{x_s}) \times V \subset \cup_{A \in \mathscr{A}_0} A$. Thus, $\mathscr{A}_0 \subset \mathscr{A}$ is a finite semi-open cover of $U \times V$, and $U \times V$ is semi-compact. Therefore, $X \times Y$ is a locally semi-compact space. □

Corollary 2.32. *Suppose each X_i is a locally semi-compact space with the SOP property for $i \in \mathbb{N}$. Then $X = \prod_{i \in \mathbb{N}} X_i$ is locally semi-compact when there are only finite many X_i are not semi-compact.*

Proof. Suppose $X_1, X_2, \cdots, X_r$ are locally semi-compact but not semi-compact, and $x = (x_i)_{i \in \mathbb{N}} \in X$. For $x_i \in X_i$, there exists a semi-compact open neighborhood U_i in X_i such that $x_i \in U_i$ for $i \in \mathbb{N}$. By Theorem 2.31, $\prod_{i \in \mathbb{N}} U_i$ is a semi-compact open neighborhood of x, where $U_i = X_i$ for $i > r$. Therefore, X is locally semi-compact. □

The following example shows that each $X_i (i \in \mathbb{N})$ is a locally semi-compact space, but $\prod_{i \in \mathbb{N}} X_i$ need not be locally semi-compact. Theorem 2.33 and Lemma 2.34 will be helpful to show the following example, so we introduce them first.

Theorem 2.33. *[20] If $f : X \to Y$ is an open continuous, then f is irresolute.*

Lemma 2.34. *Each discrete space is semi-compact if and only if it is finite.*

Proof. Suppose X is a semi-compact discrete. Assume that X is infinite. Since X is a discrete space, then $\{\{x\} : x \in X\}$ is an open cover of X. Then $\{\{x\} : x \in X\}$ is a semi-open cover of X. Since X is semi-compact, it follows that there exists a finite cover $\{\{x_1\}, \{x_2\}, \cdots, \{x_r\}\}$ such that $\cup_{i=1}^{r} \{x_i\} = X$. Thus, $X = \{x_1, x_2, \cdots, x_r\}$ is a finite set. It contradicts the assumption, and hence, X is a finite space. Conversely, suppose X is a discrete finite space. Let $X = \{x_1, x_2, \cdots, x_p\}$. Then each $\{x_i\}$ is open when $i = 1 \cdots p$. Thus, $\{x_i\}$ is a semi-open set. Let $\{U_\alpha : \alpha \in I\}$ be a semi-open cover of X. Then there exists a finite semi-open cover $\{\{x_1\}, \{x_2\}, \cdots, \{x_p\}\}$ of X. Therefore, X is semi-compact. □

Example 2.35. Suppose F_1 is the discrete topology of $\mathbb{N}$, and $X = \prod_{i\in\mathbb{N}} \mathbb{N}_i$ for each $\mathbb{N}_i = \mathbb{N}$ and $i \in \mathbb{N}$. Then $\mathbb{N}_i$ is a locally semi-compact space. Let $F_2 = \prod_{i\in\mathbb{N}} U_i$, where each U_i is open in $\mathbb{N}_i$ and $U_i \neq \mathbb{N}_i$ for only finitely many i. Then F_2 is a product topology of X. If $P_i : X \to \mathbb{N}_i$ is a projection, then P_i is an open continuous surjection. By Theorem 2.33, f is irresolute. Suppose V is a semi-compact and open set of X, and by definition of F_2, a projection P_i exists such that $P_i(V) = \mathbb{N}_i$ is a semi-compact set. Since $\mathbb{N}$ is a discrete set, and by Lemma 2.34, it follows that $P_i(V) = \mathbb{N}_i = \mathbb{N}$ is a finite set. It contradicts the fact, and hence, $\prod_{i\in\mathbb{N}} X_i$ is not locally semi-compact.

It is well known that discrete spaces are compact spaces if and only if it is finite spaces [19], and by Lemma 2.34, we arrive at the following theorem.

Theorem 2.36. *If X is a discrete space, then the following properties are equivalent.*
(1) *X is a finite space.*
(2) *X is a compact space.*
(3) *X is a semi-compact space.*

3 Semi-k space

This section begins by generalizing locally semi-compact spaces to semi-k spaces. Later, analogous results of locally semi-compact spaces will be presented under the generalization. We first obtain the following lemma to better introduce the definition of semi-k space.

Lemma 3.1. *[14] If A is semi-compact relative to X and B is semi-closed in X, then $A \cap B$ is semi-compact relative to X.*

Lemma 3.2. *Suppose X is a locally semi-compact semi-T_2 space with the SOP property. Then $A \subset X$ is a semi-closed set if and only if for each semi-compact set $C \subset X$, $A\cap C$ is a semi-closed set of X.*

Proof. Suppose A is not a semi-closed set. Then there is a point $x \in sclA$ and $x \notin A$. Since X is a locally semi-compact space, it follows that there is a semi-compact neighborhood B such that $x \in B$. Since $x \in sclA$, it follows that $B \cap A \neq \varnothing$ and $x \in sclA \cap sclB$. For each semi-open neighborhood C containing x. Since X has the SOP property, it follows that $C \cap B$ is semi-open and $x \in C\cap B$. Since $x \in sclA$, it follows that $C\cap B\cap A \neq \varnothing$ and $x \in scl(A\cap B)$. Since $x \notin A\cap B$, it follows that $A \cap B$ is not a semi-closed set.

Conversely, suppose A is a semi-closed set, and D is a semi-compact set. By Lemma 3.1, $A \cap D$ is semi-compact. By Lemma 2.21, $A \cap D$ is semi-closed. □

Definition 3.3. A space X is said to be a *semi-k* space if, for each $A \subset X$, A is semi-closed if and only if $A \cap B$ is a semi-closed set of X for any semi-compact B of X.

According to Lemma 3.2, we obtain the following corollary directly. Next, Example 3.5 shows that semi-k spaces need not be locally semi-compact.

Corollary 3.4. *If X is a locally semi-compact semi-T_2 space with the SOP property, then X is a semi-k space.*

Example 3.5. Let $\mathbb{R}$ be the real line, and P be the set of all irrational numbers. Let T be the usual topology of $\mathbb{R}$, and $\mathscr{A} = T \cup \{\{x\} : x \in P\}$ is a base that defines a topology on $\mathbb{R}$. The $\mathbb{R}$ with the topology generated by $\mathscr{A}$ is called the Michael line [19] and is denoted by X. It is clear that X is first countable and not locally compact. If X is locally semi-compact, then X is locally compact, a contradiction. Then X is not locally semi-compact.

Next, we prove X is a semi-k space. Suppose $A \subseteq X$ is not closed. Then there exists x in $d(A)$ such that $x \notin A$. Since X is a first countable space, there is a countable open neighborhood basis $\{V_i : i \in \mathbb{N}\}$ for x. Suppose $U_n = \cap_{i=1}^{n} V_i$ and $n \in \mathbb{N}$. Then $U_{n+1} \subset U_n$ and $\{U_n : n \in \mathbb{N}\}$ is a countable open neighborhood basis of x. Thus, $\{U_n : n \in \mathbb{N}\}$ is a countable semi-open neighborhood basis of x. Since $x \in d(A)$, it follows that $U_n \cap (A - \{x\}) \neq \varnothing$. Suppose x_n in $U_n \cap (A - \{x\})$ and $B = \{x_n : n \in \mathbb{N}\} \cup \{x\}$. Suppose $\{W_\alpha : \alpha \in I\}$ is a semi-open cover of

B. Then there exists W_α such that $x \in W_\alpha$. Thus, there exists $U_{n_0} \in \{U_n : n \in \mathbb{N}\}$ such that x in U_{n_0} and $U_{n_0} \subseteq W_\alpha$. For each $n \geq n_0$, $U_n \subset U_{n_0}$, it follows that x_n in U_n and $U_n \subset W_\alpha$. Then there exists a finite semi-open cover $\{W_1, \cdots, W_t\}$ of $\{x_n : n \leq n_0\}$. Then $\{W_\alpha, W_1, \cdots, W_t\}$ is a finite semi-open cover of B. Thus, B is a semi-compact space and $B \cap A = \{x_n : n \in \mathbb{N}\}$. Since x is a semi-closure point of $\{x_n : n \in \mathbb{N}\}$ and x is not in it, it follows that $\{x_n : n \in \mathbb{N}\}$ is not a semi-closed set in X. Therefore, X is a semi-k space.

Since the intersection of two semi-closed sets is semi-closed, and according to Definition 3.3 and Lemma 2.21, we obtain the following corollary directly.

Corollary 3.6. *If semi-T_2 space X has the SOP property, and each $A \subset X$ is semi-closed, providing $B \cap A$ is a semi-closed set of X for any B is semi-compact, then X is a semi-k space.*

Proposition 3.7. *Suppose X is semi-T_2 with the SOP property. Then the following properties are equivalent.*
(1) *X is a semi-k space.*
(2) *Each $B \subset X$ is semi-open, providing $A \cap B$ is a semi-open set of A for any A , which is semi-compact.*
(3) *Each $B \subset X$ is semi-closed, providing $A \cap B$ is a semi-closed set of X for any A , which is semi-compact.*

Proof. It is clear that $(1) \Rightarrow (3)$ by Definition 3.3 and $(3) \Rightarrow (1)$ by Corollary 3.6.

$(1) \Rightarrow (2)$. Since $A \cap B$ is a semi-open set of A, it follows that $A - (A \cap B) = A \cap (X - B)$ is a semi-closed set of A. Since X is semi-T_2 with the SOP property and A is semi-compact in X, and by Lemma 2.21, it follows that A is semi-closed in X. By Lemma 2.26, $A \cap (X - B)$ is a semi-closed set of X. Since X is a semi-k space, it follows that $X - B$ is semi-closed. Therefore, B is a semi-open set of X.

$(2) \Rightarrow (1)$. Suppose $A \subset X$ is a semi-compact set. Then A is semi-closed in X. If C is a subset of X and $A \cap C$ is a semi-closed set of X, then $A \cap C$ is a semi-closed set of A. Thus, $A - A \cap C = A \cap (X - C)$ is a semi-open set of A, and $X - C$ is a semi-open set of X. Therefore, C is a semi-closed set of X. By Corollary 3.6, X is a semi-k space. □

Here are some basic properties of semi-k spaces and show what mappings preserve semi-k spaces.

Proposition 3.8. *If each X_r is a semi-k space, then the topological sum $X = \oplus X_r$ is a semi-k space.*

Proof. Suppose A is a semi-closed set of X, and B is a semi-compact set of X. Since X is a topological sum, it follows that $A \cap X_r$ is a semi-closed set of X_r. Suppose $\{U_\alpha : \alpha \in I\}$ is a semi-open cover of $B \cap X_r$ in X_r. For each U_α, there exists a semi-open set $V_\alpha \subset X$ such that $U_\alpha = V_\alpha \cap X_r$ and $\{V_\alpha : \alpha \in I\}$ is a semi-open cover of B in X. Since B is a semi-compact set of X, it follows that there exists a finite semi-open subcover $\{V_1, V_2, \cdots, V_t\}$ of B. Then there exists a finite semi-open subcover $\{V_1 \cap X_r, V_2 \cap X_r, \cdots, V_t \cap X_r\} = \{U_1, U_2, \cdots, U_t\}$ of $B \cap X_r$ in X_r, and $B \cap X_r$ is a semi-compact set of X_r. Since X_r is a semi-k space, $(B \cap X_r) \cap (A \cap X_r) = B \cap A \cap X_r$ is a semi-closed set of X_r, and $\cup(A \cap B \cap X_r) = A \cap B$ is a semi-closed set of X. Suppose C is a subset of X and D is a semi-compact set of X. If $C \cap D$ is a semi-closed set of X, then $C \cap D \cap X_r = (C \cap X_r) \cap (D \cap X_r)$ is a semi-closed set of X_r and $D \cap X_r$ is a semi-compact set of X_r. Since X_r is a semi-k space, it follows that $C \cap X_r$ is a semi-closed set of X_r. Thus, $\cup(C \cap X_r) = C \cap X = C$ is a semi-closed set of X. Consequently, $X = \oplus X_r$ is a semi-k space. □

Proposition 3.9. *If X is a semi-k space and Y is regular-open in X, then Y is semi-k in X.*

Proof. Suppose A is a semi-closed set of Y, and C is a semi-compact subset of Y. Since Y is regular-open, it follows that $Y = Y^{-0}$. Then $Y^0 = Y^{-0}$ and $Y = Y^0$. Thus, $Y \subset Y^{0-}$ and $Y^0 \subset Y^{0-0}$. Then $Y \subset Y^{0-0}$ and Y is α-open. According to Lemma 2.14, C is a semi-compact subset of X. Since $Y = Y^{-0}$, it follows that $Y^{-0} \subset Y$. Then Y is semi-closed. By Lemma 2.26, A is a semi-closed set of X. Since X is a semi-k space, it follows that $A \cap C$ is a semi-closed set of X. Thus, $A \cap C$ is a semi-closed set of Y. Suppose $B \subset Y$, D is a semi-compact subset of Y, and $B \cap D$ is a semi-closed subset of Y, then $B \cap D$ is a semi-closed subset of X. By

Lemma 2.14, D is a semi-compact subset of X. Since X is a semi-k space, it follows that B is a semi-closed subset of X. Therefore, B is a semi-closed subset of Y, and Y is a semi-k set of X. □

Lemma 3.10. *If X is a semi-k space and $f : X \to Y$ is a bijective, irresolute and pre-semi-open mapping, then Y is a semi-k space.*

Proof. Suppose A is a semi-closed set of Y and B is a semi-compact set of Y. Since f is irresolute, it follows that $f^{-1}(Y - A) = X - f^{-1}(A)$ is a semi-open set of X and $f^{-1}(A)$ is semi-closed. Suppose $\{U_\alpha : \alpha \in I\}$ is a semi-open cover of $f^{-1}(B)$ in X. Then $f^{-1}(B) \subset \cup_{\alpha\in I} U_\alpha$ and $B \subset \cup_{\alpha\in I} f(U_\alpha)$. Then there exists a finite semi-open subcover $\{f(U_1), f(U_2), \cdots, f(U_r)\}$ such that $f^{-1}(B) \subset \cup_{i=1}^{r} U_i$. Thus, $f^{-1}(B)$ is a semi-compact set of X. Since X is a semi-k space, it follows that $f^{-1}(B) \cap f^{-1}(A)$ is a semi-closed set of X and $X - f^{-1}(B) \cap f^{-1}(A)$ is a semi-open set. Since f is bijective pre-semi-open mapping, it follows that $f(X - f^{-1}(B) \cap f^{-1}(A)) = Y - B \cap A$ is a semi-open set of Y. Thus, $B \cap A$ is a semi-closed set of Y.

Suppose C is a subset of Y, D is a semi-compact set of Y, and $C \cap D$ is a semi-closed set in Y. Then $f^{-1}(C \cap D) = f^{-1}(C) \cap f^{-1}(D)$ is semi-closed in X, and $f^{-1}(D)$ is semi-compact in X. Thus, $f^{-1}(C)$ is semi-closed, and $X - f^{-1}(C)$ is semi-open in X. Then $f(X - f^{-1}(C)) = Y - C$ is semi-open in Y, and C is semi-closed in Y. Therefore, Y is a semi-k space. □

It is well known that if $f : X \to Y$ is bijective, irresolute, and pre-semi-open, then f is said to be semi-homeomorphism, and by Lemma 3.10, we obtain the following corollary directly.

Corollary 3.11. *If X is a semi-k space and $f : X \to Y$ is a semi-homeomorphism mapping, then Y is a semi-k space.*

Since each locally semi-compact semi-T_2 space with the SOP property is a semi-k space, it follows that the following remark can be obtained directly by Lemma 3.10.

Remark 3.12. If X is a locally semi-compact semi-T_2 space with the SOP property and $f : X \to Y$ is an irresolute, pre-semi-open and bijective mapping, then Y is a semi-k space.

It is well known that we connect various class spaces using mappings as a linkage. This way, we will use irresolute and pre-semi-open mappings to investigate the relationships among locally semi-compact, semi-k, and k spaces. The following Theorem implies the connection between semi-k spaces and locally semi-compact spaces, and Theorem 3.16 implies the connection between semi-k spaces and k spaces.

Theorem 3.13. *If Y is a semi-T_2 semi-k space with the SOP property, then there exists a locally semi-compact semi-T_2 space X with the SOP property and a bijective irresolute and pre-semi-open mapping f such that $f(X) = Y$.*

Proof. Let the family formed by all semi-compact sets in Y be $Z = \{A_\alpha \times \{\alpha\} : \alpha \in I\}$, and $(A_\alpha \times \{\alpha\}) \cap (A_\beta \times \{\beta\}) = \varnothing$, for $\alpha \neq \beta$, $\alpha \in I$, and $\beta \in I$. Suppose the set $X = \oplus_{\alpha\in I} A_\alpha \times \{\alpha\}$ is a topological sum formed by all sets of Z. Each set $G \subset X$ is a semi-open(semi-closed) set if and only if $G \cap (A_\alpha \times \{\alpha\})$ is a semi-open(semi-closed) set in $A_\alpha \times \{\alpha\})$ for each $\alpha \in I$. Then X is locally semi-compact. For each $\alpha \in I$, let $f : X \to Y$ be a surjective mapping and $f|_{A_\alpha} : A_\alpha \times \{\alpha\} \to K_\alpha$ be a semi-homeomorphism mapping. If $M \subset Y$ is a semi-open set, then $(Y - M) \subset Y$ is a semi-closed set. Since Y is a semi-k space, it follows that $(Y - M) \cap A_\alpha$ is semi-closed in Y. Then $f^{-1}(Y - M) \cap A_\alpha \times \{\alpha\} = (Y - M) \cap A_\alpha \times \{\alpha\}$ is a semi-closed set of X. Thus, $f^{-1}(Y - M) = X - f^{-1}(M)$ is a semi-closed set, and $f^{-1}(M)$ is a semi-open set. Hence, f is an irresolute mapping. Suppose U is a semi-open set of X. Then $X - U$ is semi-closed. Then $(X - U) \cap A_\alpha \times \{\alpha\}$ is semi-closed. Then $f(X - U) \cap A_\alpha \times \{\alpha\} = (X - U) \cap A_\alpha \times \{\alpha\}$ is semi-closed. Since Y is a semi-k space, it follows that $f(X - U) = Y - f(U)$ is semi-closed, and $f(U)$ is a semi-open set. Therefore, f is a pre-semi-open mapping. □

We arrive at the following theorem according to Theorem 2.31, Remark 3.12, and Theorem 3.13.

Theorem 3.14. *Suppose X and Y are semi-T_2 spaces with the SOP property. Then $X \times Y$ is a semi-k space when X is a semi-k space, and Y is a locally semi-compact space.*

Definition 3.15. [21] A space X is said to be a k space if X is T_2 and each $A \subset X$ is closed, providing $B \cap A$ is closed in X for any compact set B.

Theorem 3.16. *Suppose $f : X \to Y$ is a pre-semi-open bijection, and the inverse image of each semi-open subset of Y is open in X. Then X is a k space if and only if Y is a semi-T_2 semi-k space.*

Proof. Suppose X is a k space. Let A be a subset of Y, and $A \cap C$ is a semi-closed set in Y, which C is semi-compact in Y. Then $f^{-1}(Y - A \cap C) = X - f^{-1}(A) \cap f^{-1}(C)$ is an open set in X, and $f^{-1}(A) \cap f^{-1}(C)$ is a closed set in X. Suppose $\{F_\alpha : \alpha \in I\}$ is an open cover of $f^{-1}(C)$. Since $f : X \to Y$ is a pre-semi-open mapping, it follows that $\{f(F_\alpha) : \alpha \in I\}$ is a semi-open cover of C. There is a finite subcover $\{f(F_1), f(F_2), \cdots, f(F_r)\}$ of C. Then $\{F_1, F_2, \cdots, F_r\}$ is an open cover of $f^{-1}(C)$, and $f^{-1}(C)$ is a compact set of X. Since X is a k space, it follows that $f^{-1}(A)$ is a semi-closed set of X and $X - f^{-1}(A)$ is a semi-open set. Then $f(X - f^{-1}(A)) = Y - A$ is semi-open, and A is semi-closed in Y.

Suppose $G \subset Y$ is semi-closed and $x \notin f^{-1}(G)$. Thus, $f(x) \notin G$. Then there is a semi-open set E in Y such that $E \cap G = \varnothing$ and $f(x) \in E$. Thus, $x \in f^{-1}(E)$. Since the inverse image of each semi-open subset of Y is open in X, it follows that $f^{-1}(E)$ is semi-open in X and $f^{-1}(E) \cap f^{-1}(G) = \varnothing$. Then $f^{-1}(G)$ is closed in X. Suppose H in Y is semi-compact and $\{W_\beta : \beta \in J\}$ is an open cover of $f^{-1}(H)$. Hence, $\{W_\beta : \beta \in J\}$ is a semi-open cover of $f^{-1}(H)$. Thus, $\{f(W_\beta) : \beta \in J\}$ is a semi-open cover of H. Then there is a finite cover $\{f(W_1), f(W_2), \cdots, f(W_s)\}$ of H. Thus, there is a finite cover $\{W_1, W_2, \cdots, W_s\}$ of $f^{-1}(H)$. Then $f^{-1}(H)$ is compact in X. Since X is a k space, it follows that $f^{-1}(G) \cap f^{-1}(H)$ is closed in X. Since f is pre-semi-open bijection, it follows that $G \cap H$ is semi-closed in X. Then Y is a semi-k space.

For every distinct y_1 and y_2 in Y, there exists distinct x_1 and x_2 in X such that $y_1 = f(x_1)$ and $y_2 = f(x_2)$. Since X is T_2, it follows that there exist disjoint open sets W and R such that $x_1 \in W$ and $x_2 \in R$. Then W and R are semi-open sets, and $y_1 \in f(W)$ and $y_2 \in f(R)$. Since f is a pre-semi-open bijection, it follows that $f(W)$ and $f(R)$ are disjoint semi-open sets. Then Y is semi-T_2.

Conversely, suppose Y is a semi-k space. Let $A \subset X$ and $A \cap C$ be a closed set in X, where C is compact in X. Thus, $X - A \cap C$ is an open set of X. Then $f(X - A \cap C) = Y - f(A \cap C)$ is semi-open, and $f(A \cap C) = f(A) \cap f(C)$ is semi-closed in Y. Suppose $\{E_\gamma : \gamma \in L\}$ is a semi-open cover of $f(C)$. Thus, $\{f^{-1}(E_\gamma) : \gamma \in L\}$ is an open cover of C. Then there exists a finite cover $\{f^{-1}(E_1), f^{-1}(E_2), \cdots, f^{-1}(E_p)\}$. Then $\{E_1, E_2, \cdots, E_p\}$ is a semi-open cover of $f(C)$, and $f(C)$ is semi-compact in Y. Since Y is a semi-k space, it follows that $f(A)$ is semi-closed, and $Y - f(A)$ is semi-open in Y. Then $f^{-1}(Y - f(A)) = X - A$ is open in X. Hence, A is closed in X. For every distinct x_1 and x_2 in X, there exists distinct y_1 and y_2 in Y such that $y_1 = f(x_1)$ and $y_2 = f(x_2)$. Since Y is semi-T_2, it follows that there exist disjoint semi-open sets U and V such that $y_1 \in U$, $y_2 \in V$. Then $x_1 \in f^{-1}(U)$, $x_2 \in f^{-1}(V)$, and $f^{-1}(U) \cap f^{-1}(V) = \varnothing$. Since $f^{-1}(U)$ and $f^{-1}(V)$ are open in X, it follows that X is T_2. Therefore, X is a k space. □

To complete the following proof, we need to introduce a new definition. First, we recall some basic notations. A collection $\mathscr{B} = \{B_\alpha : \alpha \in I\}$ of subsets of a topological space X is said to be *s-locally finite* [22] if each $x \in X$ has a semi-open set U containing x, and U intersects at most finitely many members of $\mathscr{B}$. A collection $\mathscr{C} = \{C_\beta : \beta \in J\}$ of subsets of a topological space X is said to be *net* [19] if for each $x \in V$, V is an open set of X, and there exists $C_\beta \in \mathscr{C}$ such that $x \in C_\beta \subset V$.

Definition 3.17. A collection $\mathscr{A} = \{A_\alpha : \alpha \in I\}$ of subsets of a topological space X is said to be *semi-net*, if U is a semi-open set of X and each $x \in U$, there exists $A_\alpha \in \mathscr{A}$ such that $x \in A_\alpha \subset U$.

Definition 3.18. A collection $\mathscr{A} = \cup_{n \in \mathbb{N}} \mathscr{A}_n$ of subsets of a topological space X is said to be a σ *s-locally finite semi-net* if each $\mathscr{A}_n$ is an s-locally finite semi-net.

Since each open set is a semi-open set and based on the definition of the net, we arrive at the following remark directly. But example 3.20 shows that the converse of the remark may not be true.

Remark 3.19. Every semi-net in a topological space is a net.

Example 3.20. Suppose $X = \{1,2,3\}$ and $F_1 = \{\varnothing, X, \{1\}, \{2\}, \{1,2\}\}$ is a topology of X. Thus, $F_2 = \{\varnothing, X, \{1\}, \{2\}, \{1,2\}, \{1,3\}, \{2,3\}\}$ is the family of all semi-open sets of X. Suppose $F_3 = \{X, \{1\}, \{2\}\}$. Then F_3 is a net of X. Notes that $\{1,3\}$ is a semi-open set and $3 \in \{1,3\}$. For each set A in F_3 does not satisfy the condition that $3 \in A \subset \{1,3\}$. Therefore, F_3 is not a semi-net of X.

According to Definition 3.17, we obtain the following corollaries directly.

Corollary 3.21. *If $\mathscr{A} = \{A_\alpha : \alpha \in I\}$ is a semi-net of X and $f : X \to Y$ is an irresolute surjection, then $\{f(A_\alpha) : \alpha \in I\}$ is a semi-net of Y.*

Corollary 3.22. *If $\mathscr{A} = \{A_\alpha : \alpha \in I\}$ is a semi-net of Y and $f : X \to Y$ is a pre-semi-open injection, then $\{f^{-1}(A_\alpha) : \alpha \in I\}$ is a net of X.*

Corollary 3.23. *If $\mathscr{A} = \{A_\alpha : \alpha \in I\}$ is a semi-net of X and $\mathscr{B} = \{B_\beta : \beta \in J\}$ is a semi-net of Y, Then $\{A_\alpha \times B_\beta : \alpha \in I, \beta \in J\}$ is a semi-net of $X \times Y$.*

Corollary 3.24. *If $\mathscr{A}$ is a semi-net of X, and Y is semi-open in X, then $\mathscr{A}$ is a semi-net of Y.*

Corollary 3.25. *If $\mathscr{A}$ is a semi-net of X, and each $A \in \mathscr{A}$ is open, then X has the SOP property.*

Theorem 3.26. *Suppose X is an s-regular space with the SOP property. Then X is s-normal when X has a countable semi-net.*

Proof. Suppose A and B are closed sets in X and $A \cap B = \varnothing$. For each $x \in A$, there exist semi-open sets U_x and V_x such that $x \in U_x$, $B \subset V_x$, and $U_x \cap V_x = \varnothing$. Let $W_x = X - V_x$. Then W_x is semi-closed. Thus, $x \in U_x \subset W_x$ and $W_x \cap B = \varnothing$. Then $\mathscr{A} = \{U_x : x \in A\}$ is a semi-open cover of A. Meanwhile, for each $y \in B$, there exists a semi-open set E_y containing y and a semi-closed set F_y such that $y \in E_y \subset F_y$ and $F_y \cap A = \varnothing$. Then $\mathscr{B} = \{E_y : y \in B\}$ is a semi-open cover of B. Thus, $\mathscr{A} \cup \mathscr{B} \cup \{X - (A \cup B)\}$ is a semi-open cover of X. Suppose $\mathscr{C} = \{C_n : n \in \mathbb{N}\}$ is a countable semi-net of X. Hence, $U_x = \cup\{C_n : x \in C_n \subset U_x, x \in U_x, C_n \in \mathscr{C}\}$ and each U_x in $\mathscr{A}$. Thus, $\mathscr{C}_1 = \{C_n : C_n \in \mathscr{C}, x \in C_n \subset U_x, x \in A\}$ is a cover of A. For each C_n in $\mathscr{C}$, there exists U_{x_n} in $\mathscr{A}$ such that $C_n \subset U_{x_n}$. Then there exists a countable semi-open cover $\mathscr{A}_1 \subset \mathscr{A}$ of A. Let $\mathscr{A}_1 = \{U_n : n \in \mathbb{N}\}$. Meanwhile, there exists a countable semi-open cover $\mathscr{B}_1 \subset \mathscr{B}$ of B. Let $\mathscr{B}_1 = \{E_n : n \in \mathbb{N}\}$. Let $G_n = U_n - \cup\{F_k : k \leq n\}$ and $H_n = E_n - \cup\{W_k : k \leq n\}$. Then $G_n \cap H_m = \varnothing$ and $G_m \cap H_n = \varnothing$ for $m \leq n$. Then $G_n \cap H_m = \varnothing$ for each n and m in $\mathbb{N}$. Since X has the SOP property, it follows that G_n and H_n are semi-open sets. Let $G = \cup_{n \in \mathbb{N}} G_n$ and $H = \cup_{n \in \mathbb{N}} H_n$. Then G and H are semi-open sets, $G \cap H = \varnothing$, and $A \subset G$ and $B \subset H$. Therefore, X is s-normal. □

According to Theorem 3.26 and Theorem 2.29, we obtain the following corollary directly.

Corollary 3.27. *Suppose X is a locally semi-compact space with the SOP property. Then X is s-normal when X is a semi-T_2 space with a countable semi-net.*

Theorem 3.28. *Suppose X is a semi-T_2 semi-countably compact space with the SOP property. Then X is a semi-k space when X has a σ s-locally finite semi-net.*

Proof. Suppose $\mathscr{A} = \cup_{n \in \mathbb{N}} \mathscr{A}_n$ is a σ s-locally finite semi-net of X. Let $\mathscr{A}_n = \{A_\alpha : \alpha \in I\}$. For each $\alpha \in I$, pick $x_\alpha \in A_\alpha$. Let $B = \cup\{scl\{x_\alpha\} : \alpha \in I\}$. Suppose $C = \cup\{scl\{x_\alpha\} : \alpha \in J\}$ is a subset of B. If $scl\{x_\alpha\} \subset C$, then $\{x_\alpha\} \subset \cup_{\alpha \in J}\{x_\alpha\}$ and $scl\{x_\alpha\} \subset scl(\cup_{\alpha \in J}\{x_\alpha\})$. Thus, $\cup_{\alpha \in J} scl\{x_\alpha\} \subset scl(\cup_{\alpha \in J}\{x_\alpha\})$. If $x \notin \cup_{\alpha \in J} scl\{x_\alpha\}$, there is a semi-open set U containing x such that U intersects at most finitely many members of $\mathscr{A}_n$. Then U intersects at most finitely many members of $\{\{x_\alpha\} : \alpha \in I\}$. Thus, U intersects at most finitely many members of $\{\{x_\alpha\} : \alpha \in J\}$. Let it be $\{\{x_1\}, \{x_2\}, \cdots, \{x_t\}\}$. Since $x \notin \cup_{\alpha \in J} scl\{x_\alpha\}$, it follows that $x \notin \cup_{i=1}^t scl\{x_i\}$. Since $x \notin \cup_{\alpha \in J} scl\{x_\alpha\}$, it follows that $x \notin \cup_{i=1}^t scl\{x_i\}$ and $x \in X - \cup_{i=1}^t scl\{x_i\}$. Since X has the SOP property, it follows that $\cup_{i=1}^t scl\{x_i\}$ is semi-closed, $X - \cup_{i=1}^t scl\{x_i\}$ is a semi-open set, and $U \cap (X - \cup_{i=1}^t scl\{x_i\})$ is semi-open. Since $(X - \cup_{i=1}^t scl\{x_i\}) \cap \{x_i\} = \varnothing$ for $i = 1, 2, \cdots, t$, it follows that $(U \cap (X - \cup_{i=1}^t scl\{x_i\})) \cap (\cup\{\{x_\alpha\} : \alpha \in J\}) = \varnothing$. Then $x \notin scl(\cup\{\{x_\alpha\} : \alpha \in J\})$ and $scl(\cup_{\alpha \in J}\{x_\alpha\}) \subset \cup_{\alpha \in J} scl\{x_\alpha\}$. Thus, $scl(\cup_{\alpha \in J}\{x_\alpha\}) = \cup_{\alpha \in J} scl\{x_\alpha\}$. Therefore, each subset of B and B are discrete semi-closed sets.

Assume that B is an infinite set. Let $\mathscr{B} = \{B_n : n \in \mathbb{N}\}$, $B_n = \cup\{scl\{x_{n+i}\}; i = 0, 1, 2, 3, \cdots\}$ and $n \in \mathbb{N}$. Then $B_n \subset B$ and $B_n \cap B_m \neq \emptyset$ for every $n, m \in \mathbb{N}$. We assume $\cap_{n\in\mathbb{N}} B_n = \emptyset$. Since B_n is semi-closed, it follows that $X - B_n$ is semi-open and $\cup_{n\in\mathbb{N}}(X - B_n)$ is a semi-open cover of X. Since X is a semi-countably compact space, it follows that there exists a finite semi-open cover $\{X - B_1, X - B_2, \cdots, X - B_p\}$ such that $\cup_{i=1}^{p}(X - B_i) = X$. Then $\cap_{i=1}^{p} B_i = \emptyset$. It is contradictory. Then there exists a point $x_0 \in \cap_{n\in\mathbb{N}} B_n$ and $x_0 \in B$. If x_0 is not a semi-accumulation point of B, then x_0 is not a semi-accumulation point of B_n, which contradicts the fact that B_n is semi-closed and $x_0 \in B_n$. Then x_0 is a semi-accumulation point of B, which contradicts that B is a discrete set. Therefore, B is a finite set. We may assume $scl\{x_\alpha\} = scl\{x_\beta\}$ when α and β are in I. Hence, $\mathscr{A}_n$ is not s-locally finite on x_α. It is contradictory. Thus, $\mathscr{A}_n$ is finite. Thus, $\mathscr{A}$ is a countably s-locally finite collection of X, and let $\mathscr{A} = \{A_n : n \in \mathbb{N}\}$.

Suppose $\mathscr{D} = \{D_\gamma : \gamma \in K\}$ is a semi-open cover of X. For each $\gamma \in K$, let $\mathscr{A}_\gamma = \{A_n : y \in A_n \subset D_\gamma, y \in D_\gamma, A_n \in \mathscr{A}\}$. Then $D_\gamma = \cup\{A_n : A_n \in \mathscr{A}_\gamma\}$, and $\cup_{\gamma\in K}\mathscr{A}_\gamma$ is a cover of X. For each $A_n \in \cup_{\gamma\in K}\mathscr{A}_\gamma$, there exists $D_{\gamma_n} \in \mathscr{D}$ such that $A_n \in D_{\gamma_n}$. Then there exists a countable semi-open cover $\{D_{\gamma_n} : n \in \mathbb{N}\}$ of X. Since X is semi-countably compact, it follows that there is a finite cover $\{D_1, D_2, \cdots, D_t\}$ of X. Then X is semi-compact. Thus, X is locally semi-compact. Since X is a semi-T_2 locally semi-compact space with the SOP property, and by Corollary 3.4, X is a semi-k space. □

According to Theorem 3.28, we obtain the following corollary directly.

Corollary 3.29. *Suppose $\mathscr{A}$ is s-locally finite in X and $\mathscr{A}_1 = \{A_\alpha : \alpha \in I\} \subset \mathscr{A}$. Then $\cup\{sclA_\alpha : A_\alpha \in \mathscr{A}_1\} = scl(\cup\{A_\alpha : A_\alpha \in \mathscr{A}_1\})$ when X has the SOP property.*

By Theorem 3.16 and Theorem 3.28, we obtain the following corollary.

Corollary 3.30. *Suppose $f : X \to Y$ is a pre-semi-open bijection, the inverse image of each semi-open subset of Y is open in X, and Y has a σ s-locally finite semi-net. Then X is a k space when Y is a semi-countably compact semi-T_2 space with the SOP property.*

Corollary 3.31. *Suppose X is a semi-countably compact space with the SOP property. Then X is a semi-compact space when X has a σ s-locally finite semi-net.*

Proof. The proof is similar to Theorem 3.28, and it is omitted. □

Since each subset in discrete space is open and closed, it follows that discrete space has the SOP property. We obtain the following theorem by Lemma 2.34 and corollary 3.31

Theorem 3.32. *Suppose X is a semi-countably compact discrete space. Then X is a finite and compact space when X has a σ s-locally finite semi-net.*

4 The application of locally semi-compact spaces

In this section, we obtained some applications of locally semi-compact spaces and established the relations among locally semi-compact spaces, s-paracompact spaces, and s-topological groups.

Theorem 4.1. *[23] Suppose X is an s-regular space and U is semi-open in X. Then, for each $x \in U$, there exists a semi-open set V and a semi-closed set W such that $x \in V \subset W \subset U$.*

Theorem 4.2. *Suppose X is locally semi-compact and has a countable semi-net. Then X is s-paracompact when X is a semi-T_2 space with the SOP property.*

Proof. Suppose $\mathscr{A} = \{A_\alpha : \alpha \in I\}$ is an open cover of X and $\mathscr{B} = \{B_n : n \in \mathbb{N}\}$ is a countable semi-net. By Theorem 2.29, X is s-regular. For each $x \in A_\alpha$, by Theorem 4.1, there exists a semi-open set U_x and a semi-closed set V_x such that $x \in U_x \subset V_x \subset A_\alpha$. Then $\mathscr{U} = \{U_x : x \in X\}$ is a semi-open cover of X. Since $\mathscr{B}$ is a countable semi-net, it follows that there exists a set $B_n \in \mathscr{B}$ such that $y \in B_n \subset U_x$ for $y \in U_x$. Then there exists a collection $\mathscr{B}_x = \{B_n : y \in B_n \subset U_x, y \in U_x, B_n \in \mathscr{B}\} \subset \mathscr{B}$ such that $U_x = \cup\{B_n : B_n \in \mathscr{B}_x\}$. Then $\mathscr{B}_1 = \cup_{x\in X}\mathscr{B}_x$ is a cover of X. Thus, there exists a set U_{x_n} in $\mathscr{U}$ such that $B_n \subset U_{x_n}$ for each $B_n \in \mathscr{B}_1$. Then there exists a countable semi-open cover $\mathscr{U}_1 \subset \mathscr{U}$ and let $\mathscr{U}_1 = \{U_{x_n} : n \in \mathbb{N}\}$. Let

$W_1 = U_{x_1}, W_2 = U_{x_2} - U_{x_1}, \cdots, W_n = U_{x_n} - \cup_{k<n} U_{x_k}$. Then $W_n \subset U_{x_n} \subset V_{x_n}$ for each $n \in \mathbb{N}$, and $\mathscr{W} = \{W_n : n \in \mathbb{N}\}$ is an s-locally finite cover of X.

Since V_x is a semi-closed set, it follows that $\{sclW_n : n \in \mathbb{N}\}$ is a semi-closed refinement of $\mathscr{A}$. Since $\mathscr{W}$ is s-locally in X, for each $x \in X$, it follows that there exists a semi-open set C_x such that $x \in C_x$ and C_x intersects at most finitely many members of $\mathscr{W}$. Let it be $\mathscr{W}_1 = \{W_1, W_2, \cdots, W_r\}$. Then $C_x \cap sclW_r \neq \varnothing$. If $C_x \cap sclW_{r+1} \neq \varnothing$ and W_{r+1} is not in $\mathscr{W}_1$, then there exists a point $y \in C_x \cap sclW_{r+1}$. Thus, there exists a semi-open neighborhood D_y such that $y \in D_y \subset C_x$. Since $y \in sclW_{r+1}$, it follows that $D_y \cap W_{r+1} \neq \varnothing$. Then $C_x \cap W_{r+1} \neq \varnothing$. It is a contradiction. Then $\mathscr{W}_2 = \{sclW_n : n \in \mathbb{N}\}$ is an s-locally finite cover of X and $sclW_n \subset V_{x_n}$ for each $n \in \mathbb{N}$.

For each $x \in X$, there exists a semi-open set E_x such that E_x intersects at most finitely many members of $\mathscr{W}_2$. Then $\mathscr{E} = \{E_x : x \in X\}$ is a semi-open cover of X. In the same way, there exists a countable s-locally finite semi-closed cover of X, which is a refinement of $\mathscr{E}$. Let it be $\mathscr{F}$.

For each $n \in \mathbb{N}$, let $H_n = X - \cup\{F : F \in \mathscr{F}, F \cap sclW_n = \varnothing\}$ and $\mathscr{H} = \{H_n : n \in \mathbb{N}\}$. Since $\mathscr{F}$ is s-locally finite and semi-closed, and X has the SOP property, and by Corollary 3.29, it follows that $\cup\{F : F \in \mathscr{F}, F \cap sclW_n = \varnothing\}$ is semi-closed, and H_n is semi-open in X. For each $y \in sclW_n$, there exists a semi-closet set $F \in \mathscr{F}$ such that $y \in F$. Then $F \cap sclW_n \neq \varnothing$ and F is not in $\{F : F \in \mathscr{F}, F \cap sclW_n = \varnothing\}$. Then $F \subset X - \cup\{F : F \in \mathscr{F}, F \cap sclW_n = \varnothing\} = H_n$. Thus, $y \in H_n$ and $sclW_n \subset H_n$. Hence, for each $n \in \mathbb{N}$ and $F \in \mathscr{F}$, $H_n \cap F \neq \varnothing$ when $sclW_n \cap F \neq \varnothing$. If $F \cap H_n \neq \varnothing$, then there exists a point $y \in H_n \cap F$. Then $y \notin \cup\{F : F \in \mathscr{F}, F \cap sclW_n = \varnothing\}$. Thus, $y \in \cup\{F : F \in \mathscr{F}, F \cap sclW_n \neq \varnothing\}$ and $F \cap sclW_n \neq \varnothing$. Thus, for each $n \in \mathbb{N}$ and $F \in \mathscr{F}$, $F \cap H_n \neq \varnothing$ is equivalent to $F \cap sclW_n \neq \varnothing$.

For each $n \in \mathbb{N}$, there exists a set $A_{\alpha_n} \in \mathscr{A}$ such that $sclWn \subset A_{\alpha_n}$. Let $\mathscr{A}_1 = \{A_{\alpha_n} : sclWn \subset A_{\alpha_n}, n \in \mathbb{N}\}$. Then $\mathscr{A}_1$ is a semi-open cover of X. Let $G_n = A_{\alpha_n} \cap H_n$ for each $n \in \mathbb{N}$. Let $\mathscr{G} = \{G_n : n \in \mathbb{N}\}$. Since $sclWn \subset H_n$ and $\mathscr{W}_2$ is a cover of X, it follows that $\mathscr{G}$ is a cover of X and semi-open refinement of $\mathscr{A}$.

For each $x \in X$, there exists a semi-open set Q_x such that $x \in Q_x$ and Q_x intersects at most finitely many members of $\mathscr{F}$. Let it be $\{F_1, F_2, \cdots, F_s\}$. For each $F_i \in \mathscr{F}$ and $i = 1, 2, \cdots, s$, there exists a set $E_{x_i} \in \mathscr{E}$ such that $F_i \in E_{x_i}$. Since E_{x_i} intersects at most finitely many members of $\mathscr{W}_2$ for $i = 1, 2, \cdots, s$, it follows that F_i intersects at most finitely many members of $\mathscr{W}_2$ for $i = 1, 2, \cdots, s$. Then F_i intersects at most finitely many members of $\mathscr{H}$ for $i = 1, 2, \cdots, s$. Thus, Q_x intersects at most finitely many members of $\mathscr{H}$. Then Q_x intersects at most finitely many members of $\mathscr{G}$, and $\mathscr{G}$ is an s-locally finite collection of X. Therefore, X is s-paracompact. □

Definition 4.3. [15] An s-topological group is a group G with a topology $\mathscr{A}$ such that for each x, y in G and each neighborhood W of xy^{-1}, there are semi-open neighborhoods U of x and V of y such that $UV^{-1} \subset W$.

Theorem 4.4. *[15] Let G be an s-topological group. Then each left (right) translation $l_g : G \to G(r_g : G \to G)$ is a semi-homeomorphism.*

Theorem 4.5. *Suppose G is an s-topological group. Then G is a locally semi-compact space if and only if there is a semi-compact neighborhood of e in G.*

Proof. Since G is a locally semi-compact space and e in G, it follows that e has a semi-compact neighborhood. Suppose U is a semi-compact neighborhood of e. For each x in G. Since the right translation $r_g : G \to G$ is a semi-homeomorphism, it follows that Ux is a semi-compact neighborhood and x in Ux. Therefore, G is locally semi-compact. □

Corollary 4.6. *Suppose G is a locally semi-compact semi-T_2 s-topological group with the SOP property. Then H is locally semi-compact when H is an α-open subgroup.*

Proof. Since H is α-open, it follows that H is semi-open. Let $r_x : G \to G$ be the right translation. Then r_x is a semi-homeomorphism. Thus, $r_x(H) = Hx$ is semi-open, and $\cup_{x \in G-H} Hx$ is semi-open. Then $H = G - \cup_{x \in G-H} Hx$ is semi-closed. According to Corollary 2.27, H is locally semi-compact. □

We obtain the following corollary directly according to Lemma 2.22 and Lemma 2.34.

Corollary 4.7. *Suppose G is a semi-compact group and H is a semi-closed subgroup. Then H is semi-open if and only if H is finite in G.*

Theorem 4.8. *Suppose G is an s-topological group with the SOP property, H is semi-compact, and $f : G \to G/H$ is a projection. Then G is locally semi-compact when $(G/H, \mathscr{A})$ is semi-compact, where $\mathscr{A} = \{U \subset G/H : f^{-1}(U) \; is \; semi-open\}$.*

Proof. Since $f : G \to G/H$ is a projection and $\mathscr{A}$ is a topology on G/H. Then f is semi-continuous and surjective. Let $A \subset G$ be semi-closed. Then $f^{-1}(f(A)) = AH \subset G$. Let $x \in G - AH$. Then $A \cap xH^{-1} = \varnothing$. Let $\{U_\alpha : \alpha \in I\}$ be a semi-open cover of xH^{-1}. Then $xH^{-1} \subset \cup_{\alpha\in I} U_\alpha$ and $H^{-1} \subset \cup_{\alpha\in I} x^{-1}U_\alpha$. Thus, $H \subset \cup_{\alpha\in I} xU_\alpha^{-1}$. Since H is semi-compact, it follows that there exists a finite set $J \subset I$ such that $H \subset \cup_{\alpha\in J} xU_\alpha^{-1}$ and $x^{-1}H \subset \cup_{\alpha\in J} U_\alpha^{-1}$. Then $xH^{-1} \subset \cup_{\alpha\in J} U_\alpha^{-1}$ and xH^{-1} are semi-compact. Then there exists a semi-open neighborhood V of e such that $A \cap V \cap xH^{-1} = \varnothing$ and $AH \cap Vx = \varnothing$. Since Vx is a semi-open neighborhood of x, it follows that AH is semi-closed and $G/H - AH \in \mathscr{A}$. Since $\mathscr{A}$ is a topology on G/H, it follows that $f(A)$ is closed in G/H, and f is semi-closed. If $gH \in G/H$ and $f(a) = gH$ for some $a \in G$, then $f^{-1}(gH) = f^{-1}(f(a)) = aH$. Thus, aH is semi-compact by Lemma 4.4. Then $f^{-1}(gH)$ is semi-compact for each $gH \in G/H$.

Let $\{W_\beta : \beta \in L\}$ be a semi-open cover of $G = f^{-1}(G/H)$ and $y = gH \in G/H$. Then there exists a finite set $L_0 \subset L$ such that $f^{-1}(gH) \subset \cup_{\beta\in L_0} W_\beta$. Let $D_y = \cup_{\beta\in L_0} W_\beta$. Then D_y is semi-open, $E_y = G/H - f(G - D_y)$ is semi-open, and gH is in E_y. Thus, $G/H \subset \cup_{y\in G/H} E_y$. Since G/H is semi-compact, it follows that there are finite points $y_1, y_2, \cdots, y_r$ in G/H such that $G/H \subset \cup_{i=1}^r E_{y_i}$. Then $G = f^{-1}(G/H) \subset \cup_{i=1}^r f^{-1}(E_{y_i}) = \cup_{i=1}^r D_{y_i}$. Therefore, G is locally semi-compact. □

5 Conclusion remarks

In this paper, we continue the study of the properties of locally semi-compact spaces, and some new properties have been obtained. At the same time, we establish the connections between locally semi-compact spaces and semi-k spaces by irresolute and pre-semi-open mappings and obtain the relation between semi-countably compact spaces and semi-k spaces. The relation between s-topological groups and locally semi-compact spaces is also established. Several directions for future research are discussed below. The work initiated here is the starting point for continuing work towards that direction and motivating others to do so. To obtain different types of spaces in further research, we suggest adopting semi-countable compact spaces or semi-paracompact spaces instead of semi-compact spaces.

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