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RAINBOW CHROMATIC TOPOLOGICAL INDICES OF CENTRAL GRAPHS OF SOME GRAPHS

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Abstract. The chromatic topological indices concept was introduced recently. Many other variations concerning the chromatic topological indices have been studied lately. In this paper, we have calculated the first and second rainbow chromatic Zagreb indices and rainbow chromatic irregularity indices for central graph of some standard graph classes.

1 Introduction

We use simple, connected, undirected, and finite graphs throughout the study. For various definitions, parameters, and other technical terms used in this study, we refer to[1, 2, 3, 4, 5, 6]. In the field of graph theory, Graph Coloring is one of the most ever-growing fields. Graph coloring[7] means assigning colors to the elements of graphs, such as vertices/edges. Proper coloring is defined as graph coloring such that the adjacent vertices/edges get different colors. The term chromatic number[8] refers to the minimum number of colors used to obtain a proper coloring and is represented by $\chi(G)$. In this study, our focus is on vertex coloring[9]. We consider a particular type of vertex coloring known as rainbow neighbourhood coloring[10]. In this coloring, we first assign the first color to the maximum independent vertex set. The second color to the next maximum independent vertex set, and the procedure follows until all the vertices get one or the other color. Topological indices[11] are numerical values that are associated with the molecules. In Chemical graph theory, the topological indices act as a molecular descriptor. Many topological indices are divided based on degree [12, 13] and distance [14, 15], amongst which our focus is on degree-based topological indices, mainly Zagreb indices[16]. The chromatic versions of Zagreb indices have been studied recently in the literature[17]. The notion of chromatic topological indices is being discussed in the literature[18]. The chromatic topological indices play a vital role in understanding various chemical, physical and biochemical properties associated with the molecules. There are many derived graph classes [19, 20, 21] such as middle graphs, line graphs, total graphs, central graphs and so on. In this study, we consider the central graph of a graph G and it is represented by $C(G)$. By subdividing all the edges of the graph G only once and joining the graph G non-adjacent vertices, the central graph[22] of a graph G is obtained. For computational purpose, let $C = \{c_1, c_2, ..., c_l\}$ represents the set of colors used in rainbow neighbourhood coloring and $\eta_{t,s}$ denote the total number of edges with end points having the color c_t and c_s. Here, $t < s, 1 \leq t, s \leq \chi_r(G)$. The strength of a particular color in G is represented by $\theta(c_i)$, which defines the cardinality of the specific color used. Throughout the paper, we use the minimum coloring condition i.e, $|c_{i+1}| \leq |c_i| \; \forall i$ to color the vertices of graphs. Inspired by variety of studies on different kinds of graph colorings and chromatic Zagreb indices, we discuss the notion of rainbow chromatic Zagreb indices and rainbow chromatic irregularity indices for central graphs of some standard graph classes. For definitions and informations related to various graphs used in the paper we refer to[23, 24, 25, 26, 27, 28].

Definition 1.1. [18] The first rainbow chromatic Zagreb index of G, represented by $M_1^{\varphi_r t}(G)$, is provided by $M_1^{\varphi_r t}(G) = \sum_{u \in V(G)} c(u)^2$ where c follows rainbow neighbourhood coloring of graph.

Definition 1.2. [18] The second rainbow chromatic Zagreb index of G, represented by $M_2^{\varphi_r t}(G)$, is provided by $M_2^{\varphi_r t}(G) = \sum_{uv \in E(G)} c(u) \cdot c(v)$ where c follows rainbow neighbourhood coloring of graph.

Definition 1.3. [18] The rainbow chromatic irregularity index of G, represented by $M_3^{\varphi_r t}(G)$, is provided by $M_3^{\varphi_r t}(G) = \sum_{uv \in E(G)} |c(u) - c(v)|$ where c follows rainbow neighbourhood coloring of graph.

Definition 1.4. [18] The rainbow chromatic total irregularity index of G, represented by $M_4^{\varphi_r t}(G)$, is provided by $M_4^{\varphi_r t}(G) = \frac{1}{2} \sum_{uv \in V(G)} |c(u) - c(v)|$ where c follows rainbow neighbourhood coloring of graph.

The working rule for the first and second rainbow chromatic Zagreb index, rainbow chromatic irregularity index, and rainbow chromatic total irregularity index is provided by the below equations.

(i) $M_1^{\varphi_r t}(G) = \sum_{u \in V(G)} c(u)^2 = \sum_{j=1}^{l} \theta(c_j) \cdot j^2$

(ii) $M_2^{\varphi_r t}(G) = \sum_{uv \in E(G)} c(u) \cdot c(v) = \sum_{1 \leq t,s \leq \chi_r(G)}^{t<s} ts\eta_{ts}$

(iii) $M_3^{\varphi_r t}(G) = \sum_{uv \in E(G)} |c(u) - c(v)| = \sum_{1 \leq t,s \leq \chi_r(G)}^{t<s} \eta_{ts}|t - s|$

(iv) $M_4^{\varphi_r t}(G) = \frac{1}{2} \sum_{uv \in V(G)} |c(u) - c(v)| = \frac{1}{2} \sum_{t,s \in C}^{t<s} \theta(c_t) \cdot \theta(c_s)|t - s|$

2 Main Results

Theorem 2.1. *For the central graph of cycle graph $C[C_n], n \geq 4$ we have,*

$$i) M_1^{\varphi_r t}(C[C_n]) = \begin{cases} \frac{n^3+9n^2+41n+9}{12}; & n \text{ odd}, \\ \frac{n^3+9n^2+38n}{12}; & n \text{ even}. \end{cases}$$

$$ii) M_2^{\varphi_r t}(C[C_n]) = \begin{cases} \frac{n^4+8n^3+22n^2-8n-23}{32}; & n \text{ odd}, \\ \frac{n^4+8n^3+20n^2-16n}{32}; & n \text{ even}. \end{cases}$$

$$iii) M_3^{\varphi_r t}(C[C_n]) = \begin{cases} \frac{n^3+6n^2-n+18}{12}; & n \text{ odd}, \\ \frac{n^3+6n^2-4n+24}{12}; & n \text{ even}. \end{cases}$$

$$iv) M_4^{\varphi_r t}(C[C_n]) = \begin{cases} \frac{2n^3+3n^2+n}{12}; & n \text{ odd}, \\ \frac{2n^3+3n^2-2n}{12}; & n \text{ even}. \end{cases}$$

Proof. *Case-1*: Assume n to be odd.
In such case, we foremost color the even vertices say v_2, v_4, v_6,... with the color say c_1 and then we color the odd vertices say v_1, v_3, v_5,... with the color say c_2, c_3, c_4,... which appears twice based on the selection of the graph. The final vertex of the chosen graph will take the color say $c_{\frac{n+3}{2}}$.

i) To compute $M_1^{\varphi_r t}$ of $C[C_n]$, the vertices are colored in the order described above and we have $\theta(c_1) = n$, $\theta(c_2) = 2$, $\theta(c_3) = 2$,..., $\theta(c_{\frac{n+3}{2}}) = 1$. Thus, the associated first rainbow chromatic Zagreb index is provided by,

$$M_1^{\varphi_r t}(C[C_n]) = \sum_{u\in V(G)} c(u)^2 = \sum_{j=1}^{l} \theta(c_j)\cdot j^2 = \frac{n^3+9n^2+41n+9}{12}$$

ii) To compute $M_2^{\varphi_r t}$ of $C[C_n]$, the vertices are colored as described above and for
$n = 5$, we have $\eta_{12} = 4$, $\eta_{13} = 4$, $\eta_{14} = 2$, $\eta_{23} = 3$, $\eta_{24} = 1$ and $\eta_{34} = 1$.
$n = 7$, we have $\eta_{12} = 4$, $\eta_{13} = 4$, $\eta_{14} = 4$, $\eta_{15} = 2$, $\eta_{23} = 3$, $\eta_{24} = 4$, $\eta_{25} = 1$, $\eta_{34} = 3$, $\eta_{35} = 2$ and $\eta_{45} = 1$.
$n = 9$, we have $\eta_{12} = 4$, $\eta_{13} = 4$, $\eta_{14} = 4$, $\eta_{15} = 4$, $\eta_{16} = 2$, $\eta_{23} = 3$, $\eta_{24} = 4$, $\eta_{25} = 4$, $\eta_{26} = 1$, $\eta_{34} = 3$, $\eta_{35} = 4$, $\eta_{36} = 2$, $\eta_{45} = 3$, $\eta_{46} = 2$ and $\eta_{56} = 1$.
The procedure continues for rest of the vertices based on the selection of the graph. Thus, the associated second rainbow chromatic Zagreb index is provided by,

$$M_2^{\varphi_r t}(C[C_n]) = \sum_{uv\in E(G)} c(u)\cdot c(v) = \sum_{1\le t,s\le \chi_r(C[C_n])}^{t<s} ts\eta_{ts} = \frac{n^4+8n^3+22n^2-8n-23}{32}$$

iii) To compute $M_3^{\varphi_r t}$ of $C[C_n]$, the vertices are colored as described above and we have $\eta_{12} + \eta_{23} + \eta_{34} + ... + \eta_{(\frac{n+1}{2})\frac{n+3}{2}}$ edges which contributes to 1 based on the color distance and $\eta_{13} + \eta_{24} + \eta_{35} + ... + \eta_{(\frac{n-1}{2})\frac{n+3}{2}}$ edges contributes to 2 based on the color distance and the procedure continues based on the selection of the graph. Thus, the associated rainbow chromatic irregularity index is provided by,

$$M_3^{\varphi_r t}(C[C_n]) = \sum_{uv\in E(G)} |c(u) - c(v)| = \sum_{1\le t,s\le \chi_r(C[C_n])}^{t<s} \eta_{ts}|t-s| = \frac{n^3+6n^2-n+18}{12}$$

iv) To compute $M_4^{\varphi_r t}$ of $C[C_n]$, the vertices are colored as described above. Then, we have to take into consideration the vertex pairs as well as all the color combinations which contributes non zero distances. The combinations $\{1,2\},\{2,3\},...,\{\frac{n+1}{2},\frac{n+3}{2}\}$ contributes to the color distance 1 and the combination $\{1,3\},\{2,4\},...,\{\frac{n-1}{2},\frac{n+3}{2}\}$ contributes to the color distance 2 and same procedure is followed based on the selection of the graph. Also, we have $\theta(c_1) = n$, $\theta(c_2) = 2$, $\theta(c_3) = 2$,..., $\theta(c_{\frac{n+3}{2}}) = 1$. Thus, the associated rainbow chromatic total irregularity index is provided by,

$$M_4^{\varphi_r t}(C[C_n]) = \frac{1}{2}\sum_{uv\in V(G)} |c(u) - c(v)| = \frac{1}{2}\sum_{t,s\in C}^{t<s} \theta(c_t)\cdot\theta(c_s)|t-s| = \frac{2n^3+3n^2+n}{12}$$

Case-2: Assume n to be even.
In such case, we foremost color the even vertices say v_2, v_4, v_6,... with the color say c_1 and then we color the odd vertices say v_1, v_3, v_5,... with the color say c_2, c_3,..., $c_{\frac{n+2}{2}}$ which appears twice based on the selection of the graph. Then,

i) To compute $M_1^{\varphi_r t}$ of $C[C_n]$, the vertices are colored as described above and we have $\theta(c_1) = n$, $\theta(c_2) = 2$, $\theta(c_3) = 2$,..., $\theta(c_{\frac{n+2}{2}}) = 2$. Thus, the associated first rainbow chromatic Zagreb index is provided by,

$$M_1^{\varphi_r t}(C[C_n]) = \sum_{u\in V(G)} c(u)^2 = \sum_{j=1}^{l} \theta(c_j)\cdot j^2 = \frac{n^3+9n^2+38n}{12}$$

ii) To compute $M_2^{\varphi_r t}$ of $C[C_n]$, the vertices are colored as described above and for
$n = 4$, we have $\eta_{12} = 4$, $\eta_{13} = 4$, and $\eta_{23} = 2$.
$n = 6$, we have $\eta_{12} = 4$, $\eta_{13} = 4$, $\eta_{14} = 4$, $\eta_{23} = 3$, $\eta_{24} = 3$, and $\eta_{34} = 3$.
$n = 8$, we have $\eta_{12} = 4$, $\eta_{13} = 4$, $\eta_{14} = 4$, $\eta_{15} = 4$, $\eta_{23} = 3$, $\eta_{24} = 4$, $\eta_{25} = 3$, $\eta_{34} = 3$, $\eta_{35} = 4$ and $\eta_{45} = 3$.
The procedure continues for rest of the vertices based on the selection of the graph. Thus, the associated second rainbow chromatic Zagreb index is provided by,

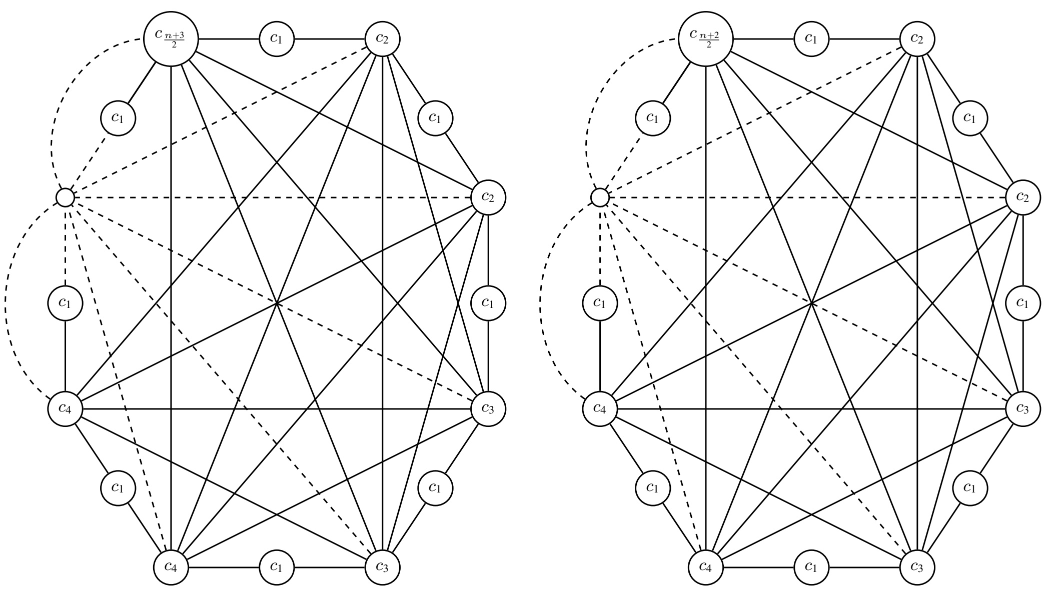
$$M_2^{\varphi_r t}(C[C_n]) = \sum_{uv\in E(G)} c(u)\cdot c(v) = \sum_{1\le t,s\le \chi_r(C[C_n])}^{t<s} ts\eta_{ts} = \frac{n^4+8n^3+20n^2-16n}{32}$$

iii) To compute $M_3^{\varphi_r t}$ of $C[C_n]$, the vertices are colored as described above and we have $\eta_{12}+\eta_{23}+\eta_{34}+...+\eta_{(\frac{n}{2})\frac{n+2}{2}}$ edges which contributes to 1 based on the color distance and $\eta_{13}+\eta_{24}+\eta_{35}+...+\eta_{(\frac{n-2}{2})\frac{n+2}{2}}$ edges contributes to 2 based on the color distance and the procedure continues based on the selection of the graph. Thus, the associated rainbow chromatic irregularity index is provided by,

$$M_3^{\varphi_r t}(C[C_n]) = \sum_{uv\in E(G)} |c(u)-c(v)| = \sum_{1\le t,s\le \chi_r(C[C_n])}^{t<s} \eta_{ts}|t-s| = \frac{n^3+6n^2-4n+24}{12}$$

iv) To compute $M_4^{\varphi_r t}$ of $C[C_n]$, the vertices are colored as described above. Then, we have to take into consideration the vertex pairs as well as all the color combinations which contributes non zero distances. The combinations $\{1,2\},\{2,3\},\{3,4\},...,\{\frac{n}{2},\frac{n+2}{2}\}$ contributes to the color distance 1 and the combination $\{1,3\},\{2,4\},\{3,5\},...,\{\frac{n-2}{2},\frac{n+2}{2}\}$ contributes to the color distance 2 and same procedure is followed based on the selection of the graph. Also, we have $\theta(c_1)=n$, $\theta(c_2)=2$, $\theta(c_3)=2$,..., $\theta(c_{\frac{n+2}{2}})=2$. Thus, the associated rainbow chromatic total irregularity index is provided by,

$$M_4^{\varphi_r t}(C[C_n]) = \frac{1}{2}\sum_{uv\in V(G)} |c(u)-c(v)| = \frac{1}{2}\sum_{t,s\in C}^{t<s} \theta(c_t)\cdot\theta(c_s)|t-s| = \frac{2n^3+3n^2-2n}{12}$$



(a) Fig. A

(b) Fig. B

Figure 1 Fig. A and B shows rainbow neighbourhood coloring for $(C[C_n])$ graph with odd and even vertices respectively

□

Theorem 2.2. *For the central graph of triangular snake graph $C[T_n], n\ge 1$ we have,*

$$i) M_1^{\varphi_r t}(C[T_n]) = \frac{2n^3+9n^2+22n+12}{3}$$

$$ii) M_2^{\varphi_r t}(C[T_n]) = \frac{3n^4+10n^3+27n^2+44n-12}{6}$$

$$iii) M_3^{\varphi_r t}(C[T_n]) = \frac{2n^3+12n^2-8n+12}{3}$$

$$iv) M_4^{\varphi_r t}(C[T_n]) = \frac{11n^3+12n^2+4n}{6}$$

Proof. We use $n+1$ colors say c_1, c_2, c_3 etc. to color the vertices of the chosen central graph of the snake graph. Primarily, we color all the middle vertices of all the triangle with the color say c_1. Further, we color the remaining vertices of the first triangle with color say c_2, then we color the remaining vertices of the second triangle with the color say c_3. Later, we color the remaining vertices of the third triangle with the color say c_4 and the pattern continues till all the remaining vertices gets one or the other color.

i) In order to calculate $M_1^{\varphi_r t}$ of $C[T_n]$, the color c_1 appears $3n$ times, the color c_2 appears thrice and the color c_3 appears twice and all the other colors say c_4, c_5, c_6,... will be appearing two times based on the graph we choose. Here, we have $\theta(c_1) = 3n$, $\theta(c_2) = 3$, $\theta(c_3) = 2$,..., $\theta(c_n) = 2$, $\theta(c_{n+1}) = 2$. Thus, the associated first rainbow chromatic Zagreb index is provided by,

$$M_1^{\varphi_r t}(C[T_n]) = \sum_{u\in V(G)} c(u)^2 = \sum_{j=1}^{l} \theta(c_j)\cdot j^2 = \frac{2n^3+9n^2+22n+12}{3}$$

ii) To compute $M_2^{\varphi_r t}$ of $C[T_n]$, the vertices are colored as described above and for
$n=1$, we have $\eta_{12} = 6$.
$n=2$, we have $\eta_{12} = 8$, $\eta_{13} = 4$ and $\eta_{23} = 4$.
$n=3$, we have $\eta_{12} = 8$, $\eta_{13} = 6$, $\eta_{14} = 4$, $\eta_{23} = 4$, $\eta_{24} = 6$ and $\eta_{34} = 2$.
$n=4$, we have $\eta_{12} = 8$, $\eta_{13} = 6$, $\eta_{14} = 6$, $\eta_{15} = 4$, $\eta_{23} = 4$, $\eta_{24} = 6$, $\eta_{25} = 6$, $\eta_{34} = 2$, $\eta_{35} = 4$ and $\eta_{45} = 2$.
.
.
.
$n = n+1$,we have $\eta_{12} = 8$, $\eta_{13} = 6$, $\eta_{14} = 6$,..., $\eta_{1(n+1)} = 4$.
$\eta_{23} = 4$, $\eta_{24} = 6$, $\eta_{25} = 6$,..., $\eta_{2(n+1)} = 6$.
$\eta_{34} = 2$, $\eta_{35} = 4$, $\eta_{36} = 4$,..., $\eta_{3(n+1)} = 4$.
,..., $\eta_{n(n+1)} = 2$.
Thus, the associated second rainbow chromatic Zagreb index is provided by,

$$M_2^{\varphi_r t}(C[T_n]) = \sum_{uv\in E(G)} c(u)\cdot c(v) = \sum_{1\le t,s\le \chi_r(C[T_n])}^{t<s} ts\eta_{ts} = \frac{3n^4+10n^3+27n^2+44n-12}{6}$$

iii) To compute $M_3^{\varphi_r t}$ of $C[T_n]$, the vertices are colored as described above and we have $\eta_{12}+\eta_{23}+\eta_{34}+...+\eta_{n(n+1)}$ edges which contributes to 1 based on the color distance and $\eta_{13}+\eta_{24}+\eta_{35}+...+\eta_{n-1(n+1)}$ edges contributes to 2 based on the color distance and the procedure continues based on the selection of the graph. Thus, the associated rainbow chromatic irregularity index is provided by,

$$M_3^{\varphi_r t}(C[T_n]) = \sum_{uv\in E(G)} |c(u)-c(v)| = \sum_{1\le t,s\le \chi_r(C[T_n])}^{t<s} \eta_{ts}|t-s| = \frac{2n^3+12n^2-8n+12}{3}$$

iv) To compute $M_4^{\varphi_r t}$ of $C[T_n]$, the vertices are colored as described above. Then, we have to take into consideration the vertex pairs as well as all the color combinations which contributes non zero distances. The combinations $\{1,2\},\{2,3\},\{3,4\},\{4,5\}$, ..., $\{n,n+1\}$ contributes to the color distance 1 and the combination $\{1,3\},\{2,4\},\{3,5\}$, ..., $\{n-1,n+1\}$ contributes to the color distance 2 and same procedure is followed based on the selection of the graph. Also, we have $\theta(c_1) = 3n$, $\theta(c_2) = 2$, $\theta(c_3) = 2$,..., $\theta(c_n) = 2$, $\theta(c_{n+1}) = 2$. Therefore, the associated rainbow chromatic total irregularity index is provided by the following formula,

$$M_4^{\varphi_r t}(C[T_n]) = \frac{1}{2}\sum_{uv\in V(G)} |c(u)-c(v)| = \frac{1}{2}\sum_{t,s\in C}^{t<s} \theta(c_t)\cdot\theta(c_s)|t-s| = \frac{11n^3+12n^2+4n}{6}$$

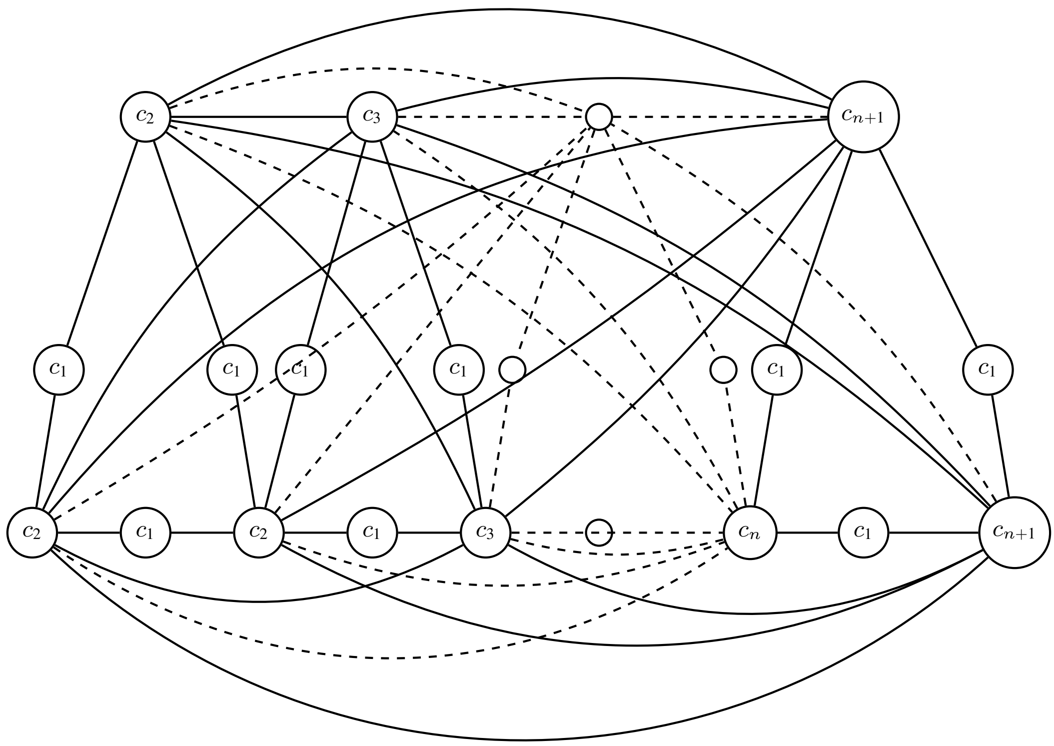


Figure 2 Rainbow neighbourhood coloring for $(C[T_n])$ graph

□

Theorem 2.3. *For the central graph of path graph $C[P_n], n \geq 3$ we have,*

$$i) M_1^{\varphi_r t}(C[P_n]) = \begin{cases} \frac{n^3+9n^2+41n-3}{12}; & n \text{ odd}, \\ \frac{n^3+9n^2+38n-12}{12}; & n \text{ even}. \end{cases}$$

$$ii) M_2^{\varphi_r t}(C[P_n]) = \begin{cases} \frac{n^4+8n^3+22n^2+8n-39}{32}; & n \text{ odd}, \\ \frac{n^4+8n^3+20n^2-32}{32}; & n \text{ even}. \end{cases}$$

$$iii) M_3^{\varphi_r t}(C[P_n]) = \begin{cases} \frac{n^3+6n^2-n-6}{12}; & n \text{ odd}, \\ \frac{n^3+6n^2-4n}{12}; & n \text{ even}. \end{cases}$$

$$iv) M_4^{\varphi_r t}(C[P_n]) = \begin{cases} \frac{4n^3+3n^2-4n-3}{24}; & n \text{ odd}, \\ \frac{4n^3+3n^2-10n}{24}; & n \text{ even}. \end{cases}$$

Proof. Case-1: Assume n to be odd.
In such case, we foremost color the even vertices say v_2, v_4, v_6,... with the color say c_1 and then we color the odd vertices say v_1, v_3, v_5,... with the color say c_2, c_3, c_4, c_5,... which appears twice based on the selection of the graph. The final vertex of the chosen graph will be getting the color say $c_{\frac{n+3}{2}}$.

i) To compute $M_1^{\varphi_r t}$ of $C[P_n]$, the vertices are colored as described above and we have $\theta(c_1) = n-1$, $\theta(c_2) = 2$, $\theta(c_3) = 2$,..., $\theta(c_{\frac{n+3}{2}}) = 1$. Thus, the associated first rainbow chromatic Zagreb index is provided by,

$$M_1^{\varphi_r t}(C[P_n]) = \sum_{u \in V(G)} c(u)^2 = \sum_{j=1}^{l} \theta(c_j) \cdot j^2 = \frac{n^3+9n^2+41n-3}{12}$$

ii) To compute $M_2^{\varphi_{rt}}$ of $C[P_n]$, the vertices are colored as described above and for
$n = 3$, we have $\eta_{12} = 3$, $\eta_{13} = 1$ and $\eta_{23} = 1$.
$n = 5$, we have $\eta_{12} = 3$, $\eta_{13} = 4$, $\eta_{14} = 1$, $\eta_{23} = 3$, $\eta_{24} = 2$ and $\eta_{34} = 1$.
$n = 7$, we have $\eta_{12} = 3$, $\eta_{13} = 4$, $\eta_{14} = 4$, $\eta_{15} = 1$, $\eta_{23} = 3$, $\eta_{24} = 4$, $\eta_{25} = 2$, $\eta_{34} = 3$, $\eta_{35} = 2$ and $\eta_{45} = 1$.
$n = 9$, we have $\eta_{12} = 3$, $\eta_{13} = 4$, $\eta_{14} = 4$, $\eta_{15} = 4$, $\eta_{16} = 1$, $\eta_{23} = 3$, $\eta_{24} = 4$, $\eta_{25} = 4$, $\eta_{26} = 2$, $\eta_{34} = 3$, $\eta_{35} = 4$, $\eta_{36} = 2$, $\eta_{45} = 3$, $\eta_{46} = 2$ and $\eta_{56} = 1$.
The procedure continues for rest of the vertices based on the selection of the graph. Thus, the associated second rainbow chromatic Zagreb index is provided by,

$$M_2^{\varphi_{rt}}(C[P_n]) = \sum_{uv\in E(G)} c(u)\cdot c(v) = \sum_{1\leq t,s\leq \chi_r(C[P_n])}^{t<s} ts\eta_{ts} = \frac{n^4+8n^3+22n^2+8n-39}{32}$$

iii) To compute $M_3^{\varphi_{rt}}$ of $C[P_n]$, the vertices are colored as described above and we have $\eta_{12} + \eta_{23} + \eta_{34} + ... + \eta_{(\frac{n+1}{2})\frac{n+3}{2}}$ edges which contributes to 1 based on the color distance and $\eta_{13} + \eta_{24} + \eta_{35} + ... + \eta_{(\frac{n-1}{2})\frac{n+3}{2}}$ edges contributes to 2 based on the color distance and the procedure continues based on the selection of the graph. Thus, the associated rainbow chromatic irregularity index is provided by,

$$M_3^{\varphi_{rt}}(C[P_n]) = \sum_{uv\in E(G)} |c(u)-c(v)| = \sum_{1\leq t,s\leq \chi_r(C[P_n])}^{t<s} \eta_{ts}|t-s| = \frac{n^3+6n^2-n-6}{12}$$

iv) To compute $M_4^{\varphi_{rt}}$ of $C[P_n]$, the vertices are colored as described above. Then, we have to take into consideration the vertex pairs as well as all the color combinations which contributes non zero distances. The combinations $\{1,2\},\{2,3\},...,\{\frac{n+1}{2},\frac{n+3}{2}\}$ contributes to the color distance 1 and the combination $\{1,3\},\{2,4\},...,\{\frac{n-1}{2},\frac{n+3}{2}\}$ contributes to the color distance 2 and same procedure is followed based on the selection of the graph. Also, we have $\theta(c_1) = n-1$, $\theta(c_2) = 2$, $\theta(c_3) = 2$,..., $\theta(c_{\frac{n+3}{2}}) = 1$. Thus, the associated rainbow chromatic total irregularity index is provided by,

$$M_4^{\varphi_{rt}}(C[P_n]) = \frac{1}{2}\sum_{uv\in V(G)} |c(u)-c(v)| = \frac{1}{2}\sum_{t,s\in C}^{t<s} \theta(c_t)\cdot\theta(c_s)|t-s| = \frac{4n^3+3n^2-4n-3}{24}$$

Case-2: Assume n to be even.
In such case, we foremost color the even vertices say v_2, v_4, v_6,... with the color say c_1 and then we color the odd vertices say v_1, v_3, v_5,... with the color say c_2, c_3,..., $c_{\frac{n+2}{2}}$ which appears twice based on the selection of the graph. Then,

i) To compute $M_1^{\varphi_{rt}}$ of $C[P_n]$, the vertices are colored as described above and we have $\theta(c_1) = n-1$, $\theta(c_2) = 2$, $\theta(c_3) = 2$,..., $\theta(c_{\frac{n+2}{2}}) = 2$. Thus, the associated first rainbow chromatic Zagreb index is provided by,

$$M_1^{\varphi_{rt}}(C[P_n]) = \sum_{u\in V(G)} c(u)^2 = \sum_{j=1}^{l} \theta(c_j)\cdot j^2 = \frac{n^3+9n^2+38n-12}{12}$$

ii) To compute $M_2^{\varphi_{rt}}$ of $C[P_n]$, the vertices are colored as described above and for
$n = 4$, we have $\eta_{12} = 3$, $\eta_{13} = 3$, and $\eta_{23} = 3$.
$n = 6$, we have $\eta_{12} = 3$, $\eta_{13} = 4$, $\eta_{14} = 3$, $\eta_{23} = 3$, $\eta_{24} = 4$, and $\eta_{34} = 3$.
$n = 8$, we have $\eta_{12} = 3$, $\eta_{13} = 4$, $\eta_{14} = 4$, $\eta_{15} = 3$, $\eta_{23} = 3$, $\eta_{24} = 4$, $\eta_{25} = 4$, $\eta_{34} = 3$, $\eta_{35} = 4$ and $\eta_{45} = 3$.
The procedure continues for rest of the vertices based on the selection of the graph. Thus, the associated second rainbow chromatic Zagreb index is provided by,

$$M_2^{\varphi_{rt}}(C[P_n]) = \sum_{uv\in E(G)} c(u)\cdot c(v) = \sum_{1\leq t,s\leq \chi_r(C[P_n])}^{t<s} ts\eta_{ts} = \frac{n^4+8n^3+20n^2-32}{32}$$

iii) To compute $M_3^{\varphi_{rt}}$ of $C[P_n]$, the vertices are colored as described above and we have $\eta_{12} + \eta_{23} + \eta_{34} + ... + \eta_{(\frac{n}{2})\frac{n+2}{2}}$ edges which contributes to 1 based on the color distance and $\eta_{13} + \eta_{24} + \eta_{35} + ... + \eta_{(\frac{n-2}{2})\frac{n+2}{2}}$ edges contributes to 2 based on the color distance and the procedure continues based on the selection of the graph. Thus, the associated rainbow chromatic irregularity index is provided by,

$$M_3^{\varphi_r t}(C[P_n]) = \sum_{uv\in E(G)} |c(u) - c(v)| = \sum_{1\leq t,s\leq \chi_r(C[P_n])}^{t<s} \eta_{ts}|t-s| = \frac{n^3+6n^2-4n}{12}$$

iv) To compute $M_4^{\varphi_r t}$ of $C[P_n]$, the vertices are colored as described above. Then, we have to take into consideration the vertex pairs as well as all the color combinations which contributes non zero distances. The combinations $\{1,2\},\{2,3\},...,\{\frac{n}{2},\frac{n+2}{2}\}$ contributes to the color distance 1 and the combination $\{1,3\},\{2,4\},...,\{\frac{n-2}{2},\frac{n+2}{2}\}$ contributes to the color distance 2 and same procedure is followed based on the selection of the graph. Also, we have $\theta(c_1) = n-1$, $\theta(c_2) = 2$, $\theta(c_3) = 2$,..., $\theta(c_{\frac{n+2}{2}}) = 2$. Thus, the associated rainbow chromatic total irregularity index is provided by,

$$M_4^{\varphi_r t}(C[P_n]) = \frac{1}{2}\sum_{uv\in V(G)} |c(u) - c(v)| = \frac{1}{2}\sum_{t,s\in C}^{t<s} \theta(c_t)\cdot\theta(c_s)|t-s| = \frac{4n^3+3n^2-10n}{24}$$

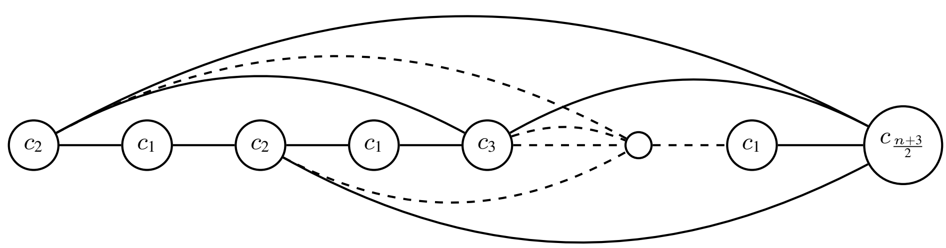


Figure 3 Rainbow neighbourhood coloring for $(C[P_n])$ graph with odd vertices

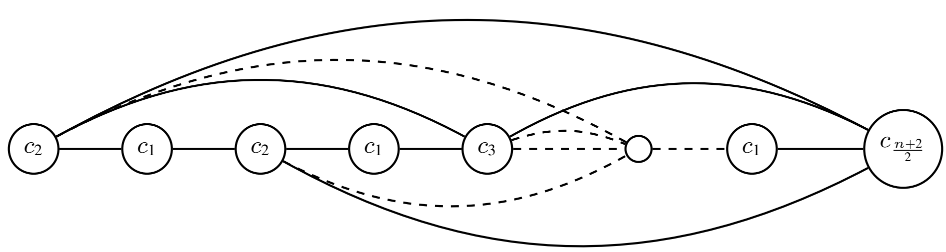


Figure 4 Rainbow neighbourhood coloring for $(C[P_n])$ graph with even vertices

□

Theorem 2.4. *For the central graph of star graph $C[S_n], n \geq 3$ we have,*

$$i) M_1^{\varphi_r t}(C[S_n]) = \frac{2n^3+3n^2+7n+12}{6}$$

$$ii) M_2^{\varphi_r t}(C[S_n]) = \frac{3n^4+2n^3-3n^2+46n-48}{24}$$

$$iii) M_3^{\varphi_r t}(C[S_n]) = \frac{n^3+5n-6}{6}$$

$$iv) M_4^{\varphi_r t}(C[S_n]) = \frac{2n^3-3n^2+n}{6}$$

Proof. We use n colors say c_1, c_2, c_3,..., c_n to color the vertices of the chosen central graph of the star graph. Primarily, we color all the middle vertices of the star graph with the color say c_1. Further, we color the central vertex of the star graph with the color say c_2. Later, all the remaining vertices of the central graph of star graph gets the color say c_2, c_3, c_4,..., c_n once in the given order as per the selection of the graph.

i) To compute $M_1^{\varphi_r t}$ of $C[S_n]$, the color c_1 appears $n-1$ times, the color c_2 appears twice and all the other colors say c_3,c_4, c_5,..., c_n appears once based on the graph we choose. Now, we have $\theta(c_1) = n-1$, $\theta(c_2) = 2$, $\theta(c_3) = 1$,..., $\theta(c_n) = 1$. Thus, the associated first rainbow chromatic Zagreb index is provided by,

$$M_1^{\varphi_r t}(C[S_n]) = \sum_{u \in V(G)} c(u)^2 = \sum_{j=1}^{l} \theta(c_j) \cdot j^2 = \frac{2n^3+3n^2+7n+12}{6}$$

ii) To compute $M_2^{\varphi_r t}$ of $C[S_n]$, the vertices are colored as described above and we have $\eta_{12} = n$, $\eta_{13} = 1$, $\eta_{14} = 1$,..., $\eta_{1n} = 1$. $\eta_{23} = 1, \eta_{24} = 1, \eta_{25} = 1, ..., \eta_{2n} = 1$ and similarly all the other combinations appears once based on the selection of the graph.
Thus, the associated second rainbow chromatic Zagreb index is provided by,

$$M_2^{\varphi_r t}(C[S_n]) = \sum_{uv \in E(G)} c(u) \cdot c(v) = \sum_{1 \le t,s \le \chi_r(C[S_n])}^{t<s} ts\eta_{ts} = \frac{3n^4+2n^3-3n^2+46n-48}{24}$$

iii) To compute $M_3^{\varphi_r t}$ of $C[S_n]$, the vertices are colored as described above and we have $\eta_{12} + \eta_{23} + \eta_{34} + ... + \eta_{n-1(n)} = 2n-2$ edges which contributes to 1 based on the color distance and $\eta_{13} + \eta_{24} + \eta_{35} + ... + \eta_{n-2(n)} = n-2$ edges contributes to 2 based on the color distance and the procedure continues based on the selection of the graph. Thus, the associated rainbow chromatic irregularity index is provided by,

$$M_3^{\varphi_r t}(C[S_n]) = \sum_{uv \in E(G)} |c(u) - c(v)| = \sum_{1 \le t,s \le \chi_r(C[S_n])}^{t<s} \eta_{ts}|t-s| = \frac{n^3+5n-6}{6}$$

iv) To compute $M_4^{\varphi_r t}$ of $C[S_n]$, the vertices are colored as described above. Then, we have to take into consideration the vertex pairs as well as all the color combinations which contributes non zero distances. The combinations $\{1,2\}, \{2,3\}$,..., $\{n-1,n\}$ contributes to the color distance 1 and the combination $\{1,3\}, \{2,4\}$,..., $\{n-2,n\}$ contributes to the color distance 2 and same procedure is followed based on the selection of the graph. Also, we have $\theta(c_1) = n-1$, $\theta(c_2) = 2$, $\theta(c_3) = 1$,..., $\theta(c_n) = 1$. Thus, the associated rainbow chromatic total irregularity index is provided by,

$$M_4^{\varphi_r t}(C[S_n]) = \frac{1}{2} \sum_{uv \in V(G)} |c(u) - c(v)| = \frac{1}{2} \sum_{t,s \in C}^{t<s} \theta(c_t) \cdot \theta(c_s)|t-s| = \frac{2n^3-3n^2+n}{6}$$

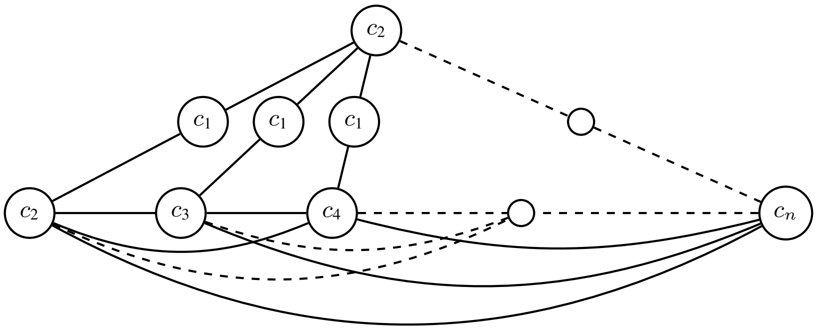


Figure 5 Rainbow neighbourhood coloring for $(C[S_n])$ graph

□

Theorem 2.5. *For the central graph of wheel graph $C[W_n], n \geq 4$ we have,*

$$i) M_1^{\varphi_r t}(C[W_n]) = \begin{cases} \frac{n^3+9n^2+53n+57}{12}; & n \text{ odd}, \\ \frac{n^3+9n^2+50n+48}{12}; & n \text{ even}. \end{cases}$$

$$ii) M_2^{\varphi_r t}(C[W_n]) = \begin{cases} \frac{n^4+8n^3+30n^2+104n-15}{32}; & n \text{ odd}, \\ \frac{n^4+8n^3+28n^2+96n}{32}; & n \text{ even}. \end{cases}$$

$$iii) M_3^{\varphi_r t}(C[W_n]) = \begin{cases} \frac{n^3+9n^2+17n+21}{12}; & n \text{ odd}, \\ \frac{n^3+9n^2+14n+24}{12}; & n \text{ even}. \end{cases}$$

$$iv) M_4^{\varphi_r t}(C[W_n]) = \begin{cases} \frac{7n^3+15n^2+23n+3}{24}; & n \text{ odd}, \\ \frac{7n^3+15n^2+14n}{24}; & n \text{ even}. \end{cases}$$

Proof. Case-1: Assume n to be odd.
In such case, we foremost color the even vertices say v_2, v_4, v_6,... of the cycle and all the vertices inside the cycle graph except the central vertex with the color say c_1 and then we color the odd vertices say v_1, v_3, v_5,... of the cycle graph and the central vertex with the color say c_2. The remaining vertices gets the rest of the colors say c_3, c_4, c_5,... which appears twice based on the selection of the graph. The final vertex of the chosen graph will be getting the color say $c_{\frac{n+3}{2}}$.

i) To compute $M_1^{\varphi_r t}$ of $C[W_n]$, the vertices are colored as described above and we have $\theta(c_1) = 2n$, $\theta(c_2) = 2$, $\theta(c_3) = 2$,..., $\theta(c_{\frac{n+3}{2}}) = 1$. Thus, the associated first rainbow chromatic Zagreb index is provided by,

$$M_1^{\varphi_r t}(C[W_n]) = \sum_{u \in V(G)} c(u)^2 = \sum_{j=1}^{l} \theta(c_j) \cdot j^2 = \frac{n^3+9n^2+53n+57}{12}$$

ii) To compute $M_2^{\varphi_r t}$ of $C[W_n]$, the vertices are colored as described above and for
$n = 5$, we have $\eta_{12} = 11$, $\eta_{13} = 6$, $\eta_{14} = 3$, $\eta_{23} = 3$, $\eta_{24} = 1$ and $\eta_{34} = 1$.
$n = 7$, we have $\eta_{12} = 13$, $\eta_{13} = 6$, $\eta_{14} = 6$, $\eta_{15} = 3$, $\eta_{23} = 3$, $\eta_{24} = 4$, $\eta_{25} = 10$, $\eta_{34} = 3$, $\eta_{35} = 2$ and $\eta_{45} = 1$.
$n = 9$, we have $\eta_{12} = 15$, $\eta_{13} = 6$, $\eta_{14} = 6$, $\eta_{15} = 6$, $\eta_{16} = 3$, $\eta_{23} = 3$, $\eta_{24} = 4$, $\eta_{25} = 4$, $\eta_{26} = 1$, $\eta_{34} = 3$, $\eta_{35} = 4$, $\eta_{36} = 2$, $\eta_{45} = 3$, $\eta_{46} = 2$ and $\eta_{56} = 1$.
The procedure continues for rest of the vertices based on the selection of the graph. Thus, the associated second rainbow chromatic Zagreb index is provided by,

$$M_2^{\varphi_r t}(C[W_n]) = \sum_{uv \in E(G)} c(u) \cdot c(v) = \sum_{1 \leq t,s \leq \chi_r(C[W_n])}^{t<s} ts\eta_{ts} = \frac{n^4+8n^3+30n^2+104n-15}{32}$$

iii) To compute $M_3^{\varphi_r t}$ of $C[W_n]$, the vertices are colored as described above and we have $\eta_{12} + \eta_{23} + \eta_{34} + ... + \eta_{(\frac{n+1}{2})\frac{n+3}{2}}$ edges which contributes to 1 based on the color distance and $\eta_{13} + \eta_{24} + \eta_{35} + ... + \eta_{(\frac{n-1}{2})\frac{n+3}{2}}$ edges contributes to 2 based on the color distance and the procedure continues based on the selection of the graph. Thus, the associated rainbow chromatic irregularity index is provided by,

$$M_3^{\varphi_r t}(C[W_n]) = \sum_{uv \in E(G)} |c(u) - c(v)| = \sum_{1 \leq t,s \leq \chi_r(C[W_n])}^{t<s} \eta_{ts}|t - s| = \frac{n^3+9n^2+17n+21}{12}$$

iv) To compute $M_4^{\varphi_r t}$ of $C[W_n]$, the vertices are colored as described above. Then, we have to take into consideration the vertex pairs as well as all the color combinations which contributes non zero distances. The combinations $\{1, 2\}, \{2, 3\}, ..., \{\frac{n+1}{2}, \frac{n+3}{2}\}$ contributes to the color distance 1 and the combination $\{1, 3\}, \{2, 4\}, ..., \{\frac{n-1}{2}, \frac{n+3}{2}\}$ contributes to the color distance 2 and same procedure is followed based on the selection of the graph. Also, we have $\theta(c_1) = 2n$,

$\theta(c_2) = 2$, $\theta(c_3) = 2$,..., $\theta(c_{\frac{n+3}{2}}) = 1$. Thus, the associated rainbow chromatic total irregularity index is provided by,

$$M_4^{\varphi_r t}(C[W_n]) = \frac{1}{2}\sum_{uv\in V(G)} |c(u) - c(v)| = \frac{1}{2}\sum_{t,s\in C}^{t<s} \theta(c_t)\cdot\theta(c_s)|t-s| = \frac{7n^3+15n^2+23n+3}{24}$$

Case-2: Assume n to be even.

In such case, we foremost color the even vertices say v_2, v_4, v_6,... of the cycle and all the vertices inside the cycle graph except the central vertex with the color say c_1 and then we color the odd vertices say v_1, v_3, v_5,... of the cycle graph and the central vertex with the color say c_2. The remaining vertices gets the rest of the colors say c_3, c_4, c_5,..., $c_{\frac{n+2}{2}}$ which appears twice based on the selection of the graph. Then,

i) To compute $M_1^{\varphi_r t}$ of $C[W_n]$, the vertices are colored as described above and we have $\theta(c_1) = 2n$, $\theta(c_2) = 2$, $\theta(c_3) = 2$,..., $\theta(c_{\frac{n+2}{2}}) = 2$. Thus, the associated first rainbow chromatic Zagreb index is provided by,

$$M_1^{\varphi_r t}(C[W_n]) = \sum_{u\in V(G)} c(u)^2 = \sum_{j=1}^{l} \theta(c_j)\cdot j^2 = \frac{n^3+9n^2+50n+48}{12}$$

ii) To compute $M_2^{\varphi_r t}$ of $C[W_n]$, the vertices are colored as described above and for
$n = 4$, we have $\eta_{12} = 10$, $\eta_{13} = 6$, and $\eta_{23} = 2$.
$n = 6$, we have $\eta_{12} = 12$, $\eta_{13} = 6$, $\eta_{14} = 6$, $\eta_{23} = 3$, $\eta_{24} = 3$, and $\eta_{34} = 3$.
$n = 8$, we have $\eta_{12} = 14$, $\eta_{13} = 6$, $\eta_{14} = 6$, $\eta_{15} = 6$, $\eta_{23} = 3$, $\eta_{24} = 4$, $\eta_{25} = 3$, $\eta_{34} = 3$, $\eta_{35} = 4$ and $\eta_{45} = 3$.
The procedure continues for rest of the vertices based on the selection of the graph. Thus, the associated second rainbow chromatic Zagreb index is provided by,

$$M_2^{\varphi_r t}(C[W_n]) = \sum_{uv\in E(G)} c(u)\cdot c(v) = \sum_{1\le t,s\le\chi_r(C[W_n])}^{t<s} ts\eta_{ts} = \frac{n^4+8n^3+28n^2+96n}{32}$$

iii) To compute $M_3^{\varphi_r t}$ of $C[W_n]$, the vertices are colored as described above and we have $\eta_{12} + \eta_{23} + \eta_{34} + ... + \eta_{(\frac{n}{2})\frac{n+2}{2}}$ edges which contributes to 1 based on the color distance and $\eta_{13} + \eta_{24} + \eta_{35} + ... + \eta_{(\frac{n-2}{2})\frac{n+2}{2}}$ edges contributes to 2 based on the color distance and the procedure continues based on the selection of the graph. Thus, the associated rainbow chromatic irregularity index is provided by,

$$M_3^{\varphi_r t}(C[W_n]) = \sum_{uv\in E(G)} |c(u) - c(v)| = \sum_{1\le t,s\le\chi_r(C[W_n])}^{t<s} \eta_{ts}|t-s| = \frac{n^3+9n^2+14n+24}{12}$$

iv) To compute $M_4^{\varphi_r t}$ of $C[W_n]$, the vertices are colored as described above. Then, we have to take into consideration the vertex pairs as well as all the color combinations which contributes non zero distances. The combinations $\{1,2\}, \{2,3\}, ..., \{\frac{n}{2}, \frac{n+2}{2}\}$ contributes to the color distance 1 and the combination $\{1,3\}, \{2,4\}, ..., \{\frac{n-2}{2}, \frac{n+2}{2}\}$ contributes to the color distance 2 and same procedure is followed based on the selection of the graph. Also, we have $\theta(c_1) = 2n$, $\theta(c_2) = 2$, $\theta(c_3) = 2$,..., $\theta(c_{\frac{n+2}{2}}) = 2$. Thus, the associated rainbow chromatic total irregularity index is provided by,

$$M_4^{\varphi_r t}(C[W_n]) = \frac{1}{2}\sum_{uv\in V(G)} |c(u) - c(v)| = \frac{1}{2}\sum_{t,s\in C}^{t<s} \theta(c_t)\cdot\theta(c_s)|t-s| = \frac{7n^3+15n^2+14n}{24}$$

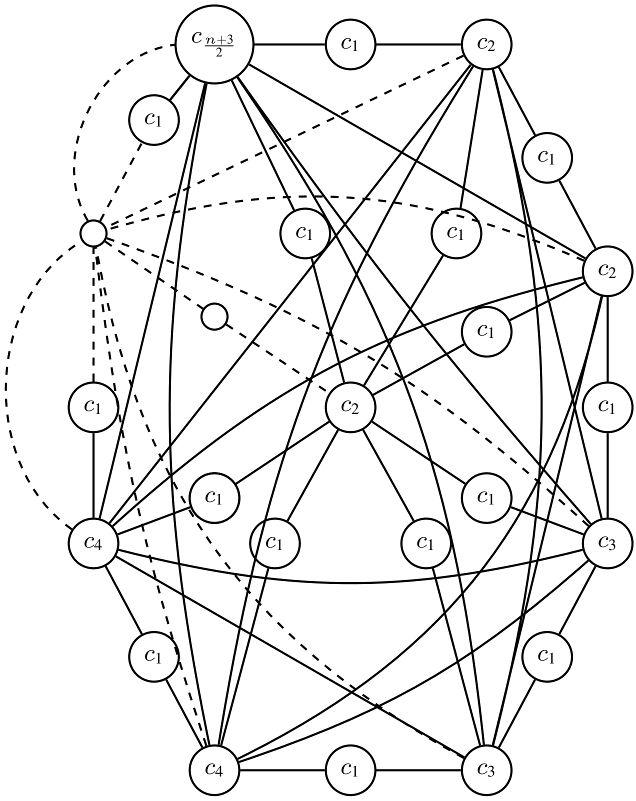


Figure 6 Rainbow neighbourhood coloring for $(C[W_n])$ graph with odd vertices

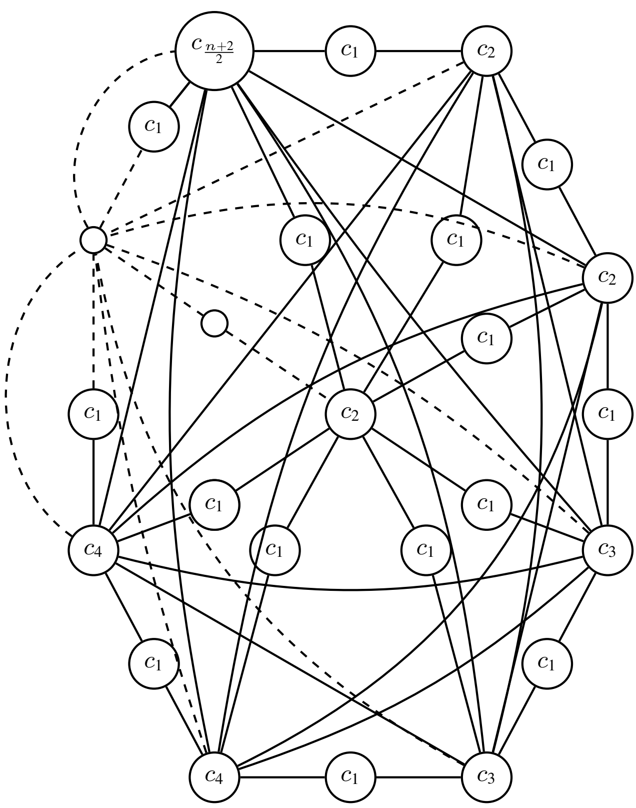


Figure 7 Rainbow neighbourhood coloring for $(C[W_n])$ graph with even vertices

□

3 Conclusion

This paper discusses the parameter known as the rainbow chromatic Zagreb indices and rainbow chromatic irregularity indices of a graph G. This paper contains the calculations of the rainbow chromatic irregularity and Zagreb indices for the central graphs of some graph classes, including the path, triangular snake, cycle, star and wheel graphs. We can extend this paper by trying various other types of graphs. This study can also be extended by determining the rainbow chromatic irregularity indices and rainbow chromatic Zagreb indices for various graph operations, including the cartesian product of graphs, join of graphs, corona product of graphs, and union of graphs, etc. Finding an algorithm to compute the rainbow chromatic irregularity and rainbow chromatic Zagreb indices of graphs will also be a significant contribution. This idea might aid in researching many properties connected to chemical compounds.

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