Palestine Journal of Mathematics

Vol 13(2)(2024) , 315–326

APPROXIMATE TO EXACT METHOD FOR STUDY RADIATIVE STEADY MHD HEAT AND MASS TRANSFER FLOW PAST A VERTICAL POROUS PLATE

Hussein Abd Allah Soliman, Muhammad Sohail and Karem M. Ewis

Communicated by Fathalla Ali Rihan

MSC 2010 Classifications: Primary 20M99, 13F10; Secondary 13A15, 13M05.

Keywords and phrases: Radiation, Induced magnetic field, Heat and Mass transfer, AEM.

The authors would like to thank the reviewers and editor for their constructive comments and valuable suggestions that improved the quality of our paper.

Praveen Agarwal was very thankful to the NBHM (project 02011/12/ 2020NBHM(R.P)/R&D II/7867) for their necessary support and facility.

Abstract In the present study, we introduce a new method with a high-accurate to solve a system of equations of boundary value problems. This method is a mixture of numerical methods (Runge-Kutta and finite difference) and exact methods (Laplace transforms), each having a role in studying. The novelty of the present method is converting boundary value problems to initial value problems using accurate numerical methods and then using Laplace transforms method to find an approximate to the exact solution. Approximate to exact method (AEM) is a new algorithm, with a very strong accuracy that approaches the exact solution. (AEM) applied in studying radiative steady MHD heat and mass transfer flow past a vertical porous plate. (AEM) applied in a system of four non-dimensional equations using suitable dimensional quantities on the governing equations. The new technique's uniqueness, convergence, and stability are verified and tested by comparisons with the previous exact solution. The present governing equations which solved using (AEM) designed by using the Wolfram Mathematica algorithms version 12.3.

1 Introduction

The most popular technique for solving differential equations numerically is the Runge-Kutta method. Three new Runge–Kutta methods are presented for numerical integration of systems of linear inhomogeneous ordinary differential equations (ODEs) with constant coefficients. New approach of combination of Runge-Kutta, finite difference, and Laplace transforms for solution linear boundary value problems ([1]−[2]). A numerical technique built on applying the shifted Jacobi Galerkin method (SJGM) for obtaining approximate solutions of the one-dimensional linear second-order hyperbolic telegraph differential equations (HTDEs) [3]. Numerical solutions are widely used to solve many linear and nonlinear differential equations in their various forms ([18]−[20]).

A novel second-order prediction differential model is designed, and numerical solutions of this novel model are presented using the integrated strength of the Adams and explicit Runge–Kutta schemes [4]. A new special two-derivative Runge-Kutta type (STDRKT) methods involving the fourth derivative of the solution for solving third-order ordinary differential equations [5]. Numerical Treatment of MHD Rotating Flow of Nano-Micropolar Fluid with Impact of Temperature-Dependent Heat Generation and Variable Porous Matrix [6]. Radiative MHD Flow of Rivlin-Ericsen nanofluid of grade three through porous medium with uniform heat source [7].

Finite difference method proposed for the solution of two-point boundary value problems has been widely applied [8-9]. They used the finite difference method (FDM) of second-order accuracy to solve the nonlinear system of differential equations. they observed that the velocity reached the steady-state faster than temperature and nanoparticles concentration. Attia et al. [10] studied the effects of the-Drcian Forchheimer and Hall current resistances on the unsteady flow and heat transfer between two porous plates. they solved the governing partial differential equations, numerically, by the finite difference method FDM. Joule and viscous dissipations are considered in the energy equation. Ewis [11] used a second-order accurate finite difference method to solve the governing equations of natural convection of non-Newtonian (RivlinEricksen) fluid flow and heat transfer under the influences of non-Darcy resistance force, constant pressure gradient, dissipation, and radiation. MHD Natural Convection Nano-fluid Flow between two Vertical Flat Plates through Porous Medium considering effects of viscous dissipation, non-Darcy, and Heat Generation/Absorption [12]. New Investigation of Asymmetric Wall Temperature and Fluid-Wall Interaction on Radiative Steady MHD Fully Developed Natural Convection in Vertical Micro-Porous–Channel [13].

In this paper, (AEM) is new approach of combination of Runge-Kutta, finite difference, and Laplace transforms applied successfully to find the solution of studying radiative steady MHD heat and mass transfer flow past a vertical porous plate. Tables and graphs of the results are very useful in showing the efficiency and accuracy of the (AEM) for the problem presented in this paper. In order to ensure that the current results are accurate, we compared these results with the previously published work ([14]-[17]).

2 Mathematical formulation of the problem

In Figure 1, a Newtonian, electrically conducting, and viscous incompressible fluid flow over a porous, vertical infinite plate with an induced magnetic field and conduction radiation has been taken into consideration. The x-axis is taken vertically up the plate, and the y-axis is normal to it. The wall is kept at a constant temperature $\breve{T}_w$, and both this temperature and the concentration near the plate $\breve{C}_w$ are higher than the ambient temperature $\breve{T}_\infty$ and concentration $\breve{C}_\infty$, respectively. The fluid is a grey gas that is optically thin. Except for density, which varies with temperature and is only considered in the body force term, all gas properties are assumed to be constant. The suction velocity on the plate is constant. The problem's governing equations correspond to [17]:

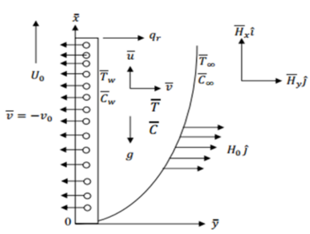

Figure 1. Geometry of the problem.

Conservation of Momentum:

$$\breve{v}\frac{\partial \breve{u}}{\partial \breve{y}} = \nu\frac{\partial^2 \breve{u}}{\partial \breve{y}^2} + \beta g(\breve{T} - \breve{T}_\infty) + \breve{\beta} g(\breve{C} - \breve{C}_\infty) + \frac{\mu_O H_O}{\rho}\frac{\partial \breve{H}}{\partial \breve{y}}, \tag{2.1}$$

Conservation of Energy:

$$\breve{v}\frac{\partial \breve{u}}{\partial \breve{y}} = \frac{\kappa}{\rho c_p}\frac{\partial^2 \breve{T}}{\partial \breve{y}^2} - \frac{1}{\rho c_p}\frac{\partial q_r}{\partial \breve{y}}, \tag{2.2}$$

Conservation of Magnetic Induction:

$$\breve{v}\frac{\partial \breve{H}_x}{\partial \breve{y}} = \frac{1}{\sigma \mu_O}\frac{\partial^2 \breve{H}_x}{\partial \breve{y}^2} + H_O\frac{\partial \breve{u}}{\partial \breve{y}}, \tag{2.3}$$

Conservation of Mass Diffusion:

$$\breve{v}\frac{\partial \breve{C}}{\partial \breve{y}} = D\frac{\partial^2 \breve{C}}{\partial \breve{y}^2}, \tag{2.4}$$

The boundary conditions are:

$$\begin{cases} \breve{y} = 0\text{: } \breve{u} = 0,\ \breve{v} = -v_0\ , \breve{T} = \breve{T}_w,\ \breve{H}_x = 0 \text{ and } \breve{C} = \breve{C}_w, \\ \breve{y} \to \infty\text{: } \breve{u} \to U_0,\ \breve{T} \to \breve{T}_\infty,\ \breve{H}_x \to 0,\ \breve{C} \to \breve{C}_\infty. \end{cases} \tag{2.5}$$

Where,

$$\frac{-\partial q_r}{\partial \breve{y}} = 64a\sigma^*(\breve{T}_\infty^4 - \breve{T}^4)\ \ and\ \ \breve{T}^4 \cong 4\breve{T}^3{}_\infty \breve{T} - 3\breve{T}_\infty^4. \tag{2.6}$$

The non-dimensional quantities in Eqs. (2.1–2.4):
$\{(\eta = \frac{\breve{y}v_0}{\nu})$ is the non-dimensional distance, $(u = \frac{\breve{u}}{U_0})$ is the dimensionless velocity of the fluid, $(\theta = \frac{\breve{T}-\breve{T}_\infty}{\breve{T}_w-\breve{T}_\infty}$ is the dimensionless temperature of the fluid, $(H = (\frac{\mu_0}{\rho})^{\frac{1}{2}}\frac{\breve{H}_x}{U_0})$ is the magnetic field, $(\varphi = \frac{\breve{C}-\breve{C}_\infty}{\breve{C}_w-\breve{C}_\infty})$ is the dimensionless species concentration, $(M = (\frac{\mu_0}{\rho})^{\frac{1}{2}}\frac{\breve{H}_0}{v_0})$ is the magnetic field parameter, $(P_m = \frac{v\sigma}{\mu_0})$ is the magnetic Prandtl number, $(G_r = \frac{vg\beta(\breve{T}_w-\breve{T}_\infty)}{U_0 v_0^2})$ is the thermal Grashof number, $(G_m = \frac{vg\beta(\breve{C}_w-\breve{C}_\infty)}{U_0 v_0^2})$ is the mass Grashof number, $(P_r = \frac{\rho v c_p}{k})$ is the Prandtl number, $(R_d = -\frac{64a\sigma^* T_\infty^3 v}{\rho v_0^2 c_p})$ is the Radiation parameter, $(S_c = \frac{v}{D})$ is the Schmidt number$\}$.

The non-dimensional form of Eqs. (2.1–2.4) through 2.5, 2.6 and non-dimensional quantities are:

$$\frac{d^2u}{d\eta^2} + \frac{du}{d\eta} + M\frac{dH}{d\eta} + G_r\theta + G_m\varphi = 0, \tag{2.7}$$

$$\frac{d^2\theta}{d\eta^2} + P_r\frac{d\theta}{d\eta} + \frac{P_rR_d}{4}\theta = 0, \tag{2.8}$$

$$\frac{d^2H}{d\eta^2} + MP_m\frac{du}{d\eta} + P_m\frac{dH}{d\eta} = 0, \tag{2.9}$$

$$\frac{d^2\varphi}{d\eta^2} + S_c\frac{d\varphi}{d\eta} = 0, \tag{2.10}$$

The non-dimensional boundary conditions 2.5 become:

$$\begin{cases} \eta = 0\text{: } u = 0,\ \theta = 1,\ H = 0,\ \varphi = 1, \\ \eta \to \infty\text{: } u = 1,\ \theta = 0,\ H = 0,\ \varphi = 0. \end{cases} \tag{2.11}$$

The main objective of the present investigation is to study the effects of radiation and induced magnetic field on a steady mixed convective heat and mass transfer past an infinite vertical permeable plate with constant suction taking into account the induced magnetic field. The fluid considered is an optically thin gray gas. The present study may have useful applications in several transport processes as well as in processing magnetic materials.

3 Approximate to Exact Method (AEM)

Using the fourth-order Runge-Kutta method to find u_i, θ_i, H_i, and φ_i, $1 \leq i \leq 4$, then applying the finite difference formulae from fourth-order Eq. (13) for the first derivate to find $u_0^{'}$, $\theta_0^{'}$,$H_0^{'}$, and $\varphi_0^{'}$ as the following ([21]–[22]):

$$f_i^{'} = \frac{-25f_i + 48f_{i+1} - 36f_{i+2} + 16f_{i+3} - 3f_{i+4}}{12h} + O(h^4), then \tag{3.1}$$

$$u_0^{'} = \frac{-25u_0 + 48u_1 - 36u_2 + 16u_3 - 3u_4}{12h} = \alpha. \tag{3.2}$$

$$\theta_0^{'} = \frac{-25\theta_0 + 48\theta_1 - 36u_2 + 16\theta_3 - 3\theta_4}{12h} = \beta. \tag{3.3}$$

$$H_0^{'} = \frac{-25H_0 + 48H_1 - 36H_2 + 16H_3 - 3H_4}{12h} = \gamma. \tag{3.4}$$

$$\varphi_0^{'} = \frac{-25\varphi_0 + 48\varphi_1 - 36\varphi_2 + 16\varphi_3 - 3\varphi_4}{12h} = \delta. \tag{3.5}$$

From Eq. 2.11 and Eqs. (3.2-3.5), then the problem converts from BVP to IVP, then take Laplace transform of both sides of Eqs. (2.7-2.10):

$$-u_0 - su_0 + s\ell\{u\} + s^2\ell\{u\} + M(-H_0 + s\ell\{H\}) + G_r\ell\{\theta\} + G_m\ell\{\varphi\} - \alpha = 0. \tag{3.6}$$

$$\frac{1}{4}P_rR_d\ell\{\theta\} + s^2\ell\{\theta\} + P_r(s\ell\{\theta\} - \theta_0) - s\theta_0 - \beta = 0. \tag{3.7}$$

$$-sH_0 + MP_m(-u_0 + s\ell\{u\}) + s^2\ell\{H\} + P_m(-H_0 + s\ell\{H\}) - \gamma = 0. \tag{3.8}$$

$$s^2\ell\{\varphi\} + S_c(s\ell\{\varphi\} - \varphi_0) - s\varphi_0 - \delta = 0. \tag{3.9}$$

At fixed values of different parameters,$M = 0.25$, $G_m = 5$, $G_r = 5$, $P_r = 0.71$, $R = 1$,$P_m = 0.1$ and $S_c = 0.6$ the computed values of α, β, γ and δ are;

- $\alpha = 13.985118262413925$,
- $\beta = -0.7095035741850294$,
- $\gamma = 0.08244753839826709$,
- $\delta = -0.6314374079026279$.

After substitution by computed values of (α, β, γ and δ), then Laplace form of Eqs. (3.6–3.9) take the following form:

$$\ell\{u\} = \frac{0.000279 + 0.019601s + 0.142218s^2 + 0.61565s^3 + 9.698404s^4 + 13.985118s^5}{s^2(0.6+s)(0.01775 + 0.71s + s^2)(0.09375 + 1.1s + s^2)}. \tag{3.10}$$

$$\ell\{\theta\} = \frac{0.000496425814970558 + s}{0.01775 + 0.71s + s^2}. \tag{3.11}$$

$$\ell\{\varphi\} = \frac{0.000069 + 0.003379s - 0.042199s^2 + 0.06342s^3 + 0.159174s^4 - 0.082447s^5}{s^2(0.6+s)(-0.09375 - 1.1s - s^2)(0.01775 + 0.71s + s^2)}. \tag{3.12}$$

$$\ell\{\varphi\} = \frac{-0.031437407902627945 + s}{s(0.6+s)}. \tag{3.13}$$

Then the approximate to exact form of $u(\eta)$, $\theta(\eta)$, $H(\eta)$ and $\varphi(\varphi)$ are:

$$\begin{aligned}\Rightarrow u(\eta) = {} & 4.709441352642907 - 41.95896970603135e^{-1.0068916720624268\eta} + \\ & 23.242278779621458e^{-0.6840516676754579\eta} + 21.26051878460026e^{-0.6\eta} + \\ & 1.123316602468682e^{-0.09310832793757336\eta} - 8.376585813301954e^{-0.02594833232454208\eta} + \\ & 0.2794436258012079\eta.\end{aligned} \tag{3.14}$$

$$\Rightarrow \theta(\eta) = 1.038674635338233e^{-0.6840516676754579\eta} - 0.03867463533823298 \\ e^{-0.02594833232454208\eta}. \tag{3.15}$$

$$\Rightarrow H(\eta) = 0.34572411032437955 - 1.156669837165049e^{-1.0068916720624268\eta} + \\ 0.9948725457854857e^{-0.6840516676754579\eta} + 1.0630259392300105e^{-0.6\eta} - \\ 4.074905887473569e^{-0.09310832793757336\eta} + 2.8279531292987405e^{-0.02594833232454208\eta} - \\ 0.06986090645029289\eta. \tag{3.16}$$

$$\Rightarrow \varphi(\eta) = -0.05239567983771325 + 1.0523956798377132e^{-0.6\eta}. \tag{3.17}$$

4 Results and discussion

In this paper, (AEM) is applied successfully to find the solution of studying radiative steady MHD heat and mass transfer flow past a vertical porous plate. Tables and graphs of the results are very useful in showing the efficiency and accuracy of the (AEM) for the problem presented in this paper. In order to ensure that the current results are accurate, we compared these results with the previously published work [14-17]. The graphs of $u(\eta)$, $\theta(\eta)$, $H(\eta)$, and $\varphi(\eta)$ under the effect of various parameters (M, G_r, G_m, P_r, R_d, P_m and S_c) are shown in Figures (2–14). and through it, we made sure that:

Figures 2–6 show that:

- The fluid velocity u increases with an increase in any parameter of G_r, G_m and R_d.
- The fluid velocity u decreases with an increase P_r and S_c.

Figures 7–11 show that:

- Increasing in G_r, G_m, M and P_m parameters lead to a decrease in $H(\eta)$.
- Increasing in P_r parameters leads to an increase in $H(\eta)$.

Figures 12–13 display:

- The influence of P_r and R_d on $\theta(\eta)$, it is noticed that $\theta(\eta)$ increases with an increase in R_d but It should be observed that the increase in P_r leads to drop in temperature on the fluid.

Figure 14 displays:

- The impact of $\varphi(\eta)$, it is observed that a reduction in $\varphi(\eta)$ on increasing S_c.

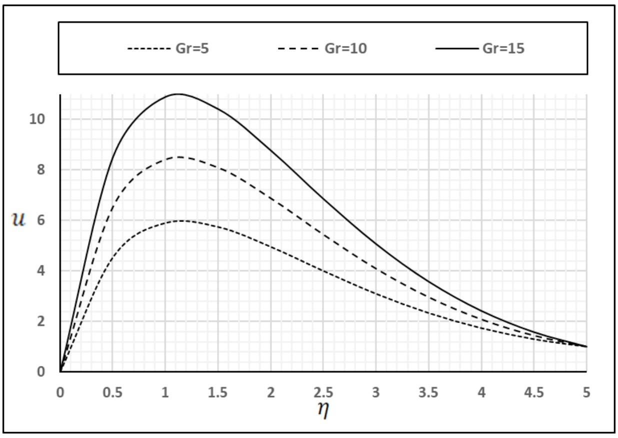

Figure 2. Action of G_r on u.

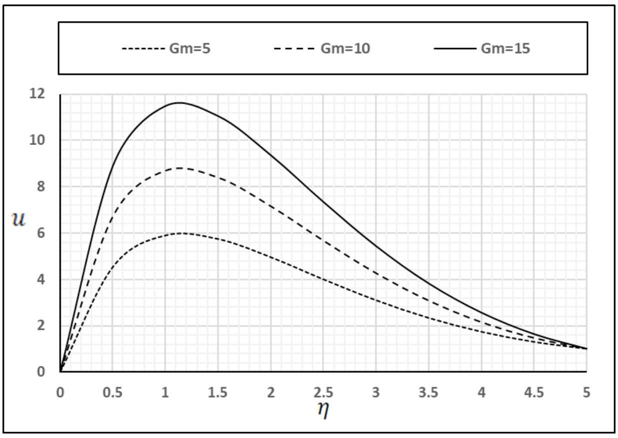

Figure 3. Action of G_m on u.

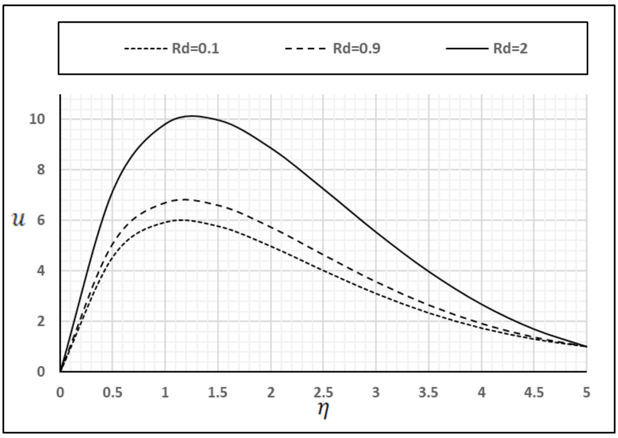

Figure 4. Action of R_d on u.

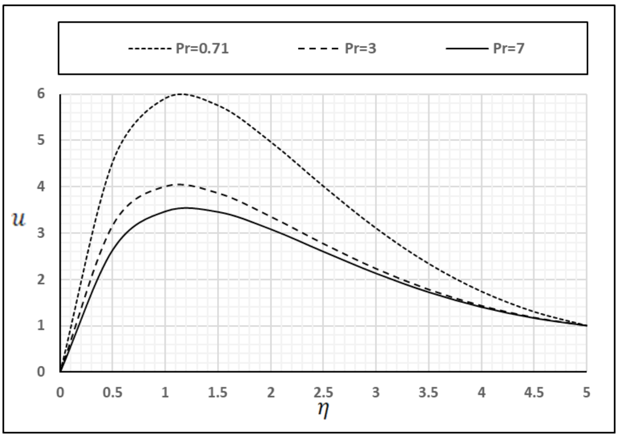

Figure 5. Action of P_r on u.

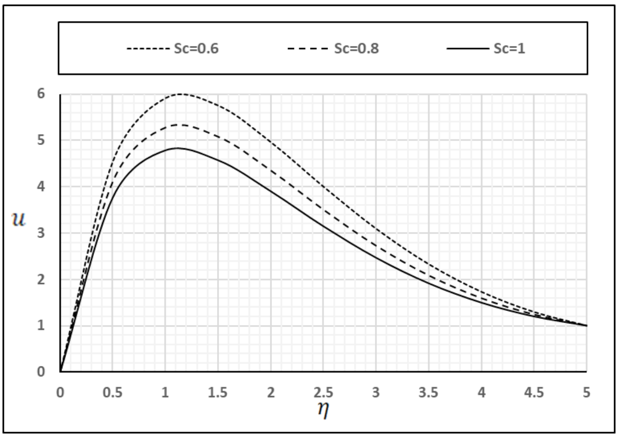

Figure 6. Action of S_c on u.

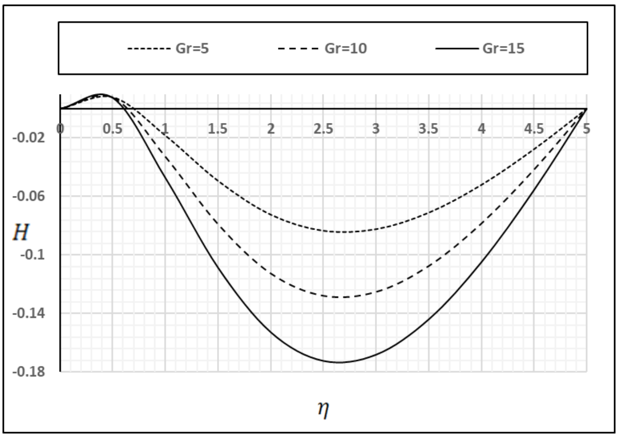

Figure 7. Action of G_r on H.

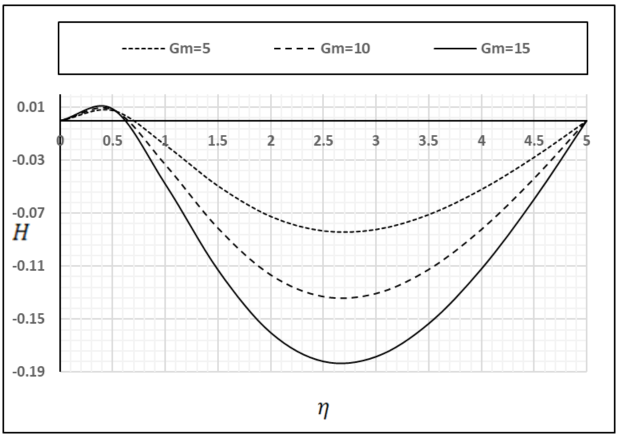

Figure 8. Action of G_m on H.

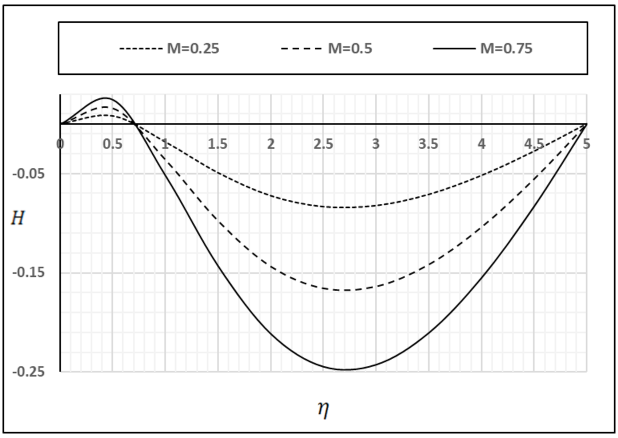

Figure 9. Action of M on H.

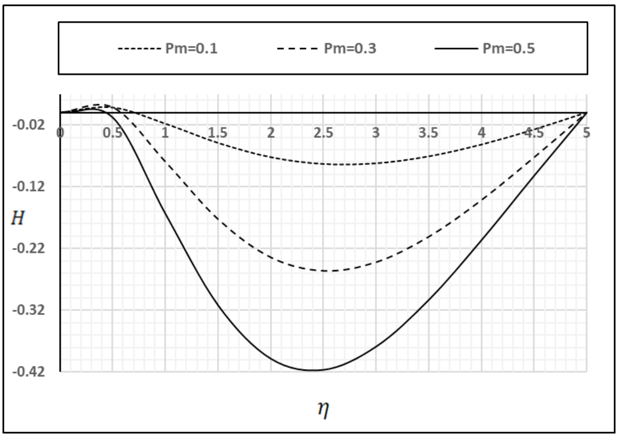

Figure 10. Action of P_m on H.

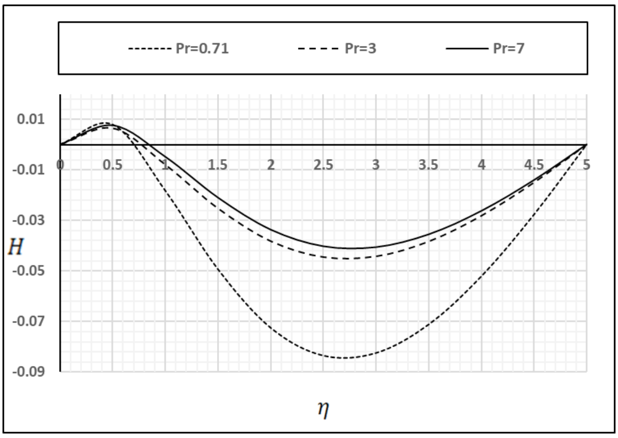

Figure 11. Action of P_r on H.

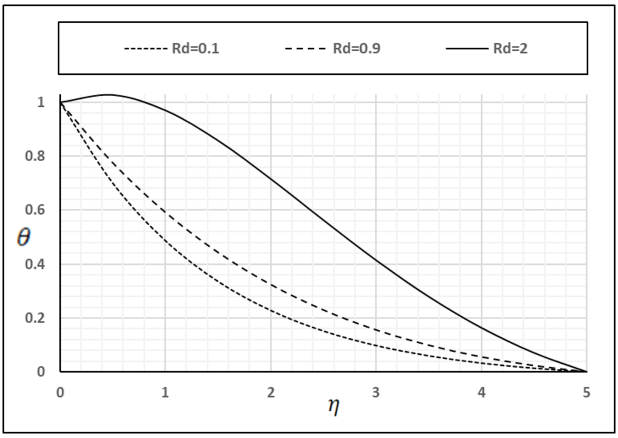

Figure 12. Action of R_d on θ.

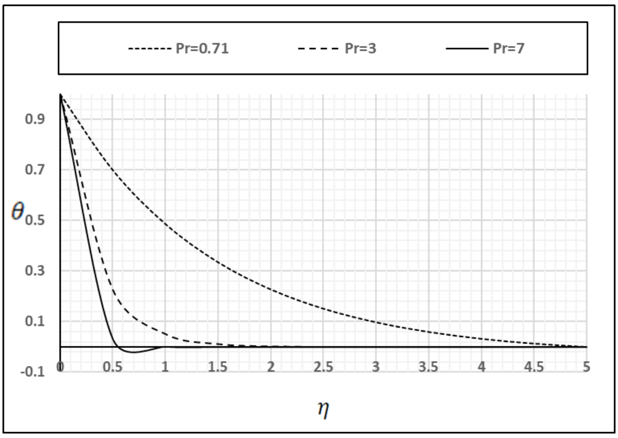

Figure 13. Action of P_r on θ.

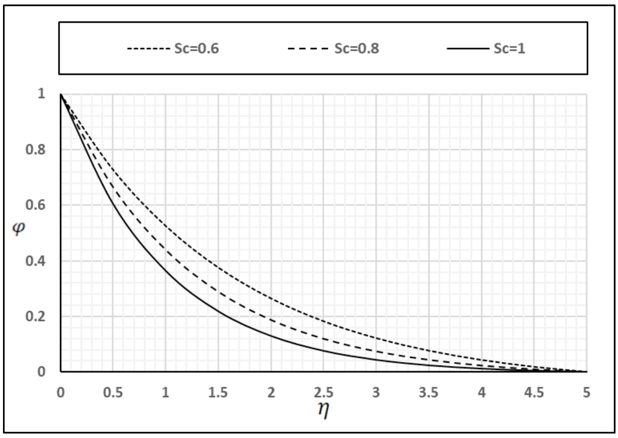

Figure 14. Action of S_c on φ.

Tables 1−2, to verify and test the accuracy of a new method and proved that the new method approaches the exact solution by comparing the results obtained with exact solution of $\theta(\eta)$ and $\varphi(\eta)$ in Eq. 2.8 and Eq. 2.10 using Laplace transform method which obtained as ([1]−[2]):

Exact solution of $\theta(\eta)$ and $\varphi(\eta)$:

$$\Rightarrow \theta(\eta) = -0.038674641203895625e^{-0.3290516676754579(-10+\eta)-0.355\eta} \\ (-1 + e^{0.6581033353509158(-5+\eta)}), \tag{4.1}$$

$$\Rightarrow \varphi(\eta) = -0.05239569649125595(1 - e^{-0.6(-5+\eta)}). \tag{4.2}$$

Table 1. Comparison (AEM) with the exact solution of $\theta(\eta)$.

η	AEM	Exact (4.1)	A.E.
0	1	1	0
1	0.4863985001343503	0.4863985001343504	-5.5×10^{-17}
2	0.22771677776404398	0.22771677776404403	-5.5×10^{-17}
3	0.09764762404825836	0.09764762404825839	-2.7×10^{-17}
4	0.03246070527161624	0.03246070527161626	-1.3×10^{-17}
5	0	0	0

Table 2. Comparison (AEM) with the exact solution of $\varphi(\eta)$.

η	AEM	Exact (4.2)	A.E.
0	1	1	0
1	0.5251713150323075	0.5251713075184227	7.5×10^{-9}
2	0.26457980757081295	0.26457979593322084	1.1×10^{-8}
3	0.12156415600866151	0.12156414210793086	1.3×10^{-8}
4	0.04307550228778395	0.043075487145016536	1.5×10^{-8}
5	$1.582441163522 \times 10^{-8}$	0	1.5×10^{-8}

5 Conclusion

In the present paper, we have applied (AEM) to compute radiative steady MHD heat and mass transfer flow past a vertical porous plate. This paper presented a new method with more accuracy approach to exact solution. (AEM) is a mixture of numerical methods (Runge-Kutta and finite difference) with a closed-form method (Laplace-transform) to solve system of linear boundary value problems. The actions of different parameters (M, G_r, G_m, P_r, R_d, P_m and S_c) on $u(\eta)$, $\theta(\eta)$, $H(\eta)$, and $\varphi(\eta)$ has been studied graphically and numerically. In particular, results for different parameters are summarized in the next paragraphs:

The fluid velocity u:

- Increases with an increase in G_r, G_m and R_d .
- Decreases with an increase P_r and S_c.

The induced magnetic field H:

- Increases with an increase in P_r .
- Decreases with an increase in G_r, G_m, M and P_m .

The fluid temperature θ:

- Increases with an increase in R_d and decreases with an increase in P_r .

The fluid mass diffusion φ:

- Decreases with an increase in S_c.

Furthermore, Comparisons with previously published works are performed and showed that the present results have high accuracy and are found to be in excellent agreement. The findings of ([14]−[17]) are backed up by this research.

References

[1] Soliman, H. A. A. (2022). Exact and Approximate to Exact Methods for Solution Linear Boundary Value Problems Using Laplace Transform. MSA Engineering Journal, 1(4), 113-125.

[2] Ibrahim, M. G., Soliman, H. A., & Elmasry, L. E. (2023). Exact and More Accurate Solution for Boundary Value Problems using Closed-Form and Numerical Methods. *International Journal of Advanced Engineering and Business Sciences*, 4(1).

[3] Ashry, H., Abd-Elhameed, W. M., Moatimid, G. M., & Youssri, Y. H. (2022). Robust Shifted Jacobi-Galerkin Method for Solving Linear Hyperbolic Telegraph Type Equation. *Palestine J. Math*, 11, 504-518.

[4] Sabir, Z., Guirao, J. L., Saeed, T., & Erdogan, F. (2020). Design of a novel second-order prediction differential model solved by using adams and explicit Runge–Kutta numerical methods. *Mathematical Problems in Engineering*, 2020, 1-7.

[5] Lee, K. C., Senu, N., Ahmadian, A., Ibrahim, S. N. I., & Baleanu, D. (2020). Numerical study of third-order ordinary differential equations using a new class of two derivative Runge-Kutta type methods. Alexandria Engineering Journal, 59(4), 2449-2467.

[6] Soliman, H. A. A. (2022). Numerical Treatment of MHD Rotating Flow of Nano-Micropolar Fluid with Impact of Temperature-Dependent Heat Generation and Variable Porous Matrix. *MSA Engineering Journal*, 1(4), 98-112.

[7] Soliman, H. A. A. (2022). Radiative MHD flow of Rivlin–Ericksen nanofluid of grade three through porous medium with uniform heat source. Beni-Suef University Journal of Basic and Applied Sciences, 11(1), 81

[8] Agom, E. U., & Badmus, A. M. (2015). Correlation of Adomian decomposition and finite difference methods in solving nonhomogeneous boundary value problem. The Pacific Journal of Science and Technology, 16(1), 104-109.

[9] Dhumal, M. L., & Kiwne, S. B. (2014). Finite difference method for Laplace equation. *International Journal of Statistics and Mathematics*, 9, 11-13.

[10] Attia, H. A., Abdeen, M. A., & Ewis, K. M. (2016). Non-Darcy unsteady MHD Hartmann flow in a porous medium with heat transfer. *Mechanics & Industry*, 17(1), 112.

[11] Ewis, K. M. (2019). Natural convection flow of Rivlin–Ericksen fluid and heat transfer through non-Darcy medium with radiation. Advances in Mechanical Engineering, 11(8), 1687814019866033.

[12] Soliman, H. A. A. (2022). MHD Natural Convection Nanofluid Flow between two Vertical Flat Plates through Porous Medium considering effects of viscous dissipation, non-Darcy, and Heat Generation/Absorption. *International Journal of Advanced Engineering and Business Sciences*, 3(3), 59-75.

[13] Soliman, H. A. (2022). New Investigation of Asymmetric Wall Temperature and Fluid-Wall Interaction on Radiative Steady MHD Fully Developed Natural Convection in Vertical Micro-Porous–Channel. *Al-Azhar Bulletin of Science*, 33(2-B), 45-58.

[14] Sahin, A. H. M. E. D., & Chamkha, A. J. (2010). Effects of chemical reaction, heat and mass transfer and radiation on the MHD flow along a vertical porous wall in the presence of induced magnetic field. *Int. Journal of Industrial Mathematics*, 2(4), 245-261.

[15] Ahmed, S. (2011). A study of induced magnetic field with chemically reacting and radiating fluid past a vertical permeable plate. Journal of Engineering Physics and Thermophysics, 84(6), 1360-1368.

[16] Kwanza, J. K., Balakiyema, J. A. (2012). Magnetohydrodynamic free convective flow past an infinite vertical porous plate with magnetic induction. Journal of fusion energy, 31(4), 352-356.

[17] Ahmed, S. (2010). Induced magnetic field with radiating fluid over a porous vertical plate: analytical study. *Journal of Naval Architecture and Marine Engineering*, 7(2), 61-72.

[18] Sahu, P. K., & Routaray, M. (2023). NUMERICAL SOLUTION OF VARIABLE ORDER FRACTIONAL INTEGRO-DIFFERENTIAL EQUATIONS USING ORTHONORMAL FUNCTIONS. *Palestine Journal of Mathematics*, 12(1).

[19] Aruldoss, R., & Balaji, K. (2023). A NOVEL ITERATIVE ALGORITHM FOR SOLVING FRACTIONAL DIFFERENTIAL EQUATIONS. *Palestine Journal of Mathematics*, 12(1).

[20] Daraghmeh, A. (2022). NUMERICAL ERROR BOUND FOR NON-HOMOGENEOUS STABLE AND UNSTABLE LINEAR DYNAMICAL SYSTEMS. *Palestine Journal of Mathematics*, 11(4).

[21] Katz, J. (2010). Introductory fluid mechanics. Cambridge University Press.

[22] Anderson, D., Tannehill, J. C.,& Pletcher, R. H.(2016). Computational fluid mechanics and heat transfer. *Taylor & Francis*.

Author information

Hussein Abd Allah Soliman, Department of Basic Science, Engineering Division, International Academy for Engineering and Media Science, 6th October, Egypt.
E-mail: Hussein.AbdAllah.Soliman@iaems.edu.eg

Muhammad Sohail, Institute of Mathematics, Khwaja Fareed University of Engineering and Information Technology, Rahim YarKhan, Pakistan.
E-mail: muhammad_sohail111@yahoo.com

Karem M. Ewis, Department of Engineering Mathematics and Physics, Faculty of Engineering, El−Fayoum University, Egypt.
E-mail: kme00@fayoum.edu.eg

Received: 2023-08-10
Accepted: 2023-10-20