A Note on Badawi-Dobbs paper "Some examples of locally divided rings."

Abdelkbir Riffi

Communicated by Ayman Badawi

Abstract We produce a counterexample to Theorem 3.1 (b) "If R is a PVD and E is a divisible R-module, then $A := R(+)E$ is a PVR." in the paper "Some examples of locally divided rings" that A. Badawi and D. E. Dobbs published in 2001 [1].

After we have pointed out, to A. Badawi and D. E. Dobbs, that [1, Theorem 3.1 (b)] is false, they kindly ask the author to write a note about their paper.

The assertion of [1, Theorem 3.1 (b)] "If R is a PVD and E is a divisible R-module, then $A := R(+)E$ is a PVR." is false. To see this, take any integral domain R with proper quotient field K and X an indeterminate over K. Then $E := K[X]$ is a divisible R-module. But $A := R(+)E$ is not a PVR. Indeed, for $u := (0, X), v := (0, X - 1)$ and $Q := P(+)E$ with P a nonzero prime ideal of R, we have neither $u \in Qv$ nor $Qv \subseteq Au$.

In fact, in the "proof" of Theorem 3.1 (b), the mistake occurs in the next-to-last sentence of the "proof". The point is that the R-module E being divisible and $e \notin Pf$ do NOT imply that $P = 0$. The divisibility of E only gives us that $E = rE$ for each nonzero element r of the domain R, but the elements e and f are given early (and not to be inferred from the equation $E = rE$, regardless of which nonzero element $r \in P$ one may try to choose).

References

[1] A. Badawi and D. E. Dobbs, *Some examples of locally divided rings*, Ideal Theoretic Methods in Commutative Algebra, Marcel Dekker, New York/Basel, 2001, pp. 73–83.

Author information

Abdelkbir Riffi, Laboratory of Mathematics and applications (LMA), Department of Mathematics, Faculty of Sciences, Ibn Zohr University, Agadir, Morocco.
E-mail: riffiabdelkbir@gmail.com