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Computation of SK Group Topological Indices on Certain Graphs

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Abstract. In this study , newly defined topological indices such as SK topological index, SK_1 topological index and SK_2 topological index [18] are derived for three graphs such as double graph, extended double cover graphs and strong double graph.

1 Introduction

Topological indices are mathematical measures that compute the structural properties of graphs, providing insights into their connectivity and behaviour. Researchers are always interested in experimenting the properties of different topological indices on different classification of graphs. Various Topological indices have demonstrated significant contribution in studies related to QSAR(Quantitative structure Activity Relationships) and QSPR (Quantitative structure Activity Relationships).The topological index values are used to predict physicochemical properties of chemical compounds, such as boiling point, molecular weight, density, and refractive index[5].

One of the oldest and widely studied topological indices of a graph are first and second zagreb index indices, proposed by Gutman and Trinajstic in [10].The first and second zagreb indices of a graph G, are defined as

$$M_1(G) = \sum_{v_i \in V(G)} [d(v_i) + d(v_j)]$$

and

$$M_2(G) = \sum_{(v_i,v_j) \in E(G)} [d(v_i)d(v_j)]$$

respectively, where $d(v_i)$ is the degree of the vertex v_i in G.

One can find a large number of studies related with these two indices, with respect to its mathematical and chemical properties. Many degree based topological indices have been proposed thereafter.

SK topological index, SK_1 topological index and SK_2 topological index are the new entrants in this category. These indices are introduced by Shigehalli et al.[18] in 2016. These indices are defined as follows.

$$SK(G) = \frac{1}{2} \sum_{(v_i,v_j) \in E(G)} [d(v_i) + d(v_j)]$$

$$SK_1(G) = \frac{1}{2} \sum_{(v_i,v_j) \in E(G)} [d(v_i)d(v_j)]$$

and

$$SK_2(G) = \frac{1}{4} \sum_{(v_i,v_j)\in E(G)} [d(v_i)d(v_j)]^2$$

In [18], the authors derived explicit formulae for theses indices for graphene.In [14], the authors defined the SK, SK_1 and SK_2 indices on weighted graphs and on interval weighted graphs and behaviors of these indices are investigated under some graph operations. Sheeja et al.derived expressions for SK index over different types of corona products of graphs in [17].

Researchers consistently display interest in examining the properties of various topological indices across different categories of graphs. Double graphs and their variations have acquired considerable interest among researchers. Imran et al. computed the general sum-connectivity index, general Randić index, geometric–arithmetic index, general first Zagreb index, first and second multiplicative Zagreb indices for double graphs and strong double graphs and derived the exact expressions for these degree-base topological indices for double graphs and strong double graphs with respect to the corresponding indices of the original graph G [11]. In [15], the authors studied strong double graphs to compute different Zagreb indices and coindices. In [2], the authors derived expressions of Eccentric Connectivity index and Eccentric Connectivity Polynomial for the double graph and extended double cover graphs.

While there have been numerous studies conducted on SK group indices and double graphs individually, no existing literature explores the relationship between SK group indices and double graphs. Therefore, the objective of this study is to establish the expression of three SK topological indices in terms of double graphs and its two variations.

2 Preliminaries

This section provides the required definitions.

Definition 2.1. For a single graph G with vertex set $v_1, v_2, v_3, ..., v_n$, the double graph of G, denoted by G_1 for convenience, contains two sets of vertex sets $a_1, a_2, a_3, ..., a_n$ and $b_1, b_2, b_3, ..., b_n$[8] . Corresponding to every edge (v_i, v_j) in $E(G)$, there are four edges in G_1, such as (a_i, a_j),(b_i, b_j), (a_i, b_j) and (a_j, b_i) . For every vertex a_i, b_i in G_1, the degrees of the vertices satisfies the relation, $d(a_i) = d(b_i) = 2d(v_i)$, where v_i is in G. Moreover, if there are n vertices and m edges in G, there will be $2n$ vertices and $4m$ edges in G_1.

Definition 2.2. For a single graph G, consider $v_1, v_2, v_3, ..., v_n$ are the set of vertises. The extended double cover graph of G, denoted by G_2 for convenience, contains two sets of vertex sets $a_1, a_2, a_3, ..., a_n$ and $b_1, b_2, b_3, ..., b_n$[1] . Corresponding to every edge (v_i, v_j) in $E(G)$, there are three edges in G_2, such as (a_i, b_i), (a_i, b_j) and (a_j, b_i) . For every vertex a_i, b_i in G_2, the degrees of the vertices satisfies the relation, $d(a_i)$= $d(b_i)$ = $d(v_i)$ +1, where v_i is in G. Moreover, if G is a (n, m) graph, $2n$ and $n + 2m$ will be the number of vertices and edges in G_2.

Definition 2.3. The Strong Double graph $SD(G)$ is the result of making two models of the graph $G(n, m)$ and linking every vertex v_i in one model with the closed neighborhood $N[V_i] = N(v_i)U\{v_i\}$ of the corresponding vertex in the other model[4]. For a graph, $G(n, m)$, its strong double graph will have vertices equal to $2n$ and edges equal to $4m + n$ edges. For convenience, the strong double graph is denoted by G_3 . Corresponding to every edge (v_i, v_j) in $E(G)$, there are five edges in G_3, such as (a_i, a_j), (b_i, b_j), (a_i, b_i), (a_i, b_j) and (a_j, b_i). Moreover, for every vertex v_i in G, the vertices in G_3 satisfies the relation $d(a_i)$= $d(b_i)$ = $2d(v_i)$ +1.

3 Main Results

The graphs G considered in this study are undirected and without parallel edges and self loops. The vertex sets and edge sets are denoted by $V(G)$ and $E(G)$ respectively with respective cardinalities n and m.The number of edges incident to a vertex v , degree of v, is denoted by $d_G(v)$ or simply $d(v)$. In this section, the expressions for SK index, SK_1 index and SK_2 indices are derived for double graphs.

Theorem 3.1. *Let G_1 be the double graph of G. Then the SK index of G_1 is given by*

$$SK(G_1) = 8SK(G)$$

.

Proof. For two vertices v_i, v_j in $V(G)$, and if (v_i, v_j) is an edge in $E(G)$, the double graph of G, G_1 contains edges such as (a_i, a_j), (b_i, b_j), (a_i, b_j) and (a_j, b_i) . Hence

$$\begin{aligned} SK(G_1) &= \sum_{(a_i,a_j)\in E(G_1)} \frac{d(a_i)+d(a_j)}{2} \\ &+ \sum_{(b_i,b_j)\in E(G_1)} \frac{d(b_i)+d(b_j)}{2} \\ &+ \sum_{(a_i,b_j)\in E(G_1)} \frac{d(a_i)+d(b_j)}{2} \\ &+ \sum_{(a_j,b_i)\in E(G_1)} \frac{d(a_j)+d(b_i)}{2} \\ &= 8 \sum_{(v_i,v_j)\in E(G)} \frac{d(v_i)+d(v_j)}{2} \\ &= 8SK(G) \end{aligned}$$

□

Theorem 3.2. *The SK_1 index of G_1 is given by*

$$SK_1(G_1) = 16SK_1(G)$$

.

Proof.

$$\begin{aligned} SK_1(G_1) &= \sum_{(a_i,a_j)\in E(G_1)} \frac{d(a_i)*d(a_j)}{2} \\ &+ \sum_{(b_i,b_j)\in E(G_1)} \frac{d(b_i)*d(b_j)}{2} \\ &+ \sum_{(a_i,b_j)\in E(G_1)} \frac{d(a_i)*d(b_j)}{2} \\ &+ \sum_{(a_j,b_i)\in E(G_1)} \frac{d(a_j*d(b_i)}{2} \\ &= 16 \sum_{(v_i,v_j)\in E(G)} \frac{d(v_i)*d(v_j)}{2} \\ &= 16SK_1(G) \end{aligned}$$

□

Theorem 3.3. *The relationship between SK_2 index of G_1 and SK_2 index of G is given by*

$$SK_2(G_1) = 16SK_2(G)$$

.

Proof.

$$\begin{aligned} SK_2(G_1) &= \sum_{(a_i,a_j)\in E(G_1)} \left[\frac{d(a_i)*d(a_j)}{2}\right]^2 \\ &+ \sum_{(b_i,b_j)\in E(G_1)} \left[\frac{d(b_i)*d(b_j)}{2}\right]^2 \\ &+ \sum_{(a_i,b_j)\in E(G_1)} \left[\frac{d(a_i)*d(b_j)}{2}\right]^2 \\ &+ \sum_{(a_j,b_i)\in E(G_1)} \left[\frac{d(a_j*d(b_i)}{2}\right]^2 \\ &= 16\sum_{(v_i,v_j)\in E(G)} \left[\frac{d(v_i)*d(v_j)}{2}\right]^2 \\ &= 16SK_2(G) \end{aligned}$$

□

4 SK index, SK_1 index and SK_2 index on Extended Double Cover graphs

In this section, the expressions for SK index, SK_1 index and SK_2 indices are derived for Extended Double cover graphs.

Theorem 4.1. *Let G_2 be the extended double cover graph of G. Then the SK index of G_2 is*

$$SK(G_2) = 2SK(G) + 4m + n$$

.

Proof. According to the definitions of the above mentioned topological indices and extended double cover graph, we have

$$\begin{aligned} SK(G_2) &= \sum_{(a_i,b_i)\in E(G_2)} \frac{d(a_i)+d(b_i)}{2} \\ &+ \sum_{(a_i,b_j)\in E(G_2)} \frac{d(a_i)+d(b_j)}{2} \\ &+ \sum_{(a_j,b_i)\in E(G_2)} \frac{d(a_j)+d(b_i)}{2} \\ &= 2\sum_{(v_i,v_j)\in E(G)} \frac{d(v_i)+d(v_j)}{2} + 4m + n \\ &= 2SK(G) + 4m + n \end{aligned}$$

□

Theorem 4.2. *The SK_1 topological index of G_2 is derived as*

$$SK_1(G_2) = 2SK_1(G) + 3SK(G) + 3m + \frac{n}{2}$$

.

Proof. According to the definitions of the above mentioned topological indices and extended double cover graph, we have

$$\begin{aligned} SK_1(G_2) &= \sum_{(a_i,b_i)\in E(G_2)} \frac{d(a_i)*d(b_i)}{2} \\ &+ \sum_{(a_i,b_j)\in E(G_2)} \frac{d(a_i)*d(b_j)}{2} \\ &+ \sum_{(a_j,b_i)\in E(G_2)} \frac{d(a_j)*d(b_i)}{2} \\ &= \sum_{(v_i\in V(G)} \frac{[d(v_i)+1]*[d(v_i)+1]}{2} \\ &+ 2\sum_{(v_i,v_j)\in E(G)} \frac{[d(v_i)+1]*[d(v_j)+1]}{2} \\ &= 2SK_1(G)+3SK(G)+3m+\frac{n}{2} \end{aligned}$$

□

Theorem 4.3. *The SK_2 index of G_2 is given by*

$$SK_2(G_2) = 2SK_2(G) + 2SK_1(G)\left[1+2SK(G)\right] + 2SK(G)\left[2SK(G)+\frac{5}{2}\right] + 3m + \frac{n}{4}$$

.

Proof. According to the definitions of the above mentioned topological indices and extended double cover graph, we have

$$\begin{aligned} SK_2(G_2) &= \sum_{(a_i,b_i)\in E(G_2)} \left[\frac{d(a_i)*d(b_i)}{2}\right]^2 \\ &+ \sum_{(a_i,b_j)\in E(G_2)} \left[\frac{d(a_i)*d(b_j)}{2}\right]^2 \\ &+ \sum_{(a_j,b_i)\in E(G_2)} \left[\frac{d(a_j)*d(b_i)}{2}\right]^2 \\ &= \sum_{v_i\in V(G)} \left[\frac{(d(v_i)+1)(d(v_i)+1)}{2}\right]^2 \\ &+ 2\sum_{(v_i,v_j)\in E(G)} \left[\frac{(d(v_i)+1)(d(v_j)+1)}{2}\right]^2 \\ &= \sum_{v_i\in V(G)} \left[\frac{(d(v_i)+1)}{4}\right]^2 \\ &+ 2\sum_{(v_i,v_j)\in E(G)} \left[\frac{d(v_i)d(v_j)+(d(v_i)+d(v_j))+1}{4}\right]^2 \\ &= 2SK_2(G)+2SK_1(G)\left[1+2SK(G)\right]+2SK(G)\left[2SK(G)+\frac{5}{2}\right]+3m+\frac{n}{4} \end{aligned}$$

□

5 SK index, SK_1 index and SK_2 index on Strong Double graphs

The Strong double graph G_3 is expressed in terms of three topological indices such as SK index, SK_1 index and SK_2 index in this section.

Theorem 5.1. *For G_3, the SK index of G_3 is given by $SK(G_3) = 8SK(G) + 8m + n$.*

Proof. According to the definitions of the above mentioned topological indices and strong double cover graph, we have

$$\begin{aligned}
SK(G_3) &= \sum_{(a_i,a_j)\in E(G_3)} \frac{d(a_i)+d(a_j)}{2} \\
&+ \sum_{(b_i,b_j)\in E(G_3)} \frac{d(b_i)+d(b_j)}{2} \\
&+ \sum_{(a_i,b_i)\in E(G_3)} \frac{d(a_i)+d(b_i)}{2} \\
&+ \sum_{(a_i,b_j)\in E(G_3)} \frac{d(a_i)+d(b_j)}{2} \\
&+ \sum_{(a_j,b_i)\in E(G_3)} \frac{d(a_j)+d(b_i)}{2} \\
&= 8 \sum_{(v_i,v_j)\in E(G)} \frac{d(v_i)+d(v_j)+1}{2} \\
&+ 2 \sum_{(v_i\in E(G))} \frac{2d(v_i)+1}{2} \\
&= 8SK(G) + 8m + n
\end{aligned}$$

□

Theorem 5.2. *The SK_1 index of G_3 can be expressed as*

$$SK_1(G_3) = 16SK_1(G) + 12SK(G) + 6m + \frac{n}{2}$$

.

Proof. According to the above definitions, we have

$$\begin{aligned} SK_1(G_3) &= \sum_{(a_i,a_j)\in E(G_3)} \frac{d(a_i)*d(a_j)}{2} \\ &+ \sum_{(b_i,b_j)\in E(G_3)} \frac{d(b_i)*d(b_j)}{2} \\ &+ \sum_{(a_i,b_i)\in E(G_3)} \frac{d(a_i)*d(b_i)}{2} \\ &+ \sum_{(a_i,b_j)\in E(G_3)} \frac{d(a_i)*d(b_j)}{2} \\ &+ \sum_{(a_j,b_i)\in E(G_3)} \frac{d(a_j)*d(b_i)}{2} \\ &= 4 \sum_{(v_i,v_j)\in E(G)} \frac{(2d(v_i)+1)(2d(v_j)+1)}{2} \\ &+ \sum_{(v_i\in E(G))} \frac{(2d(v_i)+1)^2}{2} \\ &= 16SK_1(G)+12SK(G)+6m+\frac{n}{2} \end{aligned}$$

□

Theorem 5.3. *The SK_2 index of G_3 is*

$$SK_2(G_3) >= 16SK_1(G)+12SK(G)+6m+\frac{n}{2}$$

Proof. According to the above definitions, we have

$$\begin{aligned} SK_2(G_3) &= \sum_{(a_i,a_j)\in E(G_3)} \left[\frac{d(a_i)*d(a_j)}{2}\right]^2 + \sum_{(b_i,b_j)\in E(G_3)} \left[\frac{d(b_i)*d(b_j)}{2}\right]^2 \\ &+ \sum_{(a_i,b_i)\in E(G_3)} \left[\frac{d(a_i)*d(b_i)}{2}\right]^2 + \sum_{(a_i,b_j)\in E(G_3)} \left[\frac{d(a_i)*d(b_j)}{2}\right]^2 \\ &+ \sum_{(a_j,b_i)\in E(G_3)} \left[\frac{d(a_j)*d(b_i)}{2}\right]^2 \\ &= 4 \sum_{(v_i,v_j)\in E(G)} \left[\frac{(2d(v_i)+1)(2d(v_j)+1)}{2}\right]^2 + \sum_{(v_i\in E(G))} \left[\frac{(2d(v_i)+1)^2}{2}\right]^2 \\ &= 64SK_2^2(G)+32SK^2(G)+SK(G)\left[64SK_1(G)+36\right]+5m \\ &+ \frac{n}{4}+8\sum_{(v_i\in E(G))} d(v_i)^3 \\ &>= 32\left[2SK_2^2(G)+SK^2(G)\right]+4SK(G)\left[16SK_1(G)+9\right]+5m+\frac{n}{4} \end{aligned}$$

□

6 Conclusion

In this research, newly defined topological indices namely SK topological index, SK_1 topological index and SK_2 topological index [18] have been derived for three types of graphs namely double graph, extended double cover graphs and strong double graphs.

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