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Differential identities on weakly left cancellative semirings with derivations

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Abstract *In the present paper, we study the connection between the commutativity of a WLC semiring and derivations. Moreover, we establish a complete description and classification for some of these derivations. Finally, we give an example to prove that the imposed hypotheses are necessary.*

1 Introduction

A semiring is an algebraic structure consisting of a non-empty set. S provided with two binary operations, called addition (which is associative, not necessary commutative, and usually denoted by $+$) and multiplication (denoted by $\cdot$) such that the following conditions hold :

(i) $(S,+)$ *and* $(S,\cdot)$ *are semigroups.*

(ii) Multiplication distributes over addition from either side.

Recall that a semiring is said to be commutative if $(S,\cdot)$ is commutative. If there exists a neutral element $0 \in (S,+)$ (resp. $e \in (S,\cdot)$), it is called the zero of $(S,+)$ (resp. the identity of $(S,\cdot)$). Moreover, if $a \cdot 0 = 0 \cdot a = 0$ for all $a \in S$, then S is called a semiring with absorbing zero. In other words, semirings with absorbing zero are just rings without subtraction. Nontrivial examples of semirings first appeared in the work of Richard Dedekind [4] in 1894, in connection with the algebra of ideals of a commutative ring (one can add and multiply ideals, but one cannot subtract them). Nevertheless, the formal definition of semirings was introduced by H. S. Vandiver in 1934 and has since been studied by many authors. Semirings constitute a fairly natural generalization of rings and distributive lattices, with broad applications in different areas of mathematics such as combinatorics, functional analysis, topology, graph theory, ring theory, including partial ordered rings, optimization theory, automata theory, formal language theory, coding theory, and the mathematical modeling of quantum physics and parallel computing systems.

The basic reference for semirings is [6]. Other valuable results on the structure of semirings are contained in [15] and [17]. It is well known that the zero of a semiring S need not be absorbing and may even coincide with the identity of S (cf., e.g., [16]). An element $a \in S$ is said to be additively left (resp. right) cancellable if $a+b=a+c$ (resp. $b+a=c+a$) yields $b=c$. A semiring S is said to be additively left (resp. right) cancellative if all $a \in S$ are additively left (resp. right) cancellable in S. If S is both additively right and left cancellative, then S is said to be additively cancellative. A nonzero element a of S is multiplicatively left cancellable if $ab = ac$ implies $b = c$. A semiring S is said to be multiplicatively left cancellative (MLC) if all $a \in S \backslash \{0\}$ are multiplicatively left cancellable in S.

A left (right) ideal of a semiring S is a non-empty subset I of S such that $x + y \in I$ for all $x, y \in I$ and $sx \in I$ ($xs \in I$) for all $x \in I$ and $s \in S$. An ideal of a semiring S is a non-empty subset I of S such that I is both a left and right ideal of S.
An additive mapping $d : S \longrightarrow S$ is a derivation on S if $d(xy) = d(x)y + xd(y)$ for all $x, y \in S$. The recent literature contains various results that indicate how the global structure of a ring S is often tightly connected to the behavior of additive mappings defined on S. Recently, many authors have studied commutativity in prime and semiprime rings, admitting suitably constrained derivations acting on appropriate subsets of the rings (see, for example,[8, 10, 11, 12, 13]. Moreover, several authors have proved comparable results on semirings (see, for example,[1, 4, 7] for details).

Recently, in [5], D. Filippis et al. defined a new class of semirings called weakly left cancellative (WLC) semirings, and they studied the connection between the commutativity of this class of semirings and derivations. In particular, they proved that, if S is a WLC semiring, I is a nonzero ideal of S, and d is a derivation of S such that $d(xy) = d(yx)$ for all $x, y \in I$, or $d(xy) + yx = d(yx) + xy$ for all $x, y \in I$, or $d(x)x = xd(x)$ for all $x \in I$, then S is commutative. Motivated by the previous results, in the present paper, we continue this line by studying the commutativity of a WLC semiring, provided with a derivation d satisfying certain algebraic properties on an ideal of S. Moreover, we will give a complete description and classification of some of these derivations.

2 The results

Throughout this paper, semiring means additively cancellative semiring. We will denote $Z(S) = \{z \in S : zx = xz, \ for \ all \ x \in S\}$ the center of S, and $x \circ y = xy + yx$, the Jordan product of $x, y \in S$. We begin our discussions with the following definition:

Definition 2.1. ([5], Definition 1) A semiring S is said to be *weakly left cancellative* (WLC) if $axb = axc$ for all $x \in S$ implies either $a = 0$ or $b = c$.

Any left cancellative semiring is WLC, but the converse does not hold in general.

Example 2.2. ([5], Example 1) Let $S = \left\{ \begin{pmatrix} x & y \\ z & u \end{pmatrix} |x, y, z, u \in \mathbb{N} \right\}$ where $\mathbb{N}$ is the set of positive integers. It is straightforward to check that S is not a left cancellative semiring. Moreover, if $M, N, N' \in S$, where $M \neq 0$, then the relation $MAN = MAN'$ for all $A \in S$ forces $N = N'$ and therefore S is a WLC semiring.

Let S be a WLC semiring and I be an ideal of S. We leave the proofs of the following easy facts to the reader.

Fact 1 : *If $axb = axc$ for all $x \in I$, then $a = 0$ or $b = c$.*

Fact 2 : *If I is commutative, then S is commutative. In particular, if $xy = yx$ for all $y \in I$ then $x \in Z(S)$.*

Fact 3 : *If S admits a derivation d such that $d(I) = (0)$, then $d = 0$.*

The first important finding of this study is the following lemma:

Lemma 2.3. *Let S be a 2-torsion-free WLC semiring, and let I be a nonzero ideal of S. If S admits a derivation d such that $d^2(x) = 0$, for all $x \in I$, then $d = 0$.*

Proof. Assume that $d^2(x) = 0$ for all $x \in I$. Replacing x by xy, we get $d^2(xy) = 0 = d^2(x)y + 2d(x)d(y) + xd^2(y)$, for all $x, y \in I$. But $d^2(x) = 0 = d^2(y)$ by the hypothesis. Hence $2d(x)d(y) = 0$, for all $x, y \in I$. Since S is 2-torsion-free, we find that $d(x)d(y) = 0$. Now, replacing y by yz, we get $d(x)yd(z) = 0$, for all $x, y, z \in I$. In particular, for all $z_1, z_2 \in I$, $d(x)yd(z_1) = d(x)yd(z_2)$. Since S is WLC, it follows that $d(x) = 0$ or $d(z_1) = d(z_2)$, for all $x, z_1, z_2 \in I$. First, we suppose that $d(z_1) = d(z_2)$, thus, $d(z_1 + z_2) = d(z_2)$ implies $d(z_1) = 0$ for all $z_1 \in I$. Hence, in both cases, we find $d(x) = 0$ for all $x \in I$, and by Fact 3, we conclude that $d = 0$. □

Proposition 2.4. *Let d be an arbitrary additive mapping of S. Then we have $d(xy) = d(x)y + xd(y)$ for all $x, y \in S$ if and only if $d(xy) = xd(y) + d(x)y$ for all $x, y \in S$. Therefore, d is a derivation if and only if $d(xy) = xd(y) + d(x)y$.*

Proof. Suppose $d(xy) = d(x)y + xd(y)$ for all $x, y \in$ N. Since $x(y + y) = xy + xy$, we can see that

$$d(x(y + y)) = d(x)(y + y) + xd(y + y) = d(x)y + d(x)y + xd(y) + xd(y)$$

and

$$d(xy + xy) = d(xy) + d(xy) = d(x)y + xd(y) + d(x)y + xd(y).$$

So $d(x)y + xd(y) = xd(y) + d(x)y$, we conclude that $d(xy) = xd(y) + d(x)y$.
The converse is proved in a similar way. □

Theorem 2.5. *Let S be a 2-torsion-free WLC semiring and I a nonzero ideal of S. If S admits a nonzero derivation d such that $d(x)d(y) = d(y)d(x)$ for all $x, y \in I$, then S is commutative.*

Proof. Assume that

$$d(x)d(y) = d(y)d(x) \quad \text{for all} \;\; x, y \in I. \tag{2.1}$$

Replacing y by yx, we get

$$d(x)yd(x) + d(y)d(x)x = yd(x)d(x) + d(y)xd(x) \quad \text{for all} \;\; x, y \in I. \tag{2.2}$$

Substituting y by ry in (2.2), where $r \in S$, we get

$$d(x)ryd(x) + rd(y)d(x)x + d(r)yd(x)x = ryd(x)d(x) + rd(y)xd(x) + d(r)yxd(x) \tag{2.3}$$

Replacing r by $d(z)$ in (2.3), where $z \in I$, we find that

$$d(x)d(z)yd(x)+d(z)d(y)d(x)x+d^2(z)yd(x)x = d(z)yd(x)d(x)+d(z)d(y)xd(x)+d^2(z)yxd(x) \tag{2.4}$$

Left multiplying (2.2) by $d(z)$ and comparing the result with equation (2.4), we get

$$d(x)d(z)yd(x)+d(z)d(y)d(x)x+d^2(z)yd(x)x = d(z)d(x)yd(x)+d(z)d(y)d(x)x+d^2(z)yxd(x)$$

Hence

$$d^2(z)yd(x)x = d^2(z)yxd(x) \quad \text{for all} \;\; x, y \in I. \tag{2.5}$$

Since S is WLC, then equation (2.5) together with Fact 1, yield that

$$d^2(z) = 0 \;\; \text{for all} \;\; z \in I \quad or \quad d(x)x = xd(x) \quad \text{for all} \;\; x \in I.$$

In view of Lemma 2.3, we conclude that $d(x)x = xd(x)$ for all $x \in I$. Thus, by ([5], Theorem 3), S is commutative. □

We would like to recall that any prime ring R is trivially a WLC zero absorbing semiring. As a consequence of Theorem 2.5 we have:

Corollary 2.6. ([2], Theorem 3) *Let R be a prime ring. If R admits a nonzero derivation d such that $[d(x), d(y)] = 0$ for all $x, y \in R$, then R is commutative.*

Theorem 2.7. *Let S be a WLC semiring and I a nonzero ideal of S. If S admits a nonzero derivation d such that $d(x)d(y) + xy = d(y)d(x) + yx$ for all $x, y \in I$, then S is commutative.*

Proof. We are given that

$$d(x)d(y) + xy = d(y)d(x) + yx \quad \text{for all} \;\; x, y \in I. \tag{2.6}$$

Replacing y by yx in the last equation, we get

$$d(x)yd(x) + d(x)d(y)x + xyx = yd(x)d(x) + d(y)xd(x) + yx^2 \quad \text{for all} \;\; x, y \in I. \tag{2.7}$$

Right multiplying (2.6) by x and comparing the result with (2.7), we get

$$d(x)yd(x) + d(y)d(x)x = yd(x)d(x) + d(y)xd(x) \quad \text{for all} \;\; x, y \in I. \tag{2.8}$$

Writing ry instead of y in (2.8), where $r \in S$, we get

$$d(x)ryd(x) + rd(y)d(x)x + d(r)yd(x)x = ryd(x)d(x) + rd(y)xd(x) + d(r)yxd(x) \tag{2.9}$$

Taking now $r = d(x)$ in (2.9), we find that

$$d(x)d(x)yd(x)+d(x)d(y)d(x)x+d^2(x)yd(x)x = d(x)yd(x)d(x)+d(x)d(y)xd(x)+d^2(x)yxd(x) \tag{2.10}$$

Left multiplying (2.8) by $d(x)$ and comparing the result with equation (2.10), we get

$$d(x)d(x)yd(x)+d(x)d(y)d(x)x+d^2(x)yd(x)x = d(x)d(x)yd(x)+d(x)d(y)d(x)x+d^2(x)yxd(x) \tag{2.11}$$

Hence

$$d^2(x)yd(x)x = d^2(x)yxd(x) \quad \text{for all} \;\; x, y \in I. \tag{2.12}$$

Since S is WLC, then equation (2.12) together with Fact 1, yield that

$$d^2(x) = 0 \quad \textit{or} \quad d(x)x = xd(x) \quad \text{for all} \;\; x \in I.$$

Let $u \in I$ such that $d^2(u) = 0$. By equation (2.6), we have

$$d(x)d(u) + xu = d(u)d(x) + ux \quad \text{for all} \;\; x \in I. \tag{2.13}$$

Substituting $xd(u)$ for x in equation (2.13), we get

$$d(x)d(u)d(u) + xd(u)u = d(u)d(x)d(u) + uxd(u) \quad \text{for all} \;\; x \in I. \tag{2.14}$$

Right multiplying equation (2.13) by $d(u)$ and comparing it with equation (2.14), we find that

$$xd(u)u = xud(u) \quad \text{for all} \;\; x \in I. \tag{2.15}$$

Substituting xz for x in (2.15), we get

$$xzd(u)u = xzud(u) \quad \text{for all} \;\; x, z \in I. \tag{2.16}$$

Since S is WLC and $I \neq \{0\}$, then equation (2.16) together with Fact 1, yield that $d(u)u = ud(u)$. Thus, in both cases, we find that $xd(x) = d(x)x$ for all $x \in I$, and by ([5], Theorem 3) S is commutative. □

The following is a result of Theorem 2.7:

Corollary 2.8. ([3], Theorem 1) *Let R be a prime ring and K be an ideal of R. If R admits a nonzero derivation d such that either $[d(x), d(y)] = [x, y]$ for all $x, y \in K$ or $[d(x), d(y)] = -[x, y]$ for all $x, y \in K$, then R is commutative.*

Theorem 2.9. *Let S be a 2-torsion-free WRC semiring, and let I be a nonzero ideal of S. If S admits a derivation d such that $d(x) \circ d(y) = 0$ for all $x, y \in I$, then $d = 0$.*

Proof. Assume that there exists a nonzero derivation d satisfying

$$d(x)d(y) + d(y)d(x) = 0 \quad \text{for all} \quad x, y \in I. \tag{2.17}$$

Substituting yu for y in equation (2.17), we get

$$d(x)d(y)u + d(x)yd(u) + yd(u)d(x) + d(y)ud(x) = 0 \quad \text{for all} \quad x, y, u \in I. \tag{2.18}$$

Right multiplying equation (2.17) by u and comparing it with (2.18), we get

$$d(x)yd(u) + yd(u)d(x) + d(y)ud(x) = d(y)d(x)u \quad \text{for all} \quad x, y, u \in I. \tag{2.19}$$

Substituting ry for y in (2.19), where $r \in S$, we obtain

$$d(x)ryd(u) + ryd(u)d(x) + rd(y)ud(x) + d(r)yud(x) = rd(y)d(x)u + d(r)yd(x)u \tag{2.20}$$

Writing $d(x)$ instead of r in (2.20), we arrive at

$$d(x)d(x)yd(u)+d(x)yd(u)d(x)+d(x)d(y)ud(x)+d^2(x)yud(x) = d(x)d(y)d(x)u+d^2(x)yd(x)u \tag{2.21}$$

Left multiplying equation (2.20) by $d(x)$ and comparing the result with (2.21), then it follows that

$$d^2(x)yud(x) = d^2(x)yd(x)u \quad \text{for all} \quad x, y, u \in I. \tag{2.22}$$

Since S is WLC, then equation (2.22) together with Fact 1, yield that

$$d^2(x) = 0 \quad or \quad d(x)u = ud(x) \quad \text{for all} \quad x, u \in I.$$

Let $x_0 \in I$ such that $d(x_0)u = ud(x_0)$ for all $u \in I$. In view of Fact 2, we can see that $d(x_0) \in Z(S)$. Replacing x by x_0 in (2.17) and in light of 2-torsion freeness, we get

$$d(x_0)d(y) = 0 \quad \text{for all} \quad y \in I. \tag{2.23}$$

Replacing y by yx_0 in (2.23), we obtain

$$d(x_0)yd(x_0) = 0 \quad \text{for all} \quad y \in I. \tag{2.24}$$

This implies that $d(x_0) = 0$, so $d^2(x_0) = 0$. Thus $d^2(x) = 0$ for all $x \in I$, and Lemma 2.3 forces that $d = 0$, which contradicts our supposition. Thus, we conclude that $d = 0$. □

Theorem 2.10. *Let S be a 2-torsion-free WLC semiring and I be a nonzero ideal of S. Then there exists no derivation d such that either $d(x) \circ d(y) = x \circ y$ for all $x, y \in I$.*

Proof. Assume there exists a derivation d such that

$$d(x)d(y) + d(y)d(x) = xy + yx \quad \text{for all} \quad x, y \in I. \tag{2.25}$$

Replacing y by yx in (2.25), we get

$$d(x)d(y)x + d(x)yd(x) + yd(x)d(x) + d(y)xd(x) = xyx + yx^2 \quad \text{for all} \quad x, y \in I. \tag{2.26}$$

Right multiplying equation (2.25) by x and comparing with equation (2.26), we find

$$d(x)yd(x) + yd(x)d(x) + d(y)xd(x) = d(y)d(x)x \quad \text{for all} \quad x, y \in I. \tag{2.27}$$

Substituting $d(x)y$ for y in (2.27), we get

$$d(x)d(x)yd(x)+d(x)yd(x)d(x)+d(x)d(y)xd(x)+d^2(x)yxd(x) = d(x)d(y)d(x)x+d^2(x)yd(x)x \tag{2.28}$$

Left multiplying equation (2.27) and comparing with equation (2.28), we obtain

$$d^2(x)yxd(x) = d^2(x)yd(x)x \quad \text{for all} \quad x, y \in I. \tag{2.29}$$

In view of Fact 1, equation (2.29) implies that

$$d^2(x) = 0 \text{ or } d(x)x = xd(x) \quad \text{for all } x \in I. \tag{2.30}$$

Let $u \in I$ such that $d^2(u) = 0$, we have

$$d(x)d(u) + d(u)d(x) = xu + ux \quad \text{for all } x \in I. \tag{2.31}$$

Substituting $xd(u)$ for x in equation (2.31), we get

$$d(x)d(u)d(u) + d(u)d(x)d(u) = xd(u)u + uxd(u) \quad \text{for all } x \in I. \tag{2.32}$$

Right multiplying equation (2.31) by $d(u)$ and comparing the result with equation (2.32), we find that

$$xud(u) = xd(u)u \quad \text{for all } x \in I. \tag{2.33}$$

Substituting xz for x in (2.33), we get

$$xzd(u)u = xzud(u) \quad \text{for all } x, z \in I. \tag{2.34}$$

Since S is WLC and $I \neq \{0\}$, using equation (2.34) and Fact 1, we get $d(u)u = ud(u)$. Thus, in both cases, we find $xd(x) = d(x)x$ for all $x \in I$, and by ([5], Theorem 3) S is commutative. Consequently, and in light of 2-torsion freeness, our hypothesis forces

$$d(x)d(y) = xy \quad \text{for all } x, y \in I. \tag{2.35}$$

Replacing y by yz in (2.35) we get $d(x)yd(z) = 0$ for all $y, z \in I$, so $d(x) = 0$ for all $x \in I$, and by Fact 3 we conclude that $d = 0$, therefore equation (2.21) becomes $xy = 0$ for all $x, y \in I$. Thus, $I = \{0\}$, a contradiction. This completes the proof of our result. □

Remark 2.11. We can prove that, under the same hypothesis as in Theorem 2.10, there exists no derivation d such that either $d(x) \circ d(y) + x \circ y$ for all $x, y \in I$.

As an application of Theorem 2.10, the following corollary generalizes Proposition 1 in [9].

Corollary 2.12. ([9], Proposition 1) *Let R be a 2-torsion free prime ring and I be a nonzero ideal of R. Then there exists no derivation d such that either $d(x) \circ d(y) = x \circ y$ for all $x, y \in I$ or $d(x) \circ d(y) + (x \circ y) = 0$ for all $x, y \in I$.*

3 Examples

In this part, we go over a few illustrations that demonstrate how, in some circumstances, our findings do not hold. We start by demonstrating through the following instances that the condition "weakly left cancellative hypothesis" is required.

Example 3.1. Let $S = \left\{ \begin{pmatrix} 0 & a & b \\ 0 & 0 & c \\ 0 & 0 & 0 \end{pmatrix} | a, b, c \in \mathbb{N} \right\}$, where $\mathbb{N}$ is the set of positive integers.
If we define the map d on the set S given by
$d\left(\begin{pmatrix} 0 & a & b \\ 0 & 0 & c \\ 0 & 0 & 0 \end{pmatrix}\right) = \begin{pmatrix} 0 & 0 & c \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$, therefore, it is simple to verify that S is not a WLC-semiring and d is a nonzero derivation of S which satisfies the conditions of Theorem 2.5, Theorem 2.7 and Theorem 2.9 . However, S is noncommutative, hence the weakly left cancellative hypothesis is crucial.

In the following illustration, we show that the "primeness hypothesis" of R in Corollary 2.6 and Corollary 2.8 is not a purely theoretical construct.

Example 3.2. Let us consider $R = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \mid a,b,c,d \in Z \right\}$,
$d\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} 0 & -b \\ c & 0 \end{pmatrix}$ a derivation of R, and $\mathbb{C}$ be the field of complex numbers. If we set $R_1 = R \times \mathbb{C}$, then R_1 is a semi-prime ring. Consider the derivation $D : R_1 \to R_1$ defined as $D(x,s) = (d(x), 0)$. Furthermore D satisfies the conditions of Corollary 2.6 and Corollary 2.8, but R_1 is not commutative. Consequently, the hypothesis of primeness is required.

4 Conclusion

In this study, we have investigated the relationship between derivations and the commutativity of a WLC semiring. We also provide a thorough description and categorization for a few of these derivations. The study ends with the following intriguing unsolved issues.

Problem 1 : *Let S be a 2-torsion-free WLC semiring and I a nonzero ideal of S. If S admits derivations d_1 and d_2 such that $d_1(x)d_2(y) = d_2(y)d_1(x)$ for all $x, y \in I$, then what can we conclude about the structures of S, d_1 and d_2 ?*

Problem 2 : *Let S be a 2-torsion-free WLC semiring and I a nonzero ideal of S. If S admits derivations d_1 and d_2 such that $d_1(x)d_2(y) + xy = d_2(y)d_1(x) + yx$ for all $x, y \in I$, then what can we conclude about the structures of S, d_1 and d_2?*

Problem 3 : *Let S be a 2-torsion-free WLC semiring and I a nonzero ideal of S. If S admits derivations d_1 and d_2 such that $d_1(x) \circ d_2(y) = 0$ for all $x, y \in I$, then what can we conclude about the structures of S, d_1 and d_2?*

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