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FIXED POINT RESULTS OF CONTRACTIVE MAPPINGS VIA SIMULATION FUNCTION IN B-DISLOCATED METRIC SPACES

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Abstract *In this paper, we propose a new fixed point theorem in b-dislocated metric space which extend, generalize and unify some important results in literature. Also, we present an example to illustrate our main results.*

1 Introduction and Preliminaries

P. Hitzler and A. K. Seda in [9] introduced an extension of metric space and generalized the Banach contraction principle on this space which is called dislocated metric spaces, and proved a more general Banach contraction principle in such space. Since then, many authors have been interested in investigating fixed point theorems for single-valued and set valued mappings in dislocated metric space (see [1], [12], [14], [15], [17] and [19]). On the other hand, N. Hussain et al. in [10] introduced the b-dislocated metric spaces associated with some topological aspects and properties. Afterwards, many mathematicians obtained fixed point theorems in b-dislocated metric spaces (see [16], [5], [13], [20]). Recently, Joonaghany et al. [11] introduced simulation function in metric spaces and obtained some fixed point results. Subsequently, many scholars were interested in fixed point theorems for simulation function (see [2], [4], [6] and [7]). Throughout this paper, inspired and motivated by previous results in the existing literature, we give fixed point results for simulation functions in b-dislocated metric spaces. Moreover, we provide an example to support our main assertions.

For the sake of the completeness of the manuscript, we shall recall some basic results and concepts here

Definition 1.1 (Partial b-metric space). [18] Let X be a non-empty set and Let $s \in [1, \infty)$ be a real number. A function $\rho_b : X \times X \to [0, +\infty)$ is a partial b-metric on X if it satisfies the conditions:

1) $\forall v, w \in X : v = w$ if and only if $\rho_b(v, v) = \rho_b(w, v) = \rho_b(w, w)$,

2) $\forall v, w \in X : \rho_b(v, v) \leq \rho_b(w, v)$,

3) $\forall v, w \in X : \rho_b(v, w) = \rho_b(w, v)$,

4) $\forall v, w, z \in X : \rho_b(v, w) \leq s\Big(\rho_b(v, z) + \rho_b(w, z)\Big) - \rho_b(z, z)$.

The pair (X, ρ_b, s) is called is a partial b-metric space.

1.1 Dislocated b-metric spaces

Definition 1.2 (Dislocated b-metric space). [10] Let X be a non-empty. set and let $s \in [1, +\infty)$ be a real number. A function $\rho_b : X \times X \to [0, +\infty)$ is a dislocated b-metric on X if it satisfies the conditions:

(ρ_b1) $\forall v,w\in X:\rho_b(v,w)=0\Rightarrow v=w$,

(ρ_b2) $\forall v,w\in X:\rho_b(v,w)=\rho_b(w,v)$,

(ρ_b3) $\forall v,w,z\in X:\rho_b(v,w)\leq s\Big(\rho_b(v,z)+\rho_b(w,z)\Big)$.

The pair (X,ρ_b,s) is called dislocated b-metric space (shortly ρ_b-metric space).

Example 1.3.

Let $X=[0,+\infty)$ and $\rho_b(v,w)=(v+w)^2$ then ρ_b is a a dislocated b-metric on X with $s=2$.
Indeed,

- $\forall v,w\in[0,+\infty):\rho_b(v,w)=0\Rightarrow w+v=0\Rightarrow v=w=0$.
- $\forall v,w\in[0,+\infty):\rho_b(v,w)=(v+w)^2=(w+v)^2=\rho_b(w,v)$.
- *By the inequality $(x+y)^2\leq 2(x^2+y^2)$ which is holds for all $x,y\geq 0$, we get*

  $$\rho_b(v,w)\leq(w+z+z+v)^2\leq 2\Big((v+z)^2+(z+w)^2\Big)=2(\rho_b(v,z)+\rho_b(z,w)),\forall v,w,z\in X.$$

Remark 1.4. The dislocated b-metric space given in Example 1.3 is not a partial b-metric space. Indeed, for any $0<w<v$ we have $\rho_b(v,v)=(v+v)^2>(v+w)^2=\rho_b(v,w)$, so the condition $\rho_b(v,v)\leq\rho_b(v,w),\forall v,w\in X$ in the definition of partial b-metric space is not satisfied.

Definition 1.5. [10] Let (X,ρ_b,s) a ρ_b-metric space, a sequence (v_n) on a (X,ρ_b,s) is said to be:

- *ρ_b-convergent to $v\in X$ if and only if $\lim_{n\to+\infty}\rho_b(v_n,v)=0$.*
- *ρ_b-Cauchy if and only if $\lim_{n\to+\infty}\rho_b(v_n,v_p)$ exists and tends to be finite.*

Definition 1.6. If every ρ_b-Cauchy sequence in a ρ_b-metric space (X,ρ_b,s) is ρ_b-convergent, we say that the space (X,ρ_b,s) is a complete ρ_b-metric space .

The following Lemma is useful for us.

Lemma 1.7. *Let (X,ρ_b,s) be a ρ_b-metric space, $O:X\to X$ a mapping and $\kappa\in]0,1[$. If (v_m) is a sequence in X, where $v_m=Ov_{m-1}$ and*

$$\rho_b(v_m,v_{m+1})\leq\kappa\rho_b(v_{m-1},v_m),\text{for each } m\in\mathbb{N},\tag{1.1}$$

then (v_m) is ρ_b-Cauchy sequence.

Proof. See for example [8, Lemma 10]. □

1.2 Similation function

We denote by Γ the set of all nondecreasing and continues functions $\psi:[0,+\infty[\to[0,+\infty[$ such that $\psi(0)=0$.

Definition 1.8 (Similation function). Let (X,ρ_b,s) be a ρ_b-metric space and $\psi\in\Gamma$. A $b-\psi$-simulation function is a function $\eta_b:[0,\infty)\times[0,\infty)\to\mathbb{R}$ satisfying:

(η_b1) $\forall t,r\in\mathbb{R}_+:\eta_b(r,t)<\psi(t)-\psi(r)$,

(η_b2) if $(r_n),(t_n)$ are two sequences in $[0,+\infty)$, such that for some $p>0$

$$\limsup_{n\to\infty}t_n=s^p\lim_{n\to\infty}r_n>0,\tag{1.2}$$

then

$$\limsup_{n\to\infty}\eta_b(s^pr_n,t_n)<0.\tag{1.3}$$

Example 1.9 ([8]).

Let $\psi\in\Gamma$ and $\phi:[0,+\infty)\to[0,+\infty)$ such that $\limsup_{t\to t_0}\phi(t)\geq 0$ for all $t_0>0$ and $\phi(0)=0$ if and only if $t=0$, then $\eta(t,r)=\psi(t)-\phi(t)-\psi(r)$ is b-ψ-simulation function.

We shall denote by Z_{ψ_b} the family of all b-ψ-simulation functions.

2 Main results

We start by the following notion of rationnal contraction.

Definition 2.1. Let (X, ρ_b, s) be a ρ_b-metric space. A mapping $O : X \to X$ is called η_b-rational contraction of type A if there exists a function $\eta_b \in Z_{\psi_b}$ such that

$$\frac{1}{2s} \min\Big(\rho_b(v, Ov), \rho_b(w, Ow)\Big) \le \rho_b(v, w) \textit{ implies } \eta_b(s^p\rho_b(Ov, Ow), \mathcal{D}_A(v, w)) \ge 0, \quad (2.1)$$

where

$$\mathcal{D}_A(v, w) = \max\Big(\rho_b(v, w), \rho_b(v, Ov), \rho_b(w, Ow), \frac{\rho_b(w, Ow)(1 + \rho_b(v, Ov))}{1 + \rho_b(v, w)}\Big). \quad (2.2)$$

Theorem 2.2. *Let (X, ρ_b, s) be a complete ρ_b-metric space and $O : X \to X$ be a η_b-rational contraction of type A. Then O admits exactly one fixed point.*

Proof. We fix $v_0 \in X$ (arbitrarily chosen), and define the sequence (v_m) by the relation

$$v_{m+1} = Ov_m, \forall m \in \mathbb{N}. \quad (2.3)$$

If there exists $m_0 \in \mathbb{N}$ such that $v_{m_0+1} = v_{m_0}$ then by (2.3) we have $Ov_{m_0} = v_{m_0}$, that is v_{m_0} is a fixed point for O.
Assume that $v_{m+1} \ne v_m, \forall m \in \mathbb{N}$. We choose $v = v_{m-1}$ and $w = v_m$ in (2.2) then

$$\begin{aligned}&\mathcal{D}_A(v_{m-1}, v_m) = \\ &\max\Big(\rho_b(v_{m-1}, v_m), \rho_b(v_{m-1}, v_m), \rho_b(v_m, v_{m+1}), \frac{\rho_b(v_m, v_{m+1})(1 + \rho_b(v_{m-1}, v_m))}{1 + \rho_b(v_{m-1}, v_m)}\Big) = \\ &\max\Big(\rho_b(v_{m-1}, v_m), \rho_b(v_m, v_{m+1})\Big).\end{aligned}$$

Moreover, we have

$$\begin{aligned}\frac{1}{2s} \min\Big(\rho_b(v_{m-1}, Ov_{m-1}), \rho_b(v_m, Ov_m)\Big) &= \frac{1}{2s} \min\Big(\rho_b(v_{m-1}, v_m), \rho_b(v_m, w_{m+1})\Big) \\ &\le \rho_b(v_{m-1}, v_m),\end{aligned}$$

then, by (2.1)

$$\eta_b(s^p\rho_b(Ov_{m-1}, Ov_m), \mathcal{D}_A(v_{m-1}, v_m)) \ge 0,$$

which implies by **(η_b1)**

$$\begin{aligned}0 &\le \eta_b(s^p\rho_b(v_m, v_{m+1}), \mathcal{D}_A(v_{m-1}, v_m)) \\ &< \psi(\mathcal{D}_A(v_{m-1}, v_m)) - \psi(s^p\rho_b(v_m, v_{m+1})),\end{aligned}$$

i.e.

$$\psi(s^p\rho_b(v_m, v_{m+1})) < \psi(\max\Big(\rho_b(v_{m-1}, v_m), \rho_b(v_m, v_{m+1})). \quad (2.4)$$

Since ψ is nondecreasing, we get

$$s^p\rho_b(v_m, v_{m+1}) < \max\Big(\rho_b(v_{m-1}, v_m), \rho_b(v_m, v_{m+1})\Big), \forall m \in \mathbb{N}.$$

Now, assume that there exists $m_1 \in \mathbb{N}$ such that

$$\max\Big(\rho_b(v_{m_1-1}, v_m), \rho_b(v_{m_1}, v_{m_1+1})\Big) = \rho_b(v_{m_1}, v_{m_1+1}),$$

Thus, by (2.4) it holds

$$s^p\rho_b(v_{m_1}, v_{m_1+1}) < \rho_b(v_{m_1}, v_{m_1+1}), \quad (contraduction\ \ with\ \ s \ge 1).$$

Then, again by (2.4) we have

$$\rho_b(v_m, v_{m+1})) < \frac{1}{s^p}\rho_b(v_{m-1}, v_m), \forall m \in \mathbb{N}. \tag{2.5}$$

Thus, by Lemma 1.7 we see that the sequence (v_m) is a ρ_b-Cauchy sequence on the complete ρ_b-.metric space (X, ρ_b, s). Then, there exits $u \in X$ such that

$$\lim_{p,q\to\infty} \rho_b(v_p, v_q) = \lim_{n\to\infty} \rho_b(v_n, u) = 0. \tag{2.6}$$

Now, we have

$$\forall m \in \mathbb{N} : \frac{1}{2s}\rho_b(v_m, v_{m+1}) \le \rho_b(v_m, u) \quad \text{or} \quad \frac{1}{2s}\rho_b(v_{m+1}, v_{m+2}) \le \rho_b(v_{m+1}, u), \tag{2.7}$$

indeed, if we assume the contrary then there exists $m_0 \in \mathbb{N}$ such that

$$\rho_b(v_{m_0}, u) < \frac{1}{2s}\rho_b(v_{m_0}, v_{m_0+1}) \quad and \quad \rho_b(v_{m_0+1}, u) < \frac{1}{2s}\rho_b(v_{m_0+1}, v_{m_0+2}), \tag{2.8}$$

then, by (2.8) and (2.5) we get

$$\begin{aligned}
\rho_b(v_{m_0}, v_{m_0+1}) &\le s\Big(\rho_b(v_{m_0}, u) + \rho_b(u, v_{m_0+1})\Big) \\
&< s\Big(\frac{1}{2s}\rho_b(v_{m_0}, v_{m_0+1}) + \frac{1}{2s}\rho_b(v_{m_0+1}, v_{m_0+2})\Big) \\
&= \frac{1}{2}\Big(\rho_b(v_{m_0}, v_{m_0+1}) + \rho_b(v_{m_0+1}, v_{m_0+2})\Big) \\
&< \rho_b(v_{m_0}, v_{m_0+1}),
\end{aligned}$$

contradiction.
The relation (2.7) allows to extract a subsequence $(v_{m(\ell)})$ of (v_m) verifying

$$\frac{1}{2s}\min\Big(\rho_b(v_{m(\ell)}, Ov_{m(l)}), \rho_b(u, Ou)),\Big) \le \frac{1}{2s}\rho_b(v_{m(\ell)}, v_{m(l)+1}) \le \rho_b(v_{m(\ell)}, u),$$

which implies by (2.1)

$$\eta_b\Big(s^p\rho_b(Ov_{m(\ell)}, Ou), \mathcal{D}_A(m(\ell), u))\Big) \ge 0, \tag{2.9}$$

where

$$\begin{aligned}
&\rho_b(u, Ou) \le \mathcal{D}_A(v_{m(\ell)}, u) = \\
&\max\Big(\rho_b(v_{m(\ell)}, u), \rho_b(v_{m(\ell)}, Ov_{m(\ell)}), \rho_b(u, Ou), \tfrac{\rho_b(u,Ou)(1+\rho_b(v_{m(\ell)},Ov_{m(\ell)}))}{1+\rho_b(v_{m(\ell)},u)}\Big) = \\
&\max\Big(\rho_b(v_{m(\ell)}, u), \rho_b(v_{m(\ell)}, v_{m(\ell)+1}), \rho_b(u, Ou), \tfrac{\rho_b(u,Ou)(1+\rho_b(v_{m(\ell)},v_{m(\ell)+1}))}{1+\rho_b(v_{m(\ell)},u)}\Big).
\end{aligned}$$

By passing to the limit when $\ell \to +\infty$ we get $\rho_b(u, Ou) \le \lim_{\ell\to\infty} \mathcal{D}_A(v_{m(\ell)}, u) \le \rho_b(u, Ou)$, then

$$\lim_{\ell\to\infty} \mathcal{D}_A(v_{m(\ell)}, u) = \rho_b(u, Ou). \tag{2.10}$$

In the rest of the proof, we assume that $v_m \neq u$, for infinitely many $m \in \mathbb{N}$, because if we assume the contrary, then there exists $m_0 \in \mathbb{N}$ such that $v_m = u$ for all $m \ge m_0$ then $v_{m_0} = v_{m_0+1} = Ov_{m_0} = Ou$ and u is a fixed point of O.
By $(\eta_b 1)$ and (2.9) we get

$$\begin{aligned}
0 &\le \psi\Big(s^p\rho_b(Ov_m, Ou), \mathcal{D}_A(v_m, u)\Big) \\
&< \psi\Big(\mathcal{D}_A(v_m, u)\Big) - \psi\Big(s^p\rho_b(Ov_m, Ou)\Big),
\end{aligned}$$

then

$$\psi\Big(s^p\rho_b(Ov_m, Ou)\Big) < \psi\Big(\mathcal{D}_A(v_m, u)\Big),$$

which implies

$$s^p\rho_b(Ov_m, Ou) < \mathcal{D}_A(v_m, u) \quad \text{(because } \psi \text{ is nondecreasing.)} \tag{2.11}$$

Then, we have

$$\begin{aligned}\rho_b(u, Ou) &\leq s\Big(\rho_b(u, Ov_m) + \rho_b(Ov_m, Ou)\Big)\\ &\leq s\rho_b(u, Ov_m) + s^p\rho_b(Ov_m, Ou),\end{aligned}$$

letting $m \to +\infty$ and using (2.6), (2.10) and (2.11) we get

$$\begin{aligned}\rho_b(u, Ou) &\leq \lim_{m\to\infty} s^p\rho_b(Ov_m, Ou)\\ &\leq \lim_{m\to\infty} \mathcal{D}_A(v_m, u) = \rho_b(u, Ou),\end{aligned}$$

that is

$$\lim_{m\to\infty} s^p\rho_b(Ov_m, Ou) = \rho_b(u, Ou).$$

Now, assume that $\rho_b(u, Ou) > 0$ then by $(\eta_b 2)$ (with $t_m = \mathcal{D}_A(v_m, u)$ and $r_m = \rho_b(Ov_m, Ou)$ it holds

$$\lim_{m\to\infty} \eta_b(s^p\rho_b(Ov_m, Ou), \mathcal{D}_A(v_m, u)) < 0$$

which is contradiction with (2.9) then $\rho_b(u, Ou) = 0$ and u is a fixed point of O.

Now, we proof the uniqueness of fixed point of O, if there exists $A \ni z \neq u$ verifying $Oz = z$ then

$$0 = \frac{1}{2s}\min\Big(\rho_b(z, Oz), \rho_b(u, Ou)\Big) \leq \rho_b(z, u),$$

which implies

$$\begin{aligned}0 &\leq \eta_b\Big(s^p\rho_p(Ou, Oz), \mathcal{D}_A(z, u)\Big) < \psi(\mathcal{D}_A(z, u)) - \psi(s^b\rho_b(Ou, Oz))\\ &= \psi(\rho_b(Ou, Oz)) - \psi(s^p\rho_b(Ou, Oz)),\end{aligned}$$

which is contradiction with the fact that ψ is nondecreasing, then $u = z$ and the fixed point of O is unique. □

Example 2.3.

Let $X = [0, 1]$ and $\rho_b(v, w) = (v + w)^2$, then $(X, \rho_b, 2)$ is a complete dislocated b-metric space, but not a partial b-metric space (see Example 1.3 and Remark 1.4), then [8, thm 3] does not work.

Let us proof that $O : X \to X$ defined by $Ov = \begin{cases} v^2, & v \in [0, 1/4]\\ 1/16, & v \in (1/4, 1]\end{cases}$, is η_b-rational contraction of type A, where

$$\eta_b(r, t) = \psi(t) - \phi(t) - \psi(r) = \sqrt{t} - 2\sqrt{r},$$

(see Example 1.9, with $\psi, \phi : [0, +\infty) \to [0, +\infty)$ given by $\psi(t) = 2\sqrt{t}$ and $\phi(t) = \sqrt{t}$). We will distinguish several cases with respect to v and w.

Case 1: $0 \leq v, w \leq 1/4$. *Then*

$\rho_b(v, w) = (v + w)^2, \quad \rho_b(v, Ov) = (v + v^2)^2, \quad \rho_b(w, Ow) = (w + w^2)^2$, *and*

$$\rho_b(Ou, Ow) = (v^2 + w^2)^2.$$

If $v \leq w$, then $\min(\rho_b(v, Ov), \rho_p(w, Ow)) = \rho_b(v, Ov) = (v + v^2)^2$ *and*

$$\begin{aligned}\frac{1}{4}\min(\rho_b(v, Ov), \rho_p(w, Ow)) &= \frac{1}{4}(v + v^2)^2\\ &\leq (v + w)^2 = \rho_b(v, w)\end{aligned}$$

which implies

$$\eta_b(4\rho_b(Ov, Ow), \mathcal{D}_A(v, w)) \geq \eta_b\Big(4(v^2 + w^2)^2, (w + v)^2\Big) \geq 0.$$

If $w \leq v$, *then* $\min(\rho_b(v, Ov), \rho_p(w, Ow)) = \rho_b(w, Ow) = (v + v^2)^2$ *and*

$$\begin{aligned} \frac{1}{4}\min(\rho_b(v, Ov), \rho_p(w, Ow)) &= \frac{1}{4}(w + w^2)^2 \\ &\leq (v + w)^2 = \rho_b(v, w) \end{aligned}$$

which implies

$$\eta_b(4\rho_b(Ov, Ow), \mathcal{D}_A(v, w)) \geq \eta_b\Big(4(v^2 + w^2)^2, (w + v)^2\Big) \geq 0.$$

Case 2: $1/4 < v, w \leq 1$. *Then*
$\rho_b(v, w) = (v + w)^2, \quad \rho_b(v, Ov) = (v + 1/16)^2, \quad \rho_b(w, Ow) = (w + 1/16)^2$, *and*

$$\rho_b(Ou, Ow) = (1/8)^2.$$

If $v \leq w$, *then* $\min(\rho_b(v, Ov), \rho_p(w, Ow)) = \rho_b(v, Ov) = (v + v^2)^2$ *and*

$$\begin{aligned} \frac{1}{4}\min(\rho_b(v, Ov), \rho_p(w, Ow)) &= \frac{1}{4}(v + 1/16)^2 \\ &\leq (v + w)^2 = \rho_b(v, w) \end{aligned}$$

which implies

$$\eta_b(4\rho_b(Ov, Ow), \mathcal{D}_A(v, w)) \geq \eta_b\Big(4(1/8)^2, (w + v)^2\Big) \geq 0.$$

If $w \leq v$, *then* $\min(\rho_b(v, Ov), \rho_p(w, Ow)) = \rho_b(w, Ow) = (w + 1/16)^2$ *and*

$$\begin{aligned} \frac{1}{4}\min(\rho_b(v, Ov), \rho_p(w, Ow)) &= \frac{1}{4}(w + 1/4)^2 \\ &\leq (v + w)^2 = \rho_b(v, w) \end{aligned}$$

which implies

$$\eta_b(4\rho_b(Ov, Ow), \mathcal{D}_A(v, w)) \geq \eta_b\Big(4(1/8)^2, (w + v)^2\Big) \geq 0.$$

Case 3: $v \leq 1/4 < w \leq 1$. *Then*
$\rho_b(v, w) = (v + w)^2, \quad \rho_b(v, Ov) = (v + v^2)^2, \quad \rho_b(w, Ow) = (w + 1/16)^2$, *and*

$$\rho_b(Ou, Ow) = (v^2 + 1/16)^2.$$

Then $\min(\rho_b(v, Ov), \rho_p(w, Ow)) = \rho_b(v, Ov) = (v + v^2)^2$ *and*

$$\begin{aligned} \frac{1}{4}\min(\rho_b(v, Ov), \rho_p(w, Ow)) &= \frac{1}{4}(v^2 + v)^2 \\ &\leq (v + w)^2 = \rho_b(v, w) \end{aligned}$$

which implies

$$\eta_b(4\rho_b(Ov, Ow), \mathcal{D}_A(v, w)) \geq \eta_b\Big(4(v^2 + 1/16)^2, (w + v)^2\Big) \geq 0.$$

Case 4: $w \leq 1/4 < v \leq 1$. *Then*
$\rho_b(v, w) = (v + w)^2, \quad \rho_b(v, Ov) = (v + 1/16)^2, \quad \rho_b(w, Ow) = (w + w^2)^2$, *and*

$$\rho_b(Ou, Ow) = (w^2 + 1/16)^2.$$

Then $\min(\rho_b(v, Ov), \rho_p(w, Ow)) = \rho_b(w, Ow) = (w + w^2)^2$ *and*

$$\begin{aligned}\frac{1}{4}\min(\rho_b(v, Ov), \rho_p(w, Ow)) &= \frac{1}{4}(w^2 + w)^2 \\ &\leq (v + w)^2 = \rho_b(v, w)\end{aligned}$$

which implies

$$\eta_b(4\rho_b(Ov, Ow), \mathcal{D}_A(v, w)) \geq \eta_b\Big(4(w^2 + 1/16)^2, (w + v)^2\Big) \geq 0.$$

The conditions of Theorem 2.2 are satisfied then O *have a unique fixed pint which is* $v = 0$.

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