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Fuzzy Analysis of a Retrial Machine Repair Problem using Gaussian Fuzzy Number

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Abstract With standby provisioning, a single repair server, and a N policy, this paper investigates the retrial machine repair problem using the Gaussian Fuzzy number. A first-come, first-served (FCFS) discipline is used by the server to allow failed units into the system. At the service completion instant, if the server does not find a failed unit in the system, it goes on vacation. In this study, the single repair server with a N-Policy is examined, which means that the single server is turned off when idle in order to maximize its efficiency. Therefore, there are no failed units in the system. If a machine fails at any time, it is sent to the server for repair, and the repairman restores the machine to its pre-failure state. If a server is stopped, it can only be restarted if there are sufficient failed units (more than or equal to N). Using Markov process theory, we present the differential-difference equations for the queueing system. Exponentially distributed unit lifetimes and repair times are assumed for machining systems. A cost optimization criterion and various performance characteristics for the queuing system are developed in order to determine optimal operating conditions. Parameters like the arrival rate and service rate are fuzzified using Gaussian fuzzy numbers. We also presented a numerical example to demonstrate the validity of the proposed approach, providing designers and practitioners with more information to more precisely evaluate general repairable systems. An analysis of critical problems related to the studied model is concluded with tables and graphs summarizing the numerical results.

1 Introduction

The queuing system has many possibilities in industrial life. To understand the nature and functioning of the queuing system, many authors have studied the queuing system. In this work, we used fuzzy logic to solve the queuing problems. So that a non-specialist can know the nature and work function of the queuing system without solving various complicated equations. In fuzzy logic, we only take three inputs called arrival rate, service rate, breakdown rate, and output called system efficiency. We choose a different range of variables in input and output. These are explained in the Experimental Setup section of the paper. Some of the work that has been done in this area is discussed below.

Kalidas et al.[53]investigated a new class of queueing models with working breakdowns. According to this model, the system can fail at any time while in operation. The service is balanced at a slower rate rather than being entirely stopped in the defective state. Kim et al.[7] analysed the $M/G/1$ queuing system while taking disasters and breakdown services into account. The system in this analysis consists of a primary server and a backup server, as well as the disasters that are expected to occur when the primary server is operational. The occurrence of a disaster forces all customers to leave the system, resulting in the failure of the main server. N-Policy $M/M/1$ queueing systems with working vacations and server failures were investigated by Yang et al. [25]. Whenever the server is empty, it goes on vacation. During a vacation period, the server operates at a lower service rate rather than suspending service entirely. It is possible for the server to fail with varying failure rates during idle, working vacation, and normal peak hours. The Yang et al.[54] study considered a finite capacity Markovian queueing system with working breakdowns, reneging, and impatient customer retention, where the server breaks down and needs to be repaired in order to serve the customer. Instead of stopping service during a breakdown, a working breakdown queue allows the server to operate at a lower service rate. Reliability analysis of trial machine systems containing M working units, W warm standby units, and a single repair service with a recovery policy was suggested by Chen [5]. This method assumes that all repair times, failure times, and recovery times follow exponential distributions. The single repair server is also vulnerable to failure, and failure and repair times are expected to follow exponential distributions.

As a multi-repairmen problem, Ke et al.(2013)[6] considered a problem with M working machines and W standbys (spares) in warm storage. An R repairman will quickly identify and repair a failed unit if there is a coverage probability of C. The stationary distribution for the entire system can be obtained using a recursive matrix technique. This is then used to distribute key performance metrics. Using the Lausanne probabilistic global search method and the quasi-Newton approach, a cost structure is proposed to find the global optimal system parameters. The threshold-based arrival control policy and server vacation for the machining system with standby provisions were studied by Kumari et al. [52][62][61]. According to the exponential distribution, the failed machines may balk and renege. Moreover, the failure and service times of the machine, as well as the vacation time of the server, are assumed to follow exponential distributions

As a result of Kim and Lee's (2014)[7] study of the $M/G/1$ queueing system with disasters and working breakdown services, the system size and sojourn time distributions were determined, and the cycle analysis was performed. In addition, numerical calculations are presented to investigate a number of factors affecting the sojourn time.

A Markovian queue with an infinite capacity and an unreliable service station was examined by Liou(2013)[8]. A matrix-analytic approach is used to compute steady-state probabilities for the rate matrix, the number of customers, and the stability condition in the system.

An investigation was conducted by Wang (1990) [9] on the problems of machine repair in steady-state situations with two types of breakdowns of a single service station. Numerical results are used to evaluate several system properties under ideal operating conditions. According to Wang and Kuo (1997) [10], one unreliable service station is servicing Nidentical automatic machines. Matrix geometric approaches are used to calculate the steady-state probability of failures in the system. In order to maximize the overall projected profit per machine per unit of time, a profit model is used to determine how many machines should be allocated to the service station. An $M/M/1$ queue is studied by Yang and Chen (2018) [11], in which a server is prone to breakdowns and maintenance. First, all consumers must receive the necessary service, while second, only a small percentage must receive the optional service. To calculate the stationary probability distributions for a system with varied performance, the matrix-geometric method is used. In order to optimize service rates, the evolutionary algorithm is used to reduce predicted expenses per unit of time. Yen et al (2016) [12] analysed the reliability and sensitivity of a controlled repair system with M operating units, S warm standby units, and an unreliable service station.

Studies of the working vacation queueing system have recently gained a lot of attention. The queuing system and its related hypotheses have been examined by many specialists from different perspectives over the years. On vacation queues, many specialists have focused their attention. It was first suggested by Doshi[47] that queue models with vacation should be developed in detail, examining some overall deterioration outcomes as well as how these results are reached. Numerous analysts have examined the working vacation queue system. Servi and Finn (2002) present Working vacation[13]. A working vacation is a period of time during which the server operates at a slower rate, but does not completely stop. A $G_1/M/1$ queue with vacations was considered by Baba (2005)[30] to allow servers to work at different rates rather than stop the service entirely. Using the $M/M/1$ queue with working vacation and vacation interruption, Li and Tian(2007)[15] calculated the stationary distribution for the number of customers in the system on arrival and at any epoch and time of stay. $G_1/M//1$ queues with working vacations and interruptions were examined by Li et al. (2008)[17]. Lin and Ke(2009)[20] studied the Multi-server system with a single working vacation and Neut's lattice mathematical methodology is used to foster the calculateable express recipe for likelihood dispersion of line length and framework qualities. Chen, Wang, and Yang (2009)[19] discussed the $M/M/1$ machine fix issue with an active excursion. Wang and Wu (2010)[21] dissected the F-policy, $M/M/1/K$ queuing framework with a great startup time and an excursion. In a solitary working excursion, Zhang and Hou (2011) dissected the $M/G/1$ line. In order to get the line length and different boundaries, valuable variable framework insight techniques are used. [24] discusses $M/G/1$ line with single working excursion get-away interference under the Bernoulli plan by Gao and Liu (2013). An input queuing system with a single worker is characterized by multiple functioning get-aways and interferences with get-aways, according to Rajadurai et al. (2017) [26]. A solitary server Markovian working quitting queue with Bernoulli interference was studied by Qingqing Ma et al. (2019)[46]. In their studies of single-worker Markovian queues with one get-away and numerous vacations, Tian and Wang(2020) [29]concerning clients' harmony and socially ideal joining-shying away conduct.

There have been many studies on fuzzy optimization over the years. The fuzzy objective value of a cost-based machine repair optimization problem with both a fuzzy cost coefficient and a fuzzy breakdown value was studied by Chen (2006) [4]. The lower and upper bounds of the fuzzy minimum and predicted total costs at the possibility level α are determined by nonlinear programs based on Zadah's extension concept. The construction and design of a fuzzy queuing model with finite input sources where both arrival and service patterns are exponential distributions were investigated by Pardo (2008). In order to reduce the predicted total cost per unit of time to the lowest possible level, the desired optimization criterion is to discover the optimal number of servers. To reduce the estimated total cost function per unit of time to the minimum possible value, the optimization criterion entails finding the optimal number of servers. Fuzzy simulation experiments were conducted on the differential equations using the Zadeh extension technique for generating continuous fuzzy probabilities. Shekhar, Jain, and Bhatia (2014) [3] investigated the availability aspects of a multi-active and multi-standby processing system. In Markov's machine maintenance model, switching failure and reboot were included. System availability as well as stand-by system availability can be determined using the parametric nonlinear α-cut program. For sport horses, Zarasiz et al.[59] [60] [**?**] have defined fuzzy sets and assigned entropies to each P and T wave. According to Meena et al. (2019)[58], the system involved both operational and standby machines. The machine repair model is transformed from a crisp to a fuzzy environment by treating failure, repair, and vacation rates as fuzzy numbers.

Failures and routine maintenance often require the repair of machines in real-world scenarios. It aims to maximize machine utilization while minimizing repair costs and downtime. In order to maximize system performance, the goal is to find the most efficient policy. When servers are on vacation or unavailable, a N-policy vacation is used to manage machine repairs in the N policy machine repair problem. Maintaining an optimal repair system during vacation periods is the goal of this policy. Based on the queue threshold, Maheswar et al. [55] proposed a new scheme to quantify the energy consumption of nodes during packet transmission. A cluster-based sensor network analytical model is developed using the $M/G/1$ queue model and performance characteristics such as average power consumption and average delay are evaluated. Additionally, the best threshold value expression was derived. LIN and KE [56] established the membership functions of the fuzzy objective values of a controlled queuing model with cost elements, arrival rate, and service rate all being fuzzy numbers. Based on Zadeh's extension concept, parametric nonlinear algorithms are devised to identify the upper and lower bounds of the minimum average total cost per unit of time. The membership functions of the minimum total average cost are further developed with varying values of the possible level. Jharotia [**?**] studied the repairable system with M running machines, S warm standby machines, and R heterogeneous servers with warm standby machines. q represents probability. Once all reserves have been exhausted, the system begins to run in a degraded manner. It is assumed that machine failures and repair times follow an exponential distribution. To determine the steady-state probability, successive over-relaxation (SOR) is used. In order to maintain system availability at a minimum set level, a cost model is built in order to determine the appropriate number of repairmen and replacement machines.

2 Model Description

Model assumptions and key components are outlined below. By adjusting the arrival rate and service rate, this model aims to analyze and optimize the system's performance. In this system, we used M operating units, S warm standby units, and a single repair server. Operating and standby units have exponential failure rates, respectively λ and η ($\eta \leq \lambda$). Whenever an operating unit fails, it is replaced immediately by a standby unit (if any are available). Warm standby machines have the same failure characteristics as operating machines after switching on successfully. This system uses a single repair server to repair defective units, and there is no queue. With an exponential distribution with a rate of γ, failed units are sent to the retrial queue. If the single repair server is idle and the retrial queue period has expired, the failed unit will receive repair service; otherwise, it will be placed in the retrial queue for another random period of time. In this study, the individual repair server with N-Policy is examined, which implies that it is shut down when not in use to maximize its efficiency. The system has no failed units. In order to restart a server after it has been turned off, there must be enough failed units (more than or equal to N). There is a possibility that the server will break and require maintenance at a different facility when it is turned on. A single server's time to failure and time to repair follow exponential distributions with rates of α and β, respectively.

When the server is broken, it can provide partial repair service. We assume that the repair times for failed units have exponential distributions with rates mu_1 and mu_2, where mu_2 is less than mu_1. At time t, the state of the system is (i, n, t), where i represents the state of the single repair server and n represents the number of failed units in the retrial orbit.
There are five possible states for the single repair server: $i = 0$ indicates it's turned off, $i = 1$ indicates it's switched on, in progress, and idle, $i = 2$ indicates it's on, working, and busy, $i = 3$ indicates it's on, broken, and busy, and $i = 4$ indicates it's on, broken, and idle. A Fuzzified exponential distribution describes the lifetime of identical active and standby units. Fuzzified exponential distributions also determine the repair time of failed units. The extension of queueing decision models to fuzzy environments allows the decision maker to obtain more meaningful results and a broader understanding of the system's behavior since the results obtained in the fuzzy queueing model are fuzzy subsets of all of the initial information, so the finite input source queue models insecure data can be more useful and have a broader range of applications.

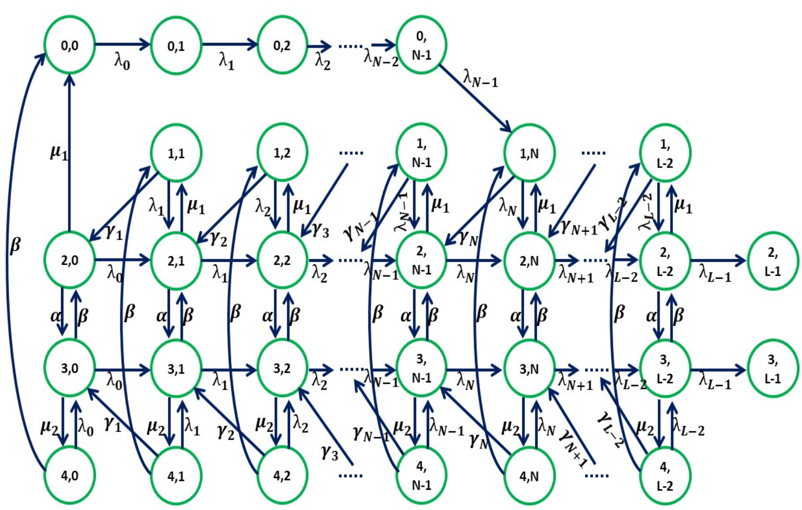

Figure 1. Queuing System

Let $n(t)$ be the number of machines in the framework at time t,

$$i(t)\begin{cases} 0, & i=0 \text{ signifies that the worker is turned off} \\ 1, & i=1 \text{ reflects the worker being on, working, and idle} \\ 2, & i=2 \text{ signifies that the worker is turned on, working, and busy} \\ 3, & i=3 \text{ implies that the worker is turned on, separated, and busy,} \\ 4, & i=4 \text{ implies that the worker is turned on, separated, and idle} \end{cases}$$

Addresses the condition of the framework at time t.

Then, at that point, $(X(t), J(t)), t \geq 0$ addresses a persistent time Markov chain (CTMC) with state rate $S = \{(i,n) : and$n$= 0, 1, 2, \cdots, L-1; I = 0, 1, 2, 3, 4\}$.

Let $P_{i,n}(t)$ signify the time-subordinate state likelihood, where there are n bombed machines in the framework at time t and the condition of the worker i takes its qualities in $\{0, 1, 2, 3, 4\}$. Numerically, $P_{i,n}(t) = P[X(t) = I, n(t) = n], n = 0, 1, 2, \cdots, L-1$ & $I = 0, 1, 2, 3, 4$, where n represents the quantity of machines in the framework and i is the condition of the server.

Besides, the state-dependent rates are given by

$$\lambda_i \begin{cases} M\lambda + (S-n)\eta, n = 0, 1, 2, \cdots, L-1 \\ (M+S-n)\lambda, n = S, S+1, \cdots, L-14 \\ 0, & \text{otherwise} \end{cases}$$

2.1 Steady State Equations

For the probabilities of the transient investigation, the monitoring differential contrast conditions are established for different conditions of the frame by adjusting the in-stream and out-stream birth and departure cycles as follows:

$J = 0$

$$\lambda_0 P_{0,0} = \beta P_{4,0} + \mu_1 P_{2,0} \tag{2.1a}$$

$$\lambda_1 P_{0,1} = \lambda_0 P_{0,0} \tag{2.1b}$$

$$\lambda_2 P_{0,2} = \lambda_1 P_{0,1} \tag{2.1c}$$

$$\lambda_{N-1} P_{0,N-1} = \lambda_{N-2} P_{0,N-2} \tag{2.1d}$$

$J = 1$

$$(\lambda_1 + r_1) P_{1,1} = \beta P_{4,1} + \mu_1 P_{2,1} \tag{2.1e}$$

$$(\lambda_2 + r_2) P_{1,2} = \beta P_{4,2} + \mu_1 P_{2,2} \tag{2.1f}$$

$$(\lambda_N + r_N) P_{1,N} = \beta P_{4,N} + \mu_1 P_{2,N} \quad \text{for} \quad 3 \le N \le L-2 \tag{2.1g}$$

$J = 2$

$$(\lambda_0 + \alpha) P_{2,0} + \mu_1 P_{0,0} = \beta P_{3,0} + r_1 P_{1,1} \tag{2.1h}$$

$$(\lambda_1 + \alpha) P_{2,1} + \mu_1 P_{2,1} = \beta P_{3,1} + r_2 P_{1,2} + \lambda_1 P_{1,1} \tag{2.1i}$$

$$(\lambda_{N+1} + \alpha) P_{2,N} + \mu_1 P_{2,N} = \beta P_{2,1} + r_{N+1} P_{1,N+1} + \lambda_N P_{1,N} \tag{2.1j}$$

for $2 \le N \le L-2$

$$\lambda_{L-1} P_{2,L-2} = 0 \tag{2.1k}$$

$J = 3$

$$(\mu_2 + \beta + \lambda_0) P_{3,0} = \lambda_0 P_{4,0} + \alpha P_{2,0} + r_1 P_{4,1} \tag{2.1l}$$

$$(\mu_2 + \beta + \lambda_1) P_{3,1} = \lambda_1 P_{4,1} + \alpha P_{2,1} + r_2 P_{4,2} + \lambda_0 P_{3,0} \tag{2.1m}$$

$$(\mu_2 + \beta + \lambda_{N+1}) P_{3,N} = \lambda_N P_{4,N} + \alpha P_{2,N} + r_{N+1} P_{4,N+1} + \lambda_N P_{3,N-1} \tag{2.1n}$$

for $2 \le N \le L-2$

$$\lambda_{L-1} P_{3,L-2} = 0 \tag{2.1o}$$

$J = 4$

$$(\lambda_0 + \beta) P_{4,0} = \mu_2 P_{3,0} \tag{2.1p}$$

$$(\lambda_1 + r_1 + \beta) P_{4,1} = \mu_2 P_{3,1} \tag{2.1q}$$

$$(\lambda_N + r_N + \beta) P_{4,N} = \mu_2 P_{3,N} \quad \text{for} \quad 2 \le N \le L-2 \tag{2.1r}$$

normalizing condition:

$$\sum_{j=0}^{3} \sum_{N=0}^{L-1} P_{j,N} = 1. \tag{2.1s}$$

3 Availability analysis

Availability of the system
(ρ_{00} probability of down state of the system)

$$A = 1 - \rho_{00} \tag{3.1}$$

3.1 Experimental Setups and variables

Fuzzy analysis lets us assume and fuzzified only three inputs which are arrival rate λ, service rate μ, and breakdown rate α.
suppose arrival rate (λ) approximated by fuzzy set λ^* .
" service rate (μ)" = μ^*
"breakdown rate(α)" = α^*

Table 1. Values of a and b for λ^*

	a	b
Very Less (VL)	0	2
Less (L)	5	2
Normal (N)	10	2
More (M)	15	2
Very More (VM)	20	2

λ_*, μ^*,α^* are obtained by a crisp universal set X, Y, Z.
Let $\eta_{\lambda^*}(x)$, $\eta_{\mu^*}(y), \eta_{\alpha^*}(z)$, be the membership function of $\lambda^*, \mu^*, \alpha^*$.

$$\lambda^* = \left\{ \left(x, \eta_{\lambda^*}(x) \right) | x \in X \right\} \tag{3.2}$$

$$\mu^* = \left\{ \left(y, \eta_{\mu_1^*}(y) \right) | y \in Y \right\} \tag{3.3}$$

$$\alpha^* = \left\{ \left(z, \eta_{\mu^*}(z) \right) | z \in Z \right\} \tag{3.4}$$

Let A(x,y,z) is the availability of the system.
Since λ, μ, α are fuzzy numbers so $A^*(\lambda^*, \mu^*, \alpha^*)$ is also a fuzzy number.
using Zadeh's principle the membership function for the availability characteristics equations are denoted by

$$\eta_{A^*(\lambda^*,\mu^*,\alpha^*)}(r) = Sup_{x\in X, y\in Y, z\in Z}\ min\left\{ \eta_{\lambda^*}(x), \eta_{\mu^*}(y), \eta_{\alpha^*}(z) | r = A(x, y, z) \right\} \tag{3.5}$$

$X \in [0, 20]$
$Y \in [20, 30]$
$Z \in [20, 60]$
A(x,y,z) obtained by 1 - ρ_{00} .

3.2 Numerical Results

Let arrival rate λ , working rate μ, breakdown rate α are Gaussian fuzzy number
λ is the fuzzy number in the range of [0,20], and continuous in the given interval given as the equation 3.6 as it is represented by the Gaussian membership function as the equation 3.6.

$$\lambda^* = \frac{\int_x e^{-(\frac{x-a}{b})^2}}{x} \tag{3.6}$$

fuzzy variable for arrival rate $\rightarrow \lambda^*[0, 20]$. it has the five variable values as a is the mean and b is the stander deviation of all variables of λ^* given in the table 1
the pictorial representation of λ^* is shown in the figure 2.

$$\mu_2^* = \frac{\int_y e^{-(\frac{y-a}{b})^2}}{y} \tag{3.7}$$

fuzzy variable for breakdown rate is $\alpha^* \in [20, 30]$ is also Gaussian membership function as given in the equation 3.7. The range of μ_2^* have categorise into three variables called No Breakdown, Medium Breakdown, and Frequent Breakdown. The values of each variable mean a and stander deviation b is given in the table 2 the pictorial representation of the breakdown rate is shown in figure 3.

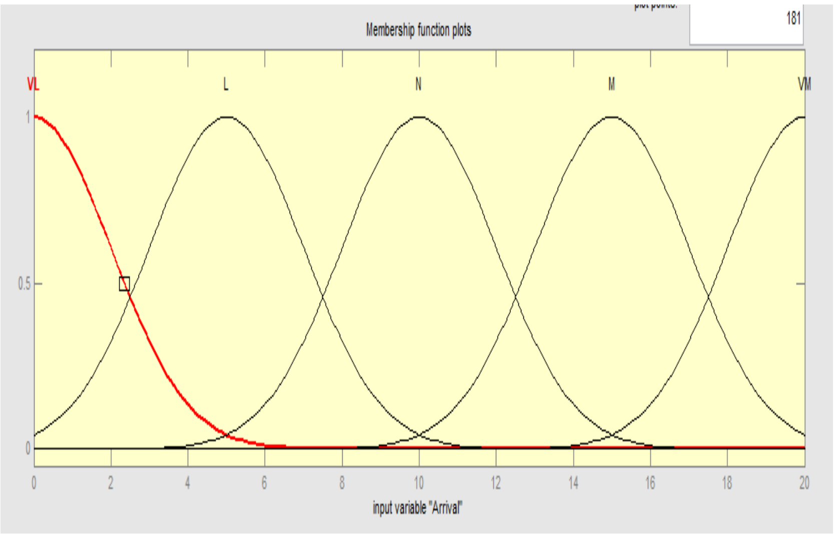

Figure 2. Gaussian membership function for arrival rate and different variable of arrival rate

Table 2. The values of a and b for μ_2^*

No Breakdown (NB)	a = 20	b = 2
Medium Breakdown (MB)	a = 25	b = 2
Frequent Breakdown (FB)	a = 25	b = 2

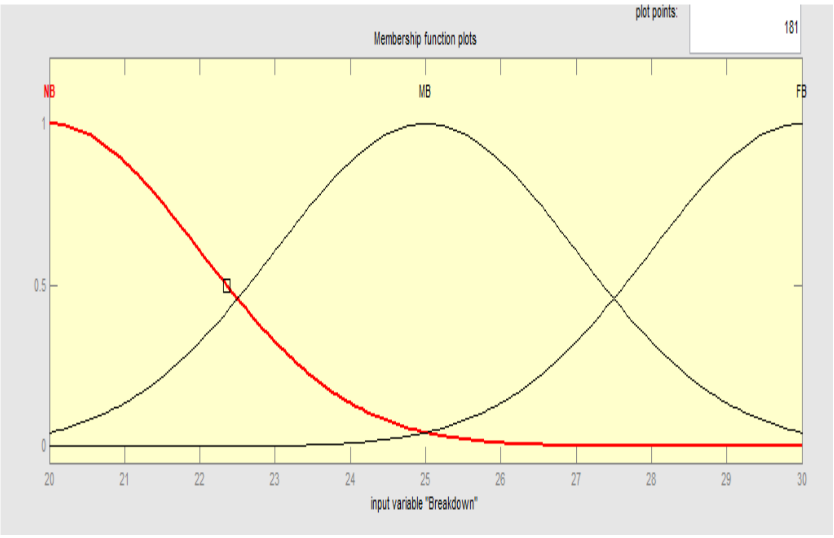

Figure 3. Gaussian membership function for breakdown and different variable of breakdown

Table 3. The values of a and b for μ_1^*

Less than Normal (LN)	a = 20	b = 3
Normal (N)	a = 30	b = 3
More than Normal (MN)	a = 40	b = 3
Efficient (EF)	a=50	b = 3
More than Efficient (B)	a= 60	b = 3

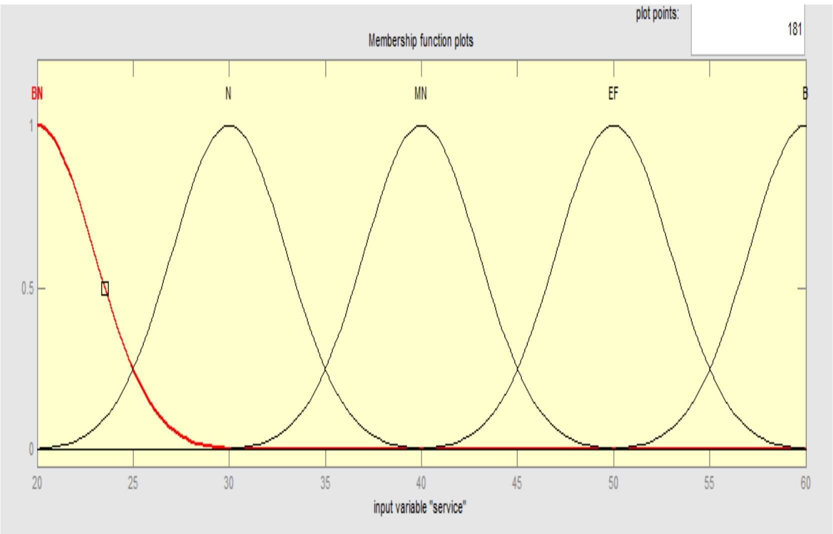

Figure 4. Gaussian membership function for service rate and their variables

$$\mu^* = \frac{\int_z e^{-(\frac{z-a}{b})^2}}{z} \tag{3.8}$$

fuzzy variable for working (service) rate is $\mu^* \in [20, 60]$ is also governed by the Gaussian membership function as shown in the equation 3.8. The whole μ^* is divided into 5 variables according to different values of mean a and stander deviation b as shown in the table 3.

Gaussian membership functions for service rate and their variables are shown in figure 4 Thus the availability or efficiency as the output of the system and represented by the symbol η^* has represented in the percentage. In the fuzzy system, it is represented by a triangular membership function. The triangular membership functions are denoted by three values [a,b,c] generally where a, b, and c are the vertices of a triangle. The general equation of the triangular membership function is denoted in the equation 3.9. The whole range of output is equally divided into five variable names. first one is worst (W) denoted by W^* [-25,0,25], the second is good (G) denoted by G^* [0,25,50], the third output variable range is very good (VG) denoted by VG^* [25,50,75], fourth variable is excellent (Ex) denoted by Ex^* in the range of [50,75,100], and final variable range is superb (Su) denoted by Su^* [75,100,125]. The output membership function is shown in figure 5 general equation of the triangular membership function is given in the equation 3.9.

$$\eta^*(x) = \begin{cases} 0 & x \leq a \\ \frac{x-a}{b-a} & x\,in\,between\,[a,b] \\ \frac{c-x}{c-b} & x\ in\ between\ [b,c] \\ 0 & x \geq c \end{cases} \tag{3.9}$$

As a result, the total fuzzy variable for the arrival rate is five, three for the breakdown rate, and five for the service rate, for a total of 5X3X5 = 75 in the rule-based system. These rule-based are defined as the system's nature, as demonstrated by numerous mathematical equations as presented in the preceding sections. The relationship between the input and output variables is represented by the rule base. These rules determine the system's final output. The input and

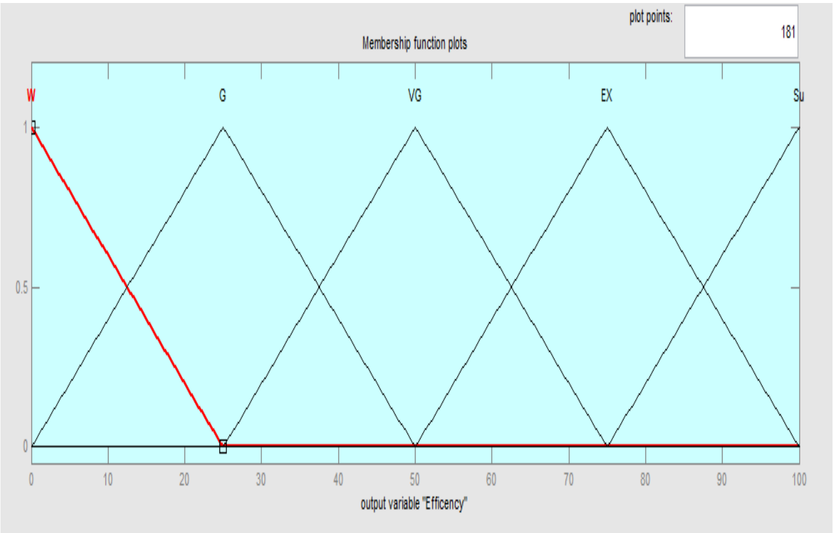

Figure 5. Triangular membership function for efficiency of system

output combinations are used to examine the variance of these rules. The output of the fuzzy will be found in this manner. However, with a crisp input, the final output should be a crisp value. As a result, defuzzification of this fuzzy output is required. For defuzzification, we use the center of gravity approach. There are several approaches available, but we choose the centered method because it outperforms others such as maximum, minimum, and mean. The weighted median value of the fuzzy output is chosen when using the centroid approach. Crisp values for all input ranges can be determined this way. In the result and discussion part, numerous graphs and curves are drawn.

4 Results and discussions

The experiment has been set up in the manner described above. The system's fuzzy variable results are produced and shown in various graphs and tables based on this input. As previously stated, a fuzzy system was created with three inputs: service rate μ, arrival rate λ, and breakdown rate α. Efficiency, represented by $\eta*$, is the only output of this fuzzy system. As mentioned in the previous section, these inputs and output have different fuzzy input ranges. Input is governed by a Gaussian membership function, and output is governed by a triangle member function. A fuzzy system was created in this fashion to analyse the behavior of a queuing system. Figure 6 depicts the relationship between arrival rate and system efficiency in the absence of a breakdown rate. The efficiency of the system is also demonstrated in figure 7 in the case of a medium breakdown rate, and in figure 8 in the case of a frequent breakdown rate.

It can be seen from these graphs that keeping the service rate higher than the arrival rate reduces the system's efficiency as the breakdown rate rises. This conclusion can be drawn from all three examples. When the arrival rate is kept constant, as in $\lambda = 10$, the system's efficiency improves as the service rate rises. As a result, the system's efficiency is proportional to the service rate while the arrival rate remains constant. As indicated in the diagram **??**. The efficiency of the system declines as the breakdown rate grows in the figure 9, while all inputs remain constant. As a result, the graph efficiency of this graph with various breakdown rates and service rates with fixed arrival rates may be examined. Figure **??** depicts the system's efficiency with breakdown rate for various service rates and a fixed arrival rate of $\lambda = 10(average)$. The effectiveness of the model decreases as the breakdown rate increases, as shown in this graph. These graphs, on the other hand, illustrate that as the service rate rises, the system's efficiency rises as well. Table 4 additionally shows the system's efficiency values with a breakdown for various service rates and a set arrival rate.

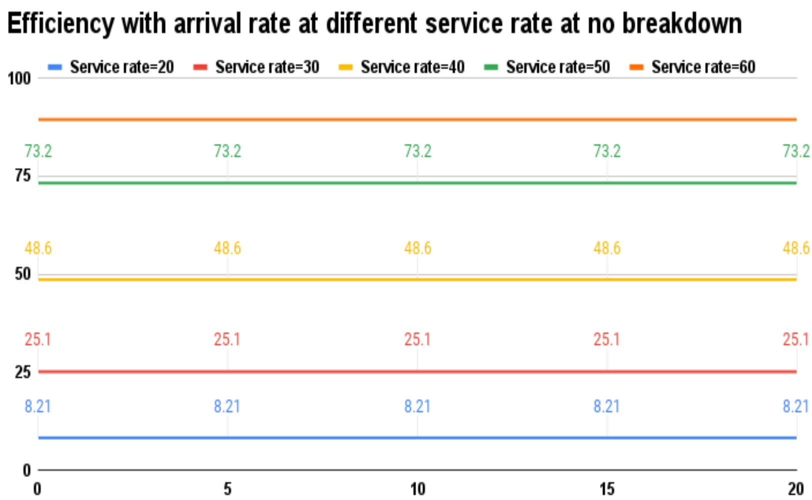

Figure 6. Efficiency of the system at No breakdown

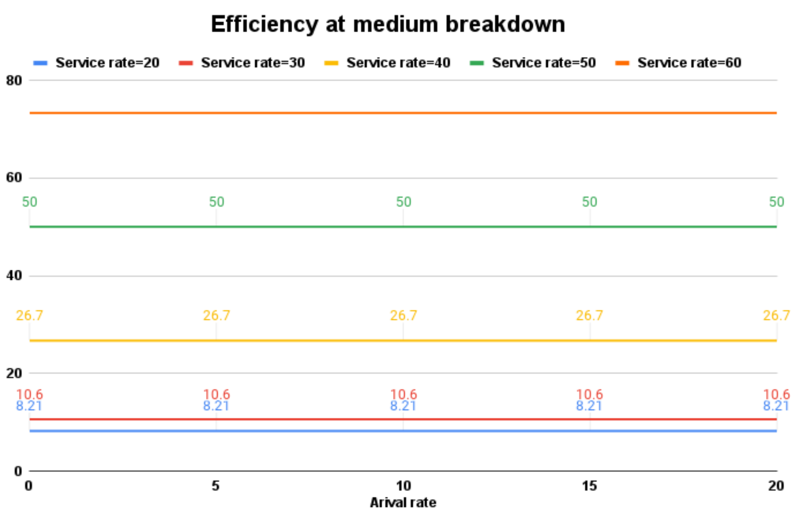

Figure 7. Efficiency of the system at medium breakdown

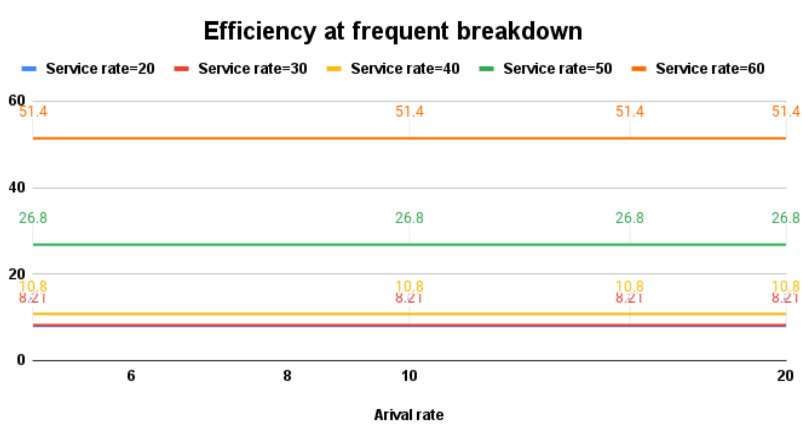

Figure 8. Efficiency of the system at frequent breakdown

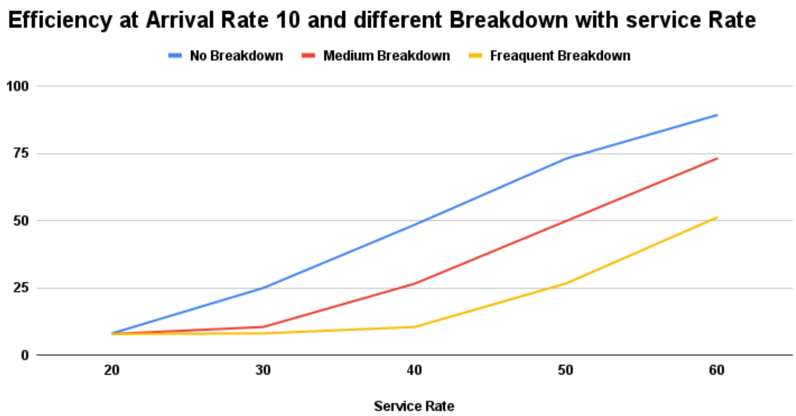

Figure 9. Efficiency of the system at the different breakdown with service rate

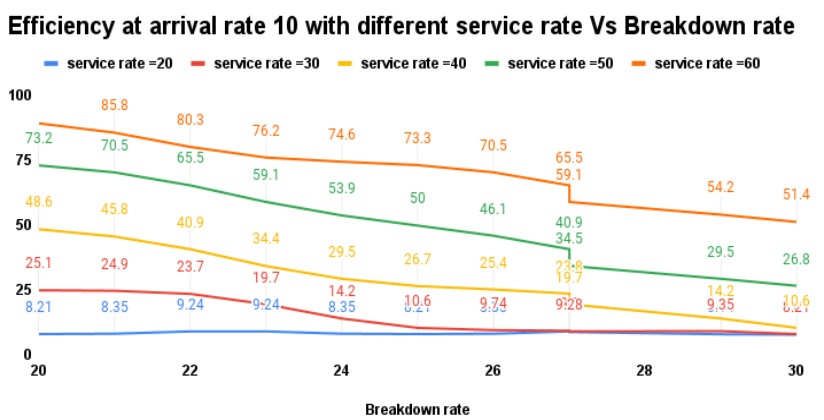

Figure 10. Efficiency of the system at different services rate with breakdown

Table 4. The system's efficiency with a breakdown rate at various service rates and a fixed arrival rate of 10

Efficiency at arrival rate 10					
Breakdown	**service =20**	**service =30**	**service=40**	**service =50**	**service =60**
20	8.21	25.1	48.6	73.2	89.4
21	8.35	24.9	45.8	70.5	85.8
22	9.24	23.7	40.9	65.5	80.3
23	9.24	19.7	34.4	59.1	76.2
24	8.35	14.2	29.5	53.9	74.6
25	8.21	10.6	26.7	50	73.3
26	8.35	9.74	25.4	46.1	70.5
27	9.23	9.5	23.8	40.9	65.5
27	9.01	9.28	19.7	34.5	59.1
29	8.14	9.35	14.2	29.5	54.2
30	8	8.21	10.6	26.8	51.4

5 conclusion

The fuzzy system was created for a server queuing system with three inputs: arrival rate, failure rate, and service rate in this study. This system is efficient in terms of output. The membership function for all inputs is a Gaussian function, but the output membership function is a triangular membership function. The output was composed of five variables: five for the arrival rate, three for the failure rate, and five for the service rate. Using various equations, a total of 75 rule bases were built based on the kind and purpose of the system. Using centroid defuzzification values, the outcome is a fuzzy number that reflects a change to a sharp value. For the fuzzy analysis of the server queuing model, we only consider three input values and one output value. A fuzzy system for the server queuing system was created in this way. In numerous visuals and tables, the relationship between various input variables and output was illustrated and displayed. Thus, we conclude that efficiency is directly related to service rate and inversely proportional to breakdown rate and arrival rate based on this study. This relationship is mathematically evaluated and shown in many figures and tables in this article.

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