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PRIME AND IRREDUCIBLE ELEMENTS OF COMMUTATIVE BE-ALGEBRAS

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Abstract The notion of divisibility is introduced in commutative BE-algebras. Some properties of the filters of multipliers are investigated in commutative BE-algebras. The concepts of prime elements and irreducible elements of commutative BE-algebras are introduced in terms of dual annihilators of BE-algebras. Finally, these elements are characterized in terms of prime filters and maximal filters respectively.

1 Introduction

The notion of BE-algebras was introduced and extensively studied by H.S. Kim and Y.H. Kim in [3]. These classes of BE-algebras were introduced as a generalization of the class of BCK-algebras of K. Iseki and S. Tanaka [2]. Some properties of filters of BE-algebras were studied by S.S. Ahn and Y.H. Kim in [1] and by B.L. Meng in [4]. In [11], A. Walendziak discussed some properties of commutative BE-algebras. He also investigate the relationship between BE-algebras, implicative algebras and J-algebras. In [5], Meng introduced the notion of prime filters in BCK-algebras, and then gave a description of the filter generated by a set, and obtained some of fundamental properties of prime filters. He also studied in [6], some properties of prime ideals in BCK-algebras. In [7], Phaneendra introduced the notion of divisibility in a distributive lattice with respect to a filter and it is proved that the set of all multipliers of an element is a filter. In [10], the authors introduced the concept of radical of filters in a BE-algebra. In [8], the author introduced the notion of prime filters in BE-algebras. In [9], the authors introduced the notion of dual annihilators of commutative BE-algebra and studied extensively the properties of these dual annihilators.

In this paper, The notion of divisibility is introduced in commutative BE-algebras. Some properties of the filters of multipliers are investigated in commutative BE-algebras. The concepts of prime elements and irreducible elements of commutative BE-algebras are introduced in terms of dual annihilators of BE-algebras. Finally, these elements are characterized in terms of prime filters and maximal filters respectively.

2 Preliminaries

In this section, we present certain definitions and results which are taken mostly from the papers [3], [4], [8] and [9] for the ready reference of the reader.

Definition 2.1. [3] An algebra $(X, *, 1)$ of type $(2, 0)$ is called a BE-algebra if it satisfies the following properties:

(1) $x * x = 1$,
(2) $x * 1 = 1$,

(3) $1 * x = x$,
(4) $x * (y * z) = y * (x * z)$ for all $x, y, z \in X$.

A BE-algebra X is called self-distributive if $x*(y*z) = (x*y)*(x*z)$ for all $x, y, z \in X$. A BE-algebra X is called transitive if $y * z \leq (x * y) * (x * z)$ for all $x, y, z \in X$. A BE-algebra X is called commutative if $(x*y)*y = (y*x)*x$ for all $x, y \in X$. Every commutative BE-algebra is transitive. For any $x, y \in X$, define $x \vee y = (y * x) * x$. If X is commutative then $(X, \vee)$ is a semilattice [11]. We introduce a relation $\leq$ on a BE-algebra X by $x \leq y$ if and only if $x * y = 1$ for all $x, y \in X$. Clearly $\leq$ is reflexive. If X is commutative, then $\leq$ is transitive, anti-symmetric and hence a partial order on X.

Theorem 2.2. [4] *Let X be a transitive BE-algebra and $x, y, z \in X$. Then*

(1) $1 \leq x$ *implies* $x = 1$,
(2) $y \leq z$ *implies* $x * y \leq x * z$ *and* $z * x \leq y * x$.

Definition 2.3. [3] A non-empty subset F of a BE-algebra X is called a *filter* of X if, for all $x, y \in X$, it satisfies the following properties:

(1) $1 \in F$,
(2) $x \in F$ and $x * y \in F$ imply that $y \in F$.

For any non-empty subset A of a transitive BE-algebra X, the set $\langle A \rangle = \{x \in X \mid a_1 * (a_2 * (\cdots * (a_n * x) \cdots)) = 1$ for some $a_1, a_2, \ldots a_n \in A\}$ is the smallest filter containing A. For any $a \in X$, $\langle a \rangle = \{x \in X \mid a^n * x = 1$ for some $n \in \mathbb{N}\}$, where $a^n * x = a*(a*(\cdots*(a*x)\cdots))$ with the repetition of a is n times, is called the principal filter generated a. If X is self-distributive, then $\langle a \rangle = \{x \in X \mid a * x = 1\}$. A proper filter P of a BE-algebra is called prime [8] if $\langle x \rangle \cap \langle y \rangle \subseteq P$ implies $x \in P$ or $y \in P$ for any $x, y \in X$. A proper filter M of a transitive BE-algebra X is called maximal [8] if there exists no proper filter Q such that $M \subset Q$. Every maximal filter of a commutative BE-algebra is prime.

Theorem 2.4. [8] *If X is self-distributive and commutative BE-algebra, then*

(1) $x \leq y$ *implies* $\langle y \rangle \subseteq \langle x \rangle$,
(2) $\langle x \rangle \cap \langle y \rangle = \langle x \vee y \rangle$ *for all* $x, y \in X$.

Theorem 2.5. [8] *Let F be a filter of a commutative BE-algebra X. For any $x, y \in X$,*

$$\langle x \rangle \cap \langle y \rangle \subseteq F \text{ if and only if } \langle F \cup \{x\} \rangle \cap \langle F \cup \{y\} \rangle = F$$

Theorem 2.6. [8] *If X is self-distributive and commutative BE-algebra, then the following assertions are equivalent:*

(1) P *is prime;*
(2) *for any* $x, y \in X$, $x \vee y \in P$ *implies* $x \in P$ *or* $y \in P$;
(3) *for any two filters* F *and* G *of* X, $F \cap G \subseteq P$ *implies* $F \subseteq P$ *or* $G \subseteq P$.

Lemma 2.7. [9] *Let X be a commutative BE-algebra. Then for any $x, y, a \in X$*

(1) $y * z \leq (z * x) * (y * x)$,
(2) $(x * y) \vee a \leq (x \vee a) * (y \vee a)$.

For any non-empty subset A of a commutative BE-algebra X, the dual annihilator [9] of A is defined as $A^+ = \{x \in X \mid x \vee a = 1$ for all $a \in A\}$. Clearly A^+ is a filter of X. Obviously $X^+ = \{1\}$ and $\{1\}^+ = X$. For $A = \{a\}$, we simply denote $\{a\}^+$ by $(a)^+$.

Proposition 2.8. [9] *For any two filters F, G of a commutative BE-algebra X, we have*

(1) $F \cap F^+ = \emptyset$,
(2) $F \subseteq F^{++}$,
(3) $F^{+++} = F^+$,
(4) $F \subseteq G$ *implies* $G^+ \subseteq F^+$,
(5) $(F \vee G)^+ = F^+ \cap G^+$,
(6) $(F \cap G)^{++} = F^{++} \cap G^{++}$.

Lemma 2.9. [9] *For any two elements a, b of a self-distributive and commutative BE-algebra X, we have*

(1) $(\langle a \rangle)^+ = (a)^+$,
(2) $\langle a \rangle \subseteq (a)^{++}$,
(3) $a \leq b$ *implies* $(a)^+ \subseteq (b)^+$,
(4) $(a \vee b)^{++} = (a)^{++} \cap (b)^{++}$.

3 Prime and Irreducible Elements

In this section, the concept of divisibility is introduced in commutative BE-algebras. Some properties of the filters of multipliers are investigated in commutative BE-algebras. The concepts of prime elements and irreducible elements of commutative BE-algebras are introduced in terms of dual annihilators of BE-algebras. Finally, these elements are characterized in terms of prime filters and maximal filters respectively.

Definition 3.1. Let X be a commutative BE-algebra and $a, b \in X$. Then we say that a is a divisor of b or a divides b if $(b)^+ = (a \vee c)^+$ for some $c \in X$. In this case, we write it as $(a|b)_\star$.

Example 3.2. Let $X = \{1, a, b, c\}$ be a set. Define a binary operation $*$ on X as follows:

$*$	1	a	b	c
1	1	a	b	c
a	1	1	1	1
b	1	b	1	1
c	1	c	c	1

$\vee$	1	a	b	c
1	1	1	1	1
a	1	a	b	c
b	1	b	b	c
c	1	c	c	c

Then clearly $(X, *, \vee, 1)$ is a commutative BE-algebra. Here $(a)^+ = (b)^+ = (c)^+ = \{1\}$. Clearly $(b)^+ = (a \vee c)^+$ for some $c \in X$. Hence a is a divisor of b.

Lemma 3.3. *Let X be a commutative BE-algebra and $a, b \in X$. Then we have*

$(a)^+ = (b)^+$ *implies that* $(a \vee x)^+ = (b \vee x)^+$ *for any* $x \in X$.

Lemma 3.4. *Let X be a commutative BE-algebra and $a, b, c \in X$. Then*

(1) $(a|a)_\star$
(2) $a \leq c \Rightarrow (a|c)_\star$
(3) $(a)^+ = (b)^+ \Rightarrow (a|b)_\star$ *and* $(b|a)_\star$
(4) $(a|b)_\star$ *and* $(b|c)_\star \Rightarrow (a|c)_\star$
(5) $(a|b)_\star \Rightarrow (a|b \vee x)_\star$ *for all* $x \in X$
(6) $(a|b)_\star \Rightarrow (a \vee x|b \vee x)_\star$ *for all* $x \in X$.

Proof. (1). Since $(a)^+ = (a \vee a)^+$, we get $(a|a)_\star$.

(2). Suppose $a \leq c$. Then $c = a \vee c$. Hence $(c)^+ = (a \vee c)^+$. Therefore $(a|c)_\star$.

(3). Suppose that $(a)^+ = (b)^+$. Then we have $(a)^+ = (b)^+ = (b \vee b)^+$. Hence $(b|a)_\star$. Similarly, we can get $(a|b)_\star$.

(4). Let $(a|b)_\star$ and $(b|c)_\star$. Then $(b)^+ = (a \vee x)^+$ and $(c)^+ = (b \vee y)^+$ for some $x, y \in X$. Now $(c)^+ = (b \vee y)^+ = (a \vee x \vee y)^+$. Therefore $(a|c)_\star$.

(5). Let $(a|b)_\star$. Then $(b)^+ = (a \vee x)^+$ for some $x \in X$. Hence, for any $y \in X$, we get $(b \vee y)^+ = (a \vee x \vee y)^+$. Therefore $(a|b \vee x)_\star$.

(6). Suppose that $(a|b)_\star$. Then $(b)^+ = (a \vee s)^+$ for some $s \in X$. Hence, for any $x \in X$, $(b \vee x)^+ = (a \vee s \vee x)^+$. Therefore $(a \vee x|b \vee x)_\star$. □

Definition 3.5. Let X be a commutative BE-algebra. For any $a \in X$, we define $(a)^\perp$ as the set of all multipliers of a. That is $(a)^\perp = \{x \in X / (a/x)_\star\}$.

Evidently, we have $(1)^{\perp} = X$.

Example 3.6. Let $X = \{1, a, b, c\}$ be a set. Define a binary operation $*$ on X as follows:

$*$	1	a	b	c
1	1	a	b	c
a	1	1	a	c
b	1	1	1	c
c	1	a	b	1

$\vee$	1	a	b	c
1	1	1	1	1
a	1	a	a	1
b	1	a	b	1
c	1	1	1	c

Then $(X, *, \vee, 1)$ is a commutative BE-algebra. Clearly $(c)^{\perp}$ = $\{x \in X/(c/x)_{\star}\} = \{1, c\}$.

Lemma 3.7. *Let X be a commutative BE-algebra. For any $x, y \in X$, we have*

$$(x)^{+} \cap (x * y)^{+} \subseteq (y)^{+}$$

Proof. Let $a \in (x)^{+} \cap (x * y)^{+}$. Then $x \vee a = 1$ and $(x * y) \vee a = 1$. Hence

$$\begin{aligned} 1 &= (x * y) \vee a \\ &\leq (x \vee a) * (y \vee a) \\ &= 1 * (y \vee a) \\ &= y \vee a \end{aligned}$$

which means $y \vee a = 1$. Hence $a \in (y)^{+}$. Therefore $(x)^{+} \cap (x * y) \subseteq (y)^{+}$. □

Proposition 3.8. *Let X be a commutative BE-algebra. For any $a \in X$, $(a)^{\perp}$ is a filter of X.*

Proof. Since $(1)^{+} = (a \vee 1)^{+}$, we get $(a|1)_{\star}$. Hence $1 \in (a)^{\perp}$. Let $x, x * y \in (a)^{\perp}$. Then $(a|x)_{\star}$ and $(a|x * y)_{\star}$. Hence $(x)^{+} = (a \vee r)^{+}$ and $(x * y)^{+} = (a \vee s)^{+}$ for some $r, s \in X$. Since $y \leq x * y$, we get $(y)^{+} \subseteq (x * y)^{+} = (a \vee s)^{+} \subseteq (a \vee (r \vee s))^{+}$. Conversely, we have

$$\begin{aligned} (a \vee (r \vee s))^{+} &= ((a \vee r) \vee (a \vee s))^{+} \\ &= (a \vee r)^{+} \cap (a \vee s)^{+} \\ &= (x)^{+} \cap (x * y)^{+} \\ &\subseteq (y)^{+} \qquad \text{By the above lemma} \end{aligned}$$

which concludes that $(y)^{+} = (a \vee (r \vee s))^{+}$. Hence $(a|y)_{\star}$, which provides $y \in (a)^{\perp}$. Therefore $(a)^{\perp}$ is a filter of X. □

Lemma 3.9. *For any $x, y \in X$, we have the following conditions.*

(1) $a \in (a)^{\perp}$;

(2) $a \in (b)^{\perp} \Rightarrow (a)^{\perp} \subseteq (b)^{\perp}$;

(3) $a \leq b \Rightarrow (b)^{\perp} \subseteq (a)^{\perp}$;

(4) $(a)^{+} = (b)^{+} \Rightarrow (a)^{\perp} = (b)^{\perp}$;

(5) $(a)^{\perp} \cap (b)^{\perp} = (a \vee b)^{\perp}$.

Proof. (1). Since $(a)^{+} = (a \vee a)^{+}$, we get $(a|a)_{\star}$. Hence $a \in (a)^{\perp}$.

(2). Let $a \in (b)^{\perp}$. Then $(b|a)_{\star}$. Hence $(a)^{+} = (b \vee s)^{+}$ for some $s \in X$. Let $x \in (a)^{\perp}$. Then $(a|x)_{\star}$ and hence $(x)^{+} = (a \vee r)^{+}$ for some $r \in X$. So we get $(x)^{+} = (a \vee r)^{+} = (b \vee s \vee r)^{+}$ for some $s \vee r \in X$. Thus we get $(b|x)_{\star}$. Therefore $x \in (b)^{\perp}$.

(3). Suppose $a \leq b$. Let $x \in (b)^{\perp}$. Then $(b|x)_{\star}$. Hence $(x)^{+} = (b \vee r)^{+} = (a \vee b \vee r)^{+}$ for some $r \in X$. Thus $(a|x)_{\star}$, which yields $x \in (a)^{\perp}$. Hence $(b)^{\perp} \subseteq (a)^{\perp}$.

(4). Let $(a)^{+} = (b)^{+}$. If $x \in (a)^{\perp}$, then $(a|x)_{\star}$. Hence $(x)^{+} = (a \vee r)^{+} = (b \vee r)^{+}$ for some $r \in X$. Thus $(b|x)_{\star}$. Hence $x \in (b)^{\perp}$. Similarly, we get $(b)^{\perp} \subseteq (a)^{\perp}$.

(5). Clearly $(a \vee b)^{\perp} \subseteq (a)^{\perp} \cap (b)^{\perp}$. Conversely, let $x \in (a)^{\perp} \cap (b)^{\perp}$. Then $(a|x)_{\star}$ and $(b|x)_{\star}$. Hence $(x)^{+} = (a \vee r)^{+}$ and $(x)^{+} = (b \vee s)^{+}$ for some $r, s \in X$. Now $(x)^{+} = (x)^{+} \cap (x)^{+} = (a \vee r)^{+} \cap (b \vee s)^{+} = ((a \vee r) \vee (b \vee s))^{+} = ((a \vee b) \vee (r \vee s))^{+}$. Thus $((a \vee b)|x)_{\star}$. Therefore $x \in (a \vee b)^{\perp}$. □

A BE-algebra X is called bounded, if there exists an element 0 satisfying $0 \leq x$ (or $0*x = 1$) for all $x \in X$. Clearly $(0)^+ = 1$.

Proposition 3.10. *Let X be a commutative BE-algebra with the smallest element 0. For any arbitrary element d of X, we have*

$$(d)^+ = (0)^+ \text{ if and only if } (d)^\perp = X.$$

Proof. Suppose $(d)^+ = (0)^+$. Hence $(d \vee 0)^+ = (0 \vee 0)^+ = (0)^+$. Thus $(d|0)_\star$. Therefore $0 \in (d)^\perp$, which yields $(d)^\perp = X$. Conversely, suppose that $(d)^\perp = X$. Then $0 \in (d)^\perp$ and hence $(d|0)_\star$. Thus we can obtain $(0)^+ = (d \vee r)^+$ for some $r \in X$. Therefore $(d)^+ = (0 \vee d)^+ = (d \vee r)^+ = (0)^+$. □

Definition 3.11. Let X be a commutative BE-algebra. An element $1 \neq a \in X$ is called a *prime element* if it satisfies the property:

$$(a|b \vee c)_\star \text{ implies that } (a|b)_\star \text{ or } (a|c)_\star.$$

Example 3.12. Let $X = \{1, a, b, c, d\}$ and define a binary operation $*$ on X as follows:

$*$	1	a	b	c	d
1	1	a	b	c	d
a	1	1	b	c	b
b	1	a	1	b	a
c	1	a	1	1	a
d	1	1	1	b	1

$\vee$	1	a	b	c	d
1	1	1	1	1	1
a	1	a	1	1	a
b	1	1	b	d	b
c	1	1	d	c	b
d	1	a	b	b	d

Clearly $(X, *, \vee, 1)$ is a commutative BE-algebra. Here $(a)^+ = \{1, b, c\}; (b)^+ = \{1, a\}$; $(c)^+ = \{1, a\}$ and $(d)^+ = \{1\}$. Observe that c is a divisor of $a \vee b$ because of $(a \vee b)^+ = (c \vee a)^+$ for some $a \in X$. Also c is a divisor of b. Hence c is a prime element.

In the following Theorem, the prime elements of commutative BE-algebras are characterized.

Theorem 3.13. *Let X be a commutative BE-algebra and $a \in X$ be such that $(a)^+ \neq \{1\}$. Then a is a prime element of X if and only if $(a)^\perp$ is a prime filter of X.*

Proof. Assume that a is a prime element. Let $x, y \in X$ be such that $x \vee y \in (a)^\perp$. Then $(a|x \vee y)_\star$. Since a is prime, we get either $(a|x)_\star$ or $(a|y)_\star$. Hence $x \in (a)^\perp$ or $y \in (a)^\perp$. Therefore $(a)^\perp$ is prime filter in X.
Conversely, assume that $(a)^\perp$ is prime filter in X. Let $x, y \in X$ and $(a|x \vee y)_\star$. Then $x \vee y \in (a)^\perp$. Since $(a)^\perp$ is a prime filter, we get either $x \in (a)^\perp$ or $y \in (a)^\perp$. Hence $(a|x)_\star$ or $(a|y)_\star$. Therefore a is a prime element of X. □

Next, the concept of irreducible elements is introduced in commutative BE-algebras.

Definition 3.14. Let X be a commutative BE-algebra. An element $1 \neq a \in X$ is called an *irreducible element* if $(a)^+ = (b \vee c)^+$, then either $(b)^+ = \{1\}$ or $(c)^+ = \{1\}$ for some $b, c \in X$.

Example 3.15. Let $X = \{1, a, b, c, d\}$ be a set. Define a binary operation $*$ on X as follows:

$*$	1	a	b	c	d
1	1	a	b	c	d
a	1	1	a	c	c
b	1	1	1	c	c
c	1	a	b	1	a
d	1	1	a	1	1

$\vee$	1	a	b	c	d
1	1	1	1	1	1
a	1	a	a	1	a
b	1	a	b	1	a
c	1	1	1	c	c
d	1	a	a	c	d

Then clearly $(X, *, \vee, 1)$ is a commutative BE-algebra. Here $(a)^+ = \{1, c\}; (b)^+ = \{1, c\}$; $(c)^+ = \{1, a, b\}$ and $(d)^+ = \{1\}$. It is easy to check that $(a)^+ = (d \vee b)^+$ and $(d)^+ = \{1\}$. Hence a is an irreducible element.

Lemma 3.16. *Let X be a self-distributive and commutative BE-algebra. Let $d \in X$ be such that $(d)^+ = \{1\}$. Then d is an irreducible element of X.*

Proof. Let $d \in X$ be such that $(d)^+ = \{1\}$. Suppose $(d)^+ = (b \vee c)^+$ for some $b, c \in X$. Then $(b \vee c)^+ = \{1\}$ and so $(b)^{++} \cap (c)^{++} = (b \vee c)^{++} = X$. Hence $(b)^+ = (1)^+ = X$. Hence $(b)^{++} = X$ and $(c)^{++} = X$. Thus $(b)^+ = \{1\}$ and $(c)^+ = \{1\}$. Therefore d is an irreducible element of X. □

Theorem 3.17. *Let X be a self-distributive and commutative BE-algebra, and $a \in X$ be such that $(a)^+ \neq \{1\}$. Then the following conditions are equivalent:*

(1) *a is irreducible;*

(2) *(i) $(a)^\perp$ is a maximal among all proper filters of the form $(x)^\perp$.*
(ii) For any $x \in X$, $(a)^+ = (a \vee x)^+$ implies $(x)^+ = \{1\}$.

Proof. $(1) \Rightarrow (2)(i)$: Assume that a is irreducible. Suppose $(a)^\perp \subseteq (b)^\perp \neq X$ for some $1 \neq b \in X$. Clearly $a \in (a)^\perp \subseteq (b)^\perp$. Then $(b|a)_\star$. Hence $(a)^+ = (b \vee c)^+$ for some $c \in X$. Since a is irreducible, we get that either $(b)^+ = \{1\}$ or $(c)^+ = \{1\}$. Since $(b)^\perp \neq X$, by Proposition 3.10, we get that $(b)^+ \neq (0)^+$. Hence $(c)^+ = \{1\}$. Now

$$
\begin{aligned}
(c)^+ = \{1\} &\Rightarrow (c)^+ = (0)^+ \\
&\Rightarrow (b \vee c)^+ = (b \vee 0)^+ = (b)^+ \\
&\Rightarrow (a)^+ = (b)^+ \\
&\Rightarrow (a)^\perp = (b)^\perp
\end{aligned}
$$

Therefore $(a)^\perp$ is maximal among all filters of the form $(x)^\perp$.

$(1) \Rightarrow (2)(ii)$: Suppose $(a)^+ = (a \vee x)^+$ for $x \in X$. Since a is irreducible, we get either $(a)^+ = \{1\}$ or $(x)^+ = \{1\}$. Since $(a)^+ \neq \{1\}$, we get $(x)^+ = \{1\}$.

$(2) \Rightarrow (1)$: Assume the conditions (2)(i) and (2)(ii). Suppose $(a)^+ = (c \vee d)^+$ for some $c, d \in X$. Hence $(d|a)_\star$. Thus $a \in (d)^\perp$. Hence $(a)^\perp \subseteq (d)^\perp$. Since the filter $(a)^\perp$ is maximal, we get that either $(a)^\perp = (d)^\perp$ or $(d)^\perp = X$.
Suppose $(a)^\perp = (d)^\perp$. Then we get $d \in (a)^\perp$. Now

$$
\begin{aligned}
d \in (a)^\perp &\Rightarrow (a|d)_\star \\
&\Rightarrow (d)^+ = (a \vee r)^+ \text{ for some } r \in X \\
&\Rightarrow (c \vee d)^+ = (c \vee a \vee r)^+ \\
&\Rightarrow (a)^+ = (c \vee a \vee r)^+ \\
&\Rightarrow (c \vee r)^+ = \{1\} \quad \text{by (2)(ii)} \\
&\Rightarrow (c)^+ = \{1\} \quad \text{since } c \leq c \vee r.
\end{aligned}
$$

Suppose $(d)^\perp = X$. Then $0 \in (d)^\perp$. Hence $(d|0)_\star$. Then there exists some $s \in X$ such that $(0)^+ = (d \vee s)^+$. Hence $(d \vee s)^+ = \{1\}$. Thus $(d)^{++} \cap (s)^{++} = (d \vee s)^{++} = (1)^+ = X$. Hence $(d)^{++} = X$, which gives $(d)^+ = \{1\}$. Therefore a is irreducible. Hence the proof is completed. □

4 Conclusion remarks

In this paper, we introduced the notion of divisibility in commutative BE-algebras and investigated some properties of the filters of multipliers in commutative BE-algebras. In addition, the concepts of prime elements and irreducible elements of commutative BE-algebras are introduced in terms of dual annihilators of BE-algebras. Finally, these elements are characterized in terms of prime filters and maximal filters respectively. We think such results are very useful for the further characterization of prime and irreducible elements in terms of congruences of this structure.

In future work, we intend to derive some significant properties of minimal prime filters and regular filters with the help of prime and irreducible elements. It is also proposed to establish the characterizations of commutative BE-algebras and quasi-complemented BE-algebras in terms of prime and irreducible elements.

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