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THE PRIME GRAPH OF RING $\mathbb{Z}_{p_1} \times \mathbb{Z}_{{p_2}^2}$

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Abstract Let R be a commutative ring. A graph $G = (V(G), E(G))$ is said to be a prime graph of a ring R if set of vertices constitutes of all elements of a ring and two distinct vertices x, y are adjacent if $xRy = \{0\}$. The prime graph of a ring denoted by $PG(R)$. The number of triangles in $PG(R)$ is a number of subgraph of $PG(R)$ which forms a triangle. The line graph of prime graph of a ring defined as a graph whose set of vertices constitutes of all edges of $PG(R)$, and two distinct vertices are adjacent if the corresponding edges are adjacent in $PG(R)$. The line graph of a prime graph of a ring denoted by $L(PG(R))$. The shortest path between all pair of vertices in $PG(R)$ called by distance. Wiener index is the sum of the distances between all pair of vertices of $PG(R)$. In this paper, we study the prime graph of ring $\mathbb{Z}_{p_1} \times \mathbb{Z}_{{p_2}^2}$, where p_1 and p_2 are prime numbers. We found that some properties of $PG(\mathbb{Z}_{p_1} \times \mathbb{Z}_{{p_2}^2})$ such as order, size, number of triangles, line graph, and Wiener index. Moreover, we calculated the order and size in line graph.

1 Introduction

The preliminary definitions and notations about graph theory, we referred to [1]–[3]. In 2010, the concept of prime graph of a ring was introduced by Satyanarayana et al (2010) [4], denoted by $PG(R)$. Satyanarayana et al (2010) [4] presented some examples of $PG(\mathbb{Z}_p)$, for any prime p and their properties. Prime graph of finite rings studied by Rajendra et al (2019) [5]. They found that correction of the first theorem of Satyanarayana et al (2010) [4], that is not true when $n = p^2$, for some prime p. Pawar and Joshi (2019) [6] give some result of the number of triangles of prime graph of a ring Z_p, for p is prime. In 2013, the concept of prime graph of cartesian product of rings was introduced by Kalita and Patra (2013) [7]. Then, Satyanarayana and Devanaboina (2015) [8] compared $PG(R) \times PG(R)$ and $PG(R \times R)$. In 2018, Joshi and Pawar (2019) [9] introduced the concept of line graph and Wiener index of prime graph of a ring. Joshi and Pawar (2019) [9] discuss the line graph and Wiener index of $PG(\mathbb{Z}_p)$ and give some properties. Study of prime graph of a ring are expanded in [10]–[13]. Krisnawati et al (2023) [14] give some result of the number of vertex, edge, and triangles in $PG(\mathbb{Z}_{p_1} \times \mathbb{Z}_{p_2})$, where p_1, p_2 are primes. Krisnawati et al (2023) [14] also established line graph of prime graph of ring $\mathbb{Z}_{p_1} \times \mathbb{Z}_{p_2}$ and found that the number of vertex and edge. Wiener index of $PG(\mathbb{Z}_{p_1} \times \mathbb{Z}_{p_2})$ also have been studied. Motivated by Krisnawati et al (2023) [14], we discuss some properties of prime graph of ring $\mathbb{Z}_{p_1} \times \mathbb{Z}_{{p_2}^2}$, where p_1, p_2 are primes such as number of vertex, number of edges, number of triangles, line graph, and Wiener index of $PG(\mathbb{Z}_{p_1} \times \mathbb{Z}_{{p_2}^2})$. This paper divided into four section. In section 2, we give some properties in $PG(\mathbb{Z}_{p_1} \times \mathbb{Z}_{{p_2}^2})$ such as order, size, and the number of triangles. In section 3, we establish a line graph of $PG(\mathbb{Z}_{p_1} \times \mathbb{Z}_{{p_2}^2})$ and discuss some properties about order and size. In the last section, we calculate the Wiener index of $PG(\mathbb{Z}_{p_1} \times \mathbb{Z}_{{p_2}^2})$.

First, we review the definition of prime graph of cartesian product of rings [7], line graph, and Wiener index of a prime graph of a ring [9].

Definition 1.1. [7] Let R be a ring, where $R = R_1 \times \cdots \times R_n$. Let $x = x_1, x_2, \ldots, x_n$ and $y = y_1, y_2, \ldots, y_n \in R$. A graph $G = (V, E)$is said to be prime graph of a ring R (denoted by $PG(R_1 \times \cdots \times R_n)$) if $V(PG(R_1 \times \cdots \times R_n)) = R_1 \times \cdots \times R_n$ and two distinct vertices (x_1, x_2) and (y_1, y_2) are adjacent if $(x_1, x_2, \ldots, x_n)(R_1 \times \cdots \times R_n)(y_1, y_2, \ldots, y_n) = (0, 0, \ldots, 0)$.
Definition 1.2. [9] The line graph of the graph $PG(R)$ (denoted by $L(PG(R))$) is defined to

graph whose set of vertices constitutes of the edges of $PG(R)$, where two vertices are adjacent if the corrensponding edges have a common vertex in $PG(R)$.

Definition 1.3. [9] Let $PG(R)$ be a prime graph of a ring with vertex set $V(G)$. We denote the length of the shortest path between every pair of vertices $x, y \in V(G)$ with $d(x, y)$. Then, the Wiener index of $PG(R)$ is the sum of the distances between all pair of vertices of $PG(R)$, i.e.

$$W(PG(R)) = \sum_{x,y \in V(G)} d(x, y).$$

2 Prime Graph of Ring $\mathbb{Z}_{p_1} \times \mathbb{Z}_{{p_2}^2}$

In this section, we show the order, size, and number of triangles in $\mathbb{Z}_{p_1} \times \mathbb{Z}_{{p_2}^2}$.

Theorem 2.1 Let $PG(\mathbb{Z}_{p_1} \times \mathbb{Z}_{{p_2}^2})$ be a prime graph $\mathbb{Z}_{p_1} \times \mathbb{Z}_{{p_2}^2}$, where p_1 and p_2 are prime numbers. Then,

$$|V(PG(\mathbb{Z}_{p_1} \times \mathbb{Z}_{{p_2}^2})) = p_1 {p_2}^2|, \text{and}$$

$$|E(PG(\mathbb{Z}_{p_1} \times \mathbb{Z}_{{p_2}^2})) = \frac{1}{2}(p_2(6p_1p_2 - 4p_1 - 3p_2 + 1)).$$

Proof. By Definition 1.1, it is clear that $|V(PG(\mathbb{Z}_{p_1} \times \mathbb{Z}_{{p_2}^2}))| = p_1{p_2}^2$. Then, neighbourhood of vertex in $PG(\mathbb{Z}_{p_1} \times \mathbb{Z}_{{p_2}^2})$ we consider in three cases as follows: Firstly, the neighborhood of vertex containing $(\bar{0}, \bar{0})$. It is clear that the number of those neighborhood is $(p_1{p_2}^2 - 1)$. Then, the neighborhood of vertex not containing $(\bar{0}, \bar{0})$. Since $(\bar{x}, \bar{0})$ is adjacent to $(\bar{0}, \bar{y})$, where $x = \bar{1}, \bar{2}, \ldots, \overline{p_1 - 1}$ and $y = \bar{1}, \bar{2}, \ldots, \overline{p_2 - 1}$, then, the number of those neighbourhood is $(p_1 - 1)({p_2}^2 - 1)$. Lastly, the neighborhood of vertex not containing $(\bar{0}, \bar{0})$, but at least one of the vertex's containing zero divisors. Probability of those neighborhood of vertex as follows: $(\bar{0}, k_1, \bar{p_2})$ adjacent to $(\bar{0}, k_2, \bar{p_2})$, for $k_1 \in 1, 2, \ldots, (p_2 - 1)$ and $k_2 \in 2, \ldots, (p_2 - 1)$ and $(\bar{0}, k_3, \bar{p_2})$ adjacent to $(\bar{x}, k_4, \bar{p_2})$, for $k_3, k_4 \in 1, 2, \ldots, (p_2 - 1)$ and $\bar{x} \in 1, 2, \ldots, (p_1 - 1)$. Therefore, the number of those neighborhood is

$$\frac{1}{2}(p_2 - 1)(p_2 - 2) + (p_2 - 1)^2(p_1 - 1). \blacksquare$$

Thus, by that three cases, we obtain that

$$|E(PG(\mathbb{Z}_{p_1} \times \mathbb{Z}_{{p_2}^2}))| = (p_1{p_2}^2 - 1) + (p_1 - 1)({p_2}^2 - 1) + \frac{1}{2}(p_2 - 1)(p_2 - 2) + (p_2 - 1)^2(p_1 - 1)$$
$$= \frac{1}{2}(p_2(6p_1p_2 - 4p_1 - 3p_2 + 1)). \blacksquare$$

Example 2.2. Contruct $(PG(\mathbb{Z}_{p_1} \times \mathbb{Z}_{{p_2}^2}))$, for $p_1 = 2$ and $p_2 = 3$. By Definition 1.1, $V(PG(\mathbb{Z}_2 \times \mathbb{Z}_9)) = \mathbb{Z}_2 \times \mathbb{Z}_9$ and the edges satisfy $(x_1, x_2)(\mathbb{Z}_2 \times \mathbb{Z}_9)(y_1, y_2) = (\bar{0}, \bar{0})$. Therefore, $PG(\mathbb{Z}_{p_1} \times \mathbb{Z}_{{p_2}^2})$ is shown in Fig 1. We can see that, based on Theorem 2.1, Fig.1 has $|V(PG(\mathbb{Z}_2 \times \mathbb{Z}_9))| = 18$ and $|E(PG(\mathbb{Z}_2 \times \mathbb{Z}_9))| = 30$. Similar to [14], $PG(\mathbb{Z}_2 \times \mathbb{Z}_9) \cong PG(\mathbb{Z}_9 \times \mathbb{Z}_2)$. $\blacksquare$ Then, we calculate the number of triangles in $PG(\mathbb{Z}_{p_1} \times \mathbb{Z}_{{p_2}^2})$.

Theorem 2.3. If p_1 and p_2 are prime numbers, then $PG(\mathbb{Z}_{p_1} \times \mathbb{Z}_{{p_2}^2})$ has the number of triangles $\frac{1}{2}(p_2(6p_1p_2 - 4p_1 - 3p_2 + 1)) - (p_1{p_2}^2 - 1)$.

Proof. By Definition 1.1, a vertex $(\bar{0}, \bar{0})$ is adjacent to all other vertices of $PG(\mathbb{Z}_{p_1} \times \mathbb{Z}_{{p_2}^2})$. So, it is clear that one of the vertex in triangles is $(\bar{0}, \bar{0})$. Hence $\mathbb{Z}_{p_1} \times \mathbb{Z}_{{p_2}^2}$ containing zero divisors, then we get the probability of other vertices as follows: First, the neighborhood of vertex not containing $(\bar{0}, \bar{0})$. Based on Theorem 2.2, the number of those neighborhood is $(p_1 - 1)({p_2}^2 - 1)$. Second, the neighborhood of vertex not containing $(\bar{0}, \bar{0})$, but at least one of the vertex's containing zero divisors. Based on Theorem 2.2, the number of those neighborhood is

$$\frac{1}{2}(p_1 - 1)({p_2}^2 - 2) + (p_2 - 1)^2(p_1 - 1).$$

Therefore, these all together forms $\frac{1}{2}(p_2(6p_1p_2 - 4p_1 - 3p_2 + 1)) - (p_1{p_2}^2 - 1)$ number of triangles. $\blacksquare$

Example 2.4. Based on Theorem 2.3, we can see that in Fig 1 has the number of triangles 12.

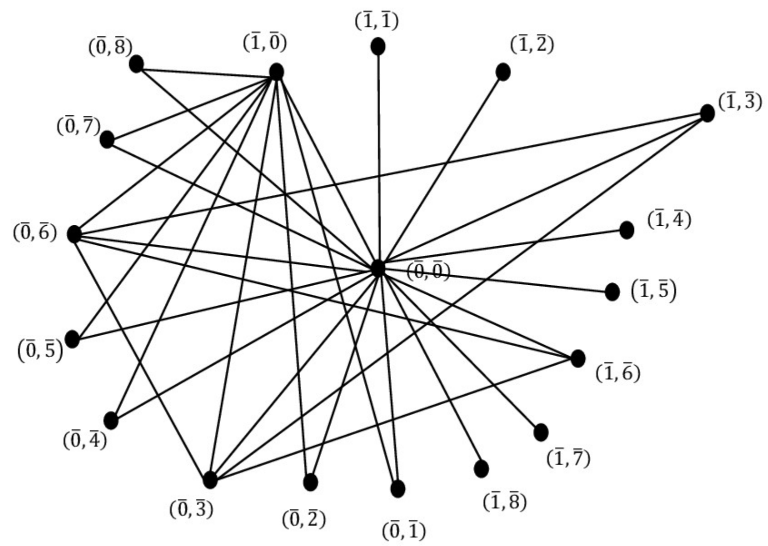

Figure 1. $PG(\mathbb{Z}_2 \times \mathbb{Z}_9)$

3 Line Graph of Prime Graph of Ring $\mathbb{Z}_{p_1} \times \mathbb{Z}_{p_2^2}$

In this section, we establish the line graph of ring $\mathbb{Z}_{p_1} \times \mathbb{Z}_{p_2^2}$ and discuss some properties about order and size in $L(PG(\mathbb{Z}_{p_1} \times \mathbb{Z}_{p_2^2}))$ by using a simple combinatorial approach.

Theorem 3.1. Let $PG(\mathbb{Z}_{p_1} \times \mathbb{Z}_{p_2^2})$ with vertex $v_1, v_2, \ldots, v_x$, be a prime graph of $\mathbb{Z}_{p_1} \times \mathbb{Z}_{p_2^2}$ of size x and order y. Then, $L(PG(\mathbb{Z}_{p_1} \times \mathbb{Z}_{p_2^2}))$ has order y and size

$$\tfrac{1}{2}(2{p_1}^2 + p_1){p_2}^4 + \tfrac{1}{2}(-4p_1 + 5){p_2}^3 + \tfrac{1}{2}(2{p_1}^2 + 3p_1 - 12)p_2^2 + \\ \tfrac{1}{2}(-14{p_1}^2 + 19)p_2 - \tfrac{1}{2}(3{p_1}^2) + \tfrac{1}{2}(13p_1) - 5.$$

Proof. The order of $L(PG(\mathbb{Z}_{p_1} \times \mathbb{Z}_{p_2^2}))$ is obvious by Definition 1.2, and Theorem 2.1. Hence, we get the order of $L(PG(\mathbb{Z}_{p_1} \times \mathbb{Z}_{p_2^2}))$ is $y = \frac{1}{2}(p_2(6p_1p_2 - 4p_1 - 3p_2 + 1))$. Then, the number of neighbourhood of vertex in $L(PG(\mathbb{Z}_{p_1} \times \mathbb{Z}_{p_2^2}))$, we consider some cases as follows:

$$\text{where } \bar{x} = \bar{1}, \bar{2}, \ldots, \overline{p_1 - 1},\ \bar{y} = \bar{1}, \bar{2}, \ldots, \overline{p_2 - 1},\ k_1 \in 1, 2, \ldots, (p_2 - 1), \\ k_2 \in 2, \ldots, (p_2 - 1),\ k_3, k_4 \in 1, 2, \ldots, (p_2 - 1), \text{ and } \bar{x} \in 1, 2, \ldots, (p_1 - 1)$$

(i) $((\bar{0}, \bar{0}), (\bar{x}, \bar{0}))$ adjacent to $((\bar{0}, \bar{0}), (\bar{x}, \bar{0}))$
Hence the line graph not containing an edges joining a vertex to itself, then we obtain that the number of these neighborhood is $\binom{p_1 - 1}{2}$ vertices.

(ii) $((\bar{0}, \bar{0}), (\bar{x}, \bar{0}))$ adjacent to $((\bar{0}, \bar{0}), (\bar{0}, \bar{y}))$
It is clear that the number of these neighborhood is $(p_1 - 1)({p_2}^2 - 1)$ vertices.

We get the same result for other cases as follow:

(iii) $((\bar{0}, \bar{0}), (\bar{x}, \bar{0}))$ adjacent to $((\bar{0}, \bar{0}), (\bar{x}, \bar{y}))$
The number of these neighborhood is $(p_1 - 1)^2({p_2}^2 - 1)$ vertices.

(iv) $((\bar{0}, \bar{0}), (\bar{x}, \bar{0}))$ adjacent to adjacent to $((\bar{x}, \bar{0}), (\bar{x}, \bar{y}))$
The number of these neighborhood is $\binom{(p_1-1)({p_2}^2-1)}{2}$ vertices.

(v) $((\bar{0}, \bar{0}), (\bar{0}, \bar{y}))$ adjacent to $((\bar{0}, \bar{0}), (\bar{0}, \bar{y}))$
The number of these neighborhood is $\binom{(p_1-1)({p_2}^2-1)}{2}$ vertices.

(vi) $((\bar{0},\bar{0}),(\bar{0},\bar{y}))$ adjacent to $((\bar{0},\bar{0}),(\bar{x},\bar{y}))$
The number of these neighborhood is $(p_1-1)^2({p_2}^2-1)$ vertices.

(vii) $((\bar{0},\bar{0}),(\bar{0},\bar{y}))$ adjacent to $((\bar{x},\bar{0}),(\bar{0},\bar{y}))$
The number of these neighborhood is $\binom{{p_2}^2-1}{2}$ vertices.

(viii) $((\bar{x},\bar{0}),(\bar{0},\bar{y}))$ adjacent to $((\bar{x},\bar{0}),(\bar{0},\bar{y}))$
The number of these neighborhood is $({p_2}^2-1)^2(p_1-1)$ vertices.

(ix) $((\bar{0},\bar{0}),(\bar{x},\bar{y}))$ adjacent to $((\bar{0},\bar{0}),(\bar{x},\bar{y}))$
The number of these neighborhood is $(p_1-1)^2({p_2}^2-1)$ vertices.

(x) $((\bar{0},\bar{0}),(\bar{0},\bar{y}))$ adjacent to $((\bar{0},k_3\bar{p_2}),(\bar{x},k_4\bar{p_2}))$
The number of these neighborhood is $(p_2-1)^3(p_1-1)$.

(xi) $((\bar{0},\bar{0}),(\bar{0},\bar{y}))$ adjacent to $((\bar{0},k_1\bar{p_2}),(\bar{0},k_2\bar{p_2}))$
The number of these neighborhood is $\frac{1}{2}(p_2-1)^2(p_2-2)$.

(xii) $((\bar{0},\bar{0}),(\bar{x},\bar{y}))$ adjacent to $((\bar{0},k_3\bar{p_2}),(\bar{0},k_4\bar{p_2}))$
The number of these neighborhood is $(p_2-1)^2(p_1-1)$.

(xiii) $((\bar{x},\bar{0}),(\bar{0},\bar{y}))$ adjacent to $((\bar{0},k_3\bar{p_2}),(\bar{0},k_4\bar{p_2}))$
The number of these neighborhood is $(p_2-1)^2(p_1-1)$.

(xiv) $((\bar{x},\bar{0}),(\bar{0},\bar{y}))$ adjacent to $((\bar{0},k_1\bar{p_2}),(\bar{0},k_2\bar{p_2}))$
The number of these neighborhood is $\frac{1}{2}(p_2-1)^2(p_2-2)(p_1-1)$.

(xv) $((\bar{0},k_3\bar{p_2}),(\bar{0},k_4\bar{p_2}))$ adjacent to $((\bar{0},k_3\bar{p_2}),(\bar{x},k_4\bar{p_2}))$
The number of these neighborhood is $\frac{1}{2}(p_2-1)^3(p_2-2)(p_1-1)$.

Therefore, the size in $L(PG(\mathbb{Z}_{p_1} \times \mathbb{Z}_{{p_2}^2}))$ is the sum of the number of neighborhood (i)-(xv). ■

Example 3.2. $PG(\mathbb{Z}_2 \times \mathbb{Z}_4)$ is shown in Fig 2.

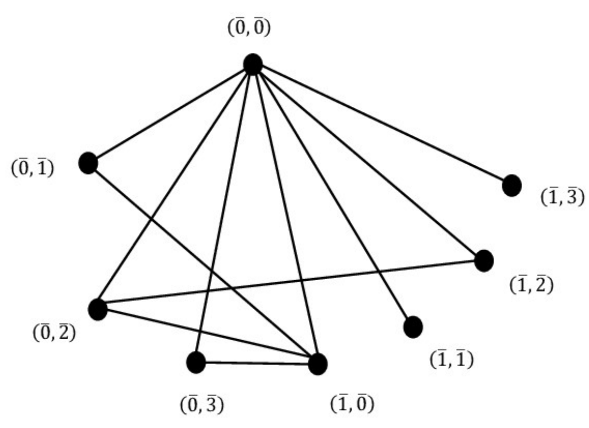

Figure 2. $PG(\mathbb{Z}_2 \times \mathbb{Z}_4)$

From Fig. 2, we can establish the line graph of prime graph of ring $\mathbb{Z}_2 \times \mathbb{Z}_4$ by Definition 1.2. The set of vertices of $L(PG(\mathbb{Z}_2 \times \mathbb{Z}_4))$ constitutes of the edges of $PG(\mathbb{Z}_2 \times \mathbb{Z}_4)$ and two distinct vertices are adjacent if the corresponding edges are adjacent in $PG(\mathbb{Z}_2 \times \mathbb{Z}_4)$. Therefore, $L(PG(\mathbb{Z}_2 \times \mathbb{Z}_4))$ is shown in Figure 3.
Based on Theorem 3.1, we can see that in Fig 3, $L(PG(\mathbb{Z}_2 \times \mathbb{Z}_4))$ has order 11 and size 33.

4 Wiener Index of Prime Graph of Ring $\mathbb{Z}_{p_1} \times \mathbb{Z}_{{p_2}^2}$

In this section, we calculate the Wiener index of $\mathbb{Z}_{p_1} \times \mathbb{Z}_{{p_2}^2}$.

Theorem 4.1. If p_1 and p_2 are prime numbers, then the Wiener index of $PG(\mathbb{Z}_{p_1} \times \mathbb{Z}_{{p_2}^2})$ is

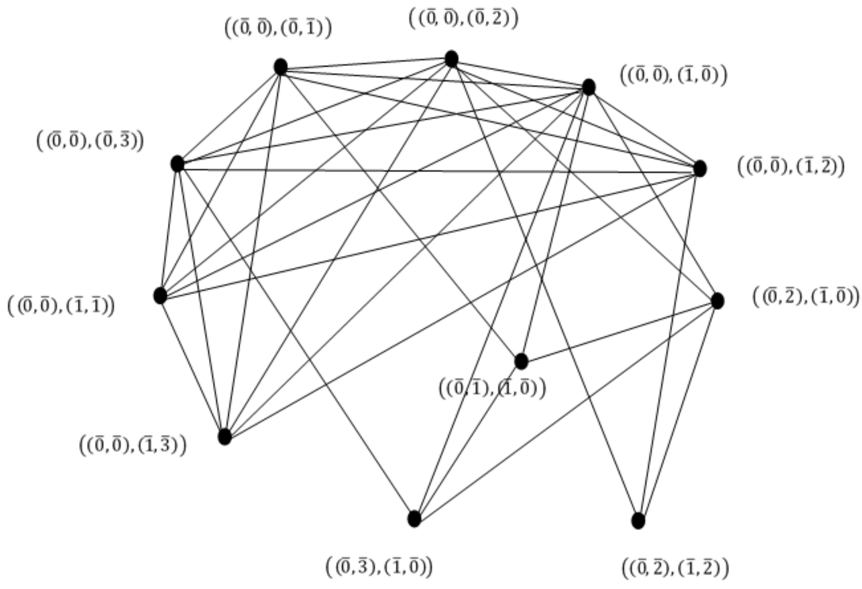

Figure 3. $L(PG(\mathbb{Z}_2 \times \mathbb{Z}_4))$

$\frac{1}{2}(p_2(6p_1p_2 - 4p_1 - 3p_2 + 1)) + p_2({p_2}^3{p_1}^2 + (3 - 7p_1)p_2 + 4p_1 - 1)$.

Proof. The distances between all pair vertices of $PG(\mathbb{Z}_{p_1} \times \mathbb{Z}_{{p_2}^2})$ divided into two cases, that is the distance between two adjacent vertices and not adjacent vertices.

(i) The distances between two adjacent vertices
By Theorem 2.1, the number of adjacent vertex in $PG(\mathbb{Z}_{p_1} \times \mathbb{Z}_{{p_2}^2})$ is

$$(p_1{p_2}^2 - 1) + (p_1 - 1)({p_2}^2 - 1) + \tfrac{1}{2}((p_2 - 1)(p_2 - 2)) + (p_2 - 1)^2(p_1 - 1)$$

where $(p_1{p_2}^2 - 1)$ is the number of vertex adjacent to $(\bar{0}, \bar{0})$, $(p_1 - 1)({p_2}^2 - 1)$ is the number of vertex not adjacent to $(\bar{0}, \bar{0})$, and $\frac{1}{2}((p_2 - 1)(p_2 - 2)) + (p_2 - 1)^2(p_1 - 1)$ is the number of vertex not adjacent to $(\bar{0}, \bar{0})$, but at least one of the vertex's containing zero divisors. Since $d((x_1, x_2), (y_1, y_2)) = 1$, for two vertices (x_1, x_2) and y_1, y_2 are adjacent, we get the distance between two adjacent vertex is

$$\tfrac{1}{2}(p_2(6p_1p_2 - 4p_1 - 3p_2 + 1)).$$

(ii) The distances between two not adjacent vertices
Suppose the number of edges in complete graph with $p_1{p_2}^2$ vertex is

$$\tfrac{1}{2}(p_1{p_2}^2(p_1{p_2}^2 - 1)).$$

Therefore, by (i), the number of not adjacent vertex is given by

$$\frac{1}{2}(p_1{p_2}^2(p_1{p_2}^2 - 1)) - \left(\frac{1}{2}(p_2(6p_1p_2 - 4p_1 - 3p_2 + 1))\right) = \frac{p_2({p_2}^3{p_1}^2 + (3 - 7p_1)p_2 + 4p_1 - 1)}{2}.$$

Since $d((x_1, x_2), (y_1, y_2)) = 2$, for two vertices (x_1, x_2) and (y_1, y_2) are not adjacent, we conclude that the distance between two not adjacent vertices is

$$p_2({p_2}^3{p_1}^2 + (3 - 7p_1)p_2 + 4p_1 - 1).$$

Thus, by (i) and (ii), we obtain that the Wiener index of $PG(\mathbb{Z}_{p_1} \times \mathbb{Z}_{{p_2}^2})$ is (i)+(ii), that is $\frac{1}{2}(p_2(6p_1p_2 - 4p_1 - 3p_2 + 1)) + p_2({p_2}^3{p_1}^2 + (3 - 7p_1)p_2 + 4p_1 - 1)$. ■

Example 4.2. Based on $PG(\mathbb{Z}_2 \times \mathbb{Z}_9)$ in Fig 1, we get the Wiener index of $PG(\mathbb{Z}_2 \times \mathbb{Z}_9)$ is 276, by Theorem 4.1.

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