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Studying new Revan indices by using M-polynomial approach on some dendrimers

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Abstract *Dendrimers have become a significant focus of research in various biomedical fields due to their unique nanoparticle properties. These branched macromolecules grow from the central core and form an intricate three-dimensional structure.*
In this article, first we explore the creation of new Revan indices using dendrimers based on triazines and porphyrins , phthalocyanine-containing dendrimers, porphyrin cores, and phosphorus-containing dendrimers. In the following by utilizing M-polynomials derived from molecular graphs and MATLAB, we will present the results obtained from our investigation.

1 Introduction

In recent years, dendrimers have gained significant attention as a new class of polymer materials, particularly in drug delivery systems due to their unique properties. Dendrimers can deliver drugs directly to the affected area of a patient's body. Dendrimers are macromolecular structures that offer multiple advantages that may vary depending on the chemical nature of the drug being delivered [14].
The reason why dendrimers are highly regarded in drug delivery is that they possess features such as uniform size, water solubility, changeable surface performance, high branching degree, multi-capacity, well-defined molecular weight, and accessible internal cavities. In addition, their high surface control over dendritic architecture makes them ideal carriers.
A dendrimer is a tree with two additional parameters, the progressive degree t*, and the radius* r*. Each internal node of the tree has a degree* $t+1$*. Like any tree, a dendrimer has one central node (single-center dendrimer) or two (double-center dendrimers). The radius represents the (maximum) distance from an outer node to the center (nearest) node. The radius r denotes the (largest) distance from an external node to the (closer) center. If all external nodes are at a distance r from the center, then the dendrimer is called homogeneous. Internal nodes are different from the central nodes are called branching nodes and are said to be on the i -th orbit if their distance to the (nearer) center is* r*[21].*
Many studies have been done on the possible use of dendrimers as drug delivery tools. One of these cases are Alzheimer's disease.
With proper design, they can target diseased areas of the brain and have multiple functions on the blood-brain barrier. Also, we have several dendrimers with therapeutic potential for Alzheimer's disease [2].
So far, many Graph polynomials have been introduced [1, 5, 9, 13, 28]. Many of which are useful in mathematical chemistry. valuable results have been obtained regarding Graph polynomials in chemical networks and consequently in topological indices. These results are used by researchers to find a useful relationship between chemical compounds and their chemical and biological properties. One of the most widely used polynomials is M-polynomial. Suppose G be

a graph, and let $m_{i,j}, i, j \geq 1$ be the number of edges uv of G such that $\{d_u, d_v\} = \{i, j\}$. The minimum and maximum vertex degree of graph G are denoted by δ and Δ respectively.

Definition 1.1. The M-polynomial of G is obtained as follows [4]:

$$M(G, x, y) = \sum_{\delta \leq i \leq j \leq \Delta} m_{i,j} x^i y^j. \tag{1.1}$$

The following operators are used in the subsequent theorems.

$$D_x = x \times \frac{\partial M(G,x,y)}{\partial x} \ , \ D_y = y \times \frac{\partial M(G,x,y)}{\partial y} \ ,$$

$$Q_{x(a)} M(G, x, y) = x^a . M(G, x, y) \ , \ Q_{y(a)} M(G, x, y) = y^a . M(G, x, y) \, ,$$

$$S_x M(G, x, y) = \int_0^x \frac{M(G,t,y)}{t} dt \ , \quad S_y M(G, x, y) = \int_0^y \frac{M(G,x,t)}{t} dt \ ,$$

$$J\, M(G, x, y) = M(G, x, x).$$

One of the applications of the M-Polynomial is that it is possible to calculate topological indices with the help of mathematical relations (derivatives, integrals, etc.) for x and y. The calculation of topological indices using M-polynomial in the molecular Graphs have also been investigated [3, 11, 12, 26].

Definition 1.2. The first and second Revan indices of a Graph G were introduced as follows [15]:

$$R_1(G) = \sum_{uv \in E(G)} [r_u + r_v] \ , \qquad R_2(G) = \sum_{uv \in E(G)} [r_u . r_v].$$

Where $r_u = \Delta + \delta - d_u$.

Recently, many studies have been conducted using topological indices [8, 23, 24, 25, 27]. Numerous research studies have been carried out on the Revan indices [7, 19, 20].

Definition 1.3. The product connectivity Revan index was introduced as follows [16]:

$$PR(G) = \sum_{uv \in E(G)} \frac{1}{\sqrt{r_u . r_v}}.$$

Definition 1.4. The forgotten Revan (F-Revan) index was introduced as follows [17]:

$$FR(G) = \sum_{uv \in E(G)} [r_u^2 + r_v^2].$$

Definition 1.5. The symmetric division Revan index was introduced as follows [17]:

$$SDR(G) = \sum_{uv \in E(G)} \frac{r_u}{r_v} + \frac{r_v}{r_u}.$$

Definition 1.6. The harmonic Revan index was introduced as follows [18]:

$$HR(G) = \sum_{uv \in E(G)} \frac{2}{r_u + r_v}.$$

Definition 1.7. The inverse sum indeg Revan index was introduced as follows [18]:

$$IR(G) = \sum_{uv \in E(G)} \frac{r_u . r_v}{r_u + r_v}.$$

In this article, first, By using M-polynomial we calculate product connectivity Revan, forgotten Revan (F-Revan), symmetric division Revan, harmonic Revan, and inverse sum indeg Revan indices. In the following for molecular Graph of phthalocyanines, Pamam, triazine, phosphorus-based on dendrimers the Revan indices have been calculated and M-polynomial was plotted. Finally, by using M- polynomial derived from molecular Graphs and using MATLAB, the F-Revan index from M-polynomial coding is computed.

2 relationship between M-polynomial and some connectivity Revan indices

Theorem 2.1. *Let $M(G,x,y)$ be the M-Polynomial the Graph G. Then the product connectivity Revan index is computed as,*

$$PR(G) = [D_x^{-\frac{1}{2}} D_y^{-\frac{1}{2}} (Q_{x(-\Delta-\delta)} Q_{y(-\Delta-\delta)} M(G,x,y))] \big|_{|(x,y)=(1,1)} .$$

Proof.

$$PR(G) = \sum_{uv\in E(G)} \frac{1}{\sqrt{r_u \cdot r_v}} = \sum_{uv\in E(G)} \frac{1}{\sqrt{(\Delta+\delta-d_u)\cdot(\Delta+\delta-d_v)}} \tag{2.1}$$

$$Q_{x(-\Delta-\delta)} Q_{y(-\Delta-\delta)} M(G,x,y) = \sum_{\delta\leq i\leq j\leq\Delta} m_{ij} x^{(i-\Delta-\delta)} y^{(j-\Delta-\delta)},$$

$$D_y^{-\frac{1}{2}} (\sum_{\delta\leq i\leq j\leq\Delta} m_{ij} x^{i-\Delta-\delta} y^{j-\Delta-\delta})$$
$$= \sum_{\delta\leq i\leq j\leq\Delta} m_{ij} x^{(i-\Delta-\delta)} \frac{1}{\sqrt{(j-\Delta-\delta)}} (y^{(j-\Delta-\delta)})^{-\frac{1}{2}},$$

$$D_x^{-\frac{1}{2}} D_y^{-\frac{1}{2}} (\sum m_{i,j} x^{(i-\Delta-\delta)} y^{(j-\Delta-\delta)})$$
$$= \sum_{\delta\leq i\leq j\leq\Delta} m_{ij} \frac{1}{\sqrt{(i-\Delta-\delta)(j-\Delta-\delta)}} (x^{(i-\Delta-\delta)})^{-\frac{1}{2}} . (y^{(j-\Delta-\delta)})^{-\frac{1}{2}}$$

$$D_x^{-\frac{1}{2}} D_y^{-\frac{1}{2}} (\sum_{\delta\leq i\leq j\leq\Delta} m_{ij} x^{(i-\Delta-\delta)} y^{j-\Delta-\delta})|_{(x,y)=(1,1)} = \sum_{\delta\leq i\leq j\leq\Delta} m_{i,j} \frac{1}{\sqrt{(i-\Delta-\delta)(j-\Delta-\delta)}}, \tag{2.2}$$

Hence, according to relations (2.1) and (2.2), we have

$$PR(G) = [D_x^{-\frac{1}{2}} D_y^{-\frac{1}{2}} (Q_{x(-\Delta-\delta)} Q_{y(-\Delta-\delta)} M(G,x,y))] \ |_{(x,y)=(1,1)} .$$

□

Theorem 2.2. *Let $M(G,x,y)$ be the M-Polynomial the graph G. Then the F-Revan Revan index is computed as,*

$$FR(G) = ({D_x}^2 + D_y^2)(Q_{x(-\Delta-\delta)} Q_{y(-\Delta-\delta)} M(G,x,y))\Big|_{(x,y)=(1,1)}.$$

Proof.

$$FR(G) = \sum_{uv\in E(G)} [r_u^2 + r_v^2] = \sum_{uv\in E(G)} [(\Delta+\delta-d_u)^2 + (\Delta+\delta-d_v)^2] \tag{2.3}$$

$$Q_{x(-\Delta-\delta)} Q_{y(-\Delta-\delta)} M(G,x,y) = \sum_{\delta\leq i\leq j\leq\Delta} m_{i,j} x^{(i-\Delta-\delta)} y^{(j-\Delta-\delta)}$$
$$D_x^2(\sum m_{i,j} x^{(i-\Delta-\delta)} y^{(j-\Delta-\delta)}) + D_y^2(\sum m_{i,j} x^{(i-\Delta-\delta)} y^{(j-\Delta-\delta)})$$
$$= \sum_{\delta\leq i\leq j\leq\Delta} m_{i,j} . [(i-\Delta-\delta)^2 + (j-\Delta-\delta)^2] x^{(i-\Delta-\delta)} . \ y^{(j-\Delta-\delta)},$$

$$D_x^2(\sum_{uv\in E(G)} m_{i,j} x^{(i-\Delta-\delta)}) y^{(j-\Delta-\delta)}) + D_y^2(\sum_{uv\in E(G)} m_{i,j} x^{(i-\Delta-\delta)}) y^{(j-\Delta-\delta)})|_{(x,y)=(1,1)}$$
$$= \sum_{\delta\leq i\leq j\leq\Delta} m_{i,j} [(i-\Delta-\delta)^2 + (j-\Delta-\delta)^2], \tag{2.4}$$

Hence, according to relations (2.3) and (2.4), we have

$$FR(G) = [(D_x^2 + D_y^2)(Q_{x(-\Delta-\delta)} Q_{y(-\Delta-\delta)} M(G,x,y))]\Big|_{(x,y)=(1,1)}.$$

□

Theorem 2.3. *Let $M(G,x,y)$ be the M-Polynomial the graph G. Then the symmetric division Revan index is computed as,*

$$SDR(G) = (D_x S_y + S_x D_y)(Q_{x(-\Delta-\delta)} Q_{y(-\Delta-\delta)} \ M(G,x,y))|_{(x,y)=(1,1)}.$$

Proof.

$$SDR(G) = \sum_{uv\in E(G)} \frac{r_u}{r_v} + \frac{r_v}{r_u} = \sum_{uv\in E(G)} \left(\frac{d_u-\Delta-\delta}{d_v-\Delta-\delta} + \frac{d_v-\Delta-\delta}{d_u-\Delta-\delta}\right) \tag{2.5}$$

$$\begin{aligned}
&D_y\Big(\sum_{\delta\le i\le j\le\Delta} m_{i,j}x^{(i-\Delta-\delta)}y^{(j-\Delta-\delta)}\Big) = \sum_{\delta\le i\le j\le\Delta} m_{i,j}(j-\Delta-\delta)x^{(i-\Delta-\delta)}y^{(i-\Delta-\delta)}\\
&S_xD_y\Big(\sum_{\partial\le i\le j\le\Delta} m_{i,j}x^{(i-\Delta-\delta)}y^{(j-\Delta-\delta)}\Big) = \sum_{\partial\le i\le j\le\Delta} \Big(m_{i,j}(j-\Delta-\delta)y^{(j-\Delta-\delta)}\int_0^x \frac{t^{(i-\Delta-\delta)}}{t}dt\Big),\\
&= \sum_{\partial\le i\le j\le\Delta} m_{i,j}(j-\Delta-\delta)y^{(j-\Delta-\delta)}\frac{x^{(i-\Delta-\delta)}}{(i-\Delta-\delta)},\\
&= \sum_{\partial\le i\le j\le\Delta} m_{i,j}\frac{(j-\Delta-\delta)}{(i-\Delta-\delta)}x^{(i-\Delta-\delta)}.\,y^{(j-\Delta-\delta)},\\
&S_yD_x\Big(\sum_{\partial\le i\le j\le\Delta} m_{i,j}x^{(i-\Delta-\delta)}y^{(j-\Delta-\delta)}\Big) = \sum_{\partial\le i\le j\le\Delta} \Big(m_{i,j}(i-\Delta-\delta)x^{(j-\Delta-\delta)}\int_0^y \frac{t^{(j-\Delta-\delta)}}{t}dt\Big),\\
&= \sum_{\partial\le i\le j\le\Delta} m_{i,j}(i-\Delta-\delta)x^{(i-\Delta-\delta)}\frac{y^{(j-\Delta-\delta)}}{(j-\Delta-\partial)},\\
&= \sum_{\partial\le i\le j\le\Delta} m_{i,j}\frac{(i-\Delta-\delta)}{(j-\Delta-\delta)}x^{(i-\Delta-\delta)}.\,y^{(j-\Delta-\delta)},
\end{aligned}$$

$$(S_xD_y+S_yD_x)\Big(\sum_{\delta\le\Delta} m_{i,j}x^{(i-\Delta-\delta)}y^{(j-\Delta-\delta)}\Big)\Big|_{(x,y)=(1,1)} = \sum_{\delta\le\Delta} m_{i,j}\left(\frac{i-\Delta-\delta}{j-\Delta-\delta} + \frac{j-\Delta-\delta}{i-\Delta-\delta}\right) \tag{2.6}$$

Hence, according to relations (2.5) and (2.6), we have

$$SDR(G) = (S_xD_y+S_yD_x)\Big(\sum_{\delta\le i\le j\le\Delta} m_{i,j}x^{(i-\Delta-\delta)}y^{(j-\Delta-\delta)}\Big)\Bigg|_{(x,y)=(1,1)}.$$

□

Theorem 2.4. *Let $M(G,x,y)$ be the M-Polynomial the graph G. Then the harmonic Revan index is computed as,*

$$HR(G) = (-2S_x.J.Q_{x(-\Delta-\delta)}Q_{y(-\Delta-\delta)}M(G,x,y))|_{x=1}.$$

Proof.

$$HR(G) = \sum_{uv\in E(G)} \frac{2}{r_u+r_v} = -2\sum_{\delta\le i\le j\le\Delta} \frac{1}{(d_u-\Delta-\delta)+(d_v-\Delta-\delta)}, \tag{2.7}$$

$$\begin{aligned}
&J\Big(\sum_{\delta\le i\le j\le\Delta} m_{i,j}x^{(i-\Delta-\delta)}y^{(j-\Delta-\delta)}\Big) = \sum_{\delta\le i\le j\le\Delta} m_{i,j}x^{(i-\Delta-\delta)}y^{(j-\Delta-\delta)},\\
&S_x.\,J\Big(\sum_{\delta\le i\le j\le\Delta} m_{i,j}x^{(i-\Delta-\delta)}y^{(j-\Delta-\delta)}\Big) = \sum_{\delta\le i\le j\le\Delta} m_{i,j}\int_0^x \frac{t^{(i+j-2\Delta-2\delta)}}{t}dt,\\
&\qquad = \Big(-2S_xJ\Big(\sum_{\delta\le i\le j\le\Delta} m_{i,j}x^{(i-\Delta-\delta)}y^{(j-\Delta-\delta)}\Big)\Bigg|_{(x=1)},
\end{aligned}$$

$$= -2\sum_{\delta\le i\le j\le\Delta} m_{i,j}\frac{1}{(i-\Delta-\delta)+(j-\Delta-\delta)} \tag{2.8}$$

Hence, according to relations (2.7) and (2.8), we have

$$HR(G) = (-2S_x.J.Q_{x(-\Delta-\delta)}Q_{y(-\Delta-\delta)}M(G,x,y))|_{x=1}.$$

,

□

Theorem 2.5. *Let $M(G,x,y)$ be the M- Polynomial the graph G. Then the Inverse sum indeg Revan index is computed as,*

$$IR(G) = -(S_xJD_xD_y)Q_{x(-\Delta-\delta)}Q_{y(-\Delta-\delta)}M(G,x,y))|_{x=1}.$$

Proof.

$$IR(G) = \sum_{uv\in E(G)} \frac{r_u\cdot r_v}{r_u+r_v} = -\sum_{uv\in E(G)} \frac{(d_u-\Delta-\delta)\cdot(d_v-\Delta-\delta)}{(d_u-\Delta-\delta)+(d_v-\Delta-\delta)}, \tag{2.9}$$

$$D_x.\,D_y(\sum_{\delta\le i\le j\le\Delta} m_{i,j}x^{(i-\Delta-\delta)}y^{(j-\Delta-\delta)})$$
$$=\sum_{\delta\le i\le j\le\Delta} m_{i,j}(i-\Delta-\delta)(j-\Delta-\delta)x^{(i-\Delta-\delta)}y^{(j-\Delta-\delta)},$$
$$JD_xD_y(\sum_{\delta\le i\le j\le\Delta} m_{i,j}x^{(i-\Delta-\delta)}y^{(j-\Delta-\delta)})$$
$$=\sum_{\delta\le i\le j\le\Delta} m_{i,j}(i-\Delta-\delta)(j-\Delta-\delta)x^{(i-\Delta-\delta)}x^{(j-\Delta-\delta)},$$
$$-S_xJD_xD_y(\sum_{\delta\le i\le j\le\Delta} m_{i,j}x^{(i-\Delta-\delta)}y^{(j-\Delta-\delta)}\ )\Big|_{x=1}$$
$$=[-\sum_{\delta\le i\le j\le\Delta} m_{i,j}(i-\Delta-\delta)(j-\Delta-\delta)\int_0^x\frac{t^{i+j-2\Delta-2\delta}}{t}dt]\Big|_{x=1},$$

$$=\sum_{\delta\le i\le j\le\Delta} m_{i,j}\frac{(i-\Delta-\delta)\cdot(j-\Delta-\delta)}{(i+j-2\Delta-2\delta)} \tag{2.10}$$

Hence, according to relations (2.9) and (2.10), we have

$$IR(G)=-(S_xJD_xD_y)Q_{x(-\Delta-\delta)}Q_{y(-\Delta-\delta)}\,M(G,x,y))|_{x=1}.$$

□

According to the results of the above theorems, Table (1) is obtained to calculate Revan indices using M-polynomial.

Topological Index name	Derivation from $M(G,x,y)$
Product connectivity Revan index	$[D_x^{-\frac{1}{2}}D_y^{-\frac{1}{2}}(Q_{x(-\Delta-\delta)}Q_{y(-\Delta-\delta)}M(G,x,y))]\,\big|_{\|(x,y)=(1,1)}$
F-Revan index	$[(D_x{}^2+D_y^2)(Q_{x(-\Delta-\delta)}Q_{y(-\Delta-\delta)}M(G,x,y))]\Big|_{(x,y)=(1,1)}$
Symmetric division Revan index	$[(D_xS_y+S_xD_y)(Q_{x(-\Delta-\delta)}Q_{y(-\Delta-\delta)}\,M(G,x,y))]\|_{(x,y)=(1,1)}$
Harmonic Revan index	$[(\,-\,2S_x.J.Q_{x(-\Delta-\delta)}Q_{y(-\Delta-\delta)}\,M(G,x,y))]\|_{x=1}$
Inverse sum Revan index	$[\,-\,(S_xJD_xD_y)Q_{x(-\Delta-\delta)}Q_{y(-\Delta-\delta)}\,M(G,x,y))]\|_{x=1}$

Table 1. Derivation of Revan indices from M-polynomial.

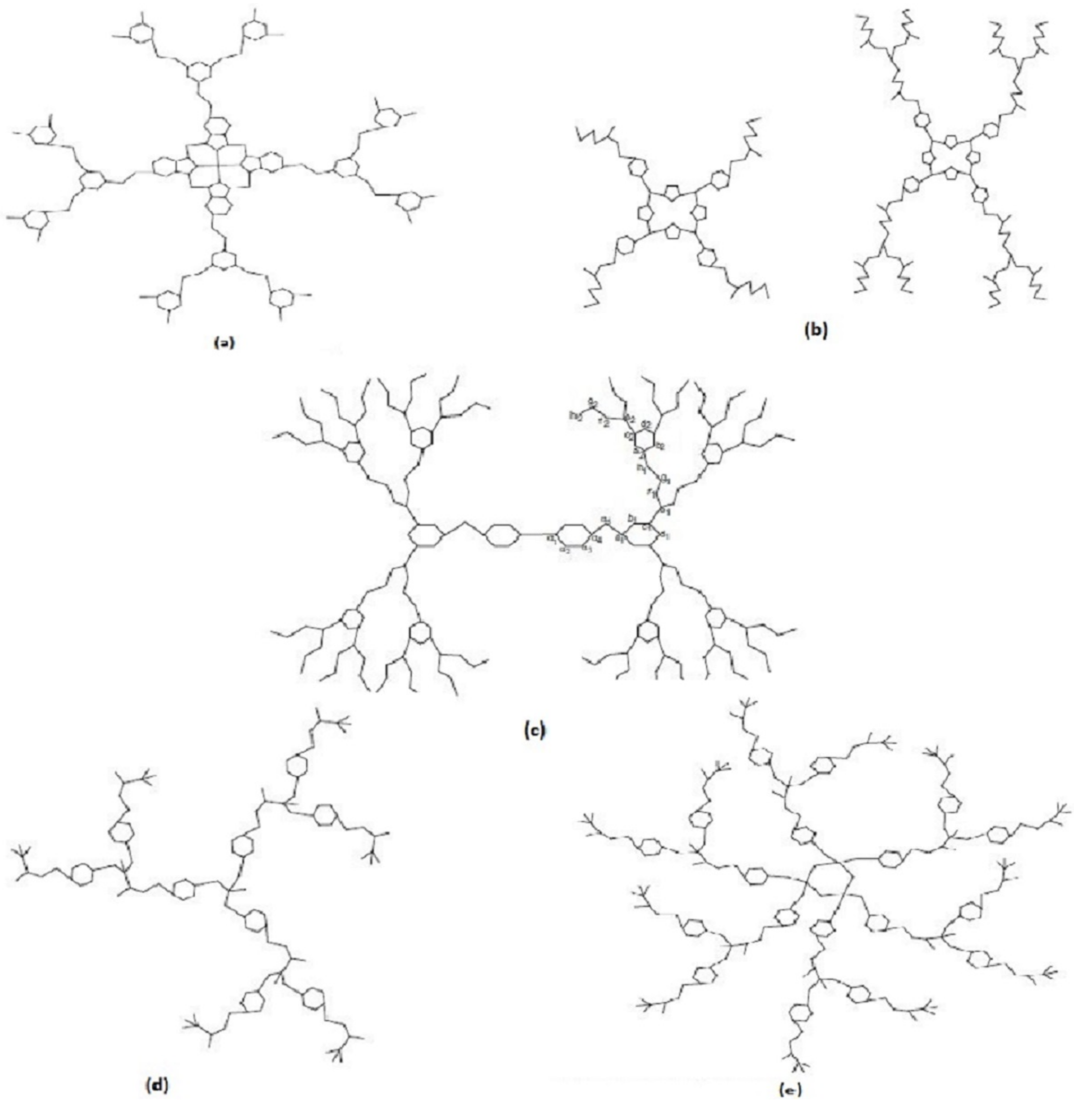

Figure 1. Molecular Graph of denderimers (a)H$_1(n), (b)H_2(n), (c)H_3(n), (d)H_4(n), (e)H_5(n),$

3 Phthalocyanines dendrimer $H_1(n)$

The proposed structure of phthalocyanines has been the subject of significant research efforts. These molecules have since been explored for their potential in various applications.
Let $H_1(n)$ be the molecular Graph of the following dendrimer, where n represents the generation stages of $H_1(n)$. Figure (1)-(a) depicts the structure of $H_1(n)$ for $n = 2$.

$$\begin{aligned}
&E_1(H_1(n)) = \{uv \in E_1(H_1(n)) : d_u = 4 \ and\, d_v = 3\}|\ E_1(H_1(n))| = 4,\\
&E_2(H_1(n)) = \{uv \in E_2(H_1(n)) : d_u = 1 \ and\, d_v = 3\}|\ E_2(H_1(n))| = 2^{n+2},\\
&E_3(H_1(n)) = \{uv \in E_3(H_1(n)) : d_u = 2 \ and\, d_v = 2\}|\ E_3(H_1(n))| = 2^{n+2},\\
&E_4(H_1(n)) = \{uv \in E_4(H_1(n)) : d_u = 3 \ and\, d_v = 2\}|\ E_4(H_1(n))| = 8(2^{n+2} - 1),\\
&E_5(H_1(n)) = \{uv \in E_5(H_1(n)) : d_u = 3 \ and\, d_v = 3\}|\ E_5(H_1(n))| = 20.
\end{aligned}$$

Theorem 3.1. *Let $H_1(n)$ be the molecular Graph structures of Phthalocyanines dendrimers, then, the M-Polynomial are:*

$$M(H_1(n, x, y)) = 4x^4y^3 + 2^{n+2}xy^3 + 2^{n+2}x^2y^2 + 8(2^{n+2} - 1)x^3y^2 + 20x^3y^3.$$

Proof. It is easily obtained by the definition of (1.1) and the structure of $H_1(n)$. □

Theorem 3.2. *Suppose $H_1(n)$, is the Graph structure of Phthalocyanines dendrimer. New Revan indices are calculated as* *follows:*

1. $PR(H_1(n)) = (\frac{3\sqrt{2}+4+16\sqrt{6}}{12})2^{n+2} + (10 + 2\sqrt{2} - 4\frac{\sqrt{6}}{3})$,

2. $Fr(H_1(n), x, y) = 142(2^{n+2}) + 76$,

3. $SDR(H_1(n), x, y) = (\frac{131}{6})2^{n+2} + \frac{98}{3}$,

4. $HR(H_1(n)) = \frac{58}{15}(2^{n+2}) + \frac{142}{15}$,

5. $IR(H_1(n)) = \frac{373}{30}(2^{n+2}) + \frac{196}{15}$.

Proof. By applying the operators of Table (1) to Theorem (3.1), we have:

$$Q_{x(-5)}Q_{y(-5)}M(H_1(n), x, y) = 4x^{-1}y^{-2} + 2^{n+2}x^{-4}y^{-2} + 2^{n+2}x^{-3}y^{-3}$$
$$+8(2^{n+2} - 1)x^{-2}y^{-3} + 20x^{-2}y^{-2},$$
$$D_x^{-\frac{1}{2}}D_y^{-\frac{1}{2}}(Q_{x(-5)}Q_{y(-5)}M(H_1(n), x, y)) = 4 \times \frac{1}{\sqrt{2}}x^{-1}.y^{-2} + 2^{n+2} \times \frac{1}{\sqrt{8}}x^{-4}.y^{-2}$$
$$+2^{n+2} \times \frac{1}{\sqrt{9}}x^{-3}.y^{-3} + \frac{8}{\sqrt{6}}(2^{n+2} - 1)x^{-2}.y^{-3} + 20 \times \frac{1}{\sqrt{4}}x^{-2}.y^{-2},$$
$$D_y^2Q_{x(-5)}Q_{y(-5)}M(H_1(n), x, y) = 4.(-2)^2x^{-1}y^{-2} + 2^{n+2}.(-2)^2x^{-4}y^{-2}$$
$$+2^{n+2}.(-3)^2x^{-3}y^{-3} + 8(2^{n+2} - 1).(-3)^2x^{-2}y^{-3} + 20.(-2)^2x^{-2}y^{-2},$$
$$D_x^2Q_{x(-5)}Q_{y(-5)}M(H_1(n), x, y) = 4.(-1)^2x^{-1}y^{-2} + 2^{n+2}.(-4)^2x^{-4}y^{-2}$$
$$+2^{n+2}.(-3)^2x^{-3}y^{-3} + 8(2^{n+2} - 1).(-2)^2x^{-2}y^{-3} + 20.(-2)^2x^{-2}y^{-2},$$
$$D_x(Q_{x(-5)}Q_{y(-5)}M(H_1(n), x, y)) = (-1) \times (4)x^{-1}y^{-2} + (-4)2^{n+2}x^{-4}y^{-2}$$
$$+(-3) \times 2^{n+2}x^{-3}y^{-3} + 8(-2)(2^{n+2} - 1)x^{-2}y^{-3} + 20(-2)x^{-2}y^{-2},$$
$$D_y(Q_{x(-5)}Q_{y(-5)}M(H_1(n), x, y)) = 4 \times (-2)x^{-1}y^{-2} + 2^{n+2} \times (-2)x^{-4}y^{-2}$$
$$+2^{n+2} \times (-3)x^{-3}y^{-3} + 8 \times (-3)(2^{n+2} - 1)x^{-2}y^{-3} + 20 \times (-2)x^{-2}y^{-2},$$
$$S_y(Q_{x(-5)}Q_{y(-5)}M(H_1(n), x, y)) = \int_0^y \frac{4x^{-1}t^{-2}}{t}dt + \int_0^y \frac{2^{n+2}x^{-4}t^{-2}}{t}dt + \int_0^y \frac{2^{n+2}x^{-3}t^{-3}}{t}dt$$
$$+\int_0^y \frac{8(2^{n+2}-1)x^{-2}t^{-3}}{t}dt + \int_0^y \frac{20x^{-2}t^{-2}}{t}dt$$
$$= \frac{4}{-2}x^{-1}y^{-2} + \frac{2^{n+2}}{-2}x^{-4}y^{-2} + \frac{2^{n+2}}{-3}x^{-3}y^{-3} + \frac{8}{-3}(2^{n+2} - 1)x^{-2}y^{-3} + \frac{20}{-2}x^{-2}y^{-2},$$

$$S_xD_y(Q_{x(-5)}Q_{y(-5)}M(H_1(n), x, y)) = \int_0^x \frac{-8t^{-1}y^{-2}}{t}dt + \int_0^x \frac{(-2)2^{n+2}t^{-4}y^{-2}}{t}dt$$
$$+\int_0^x \frac{(-3)2^{n+2}t^{-3}y^{-3}}{t}dt + \int_0^x \frac{-24(2^{n+2}-1)t^{-2}y^{-3}}{t}dt + \int_0^x \frac{-40t^{-2}y^{-2}}{t}dt$$
$$= 8x^{-1}y^{-2} + \frac{1}{2}(2^{n+2})x^{-4}y^{-2} + 2^{n+2}x^{-3}y^{-3} + 12(2^{n+2} - 1)x^{-2}y^{-3} + 20x^{-2}y^{-2},$$
$$D_xS_y(Q_{x(-5)}Q_{y(-5)}M(H_1(n), x, y)) = \frac{4}{-2}x^{-1}y^{-2} + \frac{2^{n+2}}{-2}x^{-4}y^{-2} + \frac{2^{n+2}}{-3}x^{-3}y^{-3}$$
$$+\frac{8}{-3}(2^{n+2} - 1)x^{-2}y^{-3} + \frac{20}{-2}x^{-2}y^{-2}$$
$$= 2x^{-1}y^{-2} + 2(2^{n+2})x^{-4}y^{-2} + 2^{n+2}x^{-3}y^{-3} + \frac{16}{3}(2^{n+2} - 1)x^{-2}y^{-3} + 20x^{-2}y^{-2},$$
$$J(Q_{x(-5)}Q_{y(-5)}M(H_1(n), x, y))) = 4x^{-1}x^{-2} + 2^{n+2}x^{-4}x^{-2} + 2^{n+2}x^{-3}x^{-3}$$
$$+8(2^{n+2} - 1)x^{-2}x^{-3} + 20x^{-2}x^{-2},$$
$$S_xJ(Q_{x(-5)}Q_{y(-5)}M(H_1(n), x, y))) = \int_0^x \frac{4t^{-3}}{t}dt + \int_0^x \frac{2^{n+2}t^{-6}}{t}dt + \int_0^x \frac{2^{n+2}t^{-6}}{t}dt$$
$$+\int_0^x \frac{8(2^{n+2}-1)t^{-5}}{t}dt + \int_0^x \frac{20t^{-4}}{t}dt$$
$$= \frac{4x^{-3}}{-3} + \frac{2^{n+2}x^{-6}}{-6} + \frac{2^{n+2}x^{-6}}{-6} + \frac{8(2^{n+2}-1)x^{-5}}{-5} + \frac{20x^{-4}}{-4},$$
$$D_xD_y(Q_{x(-5)}Q_{y(-5)}M(H_1(n), x, y)) = 8x^{-1}y^{-2} + 8(2^{n+2})x^{-4}y^{-2} + 9(2^{n+2})x^{-3}y^{-3}$$
$$+48(2^{n+2} - 1)x^{-2}y^{-3} + 80x^{-2}y^{-2},$$
$$J\,D_xD_y(Q_{x(-5)}Q_{y(-5)}M(G, x, y)) = 8x^{-3} + 8(2^{n+2})x^{-6} + 9(2^{n+2})x^{-6}$$
$$+48(2^{n+2} - 1)x^{-5} + 80x^{-4},$$
$$S_x\,J\,D_xD_y(Q_{x(-5)}Q_{y(-5)}M(G, x, y)) = \int_0^x \frac{8t^{-3}}{t}dt + \int_0^x \frac{8(2^{n+2})t^{-6}}{t}dt + \int_0^x \frac{9(2^{n+2})t^{-6}}{t}dt$$
$$+\int_0^x \frac{48(2^{n+2}-1)t^{-5}}{t}dt + \int_0^x \frac{80t^{-4}}{t}dt$$
$$= \frac{8}{-3}x^{-3} + \frac{8(2^{n+2})}{-6}x^{-6} + \frac{9(2^{n+2})}{-6}x^{-6} + \frac{48(2^{n+2}-1)}{-5}x^{-5} + \frac{80}{-4}x^{-4},$$

Now we have:

1. $[D_x^{-\frac{1}{2}} D_y^{-\frac{1}{2}}(Q_{x(-5)}Q_{y(-5)}M(H_1(n),x,y))]\Big|_{(x,y)=(1,1)} = (\frac{3\sqrt{2}+4+16\sqrt{6}}{12})2^{n+2}$
$+(10+2\sqrt{2}-4\frac{\sqrt{6}}{3})$,
2. $[(D_x^2+D_y^2)(Q_{x(-5)}Q_{y(-5)}M(H_1(n),x,y))]\big|_{(x,y)=(1,1)} = 142(2^{n+2})+76$,
3. $[(D_xS_y+S_xD_y)(Q_{x(-5)}Q_{y(-5)}\,M(H_1(n),x,y))]|_{(x,y)=(1,1)} = \frac{119}{6}(2^{n+2})+2^{n+1}+\frac{98}{3}$,
4. $[-2S_xJ(Q_{x(-5)}Q_{y(-5)}M(H_1(n),x,y))]\big|_{x=1} = \frac{58}{15}(2^{n+2})+\frac{142}{15}$,
5. $[-S_x\,J\,D_xD_y(Q_{x(-5)}Q_{y(-5)}M(H_1(n),x,y))]\big|_{x=1} = \frac{373}{30}(2^{n+2})+\frac{196}{15}$.

□

4 Pamam dendrimers with poryphyrin core ($H_2(n)$)

One of the important applications of PAMAM dendrimers with porphyrin core is as a potential photosensitizer for photodynamic therapy applications [22]

$$\begin{aligned}
E_1(H_2(n)) &= \{uv \in E_1(H_2(n)):\ d_u=2\ and\, d_v=2\}|\ E_1(H_2(n))| = 4(3\times 2^n+1),\\
E_2(H_2(n)) &= \{uv \in E_2(H_2(n)):\ d_u=2\ and\, d_v=3\}|\ E_2(H_2(n))| = 4(3\times 2^n+4),\\
E_3(H_2(n)) &= \{uv \in E_3(H_2(n)):\ d_u=3\ and\, d_v=3\}|\ E_3(H_2(n))| = 12,\\
E_4(H_2(n)) &= \{uv \in E_4(H_2(n)):\ d_u=2\ and\, d_v=1\}|\ E_4(H_2(n))| = 2^{n+1},\\
E_5(H_2(n)) &= \{uv \in E_5(H_2(n)):\ d_u=3\ and\, d_v=1\}|\ E_5(H_2(n))| = 4(2^n-1).
\end{aligned}$$

Theorem 4.1.

$$M(H_2(n,x,y)) = 4(3\times 2^n+1)x^2y^2+4(3\times 2^n+4)x^2y^3+12x^3y^3+2^{n+1}x^2y+4(2^n-1)x^3y^3.$$

Proof. It is easily obtained by the definition of (1.1) and the structure of $H_2(n)$. □

Theorem 4.2. *Suppose $H_2(n)$, is the Graph structure of Pamam dendrimer. New Revan indices are calculated as follows:*

1. $PR(H_2(n)) = \frac{\sqrt{6}}{6}2^{n+1}+(\frac{4\sqrt{3}}{3}+6\sqrt{2}+6)2^n+(8\sqrt{2}+2-\frac{4\sqrt{3}}{3})$,
2. $Fr(H_2(n)) = 13(2^{n+1})+196(2^n)+96$,
3. $SDR(H_2(n)) = \frac{13(2^{n+1})}{6}+\frac{202(2^n)}{3}+\frac{176}{3}$,
4. $HR(H_2(n)) = \frac{2^{n+2}}{5}+(16)2^n+\frac{68}{3}$,
5. $IR(H_2(n)) = \frac{12(2^{n+1})}{5}+46(2^n)+\frac{34}{3}$.

Proof. By applying the operators of Table (1) to Theorem (4.1), we have:

1. $[D_x^{-\frac{1}{2}} D_y^{-\frac{1}{2}}(Q_{x(-5)}Q_{y(-5)}M(H_2(n),x,y))]\Big|_{(x,y)=(1,1)} = \frac{\sqrt{6}}{6}2^{n+1}+(\frac{4\sqrt{3}}{3}+6\sqrt{2}+6)2^n$
$+(8\sqrt{2}+2-\frac{4\sqrt{3}}{3})$,
2. $[(D_x^2+D_y^2)(Q_{x(-5)}Q_{y(-5)}M(H_2(n),x,y))]\big|_{(x,y)=(1,1)} = 13(2^{n+1})+196(2^n)+96$,
3. $(D_xS_y+S_yD_x)(Q_{x(-5)}Q_{y(-5)}M(H_2(n),x,y))\big|_{(x,y)=(1,1)} = \frac{13(2^{n+1})}{6}+\frac{202(2^n)}{3}+\frac{176}{3}$,
4. $-2S_X\,J(Q_{x(-5)}Q_{y(-5)}M(H_2(n),x,y))\big|_{(x=1)} = \frac{2^{n+2}}{5}+(16)2^n+\frac{68}{3}$,
5. $[-S_X\,JD_xD_y(Q_{x(-5)}Q_{y(-5)}M(H_2(n),x,y))]\,\big|_{(x=1)} = \frac{6(2^{n+1})}{5}+23(2^n)+\frac{53}{3}$.

□

5 Triazine- based dendrimer ($H_3(n)$)

Triazine dendrimers have been investigated by scientists for a variety of pharmaceutical applications, including DNA delivery systems, especially in anticancer drugs.
The molecular Graph of the triazine-based dendrimer with production step n for $n=1$ and $n=2$ is shown in (1)-(c). suppose $V(H_3(n))$ has two representive. Say $Z_1=\{\alpha_1,\alpha_2,\alpha_3,\alpha_4,\alpha_5\}$, $Z_2=\{a_j,b_j,c_j,d_j,e_j,f_j,g_j,h_j\}$ where $1\le j\le n$, The corresponding edges of sets are $E(Z_1)=\{\alpha_1\alpha_1,\alpha_1\alpha_2,\alpha_2\alpha_3,\alpha_3\alpha_4,\alpha_4\alpha_5,\alpha_5\alpha_1\}$ and $E(Z_2)=\{a_jb_j,b_jc_j,c_jd_j,c_je_j,e_jf_j,f_jg_j,g_jh_j,h_ja_{j+1}\}$ where $1\le j\le n$.The partition of edges of set Z_1 and Z_2 for Triazine-based dendrimer with respect to the representative end vertices and its frequency is presented in Table (2).

Theorem 5.1. *Let $H_3(n)$, is the Graph structure of Triazine- based dendrimer, then, the M-Polynomial is:*

$$M(H_3(n,x,y)) = (1+\frac{4}{3}(4^n-1))x^3y^3+(8+4(\frac{4}{3}(4^n-1)+\frac{2}{3}(4^n-4))x^3y^2$$
$$+(4+\frac{2}{3}(4^n-4))x^2y^2+(4+\frac{4}{3}(4^n-1))x^2y^3+2^{2n+1}x^2y.$$

Representative pairs edges	Frequency	Representative pairs edges	Frequency
(α_1, α_1)	1	(c_j, d_j)	2^{2j}
(α_1, α_2)	4	(c_j, e_j)	2^{2j}
(α_2, α_3)	4	(e_j, f_j)	2^{2j+1}
(α_3, α_4)	4	(f_j, g_j)	2^{2j+1}
(α_4, α_5)	2	(g_j, h_j)	2^{2n+1}
(α_5, α_1)	2	(g_j, h_j) when $j \neq n$	2^{2j+1}
(a_j, b_j)	2^{2j}	(h_j, a_{j+1}) when $j \neq n$	2^{2j+1}
(a_j, b_j)	2^{2j}		

Table 2. The edges partition for Triazine- based dendrimer based on set edges Z_1 and Z_2 concerning, for, to the representative end vertices and its frequency [10]

Proof. It is easily obtained by the definition of (1.1) and the structure of $H_3(n)$. □

Theorem 5.2. *Suppose $H_3(n)$ for $n = 1$ and $n = 2$, is the Graph structure of Triazine- based dendrimer. New Revan indices are calculated as follows:*

1. $PR(H_3(n)) = \frac{\sqrt{2}}{6}(8 + 4^{n+1}) \times +\frac{1}{3}(-1 + 4^{n+1}) + \frac{\sqrt{2}}{6}(2(4^n) + 4^{n+2})$
$+\frac{1}{6}(2(4^n - 8)) + \frac{\sqrt{6}}{6}2^{2n+1}$,
2. $Fr(H_3(n)) = \frac{5}{3}(4^{n+2}) + \frac{7}{3}(4^{n+1}) + \frac{26}{3}(4^n) + 13(2^{2n+1}) - 122$,
3. $SDR(H_3(n)) = \frac{3}{2}(4^{n+1}) + \frac{5}{6}(4^{n+2}) + 3(4^n) + \frac{13}{12}(2^{2n+1}) + \frac{2}{3}$,
4. $HR(H_3(n)) = 2^{2n}(\frac{167}{80}) + 2^n(\frac{4}{3}) + \frac{57}{5}$,
5. $IR(H_3(n)) = \frac{2}{9}(8 + 4^{n+1}) + \frac{1}{6}(-1 + 4^{n+1}) + \frac{2}{9}(2(4^n) + 4^{n+2})$
$+\frac{4}{3}(2(4^n - 8)) + \frac{6}{5}(2^{2n+1})$.

Proof. By applying the operators of Table (1) to Theorem (5.2), we have:

1. $[D_x^{-\frac{1}{2}} D_y^{-\frac{1}{2}} Q_{x(-5)} Q_{y(-5)} M(H_3(n), x, y)]\Big|_{(x,y)=(1,1)} = \frac{1}{3}(8 + 4^{n+1}) \times \frac{1}{\sqrt{2}}$
$+\frac{1}{3}(-1 + 4^{n+1}) \times \frac{1}{\sqrt{1}} + \frac{1}{3}(2(4^n) + 4^{n+2}) \times \frac{1}{\sqrt{2}} + \frac{1}{3}(2(4^n - 8)) \times \frac{1}{\sqrt{4}} + 2^{2n+1} \times \frac{1}{\sqrt{6}}$,
2. $[(D_x^2 + D_y^2)Q_{x(-5)} Q_{y(-5)} M(H_3(n), x, y)]\big|_{(x,y)=(1,1)} = \frac{5}{3}(4^{n+2}) + \frac{7}{3}(4^{n+1}) + \frac{26}{3}(4^n)$
$+13(2^{2n+1}) - 122$,
3. $[(D_x S_y + S_x D_y)Q_{x(-5)} Q_{y(-5)} M(H_3(n), x, y)]\big|_{(x,y)=(1,1)} = \frac{3}{2}(4^{n+1}) + \frac{5}{6}(4^{n+2})$
$+3(4^n) + \frac{13}{12}(2^{2n+1}) + \frac{2}{3}$,
4. $[-2\, S_x\, J(Q_{x(-5)} Q_{y(-5)} M(H_3(n), x, y))]\Big|_{(x=1)} = 2^{2n}(\frac{167}{80}) + 2^n(\frac{4}{3}) + \frac{57}{5}$,
5. $[-S_x J D_x D_y Q_{x(-5)} Q_{y(-5)} M(H_3(n), x, y)]\big|_{x=1} = \frac{2}{9}(8 + 4^{n+1}) + \frac{1}{6}(-1 + 4^{n+1})$
$+\frac{2}{9}(2(4^n) + 4^{n+2}) + \frac{4}{3}(2(4^n - 8)) + \frac{6}{5}(2^{2n+1})$.

□

6 Phosphorus-containing dendrimers ($H_4(n)$ and $H_5(n)$)

Phosphorus-containing dendrimers are a class of dendrimers known for their unique properties, including the ability to form bifunctional structures. These dendrimers have found use as recyclable and potent catalysts. Suppose $H_4(n)$ and $H_5(n)$, are the molecular Graphs of Phosphorus dendrimers with the generation stage n. Figure (1)-(d) and -(e) respectively depicts the structure of Phosphorus-containing dendrimer $H_4(n)$ and $H_5(n)$ for $n = 2$. From a Graph structure of $H_4(n)$ the edges partition are as follows [10]

$$E_1(H_4(n)) = \{uv \in E_1(H_4(n)) : d_u = 1 \; and \, d_v = 3\} |\, E_1(H_4(n))| = 3(2^n - 1),$$
$$E_2(H_4(n)) = \{uv \in E_2(H_4(n)) : d_u = 2 \; and \, d_v = 4\} |\, E_2(H_4(n))| = 3(2^n - 1),$$
$$E_3(H_4(n)) = \{uv \in E_3(H_4(n)) : d_u = 2 \; and \, d_v = 2\} |\, E_3(H_4(n))| = 9(2^n - 1),$$
$$E_4(H_4(n)) = \{uv \in E_4(H_4(n)) : d_u = 2 \; and \, d_v = 3\} |\, E_4(H_4(n))| = 12(2^n - 1),$$
$$E_5(H_4(n)) = \{uv \in E_5(H_4(n)) : d_u = 3 \; and \, d_v = 4\} |\, E_5(H_4(n))| = 3(2^n - 1),$$
$$E_6(H_4(n)) = \{uv \in E_6(H_4(n)) : d_u = 1 \; and \, d_v = 4\} |\, E_5(H_4(n))| = 2(3 \times 2^n - 1).$$

From a Graph structure of $H_5(n)$ the edges partition are as follows [10]

$$E_1(H_5(n)) = \{uv \in E_1(H_4(n)) : d_u = 1 \; and \, d_v = 4\} |\, E_1(H_5(n))| = 6(2^{n+1} - 1),$$
$$E_2(H_5(n)) = \{uv \in E_2(H_4(n)) : d_u = 1 \; and \, d_v = 3\} |\, E_2(H_5(n))| = 6(2^n - 1),$$
$$E_3(H_5(n)) = \{uv \in E_3(H_4(n)) : d_u = 3 \; and \, d_v = 4\} |\, E_3(H_5(n))| = 6(2^n - 1),$$
$$E_4(H_5(n)) = \{uv \in E_4(H_4(n)) : d_u = 2 \; and \, d_v = 3\} |\, E_4(H_5(n))| = 42(2^n - 1),$$
$$E_5(H_5(n)) = \{uv \in E_5(H_4(n)) : d_u = 2 \; and \, d_v = 2\} |\, E_5(H_5(n))| = 18(2^n - 1),$$
$$E_6(H_5(n)) = \{uv \in E_6(H_4(n)) : d_u = 2 \; and \, d_v = 4\} |\, E_5(H_5(n))| = 12.$$

Theorem 6.1. *Let $H_4(n)$ and $H_5(n)$ are the molecular Graph structures of Phosphorus-containing dendrimers, then, the M-Polynomial are:*

$$i)\ M(H_4(n,x,y)) = 3(2^n-1)xy^3 + 3(2^n-1)x^2y^4 + 9(2^n-1)x^2y^2$$
$$+12(2^n-1)x^2y^3 + 3(2^n-1)x^3y^4 + 2(3\times 2^n-1)xy^4,$$
$$ii)\ M(H_5(n,x,y)) = 6(2^{n+1}-1)xy^4 + 6(2^n-1)xy^3 + 6(2^n-1)x^3y^4$$
$$+42(2^n-1)x^2y^3 + 18(2^n-1)x^2y^2 + 12x^2y^4.$$

Proof. It is easily obtained by the definition of (1.1) and the structure of $H_4(n)$ and $H_5(n)$ □

Theorem 6.2. *Let $H_4(n)$ and $H_5(n)$ are the molecular Graph structures of Phosphorus-containing dendrimers, New Revan indices are calculated as follows:*

1. $PR(H_4(n)) = (6+\frac{9\sqrt{2}}{4}+\sqrt{3}+2\sqrt{6})2^n + (-\frac{9\sqrt{2}}{4}-\sqrt{3}-2\sqrt{6}-4),$
2. $Fr(H_4(n),x,y)) = 525(2^n) - 457,$
3. $SDR(H_4(n)) = \frac{189}{2}(2^n) - (\frac{155}{2}),$
4. $HR(H_4(n)) = \frac{147}{10}(2^n) - \frac{131}{10},$
5. $IR(H_4(n)) = \frac{819}{20}(2^n) - \frac{851}{20},$
6. $PR(H_5(n)) = 3(2^{n+1}) + (\frac{9}{2}\sqrt{2}+7\sqrt{6}+9)2^n + (-12-\frac{9}{2}\sqrt{2}-7\sqrt{6}-4\sqrt{3}),$
7. $Fr(H_5(n),x,y) = 102(2^{n+1}) + 1020(2^n-1) + 120,$
8. $SDR(H_5(n)) = \frac{51}{2}(2^{n+1}) + (137)(2^n) - \frac{215}{2},$
9. $HR(H_5(n)) = \frac{12}{5}(2^{n+1}) + (\frac{144}{5})(2^n) - \frac{186}{5},$
10. $IR(H_5(n)) = \frac{24}{5}(2^{n+1}) + \frac{447}{5}(2^n) - \frac{516}{5}.$

Proof. By applying the operators of Table (1) to Theorem (6.1), we have:

$$1.\ [D_x^{-\frac{1}{2}}D_y^{-\frac{1}{2}}(Q_{x(-5)}Q_{y(-5)}M(H_4(n),x,y))]\Big|_{(x,y)=(1,1)} = (6+\frac{9\sqrt{2}}{4}+\sqrt{3}+2\sqrt{6})2^n$$
$$+(-\frac{9\sqrt{2}}{4}-\sqrt{3}-2\sqrt{6}-4),$$
$$2.\ [(D_x^2+D_y^2)(Q_{x(-5)}Q_{y(-5)}M(H_4(n),x,y))]\big|_{(x,y)=(1,1)} = 525(2^n) - 472,$$
$$3.\ [(D_xS_y+S_xD_y)(Q_{x(-5)}Q_{y(-5)}M(H_4(n),x,y))]\big|_{(x,y)=(1,1)} = \frac{189}{2}(2^n) - (\frac{155}{2}),$$
$$4.\ [-2\,S_x\,J(Q_{x(-5)}Q_{y(-5)}M(H_4(n),x,y))]\big|_{(x=1)} = \frac{147}{10}(2^n) - \frac{131}{10},$$
$$5.\ [-S_xJ\,D_xD_y(Q_{x(-5)}Q_{y(-5)}M(H_4(n),x,y))]\big|_{(x=1)} = \frac{819}{20}(2^n) - \frac{851}{20},$$
$$6.\ [D_x^{-\frac{1}{2}}D_y^{-\frac{1}{2}}(Q_{x(-5)}Q_{y(-5)}M(H_5(n),x,y))]\Big|_{(x,y)=(1,1)} = 3(2^{n+1})$$
$$+(\frac{9}{2}\sqrt{2}+7\sqrt{6}+9+4\sqrt{3})2^n + (-12-\frac{9}{2}\sqrt{2}-7\sqrt{6}),$$
$$7.\ [(D_x^2+D_y^2)Q_{x(-5)}Q_{y(-5)}M(H_5(n),x,y)]\big|_{(x,y)=(1,1)} = 102(2^{n+1}) + 1020(2^n-1) + 120,$$
$$8.\ [(D_xS_y+S_xD_y)(Q_{x(-5)}Q_{y(-5)}M(H_5(n),x,y))]\big|_{(x,y)=(1,1)} = \frac{51}{2}(2^{n+1})$$
$$+(137)(2^n) - \frac{215}{2},$$
$$9.\ [-2S_x\,J(Q_{x(-5)}Q_{y(-5)}M(H_5(n),x,y))]\big|_{(x=1)} = \frac{12}{5}(2^{n+1}) + (\frac{144}{5})(2^n) - \frac{186}{5},$$
$$10.\ [-S_x\,J\,D_xD_y(Q_{x(-5)}Q_{y(-5)}M(H_5(n),x,y))]\big|_{(x=1)} = \frac{24}{5}(2^{n+1}) + \frac{447}{5}(2^n) - \frac{516}{5}.$$

7 Plotting and comparing M-polynomial for dendrimers

The graphical representation of the M-polynomial of $H_1(n)$,...,$H_5(n)$ is given by Figure (2). According to Figure (2), it is clear that for positive values of x,y, the M-polynomial is always positive, therefore, Table (1) is valid for calculating Rivan's indices.

8 MATLAB coding for topological indices

In this section, we provide MATLAB coding for the calculation of topological indices using M-polynomial and the relationships proved in Section (2). The F-Revan index from M-polynomial coding is computed as,

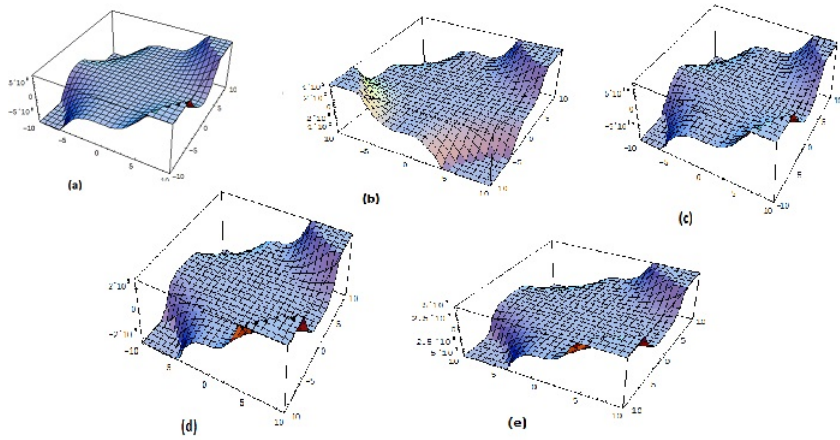

Figure 2. Molecular Graph of denderimers (a)$H_1(n)$M-polynomial, (b)$H_2(n)$M-polynomial, (c)$H_3(n)$M-polynomial, (d)$H_4(n)$M-polynomial, (e)$H_5(n)$M-polynomial,

```
clear
clc
n = input('numberofsentence :');
 A = zeros (n, 3) ;
  for i = 1 : n
  for j = 1 : 3
     A(i, j) = input('data');
       end
  end
  syms x y
  mGxy = 0;
  for i = 1 : n
           mGxy = mGxy + A(i, 1) * x^A(i, 2) * y^A(i, 3);
    end
    mGxy
       Delta = input('enter maximum degree of graph :');
    dlta = input('enter minimum degree of graph :');
    f = −Delta − dlta;
    B = zeros (n, 3) ;
    for i = 1 : n
        for j = 1 : 3
          if j == 1
         B (i, j) = A (i, j) ;
          else
                    B (i, j) = A (i, j) + f;
            end
          end
     end
     B;
     FA = 0;
     for i = 1 : n
     FA = FA + B(i, 1) * x^B(i, 2) * y^B(i, 3);
        end
     FADX = diff(FA, x) * x
     DX2 = diff(DX, x) * x
     DY = diff(FA, y) * y
     DY2 = diff(DY, y) * y
     DXDY = 0;
     for i = 1 : n
     DXDY = DXDY + B(i, 1) * B(i, 2) * B(i, 3) * x^B(i, 2) * y^B(i, 3);
     end
     DXDY
     HR1 = [(DX2 + DY2)]
     syms x y
     subs(HR1, [x, y], [1, 1])
```

9 Conclusion remarks

This article presents new Revan indices obtained through the use of M-polynomial calculations. Additionally, we apply M-polynomial and new Revan indices to five classes of dendrimers and present graphical representations of the results in Figures (2)(a)-(e). Our findings highlight the potential of these indices in predicting reactivity and other properties of dendrimers, which could have significant implications for future research in this field. In the future, one can also work on other Revan index from M-polynomial coding.

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