Palestine Journal of Mathematics

Vol 13(4)(2024) , 250–260

EXPLORING GEOMETRIC CONSTRUCTION: THE ROLE OF CUBE DUPLICATION

'A.I. Smolina

Communicated by Ayman Badawi

MSC 2010 Classifications: Primary 51M04; Secondary 68U05.

Keywords and phrases: geometric objects, solution, analysis, construction, methods.

The authors would like to thank the reviewers and editor for their constructive comments and valuable suggestions that improved the quality of our paper.

Abstract This research delves into the field of cube duplication, a significant area in modern science and engineering, focusing on enhancing algorithmic precision and efficiency. The study aims to dissect construction issues using double cubes, utilizing theorems and properties of geometric shapes. It emphasizes theoretical exploration to uncover new geometric properties and construction techniques, impacting practical realms like complex structural designs and computer graphics. The practical application of these findings is expected to revolutionize processes across various industries, aiding in complex scientific problem-solving and fostering advancements in multiple scientific disciplines.

1 Introduction

The relevance of the research lies in the need to develop new methods or improve existing ones for building doubled cubes. Solving the problem of constructing doubled cubes is an important and challenging task in mathematics and geometry. Research in the field of building doubled cubes can lead to the development of new algorithms and methods that will allow finding more efficient and accurate solutions. Such improvements can simplify the construction process and make it more accessible to researchers and engineers. Improvements in the methods of constructing doubled cubes may also influence the development of other branches of mathematics and provide new means for solving geometric problems [1, 2, 3]. In addition, the construction of doubled cubes has potential applications in scientific and engineering fields. For example, this could have important implications in number theory, cryptography, computer science, and physics. Solutions to the problem of building doubled cubes can become the basis for creating new algorithms and systems that will ensure the development of modern technologies and scientific research.

The main problem of the research is the complexity of solving cubic equations that arise when building doubled cubes. To construct doubled cubes, you need to solve a cubic equation with specific values of the coefficients, which complicates the process of finding the roots. In addition, depending on the specific construction task, it may be necessary to find complex or extractive roots of cubic equations. This also complicates the task and may require additional mathematical analysis to correctly interpret the results.

In their scientific work, S. Draganyuk and his colleagues claimed that the theory of "constructions" in Euclidean stereometry is fundamentally no different from the planimetric theories of "constructions with the help of various tools", first of all, with the help of a "compass and ruler" [4]. From a theoretical point of view, just as in Euclidean planimetry, in Euclidean stereometry we are talking about creating a corresponding canonical continuation of the chosen axiomatics of the three-dimensional Euclidean space. According to I. Lenchuk, constructive geometry is one of the branches of mathematics that deals with the development and study of methods of constructing geometric figures and objects using a ruler and a compass. It forms the basis for the development of students' geometric knowledge and skills [5]. J. Niemeyer and R. von Randow note that due to the limitations of the compass and the ruler, it turned out that the exact construction of the double cube is impossible [6].

In his work D.S. Richson covers such mathematical problems as doubling the cube, squaring the circle, triangle with given sides, and other famous ancient problems [7]. In his scientific work, Y. Barbanel considered issues related to the doubling of the cube and the construction of expressions in higher dimensions [8]. The author investigated various aspects of the double cube problem and presented various approaches to its solution. According to S.A. Morris et al., ruler and compass constructions is a classic geometric topic that deals with the use of simple geometric tools to construct various shapes and solve mathematical problems [9].

The purpose of this research is aimed at the analysis of problems on the construction of doubled cubes. Conducting an analysis of existing methods of solving problems for the construction of doubled cubes allows us to find out their advantages, limitations and opportunities. This will help to understand the strengths of previous studies and identify potential areas for improvement and optimization of methods. It should be noted that the successful research of problems on the construction of doubled cubes can have a significant impact on various aspects of scientific and technical progress.

2 Materials and methods

Geometry, trigonometry, number theory and other branches of mathematics play an important role in solving construction problems and allow finding elegant solutions to complex problems. These mathematical disciplines help to understand and analyze the geometric properties of objects, revealing their features and relationships. The geometric approach is the basis for solving many problems for the construction of doubled cubes. The use of geometric transformations and rules helps to find the appropriate relationships between the different parts of the cube and their sizes.

Doubled cubes were used in solving the problem of constructing a regular triangle using a ruler and a compass and finding a segment with a length $\sqrt[3]{2}$, corresponding to the ratio of the side of the doubled cube to the side of the original cube. One of the ways to solve such problems is to use the cubic equation. A compass and a ruler were used to construct the problems.

In the study of complex properties of doubled cubes, various theoretical methods of mathematics are used, such as number theory, combinatorics, function analysis, and others. Number theory made it possible to study the properties of numbers and establish connections between them, which helps to understand the structure of cubes and their properties. Combinatorics made it possible to analyze various possible combinations and permutations of elements of cubes, which are used in solving geometric problems. Functional analysis helped reveal the mathematical relationships between the various parts of the cubes, revealing their equations and geometric properties. The application of mathematical methods in the study of doubled cubes and other geometric problems made it possible to reveal their internal structure and find new mathematical regularities. The use of mathematical analysis and logical thinking helped to understand complex mathematical objects and find effective solutions to problems that may seem intractable.

Research in mathematical relationships and properties of doubled cubes can include the study of additional geometric shapes, finding auxiliary points and lines that simplify the solution of complex geometric problems. Understanding number theory helps to find numerical answers to geometric questions, which allows you to make more accurate and reasonable calculations. In addition, the use of auxiliary points and lines is an important element of constructions in geometric problems. They help reveal additional connections between elements and simplify the construction task, making the solution process more efficient and elegant. Such approaches make geometric construction problems interesting and exciting for researchers and scientists who are constantly studying new and interesting aspects of mathematics.

The theorems of sines and cosines are the main trigonometric tools for solving geometric problems and measuring unknown quantities in triangles. These theorems were used to find the length of the side and measure the angles of triangles, which helps to make accurate and precise constructions.

Also, the properties of regular triangles and polygons were used in the study of construction problems, which helped to more easily understand the characteristics of triangles and polygons, as well as to use these properties in various geometric problems. Due to their symmetrical properties, regular triangles have numerous advantages in solving geometric problems. For example, their sides and angles can be more easily measured and compared to other shapes, making calculations and constructions easier. Regular polygons have many interesting and unique properties that are often used in science, engineering, and art.

For a more detailed analysis, construction tasks using the doubling of cubes were considered. The analysis of these problems made it possible to find out different approaches and methods that were used in solving similar geometric problems. It is worth paying attention to the used geometric shapes, triangles, rectangles, and their relationships, as well as the properties of space and lines.

3 Results

For a better understanding of solving construction problems, one should study the theoretical aspects of using doubled cubes, as well as study and analyze the methods by which these problems are solved. The study of theoretical aspects and the analysis of methods for solving problems for the construction of doubled cubes is a step towards expanding knowledge in the field of geometry and mathematics, which, in turn, can have the potential for finding new mathematical algorithms and solving complex problems in various scientific and engineering disciplines. It should be noted that solving construction problems using doubled cubes is not limited to geometry and mathematics.

It should be noted that in the study of construction problems using the doubling of the cube, it is important to study theoretical aspects, such as the properties of regular triangles and polygons, as well as various trigonometric tools that help to refute the finding of side lengths and angle values. When studying these theoretical aspects, attention should also be paid to the use of geometric tools and methods for constructing and solving geometric problems, which helps to develop an intuitive understanding of geometric objects and their relationships. Knowledge of regular triangles and regular polygons made it possible to understand their geometric characteristics and relationships between sides and angles. Trigonometry has played an important role in the study of construction problems because it allows you to find the side lengths of triangles and the values of angles using trigonometric functions such as sines, cosines, and tangents. The use of trigonometric tools made it possible to solve geometric problems, in particular those related to doubling the cube. The first stage in the research was the analysis of theoretical aspects used in solving construction problems.

When solving geometric constructions that use doubling the cube, auxiliary points and lines were used to help simplify the task. They revealed additional connections between elements and solved constructions in a more efficient and elegant way, which allowed the use of deep mathematical connections and logical calculations to achieve more accurate and reasoned results.

In the study of complex properties of doubled cubes and their application in geometric problems, various theoretical methods of mathematics were used, which helped to reveal their internal structure and find effective solutions. One of the key approaches in the study of doubled cubes is number theory, which made it possible to investigate the properties of numbers and establish relationships between them. Number theory helped find numerical answers to geometric questions, such as determining the lengths of sides or the values of angles in complex structures. It made it possible to find numerical solutions of the equations that arose in the process of working with the doubling of the cube, which made it possible to check the correctness of the obtained results and make accurate calculations.

Combinatorics is another important tool in the study of doubled cubes. It made it possible to analyze various possible combinations and permutations of elements of cubes, which are used in solving geometric problems. This helped to understand which combinations of elements can lead to the solution of certain geometric problems, and to find optimal solutions. Function analysis has played an important role in studying the properties of doubled cubes, helping to reveal the mathematical relationships between the various parts of the cubes. Finding functions that describe various elements of cubes made it possible to understand their geometric properties and solve geometric constructions using analytical methods. The application of mathematical methods in the study of doubled cubes and other geometric problems made it possible to understand the structure of cubes, find new mathematical regularities and solve complex mathematical objects. The use of mathematical analysis and logical thinking helped to reveal additional connections between elements and solve constructions in a more efficient and elegant way, which makes it possible to solve geometric problems more accurately and reasonably.

After conducting the initial research, the next step was to study the construction problems using the doubling cube method for a more detailed analysis. The analysis of these problems gave researchers the opportunity to identify various approaches and methods that were used to solve similar geometric problems. Special attention was paid to geometric shapes, such as triangles, rectangles, and their relationships, as well as the properties of space and lines.

Task 1. Construction of a regular triangle 1. Construction algorithm:

(i) Construct a regular triangle with side 1.

(ii) Place point D on the line AB, hence AD=AB-1.

(iii) Run a direct CD.

(iv) 4) Construct the segment $KL = 1$ so that $L \in AC$; $K \in BL$; $K \in CD$.

(v) BK=$\sqrt[3]{2}$.

Proof:
$\triangle BLC$: $\angle BCK = 90° = 180° - (60° + 30°) = 90°$.
Let $BK = a$, $CK = b$, $\angle BCK = \alpha$.
Then $\cos\alpha = \frac{b}{a} \Rightarrow b = a\cos\alpha$; $BK^2 = CK^2 + CB^2$, $a^2 = b^2 + 1$, $\sin\alpha = \frac{1}{a}$.
$\triangle CKL$: $\angle KLC = 30° = \angle ACD$ (as vertical angles), $\angle CKL = 180° - \alpha$,
$\angle KLC = 180° - (\angle CKL + \angle KLC) = 180° - 30° - 180° + \alpha = \alpha - 30°$.
According to the theorem of sines $\triangle CKL$:

$$\frac{CK}{\sin(\alpha - 30°)} = \frac{KL}{\sin 30°} \implies \frac{b}{\sin(\alpha - 30°)} = \frac{1}{\sin 30°} \implies b = 2\sin(\alpha - 30°).$$

$$b = 2\sin\alpha\cos 30° - 2\sin 30°\cos\alpha$$
$$b = \sqrt{3}\sin\alpha - \cos\alpha$$
$$a\cos\alpha = \sqrt{3}\sin\alpha - \cos\alpha$$
$$(a+1)\cos\alpha = \sqrt{3}\sin\alpha$$
$$(a+1)^2\cos^2\alpha = 3\sin^2\alpha$$
$$(a+1)^2\left(1 - \frac{1}{a^2}\right) = \frac{3}{a^2}$$
$$(a^2 + 2a + 1)(a^2 - 1) = 3$$
$$a^4 - a^2 + 2a^3 - 2a + a^2 - 1 - 3 = a$$
$$a^4 + 2a^3 - 2a - 4 = 0$$
$$a^3 a + 2 - 2a + 2 = 0$$
$$(a+2)(a^3 - 2) = 0$$

It has been proven

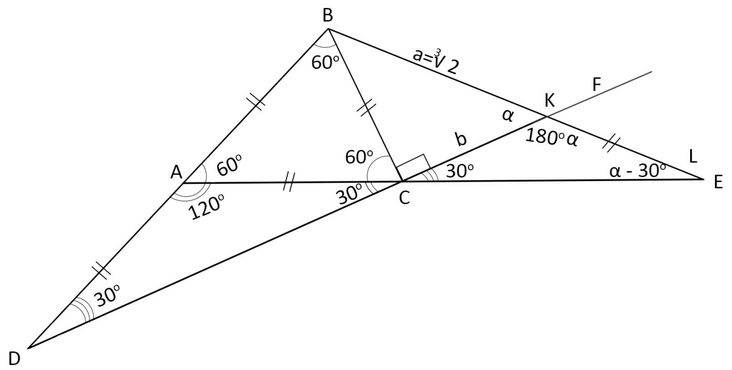

Figure 1. Construction of a regular triangle

Source: created by the author.

Analysis of the problem made it possible to understand the algorithm for constructing a correct triangle and to prove the correctness of the obtained result. The answer to the problem is obtained by successive application of various trigonometric properties and geometric rules. First, trigonometry and geometry were used to find the value of BK, using cosine and the Pythagorean theorem for right triangles BLC and BCK. Next, find the value of b, CK, and $\sin\alpha$ for triangle CKL, using the theorem of sines and the geometry of vertical angles. Then, using trigonometric identities, it is proved that a=$\sqrt[3]{2}$. Also, the analysis made it possible to understand which trigonometric properties and geometric regularities are used in the algorithm and how they interact with each other to obtain a solution. He helped to confirm the correctness of the algorithm and the correctness of the obtained mathematical equations. The analysis of the problem also showed that the solution is based on trigonometry and geometry, which allows you to understand the geometric nature of constructing a right triangle with side 1 and BK=$\sqrt[3]{2}$. The use of trigonometry and geometry helps to make calculations accurate and reasonable. Thus, the analysis of the problem makes it possible to increase the understanding of mathematical methods and the construction of geometric objects.

The analysis of the solution of the problem helped to better understand the algorithm used to construct a correct triangle, and also made it possible to understand the geometric properties of this triangle, namely to establish the relationships between the angles and sides of the triangles and to determine the value of the required side. Also, the analysis of the considered problem made it possible to improve skills in using trigonometric functions to express the relationships between the sides and angles of triangles. The analysis of the used geometry and trigonometry made it possible to find additional geometric shapes and relationships between the sides and angles of the triangle. In summary, the analysis of the solution of this problem helps to reveal the mathematical foundations of the construction of a right triangle with side 1. Such analytical approaches and geometric relationships can be useful in solving similar construction problems, as well as expanding the understanding and application of geometric and trigonometric concepts.

Finding auxiliary points and lines in geometric constructions is one of the key elements that helped simplify geometric constructions and ensured their effective solution. This made it possible to transfer the problem of building doubled cubes to a similar problem using simpler geometric elements, which helped to reduce the complexity of the solution. Understanding number theory played an important role in discovering numerical answers to geometric questions. It made it possible to connect the algebraic and geometric aspects of the problem, which made it possible to find exact numerical values and justify the solution.

Task 2. Construction of a regular polygon 2. Construction algorithm:

(i) Construct two mutually non-perpendicular lines AC and BD, so that OC=1, OD=2.

(ii) Merge CD.

(iii) With the help of two lines with a right angle, we build $\angle CBA = 90°$, $\angle DAB = 90°$ so that they lie on the diagonals AC and BD. $BC \parallel DA$.

(iv) $BO = \sqrt[3]{2}$

Proof:
ABCD is a trapezoid.

$$\angle A = 90°, \angle B = 90°$$

AD||BC is the base,BD⊥AC,O is the point of intersection of BD and AC.CO=1,DO=2.
Let BO=x,AO=y.
Let's consider △BOC is rectangular.

$$\tan\alpha = \frac{BO}{OC} = \frac{x}{1} = 1$$

△AOB – rectangular. $\angle ABO = 90° - \angle OBA = 90° - (90° - \alpha) = \alpha$

$$\tan\alpha = \frac{AO}{BO} = \frac{y}{x}$$

△AOD: $\angle OAD = \angle OCB = \alpha$,as internal versatile.

$$\tan\alpha = \frac{OD}{OA} = \frac{2}{y}$$

$$\tan\alpha = x/1 = y/x = 2/y$$

$$\begin{cases} x^2 = y \\ y^2 = 2x \end{cases} \Rightarrow \begin{cases} x^2 \cdot y^2 = 2x \\ x^4 - 2x = 0 \end{cases}$$

$$x(x^2 - 2) = 0 \; x = 0 \text{ or } x^3 - 2 = 0 \; x = \sqrt[3]{2}$$

It is proved that if CO=a,DO=2a then BO=$\sqrt[3]{2}$ a; if CO=a,DO=2a then BO=$\sqrt[3]{n}$ a.
In this way, you can construct the cube root of any number.

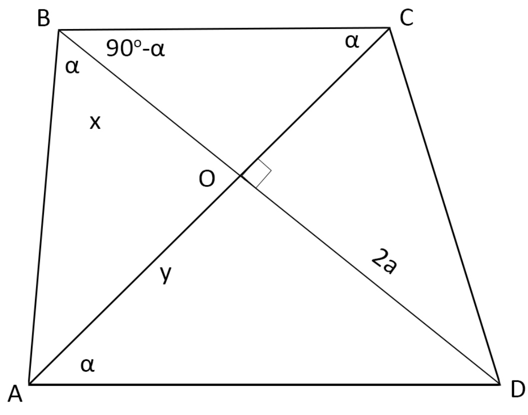

Figure 2. Construction of a regular polygon

Source: created by the author.

Analysis of the problem made it possible to obtain a solution and proof for the construction of two mutually non-perpendicular lines AC and BD, such as OC=1 and OD=2. Using the properties of trapezoids and right triangles, the authors of the algorithm find the values of BO(x) and AO(y) using the tangents of the angles of the triangles. Further solving is simplified to the solution of the cubic equation $x^3 - 2 = 0$, from where the value BO=$\sqrt[3]{2}$ is found. An important aspect of the analysis is understanding the geometric properties of trapezoids, right triangles, and the use of tangents of angles to find the values of the sides of triangles. This analysis made it possible to confirm the correctness of the algorithm and prove the correctness of the obtained results. Upon further analysis of the algorithm, it can be understood that it is general in nature and can be used to construct the cube root of any number by changing the values of OC and

DO. Thus, the analysis made it possible to understand the limitations of the algorithm and establish its potential capabilities in solving geometric problems. The analysis also showed that the use of cube roots in geometric constructions can have wide applications and create potential for finding new mathematical algorithms and solving complex problems in various scientific and engineering disciplines.

The analysis of the solution of the problem provided an extended understanding of the algorithm for constructing two mutually non-perpendicular lines and the values of BO(x) and AO(y). The authors analyze the properties of trapezoids and right triangles, using the tangents of the angles to find the values of the sides of the triangles. This allows you to find out the relationship between various elements of a geometric design and establish mathematical equations for finding BO(x) and A (y). Next, the solution of the cubic equation $x^3 - 2 = 0$ is carried out, from where the value BO=$\sqrt[3]{2}$ is found. The analysis also indicated the general nature of the algorithm and its possibility of application for finding the cube root of any number by changing the values of OC and DO. This indicates the potential capabilities of the algorithm in solving various geometric problems. Thus, the analysis made it possible to establish the limitations and advantages of the algorithm, as well as to reveal its application to a wide range of tasks in various scientific and engineering disciplines. In particular, the use of cube roots in geometric constructions can create potential for finding new mathematical algorithms and solving complex problems. This emphasized the importance of studying geometric properties and trigonometry for solving various geometric problems. The analysis helped to confirm the correctness of the algorithm and prove the correctness of the obtained results, which makes it an important tool in the study of geometry and mathematics.

The study showed that doubling the cube is an important method for solving geometric problems. The property of a right triangle formed with side 1 and doubled sides allowed us to construct two mutually non-perpendicular straight lines lying on the diagonals of the trapezium. These properties can be applied in various geometric problems, such as the construction of parallel lines, perpendicular lines, and mutually non-perpendicular lines. It was also found that the study of number theory helps to find numerical answers to geometric questions, which allows for more accurate and reasonable calculations. The use of various theorems and properties of numbers helps to establish relationships between quantities and solve various geometric problems. During the research, it was noted that the algorithm for constructing a doubled cube is general in nature and can be used to construct the cube root of any number by changing the values of certain parameters. This indicates the discovery of new mathematical regularities, which may have significance in various fields of mathematics and engineering.

Thus, the doubling of a cube is an important mathematical concept that arises in the context of problems and research related to cubes and their properties. Doubling a cube, also known as doubling the volume of a cube, is a problem that has been studied since ancient times. It consists in finding the edge of a new cube, the volume of which is twice as large as the volume of the given cube. More precisely, the problem boils down to constructing a cube with a volume of 2V, given a cube with a volume of V. In mathematics, it has been proven that doubling the cube is impossible with the help of an ordinary ruler and compass. This was an important discovery in the history of mathematics and played a role in the development of algebra and geometry. The problem of doubling the cube is also one of those that led to the formulation of the concept of an unsolvable problem or the impossibility of constructing some geometric figures using only a compass and a ruler.

4 Discussion

The study of doubling cubes is important for various aspects of scientific research and engineering applications. The study of geometric constructions related to the doubling of the cube helps to reveal the features of geometric objects and to establish connections between various elements of figures. The study of number theory and methods of solving such equations plays an important role in solving various mathematical problems. Doubling the cube has practical applications in technical fields. For example, in computational geometry, doubling the cube can be used to solve geometric problems, such as constructing complex shapes and three-dimensional models. Doubling the cube is related to issues of volume and geometry of space. The study of this concept can be used in physical research and calculations, in particular in geometric models and volumetric problems of physical systems. Some cryptographic algorithms are based on number theory and geometry. Knowledge of cube doubling can help develop more efficient cryptographic techniques, which is important for protecting information and data.

The problem of doubling the cube is one of the three unsolved problems of construction using a ruler and a compass. The use of modern approaches and techniques, such as algebraic methods, the use of additional geometric shapes and transformations can provide approximation of impossible structures [10].

For example, doubling the cube was used to create a generator of pseudo-random numbers, which is the basis for generating keys for symmetric ciphers [11]. Number theory and other mathematical approaches can be used not only in scientific fields, but also in the art of origami [12, 13]. Cubic equations of state are used to describe the behavior of real substances, such as gases and liquids, based on specific experimental data such as temperature, pressure, and volume [14]. Cubic equations are also widely used in modeling various problems [15]. Cubic equations are also used in studies of rock mechanics and hydrogeology [16].

Geometric constructions are an important part of studying geometry, as they develop the ability to construct geometric objects, solve problems and prove geometric statements [17]. The construction of straight lines and circles can be constructed only with the help of a ruler and a compass, which is a significant breakthrough in modern geometry, especially for problems on doubling the cube, since in ancient times this was considered impossible [18].

In his book A.O. Larin describes the development of various fields of science and technology, in particular mathematics, in which the development and use of doubled cubes in solving problems was described in detail [19]. An analysis of this book will allow you to find out what basic principles and methods are used to build doubled cubes and how they can be used to solve various problems. It will also allow us to understand how important solving problems for the construction of doubled cubes is in scientific and engineering research. It can highlight the contribution of doubling cubes to solving complex problems and identify its potential for future scientific research.

In his work devoted to the fractal analysis of images in medicine and morphology, N.I. Marienko and O.Yu. Stepanenko conduct a comparative analysis of fractal analysis methods used for morphomerism in medical and biological research [20]. The similarity of this study is that both studies focus on the analysis and use of mathematical and computational methods to solve complex problems in science and medicine. In both cases, analytical tools are used, which help to reveal new connections and structures in the studied objects.

In a scientific work devoted to the study of minimal tools of geometry, M. Varhadpande described the theoretical and practical aspects of the study of the possibility of using a limited set of tools to implement certain geometric constructions [21]. Analysis of this work will help reveal how effective minimal tools are in geometry. The study of minimal tools of geometry can be of great importance for mathematics pedagogy and learning. Learning new approaches and construction methods can improve the quality of geometric education and the development of students' geometric thinking.

A.B. Araujo , in his scientific work on the method of constructing virtual realistic panoramas for virtual reality using geometric constructions and techniques of descriptive geometry, described the construction method and mathematical approaches that made it possible to create a cubic spherical perspective [22]. Although this study and the work of A.B. Araujo [22] have different topics and applications, but have some common features and similarities in aspects of these studies. One such aspect is the use of geometric methods to achieve a goal.

In his work devoted to construction tasks, E.H. Haug investigated the problem of constructing a square from a given circle and the problem of doubling a cube using geometric constructions in space-time [23]. The author presented theoretical reflections, mathematical approaches and constructions that allow solving the problems of squaring the circle and doubling the cube in space-time. The analysis of this work will allow us to assess and evaluate the author's contribution to the field of geometry, in particular in the context of solving classical geometric problems, such as the problem of squaring the circle and doubling the cube. Although the problems of constructing a square and doubling a cube in space-time may be theoretical, their possible practical application in some aspects of mathematics, physics and engineering can be an important tool for investigating or solving more complex problems. Studying the problems of constructing a square and doubling a cube in space-time allows one to expand the understanding of geometric concepts to more complex spaces. This can lead to the discovery of new possibilities and approaches in geometry.

Overall, the use of doubled cubes in construction tasks has significant potential for advancing science, technology, and engineering. In the field of geometry and mathematics, the use of doubled cubes helps to solve various geometric problems, leading to the creation of new algorithms for construction and proof. This allows you to expand your knowledge in the field of geometry and find effective and elegant solutions to complex geometric problems. Studies of doubling cubes have made important contributions to the development of mathematical sciences, enriching our understanding of geometric principles and means of proof. Research results can have a far-reaching impact on various fields, such as computer graphics, architecture, engineering and others, where accurate modeling of form and geometric objects is an important component.

Also, the study of doubling cubes opens up a wide range of possibilities for the further development of science and technology. Further research and improvement of construction methods can lead to new knowledge, provide practical application and contribute to the development of various spheres of human activity. The application of doubled cubes in geometry and mathematics helps to solve various geometric problems, creating new algorithms for construction and proof. This provides an expansion of knowledge in the field of geometry and allows you to find efficient and elegant solutions to complex geometric problems. Such studies make an important contribution to the development of mathematical sciences. In addition, the doubling of cubes is used in cryptography, where mathematical methods of encryption and decoding of information are used. Understanding doubled cubes allows for the development of more efficient and secure cryptographic methods, which is of great importance for the protection of confidential information in today's world. All these aspects indicate the importance of the study of doubled cubes in various scientific disciplines. This opens up a wide range of possibilities for the further development of science and technology, which can find its application in various fields of life, from mathematics and cryptography to physics and engineering. The study of doubled cubes is important to society, helping to solve complex problems and ensuring the development of science and technology in the future.

5 Conclusions

The study showed that the solution of construction problems using doubled cubes is of great importance for various scientific, technical and practical applications. In addition, solving construction problems using doubled cubes can increase the efficiency and accuracy of 3D data analysis, allowing you to discover complex dependencies and find new ways to process information. The main task is the analysis of solved problems and the possibilities of using the solutions of problems for the construction of doubling cubes in various fields of science and technology. For example, in geometry, you can explore the properties of various geometric shapes that can be constructed using doubled cubes, and in computer graphics, you can use these solutions to create more realistic 3D models and animations. The main problems of the research are the complexity of mathematical models and algorithms used to solve construction problems using doubled cubes. Cubic equations that arise in the process of construction have their own characteristics, in particular, the impossibility of analytical solution using elementary arithmetic operations. Since construction problems are an important component of many fields of science, technology, and engineering, research in this area is of great importance for solving real-world problems and improving technologies.

The development and application of new algorithms and methods for building doubled cubes, as well as the study and analysis of mathematical properties of doubled cubes, which will help to better understand their structure and the relationships between their parameters, remain a relevant area of research. This can lead to the discovery of new regularities and useful properties that can be applied in various fields of science and technology. In addition, an urgent task is to investigate the possibility of using the solutions of construction problems using doubled cubes in new areas of science and technology. These can be applications in robotics, manufacturing, architecture, aerospace engineering, and other industrial fields. Research in this area can be of practical importance, in particular, in improving methods of visualization, processing and analysis of three-dimensional data, creating more accurate and realistic models, increasing the security and reliability of cryptographic systems, etc.

References

[1] S. Janson, B. Seidler and D. Zeilberger, *n the Statistics of the Number of Fixed-Dimensional Subcubes in a Random Subset of the n-Dimensional Discrete Unit Cube*, Palest. J. Math., **12(3)** , 512–520, (2023).

[2] J.A. Younis, A. Hasanov and M.A.A. El Salam, *Certain decomposition fo rmulas for hypergeometric Gaussianfunctions in three variables*, Palest. J. Math., **12(1)**, 101–110,(2023).

[3] S. Jain, J. Younis, P. Agarwal and M.A.A. El Salam, *Certain new results for some horns hypergeometric functions in two variables*, Palest. J. Math., **12(1)**, 125–132,(2023).

[4] S.V. Draganyuk and O.M. Sinyukova, *"Constructions" in three-dimensional Euclidean space and the expedient nature of their coverage in educational courses of Euclidean stereometry in terms of practice-oriented learning*, Scientific Notes. Series: Pedagogical Sciences, **203**, 61–68,(2022).

[5] I.G. Lenchuk, *The very first operations of constructive geometry*, Mathematics in the Native School, **2**, 7–11,(2021).

[6] J. Niemeyer and R. von Randow, *Doubling the cube – Revisited*, in Proceedings of Bridges 2021: Mathematics, art, music, architecture, culture, Tessellations Publishing, Phoenix, 313–314, (2021).

[7] D.S. Richeso, *Tales of impossibility: The 2000-year quest to solve the mathematical problems of antiquity*, Princeton University Press, Princeton. (2019).

[8] J. Barbanel, *Doubling the cube and constructability in higher dimensions*, Math. Mag., **95(5)**, 465–481,(2022).

[9] S.A. Morris, A. Jones and K.R. Pearson, *Pearson, Straightedge and compass constructions*, in Abstract algebra and famous impossibilities: Squaring the circle, doubling the cube, trisecting an angle, and solving quintic equations, Springer, Cham, 65–88, (2022).

[10] F. Wredh, *Impossible constructions within Euclidean geometry*, Uppsala University, Uppsala. (2022).

[11] S. Krishnamoorthi, P. Jayapaul, R.K. Dhanaraj, V. Rajasekar, B. Balusamy and S.K.H. Islam, *Design of pseudo-random number generator from turbulence padded chaotic map*, Nonlinear Dyn., **104**, 1627–1643,(2021).

[12] N. Budinski, *Mathematics and origami: The art and science of folds*, in B. Sriraman (Ed.), Handbook of the mathematics of the arts and sciences, Springer, Cham, 317–348, (2019).

[13] M. Ben-Ari, *Geometric constructions using origami. In: Mathematical surprises*, Springer, Cham. (2021).

[14] I.H. Bell, U.K. Deiters, *Superancillary equations for cubic equations of state*, Ind. Eng. Chem. Res., **60(27)**, 9983–9991,(2021).

[15] G. Koval and M. Lazarchuk, *Geometric modeling of a closed flat contour with the application of rational cubic curves*, Applied Geometry and Engineering Graphics, **96**, 43–50,(2019).

[16] A.M. Cook, L.R. Myer, N.G.W. Cook and F.M. Doyle, *The effects of tortuosity on flow through a natural fracture*, in W. Hustrulid, G.A. Johnson (Eds.), Rock mechanics contributions and challenges, CRC Press, London, 371–378, (2020).

[17] E. Southall, *The compass and straight edge: Alternative approaches to teaching constructions*, Mathematics in Schools, **49(4)**, 1–6, (2020).

[18] A. Kimuya, *The independence of Euclidean geometric system: A proof on the constructability of geometric magnitudes*, (2021), $http://dx.doi.org/$10.13140/RG.2.2.25019.95528.

[19] A.O. Larin, *History of science and technology*, National Technical University "Kharkiv Polytechnic Institute", Kharkiv. (2021).

[20] N.I. Maryenko and O.Yu. Stepanenko, *Fractal analysis of images in medicine and morphology: Basic principles and methodologies*, Morphologia, **15(3)**, 196–206, (2021).

[21] M. Warhadpande, *The minimal instruments of geometry – II*, At Right Angles, **13**, 12–16, (2022).

[22] A.B. Araújo, *Spherical perspective*, in B. Sriraman (Ed.), Handbook of the mathematics of the arts and sciences, Springer, Cham, 527–587, (2021).

[23] E.G. Haug, *Squaring the circle and doubling the cube in space-time*, The Mathematics Enthusiast, **18(1)**, 59–77, (2021).

Author information

'A.I. Smolina, Irshava Lyceum, Ukraine.
E-mail: al.smolina@ukr.net

Received: 2023-05-12
Accepted: 2024-02-24