Palestine Journal of Mathematics

Vol 13(4)(2024) , 327–336

Achieved Isomorphic Based on Hyperideals of $\mathbb{Z}_{p^n}$

R. Daneshpayeh and S. Jahanpanah

Communicated by Ayman Badawi

MSC 2010 Classifications: Primary N2020; Secondary N2020.

Keywords and phrases: General multiring, Boolean general multiring, (normal)hyperideal, general isomorphism theorems.

The authors would like to thank the reviewers and editor for their constructive comments and valuable suggestions that improved the quality of our paper.

Abstract *In this paper, we try to establish that generalizing the theory of algebraic structures to structures equipped with actions and hyperoperations can be effective and useful in expanding the rules on any arbitrary set and reducing algebraic limitations. This paper considers the concept of general multirings as an extension of multirings and investigates and analyzes some of their essential properties. This study defines the notation of hyperideals in general multirings consider the notation of homomorphism and prove the (general) theorems isomorphism in general multirings, via fundamental relations.*

1 Introduction

An extension of algebraic structures is playing a prominent role in the sphere of mathematics. One such generalization of algebraic structures is the notion of multigroups. Multigroups are a generalization of groups and have come into the center of interest. The theory of multigroups, multirings, and multifields was introduced by Marshall as a generalization of groups, rings, and fields in [15]. Some researchers have investigated some works in this scope such as on cyclic multigroup family [1], On some algebraic properties of order of an element of a multigroup [2], theory of multigroups [5], homomorphism of fuzzy multigroups and some of its properties [6], some properties of multigroups [7], multigroup decodable STBCs from clifford algebras [14] and neutrosophic multigroups and applications [19]. It provided a convenient framework to study the reduced theory of quadratic forms and spaces of orderings. A multiring is just a ring, which is equipped to a hyperaddition and multirings are considered in spaces of signs, also known as abstract real spectra and objects which arise naturally in the study of constructible sets in real geometry. Recently, some researchers are worked in the scope of (hyper)ring and multiring which are related to multiring structures such as 1*-absorbing prime avoidance theorem in multiplicative hyperring [9], permuting tri-derivations on hyperrings [16], functorial relationships between multirings and the various abstract theories of quadratic forms [17] and on idempotent graph of rings [18]. Fundamental relations are basic tools in algebraic hyperstructures theory and some researchers worked on fundamental relations of hypergroups and hyperrings such as commutative rings obtained from hyperrings* $(hv\text{-}rings)$ *with* α^**–relations [4], boolean rings obtained from hyperrings with* $\eta^*_{1,m}$ *relations [8], fundamental relation and the automorphism group of very thin* H_v*–groups [10] and on (2-closed) regular hypergroups [13]. Hamidi et al. constructed multigroups and multirings on every non–empty set, introduced and analyzed a special relation on multirings and extended it to the smallest strongly regular equivalence binary relation in such a way that the quotient of each given multiring on this relation is a commutative Boolean ring with identity [3]. Ameri et al. extended multirings to a novel concept as general multirings, investigated their properties, and presented a special general multiring as a notation of* (m, n)*-potent general multirings. They analyzed the differences* *between the class of multirings, general multirings, and general hyperrings and constructs the class of (in)finite general multirings based on any given nonempty set [11]. Borumand Saeid et al. introduced the concepts of very thin multigroup, non-distributive (very thin) multirings, zero-divisor elements of multirings, and zero-divisor graphs based on zero-divisor elements of multirings. They considered the relationship between finite nondistributive (very thin) multirings and multirings and constructed non-distributive (very thin) multirings based on a given ring. They introduced the Zero-divisor graph based on the zero-divisor set of (non-distributive) (very thin) multirings, so investigated some necessity and sufficiency conditions that compute of order and size of these zero-divisor graphs. Also, the notations of derivable zero-divisor graphs and derivable zero-divisor subgraphs are introduced in this study and showed that some multipartite graphs are derivable zero-divisor graphs, all complete graphs, and cyclic graphs are derivable zero-divisor subgraphs [12]. We try to generalize the concept of multirings to general multirings, to describe some of their properties and their differences with multirings and general hyperrings. This paper works on the construction of general multirings and shows that this class of hyperstructures has some identity elements while having a unique zero element. It is natural to question as to what are the relationships between elements whence are considered in the same set concerning algebraic operations. Since any operation at most connects three elements, we need to extend more elements in defined axioms. It motivates us to introduce the concept of two algebraic hyperoperations in an underlying set. So the main motivation is to introduce some identity elements concerning algebraic hyperproduct and to consider the differences between other hyperstructures and structures. We obtained some theorems and corollaries that in special conditions are similar to corresponded theorems in (non-associative)rings, so we conclude that general multirings are a generalization of (non-associative)rings. Because in general multirings the hyperproduct of any element to zero element necessarily is not zero element, so the concept of the kernel of homomorphisms and isomorphism theorems are different from corresponding results in other hyperstructures. multifields, which play an important role in tropical geometry. So we introduce the concept of tropical general multiring as an application of general multirings.*

2 Preliminaries

In what follows, we recall some results that need to in our work.

*Let $R \neq \emptyset$ and $P^*(R) = \{S \mid \emptyset \neq S \subseteq R\}$. Every map $\varrho : R \times R \longrightarrow P^*(R)$ is a hyperoperation, (R, ϱ) is a hypergroupoid and $\forall\ A, B \subseteq R, \varrho(A, B)$ $\bigcup_{a\in A, b\in B} \varrho(a, b)$. A semihypergroup is a hypergroupoid (R, ϱ), if is satisfied in associative property and semihypergroup (R, ϱ) is called a hypergroup if satisfies in reproduction axiom. A commutative hypergroup (R, ϱ) is said to be a commutative multigroup, provided that (i) $\exists!\ e \in R$ s.t $\forall x \in R, \varrho(e, x) = \varrho(x, e) = \{x\}$, (ii) $\forall\ x \in R\ \exists!\ \varsigma(x) \in R$ s.t $e \in \varrho(x, \varsigma(x)) \cap \varrho(\varsigma(x), x)$, where $\varsigma : R \to R$ is a unary operation, (iii) $x \in \varrho(y, z)$ implies $y \in \varrho(x, \varsigma(z))$ and $z \in \varrho(\varsigma(y), x)$. A hypersystem $(R, \varrho, \varsigma, e, \upsilon, \iota)$ is called a multiring if (i) $(R, \varrho, \varsigma, e)$ is a commutative multigroup, (ii) (R, υ, ι) is a commutative monoid, (iii) $\forall\ x \in R\ \upsilon(x, \iota) = \iota$, (iv) $\forall\ x, y, z \in R\ \upsilon(x, (\varrho(y, z)) \subseteq \varrho(\upsilon(x, y), \upsilon(x, z))$. A hypersystem (R, ϱ, υ) is called a general hyperring if (i) (R, ϱ) is a hypergroup, (ii) (R, υ) is a semihypergroup and (iii) $\forall\ x, y, z \in R, \upsilon(x, (\varrho(y, z)) = \varrho(\upsilon(x, y), \upsilon(x, z))$. A map $f : R \to R'$ is a multiring homomorphism if, $\forall\ x, y \in R$; $(i) f(\varrho(x, y)) \subseteq \varrho'(f(x), f(y)), (ii) f(\upsilon(x, y)) = \upsilon'(f(x), f(y)), (iii) f(\varsigma(x)) = \varsigma'(f(x)), (iv) f(e) = e'$ and $f(\iota) = \iota'$. Let (R, ϱ, υ) be a hyperring and θ be an equivalence relation on R. Consider $R/\theta = \{\theta(r) \mid r \in R\}$ and define $\overline{\varrho}$ and $\overline{\upsilon}$ by $\overline{\varrho}(\theta(a), \theta(b)) = \{\theta(c) \mid c \in \varrho(\rho(a), \rho(b))\}$ and $\overline{\upsilon}(\theta(a), \theta(b)) = \{\theta(c) \mid c \in \upsilon(\theta(a), \theta(b))\}$. In [**?**] it was proved that $(R/\theta, \overline{\varrho}, \overline{\upsilon})$ is a ring if and only θ is strongly regular. Let $\mathcal{U}$ be the set of all finite hyperaddition of finite products of elements of R. Define relation γ on R by $a\gamma b \Longleftrightarrow \exists\ u \in \mathcal{U}$ s.t $\{a, b\} \in u$. The smallest strongly regular equivalence relation γ^*, on R that $(\frac{R}{\gamma^*}, \overline{\varrho}, \overline{\upsilon})$ means a ring, is said to be a fundamental relation.*

Definition 2.1. [3] A hypersystem $(R, \varrho, \varsigma, e, \upsilon, \iota)$ is called a general multiring if

(i) $(R, \varrho, \varsigma, e)$ is a multigroup,

(ii) (R, υ) is a semihypergroup,

(iii) $\forall\ x \in R, e \in \big(\upsilon(e, x) \cap \upsilon(x, e)\big)$ and $x \in \big(\upsilon(\iota, x) \cap \upsilon(x \cdot \iota)\big)$,

(iv) $\forall\, x, y, z \in R$, $\upsilon(x, (\varrho(y, z))) \subseteq \varrho(\upsilon(x, y), \upsilon(x, z))$ and $\upsilon(\varrho(x, y), z) \subseteq \varrho(\upsilon(x, z), \upsilon(y, z))$.

Clearly, every multiring is a general multiring. A general multiring $(R, \varrho, \varsigma, e, \upsilon, \iota)$ is called a $((\upsilon)$-commutative$)(\varrho)$-commutative general multiring, if it is commutative with respect to hyperoperation $(``\upsilon")``\varrho"$ and a commutative general multiring, if it is (υ)-commutative and (ϱ)-commutative.

Theorem 2.2. *[3] Let $(R, \varrho, \varsigma, e, \upsilon, \iota)$ be a general multiring and $A, B \subseteq R$. Then*

(i) $\varsigma(A) = \{\varsigma(a) \mid a \in A\}$,

(ii) $e \in \varsigma(A, A)$,

(iii) *if* $C \subseteq \varrho(A, B)$, *then* $A \cap \varrho(C, \varsigma(B)) \neq \emptyset$,

(iv) $\varsigma((\varsigma(A) = A$ *and* $\varrho(e, A) = A = \varrho(A, e)$,

(v) *if* $e \in \varrho(A, B)$, *then* $A \cap \varsigma(B) \neq \emptyset(\varsigma(A) \cap B \neq \emptyset)$,

(vi) $e \in (\upsilon(e, A)) \cap (\upsilon(A, e))$ *and* $A \subseteq (\upsilon(\iota, A)) \cap (\upsilon(A, \iota))$,

(vii) $\varrho(e, \varsigma(A) = \varsigma(A)$ *and* $\varrho(A, \varsigma(e)) = A$,

$(viii)$ *if* $A \subseteq B$, *then* $\varsigma(A) \subseteq \varsigma(B)$.

Theorem 2.3. *[3] Let $(R, \varrho, \varsigma, e, \upsilon, \iota)$ be a general multiring and $a, b, c, d \in R$. Then $\upsilon(\varrho(a, b), \varrho(c, d)) \subseteq \varrho(\upsilon(a, c), \upsilon(a, d), \upsilon(b, c), \upsilon(b, d))$.*

Theorem 2.4. *[3] Let p be a prime and $R = \mathbb{Z}_p \cup \{\sqrt{p}\}$. Then in general multiring $(R, \varrho_{\sqrt{p}}, -, \overline{0}, \upsilon_{\sqrt{p}}, \overline{1})$, we have*

(i) $\mathcal{H}I(R) = \{R, \{\overline{0}\}, \{\overline{0}, \sqrt{p}\}\}$,

(ii) $M = \{\overline{0}, \sqrt{p}\}$ *is the only maximal hyperideal of* R.

Theorem 2.5. *[3] Let p be a prime, $k \in \mathbb{N}$ and $R = \mathbb{Z}_{p^k} \cup \{\sqrt{p}\}$. Then in the general multiring $(R, \varrho_{\sqrt{p}}, -, \overline{0}, \upsilon_{\sqrt{p}}, \overline{1})$, we have*

(i) $\mathcal{H}I(R) = \{R, \{\overline{0}\}, I_p^{(m)} \mid 1 \leq m \leq p^{k-1}\}$,

(ii) $|\mathcal{H}I(R)| = k + 2$,

(iii) $m \leq m'$ *if and only if* $I_p^{(m)} \supseteq I_p^{(m')}$, *where* $1 \leq m, m' \leq p^{k-1}$,

(iv) $I_p^{(1)}$ *is the only maximal hyperideal of* R.

3 Quotient general multiring construction

In this section, we apply the concept of (strongly) homomorphism on general multirings and with this regards (strongly)homomorphisms, generate hyperideals. It is presented the notation of kernel of homomorphisms and so is concluded the general isomorphism theorems.

Definition 3.1. Let $(R, \varrho, \varsigma, e, \upsilon, \iota)$ be a general multiring and $\emptyset \neq I \subseteq R$. We say

(i) I is a general submultiring of R, if $(I, \varrho, \varsigma, e, \upsilon, \iota)$ is a general multiring;

(ii) I is a hyperideal of R, if $\varrho(I, \varsigma(I)) = I$ and $\upsilon(R, I) \cup \upsilon(I, R) \subseteq I$.

We will denote the set of all hyperideals of general multiring R by $\mathcal{H}I(R)$.

Theorem 3.2. *Let $(R, \varrho, \varsigma, e, \upsilon, \iota)$ be a general multiring and I be a hyperideal of R. Then*

(i) $e \in I$.

(ii) *if* $\iota \in I$, *then* $I = R$,

(iii) $\forall\, r \in R, x \in I, n \in \mathbb{N}$, *we have* $\varrho\underbrace{(\upsilon(r, x), \upsilon(r, x), \ldots \upsilon(r, x))}_{n\ times} \subseteq I$,

(iv) *if $x \in I$, then $\varsigma(x) \in I$.*

Proof. Immediate. □

Theorem 3.3. *Let $(R, \varrho, \varsigma, e, \upsilon, \iota)$ be a general multiring and $\emptyset \neq I \subseteq R$. Then I is a hyperideal of R if and only if satisfies in the following conditions:*

(i) $\forall\, x, y \in I, \varrho(x, \varsigma(y)) \subseteq I$,

(ii) $\forall\, r \in R$ *and* $x \in I$, *we have* $\upsilon(r, x) \cup \upsilon(x, r) \subseteq I$.

Proof. It is obvious. □

Theorem 3.4. *Let $(R, \varrho, \varsigma, e, \upsilon, \iota)$ be a general multiring and $\emptyset \neq I \subseteq R$. Then I is a general submultiring of R if and only if satisfies in the following conditions:*

(i) $\iota \in I$,

(ii) $\forall\, x, y \in I, \varrho(x, \varsigma(y)) \subseteq I$,

(iii) $\forall\, x, y \in I, \upsilon(x, y) \subseteq I$.

Proof. The proof is obtained by definition. □

Example 3.5. Let $R = \{e, \iota, a, b\}$. Then $(R, \varrho, \varsigma, e, \upsilon, \iota)$ is a general multiring as follows.

ϱ	e	ι	a	b
e	e	ι	a	b
ι	ι	R	$\{\iota, a\}$	$\{\iota, b\}$
a	a	$\{\iota, a\}$	R	$\{a, b\}$
b	b	$\{\iota, b\}$	$\{a, b\}$	R

and

υ	e	ι	a	b
e	e	e	e	e
ι	e	ι	a	b
a	e	a	a	a
b	e	b	a	$\{a, b\}$

.

Then $\mathcal{H}I(R) = \{I = \{e\}, J = R\}$.

A map $f : R \to R'$ is called a general multiring (strongly) homomorphism if, $\forall\, x, y \in R$ we have $(i)(f(\varrho(x, y)) = \varrho'(f(x), f(y)))f(\varrho(x, y)) \subseteq \varrho'(f(x), f(y)), (ii)(f(\upsilon(x, y)) = \upsilon'(f(x), f(y)))f(\upsilon(x, y)) \subseteq \upsilon'(f(x), f(y)), (iii)f(\varsigma(x)) = \varsigma(f(x)), (iv)f(e) = e$ and $f(\iota) = \iota$. Of course in definition of strongly homomorphism, dont need to have the item (iii), because it concludes from other conditions. From now on, we will call $f : R \to R'$ as (strongly)homomorphism.

Theorem 3.6. *Let $f : R \to R'$ be a map, $J \in \mathcal{H}I(R')$ and $I \in \mathcal{H}I(R)$. Then*

(i) *if f is a homomorphism, then $f^{-1}(J)$ is a hyperideal of R,*

(ii) *if f is a strongly epimorphism, then $f(I)$ is a hyperideal of R'.*

Proof. (i) By Theorem 3.2, $e' \in J$, implies that $e \in f^{-1}(J) \neq \emptyset$. Let $x, y \in f^{-1}(J)$. Then $f(\varrho(x, \varsigma(y))) \subseteq \varrho(f(x), \varsigma(f(y))) \subseteq \varrho(J, \varsigma(J)) \subseteq J$, concludes that $\varrho(x, \varsigma(y)) \subseteq f^{-1}(J)$. Also $\forall\, x \in f^{-1}(J)$ and $\forall\, r \in R$, we have $f(\upsilon(x, r)) \subseteq \upsilon(f(x), f(r)) \subseteq J$, so $\upsilon(x, r) \subseteq f^{-1}(J)$. Thus by Theorem 3.3, $f^{-1}(J)$ is a hyperideal of R.

(ii) By Theorem 3.2, $e \in I$, implies that $e' \in f(I) \neq \emptyset$. Let $f(x), f(y) \in f(I)$. Then $\varrho(f(x), \varsigma(f(y))) = f(\varrho(x, \varsigma(y))) \subseteq f(I)$. In addition, $\forall\, f(x) \in I$ and $\forall\, r' \in R'$, there exist $r \in R$ that $\upsilon(r', f(x)) = f(\upsilon(r, x)) \subseteq f(I)$. Thus by Theorem 3.3, $f(I)$ is a hyperideal of R'. □

Theorem 3.7. *Let $(R, \varrho, \varsigma, e, \upsilon, \iota)$ be a general multiring and $x \in R$. Then*

(i) *$\gamma^*(e)$ is a hyperideal of R,*

(ii) $\gamma^*(\varsigma(x)) = \varsigma(\gamma^*(x))$,

(iii) *$\varphi : R \to R/\gamma^*$ by $\varphi(x) = \gamma^*(x)$ is a strongly homomorphism.*

Proof. (i) Since $e \in \gamma^*(e)$, we get $\gamma^*(e) \neq \emptyset$. Let $x, y \in \gamma^*(e)$. Then $x\gamma^* y$, because γ^* is a strongly regular relation on R, we have $\varrho(x, \varsigma(y))\gamma^*\varrho(y, \varsigma(y))$. It follows that $\forall\ t \in \varrho(x, \varsigma(y)), t\gamma^* e$ or $t \in \gamma^*(e)$ and so $\varrho(x, \varsigma(y)) \subseteq \gamma^*(e)$. On other hand, $\forall\ r \in R$ and $x \in \gamma^*(e)$, because of strongly regularity relation of γ^*, we have $(\upsilon(r, x)\gamma^*(\upsilon(r, e)$. It means that $\forall\ t \in \upsilon(r, x)$ we have $t \in \gamma^*(e)$ and so $\upsilon(r, x) \subseteq \gamma^*(e)$.

(ii) Let $y \in \gamma^*(\varsigma(x))$. Then there exists $u \in \mathcal{U}$ that $\{y, \varsigma(x)\} \subseteq u$. Using Theorem 2.2, we get $\{\varsigma(y), x\} = \{\varsigma(y), \varsigma((\varsigma(x)))\}) = \varsigma(\{y, \varsigma(x)\}) \subseteq \varsigma(u) \in \mathcal{U}$. Hence $\varsigma(y) \in \gamma^*(x)$ or $y \in \varsigma(\gamma^*(x))$ and so $\gamma^*(\varsigma(x)) \subseteq \varsigma(\gamma^*(x))$. In a similar a way one can see that $\varsigma(\gamma^*(x)) \subseteq \gamma^*(\varsigma(x))$.

(iii) Let $x \in R$. Then $\overline{\varrho}(\gamma^*(x), \gamma^*(e)) = \gamma^*(x), \overline{\upsilon}(\gamma^*(x), \gamma^*(e)) = \gamma^*(e), \overline{\upsilon}(\gamma^*(x), \gamma^*(\iota)) = \gamma^*(x)$ and by item (ii), we have $\varphi : R \to R/\gamma^*$ is a strongly homomorphism. $\square$

Hamidi et al. in [11], converted the ring $R = \mathbb{Z}_p \cup \{\sqrt{p}\}$ to a general multiring as follows theorems.

Theorem 3.8. *Let p be a prime and $R = \mathbb{Z}_p \cup \{\sqrt{p}\}$. Then there exist hyperoperations "ϱ", "υ", nullary operations e, ι and a unary operation ς on R that $(R, \varrho, \varsigma, e, \upsilon, \iota)$ is a (p, p)-potent general multiring, where for any $x, y \in R$:*

$$x\varrho_{\sqrt{p}}y = y\varrho_{\sqrt{p}}x = \begin{cases} \{\overline{0}, \sqrt{p}\} & x = -y \text{ or } x = y = \sqrt{p}, \\ x + y & x, y \in \mathbb{Z}_p, x \neq -y \\ y & (x = \sqrt{p} \text{ and } y \notin \{\overline{0}, \sqrt{p}\}) \text{ or } x = \overline{0} \end{cases}$$

and

$$x\upsilon_{\sqrt{p}}y = y\upsilon_{\sqrt{p}}x = \begin{cases} xy & x, y \in \mathbb{Z}_p, \\ \sqrt{p} & x \in \mathbb{Z}_p \setminus \{\overline{0}\}, y = \sqrt{p}, \\ \overline{0} & x = \overline{0}, y = \sqrt{p}, \\ \{\overline{0}, \sqrt{p}\} & x = y = \sqrt{p}. \end{cases}$$

Clearly $(R, \varrho, \varsigma, \overline{0}, \upsilon, \overline{1})$ is a (p, p)-potent general multiring.

Theorem 3.9. *Let p be a prime, $k \in \mathbb{N}$ and $R = \mathbb{Z}_{p^k} \cup \{\sqrt{p}\}$. Then there exist hyperoperations "ϱ", "υ", nullary operations e, ι and a unary operation ς on R that $(R, \varrho, \varsigma, e, \upsilon, \iota)$ is a general multiring, where for any $x, y \in R$,*

$$x\varrho_{\sqrt{p}}y = y\varrho_{\sqrt{p}}x = \begin{cases} \{\overline{0}, \sqrt{p}\} & x = -y \text{ or } x = y = \sqrt{p}, \\ x + y & x, y \in \mathbb{Z}_{p^k}, x \neq -y, \\ y & x = \overline{0} \text{ or } (\ x = \sqrt{p} \text{ and } y \notin \{\overline{0}, \sqrt{p}\}) \end{cases}$$

and

$$x\upsilon_{\sqrt{p}}y = y\upsilon_{\sqrt{p}}x = \begin{cases} x.y & x, y \in \mathbb{Z}_{p^k}, \\ \sqrt{p} & x \in \mathbb{Z}_{p^k} \setminus \{mp\}, y = \sqrt{p}(m \in \mathbb{N}), \\ \overline{0} & x = mp, y = \sqrt{p}(m \in \mathbb{N}), \\ \{\overline{0}, \sqrt{p}\} & x = y = \sqrt{p}. \end{cases}$$

Clearly $(R, \varrho, \varsigma, \overline{0}, \upsilon, \overline{1})$ is a general multiring.

Theorem 3.10. *Let p be a prime, $k \in \mathbb{N}$ and $R = \mathbb{Z}_{p^k} \cup \{\sqrt{p}\}$. Then in the general multiring $(R, \varrho_{\sqrt{p}}, -, \overline{0}, \upsilon_{\sqrt{p}}, \overline{1})$, we have*

(i) *if I is a nontrivial hyperideal of R, then $\sqrt{p} \in I$,*

(ii) $\forall\ 1 \leq m \leq p^{k-1}$, $I_p^{(m)} = \{m\overline{p}, 2m\overline{p}, \ldots, tm\overline{p}, \sqrt{p} \mid t \in \mathbb{N}$ *is the smallest s.t* $tm \equiv 0 \pmod{p^{k-1}}\}$ *is a hyperideal of R,*

(iii) $\forall\ 1 \leq m \leq p^{k-1}$, *we have* $|I_p^{(m)}| = 1\varrho\dfrac{p^{k-1}}{\gcd(m, p^{k-1})}$,

(iv) $\forall\ 1 \leq m, m' \leq p^{k-1}$, $I_p^{(m)} = I_p^{(m')}$ *if and only if* $\gcd(p^{k-1}, m) = \gcd(p^{k-1}, m')$.

Example 3.11. Consider the general multiring $R = \mathbb{Z}_{27} \cup \{\sqrt{3}\}$. Computations show that

$$I_3^{(1)} = I_3^{(2)} = I_3^{(4)} = I_3^{(5)} = I_3^{(7)} = I_3^{(8)} = \{\bar{3}, \bar{6}, \bar{9}, \overline{12}, \overline{15}, \overline{18}, \overline{21}, \overline{24}, \bar{0}, \sqrt{3}\},$$
$$I_3^{(3)} = I_3^{(6)} = \{\bar{9}, \overline{18}, \bar{0}, \sqrt{3}\}, I_3^{(9)} = \{\bar{0}, \sqrt{3}\}$$

and so $\mathcal{H}I(R) = \{I_3^{(1)}, I_3^{(3)}, I_3^{(9)}, \{\bar{0}\}, R\}$. Computations show that $\gamma^*(\bar{0}) = I_3^{(9)}$ and
$\gamma^*(\varsigma(\sqrt{3})) = \gamma^*(\{\sqrt{3}\}) = I_3^{(9)} = \varsigma(I_3^{(9)})$.

Definition 3.12. Let $(R, \varrho, \varsigma, e, \upsilon, \iota)$, $(R', \varrho', \varsigma', e', \upsilon', \iota')$ be general multirings, $e \in R, e' \in R'$, and $f : R \to R'$ be a homomorphism. Define $Ker(f) = \{r \in R \mid f(r) \in \mathcal{U}$ where $e' \in \mathcal{U}\}$.

Corollary 3.13. *Let $f : R \to R'$ be a homomorphism. Then*

(i) $Ker(f) = \{r \in R \,|f(r) \in \gamma^*(e')\}$,

(ii) $Ker(f)$ *is a hyperideal of R,*

(iii) *if* $|\gamma^*(e')| = 1$, *then* $Ker(f) = \{r \in R \,|f(r) = e'\}$ *is a hyperideal of R,*

(iv) *if f is a strongly homomorphism, then* $Im(f)$ *is a general submultiring of R,*

(v) $w_R = \{x \in R \mid \varphi(x) = \gamma^*(e)\}$ *is a hyperideal of R.*

Proof. We refer to items $(ii), (iii)$ and the other items are clear.
(ii) Since $Ker(f) = f^{-1}(\gamma^*(e'))$, by Theorem 3.6, $Ker(f)$ is a hyperideal of R.
(iii) Let $|\gamma^*(e')| = 1$. Then $\gamma^*(e') = \{e'\}$ and so $\forall\, x \in R$, we have $x \in Ker(f)$ if and only if $f(x) = e'$. □

We say that A hyperideal I of R is a normal hyperideal, if $\forall\, x \in R, \varrho(x, \varsigma(x)) \subseteq I$.

Theorem 3.14. *Let $f : R \to R'$ be a homomorphism. Then*

(i) $Ker(f)$ *is a normal hyperideal of R,*

(ii) w_R *is a normal hyperideal of R.*

Proof. (i) Let $x \in R$. Then $\forall\, t \in \varrho(x, \varsigma(x))$, we have $f(t) \in f(\varrho(x, \varsigma(x)) \subseteq \varsigma(f(x), f(x))$. Since $\varsigma((f(x), f(x)))\gamma^* e'$ and $f(t) \in \varsigma(f(x), f(x))$, we get that $f(t)\gamma^* e'$ and so $t \in Ker(f)$.
(ii) Let $x \in R$. Then $\forall\, t \in \varrho(x, \varsigma(x))$, we have $\varphi(t) \in \varphi(\varrho(x, \varsigma(x))) \subseteq \varsigma(\varphi(x), \varphi(x)) = \varsigma(\gamma^*(x), \gamma^*(x)) = \gamma^*(e)$. So we get that $\varphi(t) = \gamma^*(e)$ and so $t \in w_R$. □

Example 3.15. (i) Let $R = \{e, \iota, a, b\}$. Then $(R, \varrho, \varsigma, e, \upsilon, \iota)$ and $(R', \varrho', \varsigma', e', \upsilon', \iota')$ are general multirings as follows.

ϱ	e	ι	a	b
e	e	ι	a	b
ι	ι	R	S	S
a	a	S	R	S
b	b	S	S	R

,

υ	e	ι	a	b
e	e	e	e	e
ι	e	ι	a	b
a	e	a	$\{a, b\}$	b
b	e	b	b	b

and

ϱ'	e	ι	a	b
e	e	ι	a	b
ι	ι	R	S	S
a	a	S	R	S
b	b	S	S	R

υ'	e	ι	a	b
e	e	e	e	e
ι	e	ι	a	b
a	e	a	a	a
b	e	b	a	$\{a, b\}$

,

where $S = R \setminus \{e\}$. Define $f : R \to R$ by $f = \{(e, e), (\iota, \iota), (a, b), (b, b)\}$. Clearly f is a homomorphism, while it is not a strongly homomorphism and $Ker(f) = w_R = R$.
(ii) Let $R = \{e, \iota, a, b\}$. Then $(R, \varrho, \varsigma, e, \upsilon, \iota)$ is a multiring as follows.

ϱ	e	ι	a	b
e	e	ι	a	b
ι	ι	$\{e, b\}$	b	$\{\iota, a\}$
a	a	b	e	ι
b	b	$\{\iota, a\}$	ι	$\{e, b\}$

and

υ	e	ι	a	b
e	e	e	e	e
ι	e	$\{\iota, a\}$	$\{e, a\}$	$\{e, b\}$
a	e	$\{e, a\}$	$\{e, a\}$	e
b	e	$\{e, b\}$	e	$\{e, b\}$

.

Then $\mathcal{H}I(R) = \{\{e\}, I = \{e, a\}, J = \{e, b\}, R\}$. Define $f : R \to R$ by $f(x) = x$, whence is the only strongly homomorphism and $Ker(f) = w_R = R$.

(iii) Let $R = \{e, \iota, a_2, a_3, a_4, a_5, a_6\}$. Then $(R, \varrho, e, \upsilon, \iota)$ is a general multiring as follows.

ϱ	e	ι	a_2	a_3	a_4	a_5	a_6
e	e	ι	a_2	a_3	a_4	a_5	a_6
ι	ι	T	a_3	a_2	ι	ι	ι
a_2	a_2	a_3	T	ι	a_2	a_2	a_2
a_3	a_3	a_2	ι	T	a_3	a_3	a_3
a_4	a_4	ι	a_2	a_3	T	T'	T'
a_5	a_5	ι	a_2	a_3	T'	T	T'
a_6	a_6	ι	a_2	a_3	T'	T'	T

and

υ	e	ι	a_2	a_3	a_4	a_5	a_6
e	e	e	e	e	e	e	e
ι	e	ι	a_2	a_3	a_4	a_5	a_6
a_2	e	a_2	a_2	a_4	a_4	a_4	a_4
a_3	e	a_3	a_4	a_3	a_4	a_4	a_4
a_4	e	a_4	a_4	a_4	a_4	a_4	a_4
a_5	e	a_5	a_4	a_4	a_4	a_5	a_4
a_6	e	a_6	a_4	a_4	a_4	a_4	$\{a_6, a_4\}$

,

where $T = \{e, a_4, a_5, a_6\}$ and $T' = \{a_4, a_5, a_6\}$. Clearly $(R, \varrho, \varsigma, e, \upsilon, \iota)$ is a commutative general multiring,and since $\upsilon(a_2, (\varrho(a_6, a_4))) = \{a_4\} \subseteq \{e, a_4, a_5, a_6\} = \varrho(\upsilon(a_2, a_6), \upsilon(a_2, a_4))$, we get that $(R, \varrho, \varsigma, e, \upsilon, \iota)$ is not a general hyperring. Define $f : R \to R$ by

$f = \{(e, e), (\iota, \iota), (a_2, a_2), (a_3, a_3), (a_4, a_4),$
$(a_5, a_4), (a_6, a_4)\}$. Clearly f is a homomorphism, because $\gamma^*(e) = \{e, a_4, a_5, a_6\}$, we get $Ker(f) = w_R = \gamma^*(e)$.

Let $(R, \varrho, \varsigma, e, \upsilon, \iota)$ be a general multiring, I be a normal hyperideal of R and $x, y \in R$. Define $x \sim_I y$ if and only if $(\varrho(x, \varsigma(y))) \cap I \neq \emptyset$.

Theorem 3.16. *Let $(R, \varrho, \varsigma, e, \upsilon, \iota)$ be a general multiring and I be a normal hyperideal of R. Then*

(i) *$\sim_I$ is an equivalence relation on R,*

(ii) *if R is a (ϱ)-commutative general multiring, then $\sim_I$ is a congruence equivalence relation on R,*

(iii) *$\forall\, x \in R$, $\varrho(x, I) = I$ if and only if $x \in I$.*

Proof. (i) Let $x, y, \in R$. By Theorem 3.2, $e \in \varrho(x, \varsigma(x))$ and $e \in I$ imply that $\varrho(x, \varsigma(x)) \cap I \neq \emptyset$ and so $x \sim_I x$. If $x \sim_I y$, then there exist $a \in (\varrho(x, \varsigma(y))) \cap I$ and so $\varsigma(x) \in y\varsigma(x)$. Using Theorem 3.2, $\varsigma(x) \in y\varsigma(x)$ and so we have $y \sim_I x$. Suppose that $x \sim_I y$ and $y \sim_I z$. Then there exist $a \in (\varrho(x, \varsigma(y))) \cap I$, $b \in (\varrho(y, \varsigma(z))) \cap I$ and so $x \in \varrho(a, y)$, $\varsigma(z) \in \varrho(\varsigma(y), b)$. It follows that $\varrho(x, \varsigma(z)) \subseteq \varrho((\varrho(a, y), \varrho(\varsigma(y), b)) = \varrho(a, \varrho(y, \varsigma(y)), b)$, because of normality of I, we obtain that $\varrho(x, \varsigma(z)) \subseteq I$ and so $x \sim_I z$. Thus $\sim_I$ is an equivalence relation on R.

(ii) Now, we show that it is a congruence relation on R. Let $x \sim_I x'$ and $y \sim_I y'$. Then there exist $a \in (\varrho(x, \varsigma(x')) \cap I$, $b \in \varrho(y, \varsigma(y')) \cap I$ and so $x \in \varrho(a, x')$, $y \in \varrho(b, y')$. Applying Theorem 2.3, because R is a (ϱ)-commutative general multiring, we get that

$$\begin{aligned} & \upsilon(x, y) \subseteq \upsilon(\varrho(a, x'), \varrho(b, y')) \subseteq \varrho(\upsilon(a, b), \upsilon(a, y'), \upsilon(x', b), \upsilon(x', y')) \\ = \; & \varrho\big(\varrho(\upsilon(a, b), \upsilon(a, y'), \upsilon(x', b)), \upsilon(x', y')\big). \end{aligned}$$

By Theorem 2.2, we get that $\varrho(\upsilon(a, b), \upsilon(a, y'), \upsilon(x', b)) \cap \varrho((\upsilon(x, y), \varsigma(\upsilon(x', y')))) \neq \emptyset$.

In addition, we have $\varrho(\upsilon(a, b), \upsilon(a, y'), \upsilon(x', b)) \subseteq I$ and so $\varrho((\upsilon(x, y), \varsigma(\upsilon(x', y')))) \cap I \neq \emptyset$. Thus $\upsilon(x, y) \sim_I \upsilon(x', y')$. On other hands, $x \sim_I x'$ and $y \sim_I y'$ imply that there exist $a \in \varrho(x, \varsigma(x')) \cap I$ and $b \in \varrho(y, \varsigma(y')) \cap I$ and so $\varrho(x, y) \subseteq \varrho(\varrho(a, b), \varrho(x', y'))$. Applying Theorem 2.2, we have $\varrho(a, b) \cap \big(\varrho(\varrho(x, y), \varsigma(\varrho(x', y')))\big) \neq \emptyset$. Because $\varrho(a, b) \subseteq I$, then $\big(\varrho(\varrho(x, y), \varsigma(\varrho(x', y')))\big) \cap I \neq \emptyset$ and so $\varrho(x, y) \sim_I (\varrho(x', y')$.

(iii) Let $x \in I$. Then $x \in \varrho(x, e) \subseteq \varrho(x, I)$ and because I is a hyperideal, we get that $\varrho(x, I) \subseteq I$ and so $\varrho(x, I) = I$. If $\varrho(x, I) = I$, then $\forall\, i' \in I$ there exists $i \in I$ that $i' \in \varrho(x, i)$ and so $x \in \varrho(i', \varsigma(i)) \subseteq I$. Thus $x \in I$. □

Example 3.17. Consider the general multiring $(R, \varrho, \varsigma, e, \upsilon, \iota)$ in Example 3.11. Computations show that $\sim_{I_3^{(4)}} = R \times R$, which is the largest equivalence relation.

In case $x \sim_I y$, we say that x and y are congruent modulo I. The equivalence class of the element r in R is given by $\varrho(r, I) := \{\varrho(r, s) : s \in I\}$. This equivalence class is also sometimes written as r mod I and called the residue class of r modulo I. Consider $R/I = \{\varrho(r, I) \mid r \in R\}$, then we have the following theorem.

Theorem 3.18. *Let $(R, \varrho, \varsigma, e, \upsilon, \iota)$ be a (ϱ)-commutative general multiring and I be a normal hyperideal of R. Then*

(i) *There exist hyperoperations "ϱ'", "υ'", nullary operations e', ι' and a unary operation ς' on R/I, that $(R/I, \varrho', \varsigma', e', \upsilon', \iota')$ is a general multiring,*

(ii) *$\varphi : R \to R/I$ by $\varphi(r) = \varrho(r, I)$ is a strongly homomorphism.*

Proof. (i) Let $\overline{a} = \varrho(a, I)$ and $\overline{b} = \varrho(b, I)$. Define "$\varrho'$" and "$\upsilon'$" on R/I by $\varrho'(\overline{a}, \overline{b}) = \{\overline{c} \mid c \in \varrho(a, b)\}$, $\upsilon'(\overline{a}, \overline{b}) = \{\overline{c} \mid c \in \upsilon(a, b)\}$, $e' = \overline{e} = \varrho(e, I) = I, \iota' = \overline{\iota} = \varrho(\iota, I)$ and $\varsigma(\overline{x}) = \overline{\varsigma(x)}$. By Theorem 3.16, the above hyperoperations are well-defined. It is easy to see that $(R/I, \varrho', \varsigma', e', \upsilon', \iota')$ is a general multiring.

(ii) It is obtained by (i), obviously. □

Theorem 3.19. (*First general isomorphism theorem*) *Let $f : R \to R'$ be a strongly homomorphism. Then $R/Ker(f) \cong \gamma^*(Im(f))$.*

Proof. Let $x \in R$. Define $\varphi : R/Ker(f) \to \gamma^*(Im(f))$ by $\varphi(\varrho(x, Ker(f))) = \gamma^*(f(x))$. Let $\varrho(x, Ker(f)) = \varrho(y, Ker(f))$. Then $(\varrho(x, \varsigma(y)) \cap Ker(f) \neq \emptyset$ and so $(\varrho(f(x), \varsigma(f(y)))\gamma^* e'$. Thus $[\varrho(\varrho(f(x), \varsigma(f(y)), f(y))]\gamma^*[\varrho(e', f(y))]$. It follows that $f(x)\gamma^* f(y)$ or $\varphi(\varrho(x, Ker(f))) = \varphi(\varrho(y, Ker(f)))$ and so φ is a well-defined map. Now, we show that φ is a strongly homomorphism. Let $x, y \in R$. Then

$$\begin{aligned} & \varphi\big(\varrho((\varrho(x, Ker(f))), (\varrho(y, Ker(f))))\big) = \varphi\{\varrho(z, Ker(f)) \mid z \in \varrho(x, y)\} \\ = \;& \{\gamma^*(f(z)) \mid f(z) \in \varrho(f(x), f(y))\} = \varrho'(\gamma^*(f(x)), \gamma^*(f(y))) \\ = \;& \varrho'\big(\varphi(\varrho(x, Ker(f)), \varphi(\varrho(y, Ker(f)))\big). \end{aligned}$$

In a similar a way, one can see that
$\varphi\big(\upsilon(\varrho(x, Ker(f)), \varrho(y, Ker(f))\big) = \upsilon'\big(\varphi(\varrho(x, Ker(f)), \varphi(\varrho(y, Ker(f)))\big)$.

Clearly, $\varphi(\varrho(e, Ker(f)) = \gamma^*(f(e)) = \gamma^*(e'), \varphi(\varrho(\iota, Ker(f)) = \gamma^*(f(\iota)) = \gamma^*(\iota')$. Let $x \in R$. Then by Theorem 3.7, we get that $\varphi\big(\varsigma(\varrho(x, Ker(f))\big) = \gamma^*(f(\varsigma(x))) = \varsigma(\gamma^*(f(x))) = \varsigma(\varphi\big(\varrho(x, Ker(f))\big)$ and so φ is a strongly homomorphism. Let $x, y \in R$. Then $\varphi\big(\varrho(x, Ker(f)\big) = \varphi\big(\varrho(y, Ker(f)\big)$, implies that $\gamma^*(f(x)) = \gamma^*(f(y))$. Thus there exists $u \in \mathcal{U}$ that $\{f(x), f(y)\} \subseteq u$ and so $\varrho(\varsigma(f(x)), \{f(x), f(y)\}) \subseteq \varrho(\varsigma(f(x)), u)$. It follows that there exists $u' \in \mathcal{U}$ that $\{e', f(\varrho(x, \varsigma(y))\} \subseteq u'$ or $f(\varrho(x, \varsigma(y))\gamma^* e'$. Hence $(\varrho(x, \varsigma(y))) \cap Ker(f) \neq \emptyset$ or $\varrho(x, Ker(f)) = \varrho(y, Ker(f))$ and so φ is a monomorphism. Clearly φ is an onto map and then is an isomorphism, so $R/Ker(f) \cong \gamma^*(Im(f))$. □

Example 3.20. Consider the general multiring in Example 3.11. Then by the given homomorphism in this example,

$R/Ker(f) = R/\{e, a_4, a_5, a_6\} = \{\varrho(x, \{e, a_4, a_5, a_6\}) \mid x \in R\} = \{e, \iota, a_2, a_3, a_4, a_5, a_6\} = \gamma^*(Im(f))$. This example shows that the isomorphism, may be converts to equality.

Corollary 3.21. (*First isomorphism theorem*) *Let $f : R \to R'$ be a strongly homomorphism. If $|w_{R'}| = 1$, then $R/Ker(f) \cong Im(f)$.*

Proof. If $|w_{R'}| = 1$, then $\gamma^*(e') = \{e'\}$ and $\forall\ x \in R', \upsilon(e', x) = \upsilon(x, e') = e'$. Thus by Corollary 3.13 and Theorem 3.14, $Ker(f)$ is a normal hyperideal of R. Now, all for $x \in R$, define $\varphi : R/Ker(f) \to Im(f)$ by $\varphi(\varrho(x, Ker(f)) = f(x)$. Thus by Theorem 3.19, φ is an isomorphism and so $R/Ker(f) \cong Im(f)$. □

Example 3.22. Consider the general multiring in Example 3.11. In the example 3.20, we see that it is not necessary $|w_{R'}| = 1$, because of $w_R = \{e, a_4, a_5, a_6\}$.

Lemma 3.23. *Let $(R, \varrho, \varsigma, e, \upsilon, \iota)$ be a general multiring, I, J be hyperideals of R and A be a general submultiring of R.Then* (*i*) *$I \cap A$ is a hyperideal of A,*

(*ii*) *$I \cap J$ is a hyperideal of R,*

(*iii*) *if R is a (ϱ)-commutative general multiring and I is a hyperideal, then $\varrho(A, I)$ is a general submultiring of R,*

(*iv*) *if R is a (ϱ)-commutative general multiring and J is a hyperideal, then $\varrho(I, J)$ is a hyperideal of R.*

Proof. It is immediate by definition. □

Theorem 3.24. (*Second general isomorphism theorem*) *Let $(R, \varrho, \varsigma, e, \upsilon, \iota)$ be a (ϱ)-commutative general multiring, I be a normal hyperideal of R and A be a general submultiring of R. Then*

(*i*) *$I \cap A$ is a normal hyperideal of A and I is a normal hyperideal of $\varrho(A, I)$,*

(*ii*) *$\gamma^*(\varrho(A, I)/I) \cong A/(A \cap I)$.*

Proof. (i) It is obtained by Lemma 3.23.

(ii) Let $x \in A$. Define $f : A \to \varrho(A, I)/I$ by $f(x) = \varrho(x, I)$. One can see that φ is a strongly epimorphism. Now, we show that $Ker(f) = A \cap I$. If $x \in A \cap I$, by Theorem 3.16, $\varrho(x, I) = I$ and so $\varrho(x, I) \in \gamma^*(I)$. It concludes that $x \in Ker(f)$. Suppose $x \in A$. Then $x \in Ker(f)$ if and only if $\varrho(x, I) \in \gamma^*(I)$. Thus there exists $i \in I$ that $\varrho(x, I) = \gamma^*(i)$ and so $i \in \varrho(x, I)$. Since I is a hyperideal of R, we have $x \in I$. Therefore, by Theorem 3.19, we get that $\gamma^*(\varrho(A, I)/I) \cong A/(A \cap I)$. □

Corollary 3.25. (*Second isomorphism theorem*) *Let $(R, \varrho, \varsigma, e, \upsilon, \iota)$ be a (ϱ)-commutative general multiring, I be a normal hyperideal of R and A be a general submultiring of R. If $|w_{\frac{\varrho(A,I)}{I}}| = 1$, then $\varrho(A, I)/I \cong A/(A \cap I)$.*

Proof. Since $|w_{\frac{\varrho(A,I)}{I}}| = 1$, we get that $\gamma^*(e_{\frac{\varrho(A,I)}{I}}) = I$. Now, define $f : A \to \frac{\varrho(A,I)}{I}$ by $f(a) = \varrho(a, I)$. Applying Theorem 3.24, f is an epimorphism and $\forall\ x \in Ker(f)$ we have $\varrho(x, I) \in \gamma^*(e_{\frac{\varrho(A,I)}{I}})$ if and only if $\varrho(x, I) = I$. Using Theorem 3.24, we have $Ker(f) = A \cap I$ and so $\varrho(A, I)/I \cong A/(A \cap I)$. □

Theorem 3.26. (*Third general isomorphism theorem*) *Let $(R, \varrho, \varsigma, e, \upsilon, \iota)$ be a general multiring, I, J be hyperideals of R and $I \subseteq J$.Then*

(*i*) *if I is a normal hyperideal of R, then J is a normal hyperideal of R,*

(*ii*) *if I is a normal hyperideal of R, then J/I is a normal hyperideal of R/I,*

(*iii*) *if R is a (ϱ)-commutative general multiring and I is a normal hyperideal, then $(R/I)/(J/I) \cong \gamma^*(R/J)$.*

Proof. (i) It is easy by definition.

(ii) Let $x, y \in J$ and $r \in R$. Then $\varrho(\varrho(x, I), \varsigma(\varrho(y, I)))) = \varrho(\varrho(x, \varsigma(y)), I) \subseteq J/I$ and $\upsilon(\varrho(r, I), \varrho(y, I)) = \varrho(\upsilon(r, y), I) \subseteq J/I$ so J/I is a hyperideal of R/I. For all $r, r' \in R$, because of normality of J(by (i)), we get that $\varrho(\varrho(r, I), \varsigma(\varrho(r', I))) = \varrho(\varrho(r, \varsigma(r')), I) \subseteq J/I$ and so J/I is a normal hyperideal of R/I.

(iii) Let $x \in R$. Define $\varphi : R/I \to R/J$ by $\varphi(\varrho(x, I)) = \varrho(x, J)$. Obviously, φ is a strongly homomorphism. Now, we show that $Ker(\varphi) = J/I$. Let $x \in R$. Then $\varrho(x, I) \in Ker(\varphi)$ if and only if $\varphi(\varrho(x, I)) \in \gamma^*(e_{R/J}) = \gamma^*(J)$ if and only if there exists $j \in J$ that $j \in \gamma^*(J) = \varrho(x, J)$ if and only if $x \in J$. □

Corollary 3.27. (*Third isomorphism theorem*) *Let R be a (ϱ)-commutative general multiring, I, J be hyperideals of R and $I \subseteq J$. If $|w_{R/J}| = 1$ and I is a normal hyperideal, then $(R/I)/(J/I) \cong R/J$.*

4 Conclusion remarks

The current paper has defined the general multirings as a generalization of multirings and presented some properties in these hyperstructures. Also, using the multigroups, general multirings are constructed. The concept of a kernel of homomorphisms is defined and is shown that it is a hyperideal. It proved that the Hass tree of hyperideals of finite general multirings is a chain. The general isomorphism theorems are considered and proved, in this regard. We hope that these results are helpful for further studies in fuzzy multigroups, fuzzy multirings, fuzzy general multirings, and fuzzy graphs based on general multirings. Also, we want to work on the Plithogenics and new types of (hyper)soft sets and general multirings and their relation to real-world problems.

References

[1] *J. A. Awolola,* On cyclic multigroup family, *Ratio Mathematica,* **37**, *61-68, (2019).*

[2] *A. J. Aderemi, E. Paul,* On some algebraic properties of order of an element of a multigroup, *Quasigr. Relat. Syst.,* **1(25)**, *21-26, (2017).*

[3] *R. Ameri, M. Hamidi and A. A. Tavakoli,* Boolean Rings Based on Multirings, *J. Sci. Islam. Repub. Iran., 32(2), 159–168, (2021).*

[4] *B. Davvaz, T. Vougiouklis,* Commutative rings obtained from hyperrings (Hv-rings) with α^*–relations, *Comm. Algebra.,* **35**, *3307–3320, (2007).*

[5] *M. Dresher and O. Ore,* Theory of Multigroups, *J. Am. Math. Soc.,* **60(3)**, *705–733, (1938).*

[6] *P. A. Ejegwa,* Homomorphism of Fuzzy Multigroups and Some of its Properties, *Appl. Appl. Math.,* **13(1)**, *114–129, (2018).*

[7] *P. A. Ejegwa,* Some Properties of Multigroups, *Palest. J. Math.,* **9(1)**, *31–47, (2020).*

[8] *P. Ghiasvand, S. Mirvakili and B. Davvaz,* Boolean Rings Obtained from Hyperrings with $\eta^*_{1,m}$ Relations, *Iran. J. Sci. Technol Trans. Sci.,* **41(1)**, *69–79, (2017).*

[9] *P. Ghiasvand and F. Farzalipour,* 1-Absorbing Prime Avoidance Theorem In Multiplicative Hyperring, *Palest. J. Math.,* **12(1)**, *900–908, (2023).*

[10] *M. Hamidi and A. R. Ashrafi,* Fundamental relation and automorphism group of very thin H_v–groups, *Comm. Algebra.,* **45(1)**, *130–140, (2017).*

[11] *M. Hamidi, A. A. Tavakoli , and R. Ameri,* Valued-Potent (General) Multirings, *J. Algebr. Syst.,* **10(1)**, *49–68, (2022).*

[12] *M. Hamidi and A. Borumand Saeid,* Graphs derived from multirings, *Soft Computing,* **25**, *13921–13932, (2021).*

[13] *M. Hamidi, V. Leoreanu,* ON (2-Closed) Regular Hypergroups, *U.P.B. Sci. Bull., Series A,* **80(4)**, *173–186, (2018).*

[14] *S. Karmakar and B. S. Rajan,* Multigroup Decodable STBCs From Clifford Algebras, *IEEE Trans. Inf. Theory.,* **55(1)**, *223-231, (2009).*

[15] *M. Marshall,* Real reduced multirings and multifields, *J. Pure Appl. Algebra* **205**, *452–468, (2006).*

[16] *S. Oguzunal,* Permuting Tri-Derivitatons on Hyperrings, *Palest. J. Math.,* **9(1)**, *105–111, (2020).*

[17] *H. R. d. O. Ribeiro, K. M. d. A. Roberto and H. L. Mariano,* Functorial relationships between multirings and the various abstract theories of quadratic forms, *Sao Paulo J. Math. Sci.,* **16**, *5–42, (2022).*

[18] *P. Sharma and S. Dutta,* On Idempotent Graph of Rings, *Palest. J. Math.,* **12(1)**, *883–891, (2023).*

[19] *V. Ulucay,* Neutrosophic Multigroups and Applications, *Mathematics,* **7(1) 95**, *1-17, (2019).*

Author information

R. Daneshpayeh, Department of Mathematics, University of Payame Noor, Tehran, Iran.
E-mail: `rdaneshpayeh@pnu.ac.ir`

S. Jahanpanah, Department of Mathematics, University of Payame Noor, Tehran, Iran.
E-mail: `s.jahanpanah@pnu.ac.ir`

Received: 2023-05-19
Accepted: 2023-12-13