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ON AUTOMORPHISMS AND α-BI-SEMIDERIVATIONS OF PRIME RINGS

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Abstract In this research paper, our intension is to prove the commutative structure of a prime ring using identities involving with automorphisms and α-bisemiderivations. Moreover, we introduced the concept of α-bisemiderivations on rings for an automorphism α.

1 Introduction

The well-known Posner's result, which stipulates that the presence of a nonzero centralizing derivation on a prime ring compels the ring to be commutative, is the basis of our investigation. Mayne extends this result for automorphism. He proved that there exists a nontrivial centralizing automorphism on a prime ring forces the ring to be commutative. Number of mathematicians established a relationship between the structure of prime (semiprime) rings and their subsets including the behavior of mappings namely derivations, α-derivations, automorphisms, endomorphisms etc. satisfying algebraic identities involving such mappings. For further information and study in the related subject, one can turned to [1], [9], [10]. In the sequel, we obtained some commutativity results using identities on α-bisemiderivation on prime ring. Let us describe the basic terminology used in our study.

$\mathcal{R}$ will be called as associative ring, together with the center $\mathcal{Z}(\mathcal{R})$ throughout the paper. The expression $[b, d]$ defined as $[b, d] = bd - db$ and denoted the commutator of $b, d \in \mathcal{R}$. Remember that if $cRb = 0$ shows that either $c = 0$ or $b = 0$, $\mathcal{R}$ is said to be prime. A mapping ζ from $\mathcal{R}$ to $\mathcal{R}$ is recognized as a derivation on $\mathcal{R}$, if it satisfies $\zeta(ce) = \zeta(c)e + c\zeta(e)$, for every $c, e \in \mathcal{R}$.

A mapping $\mathcal{D} : R \times \mathcal{R} \to \mathcal{R}$ is considered to be symmetric, according to Maksa [3], if $\mathcal{D}(p, q) = \mathcal{D}(q, p)$ for every p, q in $\mathcal{R}$. If a mapping $\mathcal{D} : R \times R \to R$ is additive in both slots, it is said to be bi-additive. The idea of symmetric bi-derivations is now introduced as follows: When the map $q \mapsto \mathcal{D}(p, q)$ and the map $p \mapsto \mathcal{D}(p, q)$ are both derivations of $\mathcal{R}$, the bi-additive mapping $\mathcal{D} : \mathcal{R} \times \mathcal{R} \longrightarrow \mathcal{R}$ is said to be bi-derivation. For ideational reading in the related matter one can turned to [3]. For a symmetric mapping $\mathcal{D}$, a map $h : \mathcal{R} \to \mathcal{R}$ defined as $h(p) = \mathcal{D}(p, p)$ is called the trace of $\mathcal{D}$.

J. Bergen introduces the idea of semiderivations of a ring R in [4]. A mapping f that is additive from R to R is known as a semiderivation if there exists a function g on $\mathcal{R}$ such that $f(ab) = f(a)g(b) + af(b) = f(a)b + g(a)f(b)$ and $f(g(a)) = g(f(a))$ for each a, b in $\mathcal{R}$. All semiderivations associated with g are just normal derivations if g is an identity map of $\mathcal{R}$. However, if g is a homomorphism of R such that $g \neq I_{identity}$, then $f = g - I$ is a semiderivation rather than a derivation. Some remarkable results related to semiderivations found in [5].

A bi-additive and symmetric mapping $\mathcal{D}$ from $\mathcal{R} \times \mathcal{R}$ to $\mathcal{R}$ is recognised as a symmetric bi-semiderivation associated with a mapping $f : \mathcal{R} \longrightarrow \mathcal{R}$, if

$$\mathcal{D}(pq, r) = \mathcal{D}(p, r)f(q) + p\mathcal{D}(q, r) = \mathcal{D}(p, r)q + f(p)\mathcal{D}(q, r)$$

and $h(f) = f(h)$ for each p, q, r in $\mathcal{R}$.

If α is an automorphism on R. An additive mapping $\mathrm{d} : \mathcal{R} \longrightarrow \mathcal{R}$ is said to be an α-semiderivation with an epimorphism $\varphi : \mathcal{R} \longrightarrow \mathcal{R}$ if its fulfill the following conditions:

(i) $\mathrm{d}(pq) = \mathrm{d}(p)\varphi(q) + \alpha(p)\mathrm{d}(q) = \mathrm{d}(p)\alpha(q) + \varphi(p)\mathrm{d}(q)$.

(ii) $\mathrm{d}(\varphi(p)) = \varphi(\mathrm{d}(p))$.

Motivated by the definition of bi-semiderivation and α-semiderivation, we intend to define the concept of α-bi-semiderivation on ring as follows:

Let α be an automorphism on ring and $\mathcal{K}$ be a bi-additive mapping on $\mathcal{R}$. We say $\mathcal{K} : \mathcal{R} \times \mathcal{R} \longrightarrow \mathcal{R}$, an α-bisemiderivation on R with associated function $\hbar$ if it fulfills the following conditions:

(i) $\mathcal{K}(pq, r) = \mathcal{K}(p, r)\hbar(q) + \alpha(p)\mathcal{K}(q, r)$.

(ii) $\mathcal{K}(pq, r) = \mathcal{K}(p, r)\alpha(q) + \hbar(p)\mathcal{K}(q, r)$.

(iii) $\mathcal{K}(\hbar) = \hbar(\mathcal{K})$.

The definition above will be reduces to α-semiderivation if we take for some fixed a, $\mathcal{K}(p, a) = d(p)$ for every $p \in \mathcal{R}$. For more literature and examples about bi-semiderivations, one can look into [2] and the references therein.

Example 1.1. Let $R = \left\{ \begin{pmatrix} p & 0 \\ 0 & q \end{pmatrix} \mid p, q \in 2\mathbb{Z}_4 \right\}$ is a ring under matrix addition and matrix multiplication. Define mapping α, h from R to itself by

$$\alpha\left[\begin{pmatrix} p & 0 \\ 0 & q \end{pmatrix}\right] = \begin{pmatrix} q & 0 \\ 0 & p \end{pmatrix},$$

$$h\left[\begin{pmatrix} p & 0 \\ 0 & q \end{pmatrix}\right] = \begin{pmatrix} 0 & 0 \\ 0 & q \end{pmatrix}$$

and $\mathcal{K} : R \times R \to R$ by $\mathcal{K}\left[\begin{pmatrix} p & 0 \\ 0 & q \end{pmatrix}, \begin{pmatrix} r & 0 \\ 0 & t \end{pmatrix}\right] = \begin{pmatrix} pr & 0 \\ 0 & 0 \end{pmatrix}$ for every $p, q, r, t \in 2Z_4$. Then $\mathcal{K}$ is α-bisemiderivation on $\mathcal{R}$ with associated function $\hbar$ and automorphism α.

2 Prerequisite

The purpose of the current work is to examine the identities related to α-bisemiderivations with associated surjective map $\hbar$ on a prime ring $\mathcal{R}$, where α represents an automorphism on $\mathcal{R}$. We begin our discussion with the following lemmas:

Lemma 2.1. *[7] Let α be a nontrivial automorphism on a prime ring $\mathcal{R}$. If $[\alpha(t), t] = 0$, for every $t \in \mathcal{R}$, then $\mathcal{R}$ is a commutative ring.*

In [6], author obtained that if $pq = 0$ and $p \neq 0$ is a nonzero element in $\mathcal{R}$, then $q = 0$. Our next lemma is an analogous form of this result.

Lemma 2.2. *Consider $\mathcal{R}$ to be a prime ring with characteristic not two and α be an automorphism on R. If $\mathcal{K}$ is a nonzero α-bisemiderivation on $\mathcal{R}$ with associated function $\hbar$ such that $b\mathcal{K}(p, p) = 0$ for all $p \in \mathcal{R}$ and some fixed $b \in \mathcal{R}$, then $b = 0$.*

Proof. Given that for all $p \in \mathcal{R}$ and a fixed $b \in \mathcal{R}$

$$b\mathcal{K}(p,p) = 0 \tag{2.1}$$

Put $p+q$ for p in (2.1) to find

$$b\mathcal{K}(p,q) = 0 \text{ for all } p,q \in \mathcal{R}. \tag{2.2}$$

Substitute qw for q in above equation to get

$$b\{\mathcal{K}(p,q)\hbar(w) + \alpha(q)\mathcal{K}(p,w)\} = 0 \text{ for all } p,q,w \in \mathcal{R}. \tag{2.3}$$

Analysing equations (2.2) and (2.3), we have

$$b\alpha(q)\mathcal{K}(p,w) = 0 \text{ for all } p,q,w \in \mathcal{R}. \tag{2.4}$$

As α is an automorphism, we can put $\alpha^{-1}(q)$ for q in above equation to get $bq\mathcal{K}(p,w) = 0$ for every $p,q,w \in \mathcal{R}$. Therefore, we have $b\mathcal{R}\mathcal{K}(p,w) = 0$ for every $p,w \in \mathcal{R}$. Making use of primeness of $\mathcal{R}$ and the fact that $\mathcal{K} \neq 0$ to see $b = 0$. □

3 Main Results

We begin investigation with the following :

Theorem 3.1. *Consider $\mathcal{R}$ to be a prime ring with characteristic not two and α be an automorphism on R. If $\mathcal{K}$ is a α-bisemiderivation on $\mathcal{R}$ with associated function $\hbar$ such that $\mathcal{K}(p,p)\mathcal{K}(q,q) = 0$ for all $p,q \in \mathcal{R}$, then $\mathcal{K} = 0$.*

Proof. We are given that for each $p,q \in \mathcal{R}$

$$\mathcal{K}(p,p)\mathcal{K}(q,q) = 0. \tag{3.1}$$

Replace $q+w$ for q in (3.1) and use characteristic condition to find

$$\mathcal{K}(p,p)\mathcal{K}(q,w) = 0, \text{ for all } p,q,w \in \mathcal{R}. \tag{3.2}$$

We obtain by substituting qy for q in (3.2)

$$\mathcal{K}(p,p)\{\mathcal{K}(q,w)\hbar(y) + \alpha(q)\mathcal{K}(y,w)\} = 0 \text{ for every } p,q,w,y \in \mathcal{R}. \tag{3.3}$$

In light of (3.2), (3.3) assumes the form

$$\mathcal{K}(p,p)\alpha(q)\mathcal{K}(y,w) = 0 \text{ for each } p,q,w,y \in \mathcal{R}. \tag{3.4}$$

Reword the above equation after substituting $\alpha^{-1}(q)$ for q to have $\mathcal{K}(p,p)q\mathcal{K}(y,w) = 0$ for every $p,q,w,y \in \mathcal{R}$. Hence we observe the primeness in the equation $\mathcal{K}(p,p)\mathcal{R}\mathcal{K}(y,w) = 0$ for every $p,w,y \in \mathcal{R}$. We get $\mathcal{K} = 0$.

□

As a consequences of above theorem, we listed the following corollaries:

Corollary 3.2. *Consider $\mathcal{R}$ to be a prime ring with characteristic not two and α be an automorphism on R. If $\mathcal{K}$ is a α-bisemiderivation on $\mathcal{R}$ with associated function $\hbar$ and d is a α-semiderivation on $\mathcal{R}$ such that $\mathcal{K}(p,p)\mathrm{d}(q) = 0$ for every $p,q \in \mathcal{R}$, then either $\mathcal{K} = 0$ or $\mathrm{d} = 0$.*

Corollary 3.3. *Consider $\mathcal{R}$ to be a prime ring with characteristic not two and α be an automorphism on R. If $\mathcal{K}$ is a α-bisemiderivation on $\mathcal{R}$ with associated function $\hbar$ and d is a α-semiderivation on $\mathcal{R}$ such that $\mathrm{d}(p)\mathcal{K}(q,q) = 0$ for every $p,q \in \mathcal{R}$, then either $\mathcal{K} = 0$ or $\mathrm{d} = 0$.*

Corollary 3.4. *Consider $\mathcal{R}$ to be a prime ring with characteristic not two and α be an automorphism on R. If* d *is a α-semiderivation on $\mathcal{R}$ such that* $\mathrm{d}(p)\mathrm{d}(q) = 0$ *for all $p, q \in \mathcal{R}$, then* $\mathrm{d} = 0$.

Theorem 3.5. *Consider $\mathcal{R}$ to be a prime ring with characteristic not two and α be an automorphism on R. If $\mathcal{K}$ is a α-bisemiderivation on $\mathcal{R}$ with associated function $\hbar$ such that $[\mathcal{K}(p,p), \hbar(q)] = 0$ for all $p, q \in \mathcal{R}$, then either $\mathcal{R}$ is commutative or $\mathcal{K} = 0$.*

Proof. We have given that for each $p, q \in R$,

$$[\mathcal{K}(p,p), \hbar(q)] = 0. \tag{3.5}$$

Replace $p + w$ for p in (3.5) and use characteristic condition to obtain

$$[\mathcal{K}(p,w), \hbar(q)] = 0 \text{ for every } p, q, w \in \mathcal{R}. \tag{3.6}$$

Simplify the equation (3.6) after putting wq for w to obtain

$$[\alpha(w), \hbar(q)]\mathcal{K}(p,q) = 0 \text{ for all } p, q, w \in \mathcal{R}. \tag{3.7}$$

Since α is an automorphism, we can put $\alpha^{-1}(w)$ for w in above equation to have

$$[w, \hbar(q)]\mathcal{K}(p,q) = 0 \text{ for each } p, q, w \in \mathcal{R}. \tag{3.8}$$

Again replace p by pt in(3.8) and making use of (3.8) to acquire

$$[w, \hbar(q)]\alpha(p)\mathcal{K}(t,q) = 0 \text{ for all } p, q, t, w \in \mathcal{R}. \tag{3.9}$$

Suitable substitution for p in (3.9) implies that

$$[w, \hbar(q)]p\mathcal{K}(t,q) = 0 \text{ for every } p, q, t, w \in \mathcal{R}. \tag{3.10}$$

In view of above equation, we now designed the two subsets as $\mathbf{A} = \{t, w \in \mathcal{R} \mid \mathcal{K}(t,w) = 0\}$ and $\mathbf{B} = \{q \in \mathcal{R} \mid [w, \hbar(q)] = 0, w \in \mathcal{R}\}$. $\mathbf{A}$ and $\mathbf{B}$ are two additive subgroups of $\mathcal{R}$ that we have seen, and $\mathcal{R}$ is the union of these subgroups. Because $(\mathcal{R}, +)$ cannot be the union of two of its proper subgroups, this leads to a contradiction. Hence we arrive at the conclusion that either $\mathcal{R} = \mathbf{A}$ or $\mathcal{R} = \mathbf{B}$. From the first case $\mathcal{K} = 0$. Another case gives the commutativity of R after elementary calculation and using surjectiveness of $\hbar$. □

Theorem 3.6. *Consider $\mathcal{R}$ to be a prime ring with characteristic not two and α be an automorphism on R. If $\mathcal{K}$ is a α-bisemiderivation on $\mathcal{R}$ with associated function $\hbar$ such that $[\mathcal{K}(p,p), p] = 0$ for all $p \in \mathcal{R}$, then either $\mathcal{R}$ is commutative or $\mathcal{K} = 0$.*

Proof. We have given that for each $p \in R$,

$$[\mathcal{K}(p,p), p] = 0. \tag{3.11}$$

Replace $p + q$ for p in (3.11) and use (3.11) to obtain

$$[\mathcal{K}(q,q), p] + 2[\mathcal{K}(p,q), p] + [\mathcal{K}(p,p), q] + 2[\mathcal{K}(p,q), q] = 0 \text{ for each } p, q \in \mathcal{R}. \tag{3.12}$$

Rewrite the above equation by swapping $-q$ for q, we have

$$[\mathcal{K}(q,q), p] - 2[\mathcal{K}(p,q), p] - [\mathcal{K}(p,p), q] + 2[\mathcal{K}(p,q), q] = 0 \text{ for all } p, q \in \mathcal{R}. \tag{3.13}$$

Adding (3.12) and (3.13) to find

$$[\mathcal{K}(q,q), p] + 2[\mathcal{K}(p,q), q] = 0 \text{ for every } p, q \in \mathcal{R}. \tag{3.14}$$

Putting pt for p in (3.14) to get

$$\begin{aligned}&[\mathcal{K}(q,q), p]t + p[\mathcal{K}(q,q), t] + 2\mathcal{K}(p,q)[\hbar(t), q] + 2[\mathcal{K}(p,q), q]\hbar(t)\\ &+2\alpha(p)[\mathcal{K}(t,q), q] + 2[\alpha(p), q]\mathcal{K}(t,q) = 0 \text{ for all } p, q, t \in \mathcal{R}.\end{aligned} \tag{3.15}$$

Particularly reword (3.15) for $t = q$, we have

$$\begin{aligned}&[\mathcal{K}(q,q),p]q + 2\mathcal{K}(p,q)[\hbar(q),q] + 2[\mathcal{K}(p,q),q]\hbar(q)\\ &+2[\alpha(p),q]\mathcal{K}(q,q) = 0 \text{ for each } p,q \in \mathcal{R}.\end{aligned} \tag{3.16}$$

Using surjectivity of $\hbar$ for suitable substitution in the last expression to find

$$[\mathcal{K}(q,q),p]q + 2[\mathcal{K}(p,q),q]q + 2[\alpha(p),q]\mathcal{K}(q,q) = 0 \text{ for each } p,q \in \mathcal{R}. \tag{3.17}$$

From (3.14) and (3.17), we have

$$[\alpha(p),q]\mathcal{K}(q,q) = 0 \text{ for all } p,q \in \mathcal{R}. \tag{3.18}$$

This yields that $[\alpha(p),q]\mathcal{R}\mathcal{K}(q,q) = 0$ for every $p,q \in \mathcal{R}$. Primeness of $\mathcal{R}$ gives us either $\mathcal{K}(q,q) = 0$ or $[\alpha(p),q] = 0$ for every $p,q \in \mathcal{R}$. the last case gives the desired result by applying Lemma 2.1. □

Corollary 3.7. *Consider $\mathcal{R}$ to be a prime ring with characteristic not two and α be an automorphism on R. If $\mathcal{K}$ is a α-bisemiderivation on $\mathcal{R}$ with associated function $\hbar$ such that $[\mathcal{K}(p,p),q] = 0$ for all $q,p \in \mathcal{R}$, then either $\mathcal{R}$ is commutative or $\mathcal{K} = 0$.*

Corollary 3.8. *Consider $\mathcal{R}$ to be a prime ring with characteristic not two and α be an automorphism on R. If $\mathcal{K}$ is a α-bisemiderivation on $\mathcal{R}$ with associated function $\hbar$ such that $[\mathcal{K}(p,p),\alpha(q)] = 0$ for all $p,q \in \mathcal{R}$, then either $\mathcal{R}$ is commutative or $\mathcal{K} = 0$.*

Theorem 3.9. *Consider $\mathcal{R}$ to be a prime ring with characteristic not two and α be an automorphism on R. If $\mathcal{K}$ is a α-bisemiderivation on $\mathcal{R}$ with associated function $\hbar$ such that $\mathcal{K}(p,p) \circ \alpha(q) = 0$ for all $p,q \in \mathcal{R}$, then either $\mathcal{R}$ is commutative or $\mathcal{K} = 0$.*

Proof. Given that

$$\mathcal{K}(p,p) \circ \alpha(q) = 0 \text{ for each } p,q \in \mathcal{R}. \tag{3.19}$$

If we substitute $p + r$ for p in above equation, then we find

$$\mathcal{K}(p,r) \circ \alpha(q) = 0 \text{ for each } p,q,r \in \mathcal{R}. \tag{3.20}$$

Next, put pq for p in (3.20) to obtain

$$\{\mathcal{K}(p,r)\hbar(q) + \alpha(p)\mathcal{K}(q,r)\} \circ \alpha(q) = 0 \text{ for each } p,q,r \in \mathcal{R}. \tag{3.21}$$

This implicit that

$$(\mathcal{K}(p,r)\hbar(q)) \circ \alpha(q) + (\alpha(p)\mathcal{K}(q,r)) \circ \alpha(q) = 0 \text{ for each } p,q,r \in \mathcal{R}. \tag{3.22}$$

We use the operation $'\circ'$ and simplify (3.22) to get

$$\begin{aligned}&(\mathcal{K}(p,r) \circ \alpha(q))\hbar(q) + \mathcal{K}(p,r)[\hbar(q),\alpha(q)] + \alpha(p)(\mathcal{K}(q,r)) \circ \alpha(q))\\ &-[\alpha(p),\alpha(q)]\mathcal{K}(q,r) = 0 \text{ for each } p,q,r \in \mathcal{R}.\end{aligned} \tag{3.23}$$

Making use of (3.20) in the last equation, we arrive at

$$\mathcal{K}(p,r)[\hbar(q),\alpha(q)] = 0 \text{ for each } p,q,r \in \mathcal{R}. \tag{3.24}$$

Since $\hbar$ is surjective, we have

$$\mathcal{K}(p,r)[q,\alpha(q)] = 0 \text{ for each } p,q,r \in \mathcal{R}. \tag{3.25}$$

Last equation yields that $\mathcal{K}(p,r)\mathcal{R}[q,\alpha(q)] = 0$ for every $p,q,r \in \mathcal{R}$. We may designed the two subsets as $\mathbf{A} = \{r,p \in \mathcal{R} \mid \mathcal{K}(p,r) = 0\}$ and $\mathbf{B} = \{q \in \mathcal{R} \mid [q,\alpha(q)] = 0\}$. We observe that $\mathbf{A}$ and $\mathbf{B}$ are two additive subgroups of $\mathcal{R}$ and union of such subgroups comprises $\mathcal{R}$. Which yields a contradiction to the fact that $(\mathcal{R},+)$ can not be the union of two it's proper subgroups. Hence we can conclude that either $\mathcal{K}(p,r) = 0$ or $[q,\alpha(q)] = 0$ for all $p,r,q \in \mathcal{R}$. Second case gives us the commutativity of $\mathcal{R}$ by Lemma 2.1. □

Corollary 3.10. *Suppose that $\mathcal{R}$ is a prime ring and α is an automorphism on R. If d is a α-semiderivation on $\mathcal{R}$ with associated function h such that $\mathrm{d}(p)\alpha(q) = 0$ for all $p, q \in \mathcal{R}$, then either $\mathcal{R}$ is commutative or $\mathrm{d} = 0$.*

Proof. The step wise proof is found in [8]. □

Theorem 3.11. *Suppose that $\mathcal{R}$ is a prime ring and α is an automorphism on R. If $\mathcal{K}$ is a α-bisemiderivation on $\mathcal{R}$ with associated function $\hbar$ such that $\mathcal{K}(pq, r) = \mathcal{K}(p, r)\mathcal{K}(q, r)$ for all $p, q, r \in \mathcal{R}$, then $\mathcal{K} = 0$.*

Proof. This is given that

$$\mathcal{K}(pq, r) = \mathcal{K}(p, r)\mathcal{K}(q, r) \text{ for each } p, q, r \in \mathcal{R}. \tag{3.26}$$

Simplify the left side in above equation, we find

$$\mathcal{K}(pq, r) = \mathcal{K}(p, r)\hbar(q) + \alpha(p)\mathcal{K}(q, r) \text{ for each } p, q, r \in \mathcal{R}. \tag{3.27}$$

Put pt for p in (3.27) to obtain

$$\mathcal{K}(ptq, r) = \mathcal{K}(pt, r)\hbar(q) + \alpha(pt)\mathcal{K}(q, r) \text{ for each } p, q, r, t \in \mathcal{R}. \tag{3.28}$$

Rewrite the right side of (3.28) by using definition of $\mathcal{K}$, we observe

$$\mathcal{K}(ptq, r) = \mathcal{K}(p, r)\mathcal{K}(r, t)\hbar(q) + \alpha(p)\alpha(t)\mathcal{K}(q, r) \text{ for each } p, q, r, t \in \mathcal{R}. \tag{3.29}$$

Reword the left side of (3.28) as

$$\mathcal{K}(ptq, r) = \mathcal{K}(p, r)\mathcal{K}(tq, r) \text{ for each } p, q, r, t \in \mathcal{R}. \tag{3.30}$$

This implies that

$$\begin{aligned} \mathcal{K}(ptq, r) &= \mathcal{K}(p, r)\{\mathcal{K}(t, r)\hbar(q) + \alpha(t)\mathcal{K}(q, r)\} \\ &= \mathcal{K}(p, r)\mathcal{K}(t, r)\hbar(q) + \mathcal{K}(p, r)\alpha(t)\mathcal{K}(q, r) \text{ for each } p, q, r, t \in \mathcal{R}. \end{aligned} \tag{3.31}$$

Compare (3.29) and (3.31) to find

$$(\alpha(p) - \mathcal{K}(p, r))\alpha(t)\mathcal{K}(q, r) = 0 \text{ for each } p, q, r, t \in \mathcal{R}. \tag{3.32}$$

Substitute $\alpha^{-1}(t)$ for t in above last equation to get

$$(\alpha(p) - \mathcal{K}(p, r))t\mathcal{K}(q, r) = 0 \text{ for each } p, q, r, t \in \mathcal{R}. \tag{3.33}$$

Primeness of R facilitate us to split the last expression in two cases: (i) either $\alpha(p) = \mathcal{K}(p, r)$ or (ii) $\mathcal{K}(q, r) = 0$ for every $q, p, r \in R$. Now consider the first case if $\alpha(p) = \mathcal{K}(p, r)$ for every $p, r \in R$. Put ps for p and simplify to obtain $\mathcal{K}(p, r)\hbar(s) = 0$ for each $p, r, s \in R$. Since $\hbar$ is surjective, we have $\mathcal{K}(p, r)s = 0$ for each $p, r, s \in R$. This implies that $\mathcal{K}(p, r) = 0$ for each $p, r \in R$. Hence in both case we get $\mathcal{K} = 0$.

Corollary 3.12. *Suppose that $\mathcal{R}$ is a prime ring and α is an automorphism on R. If d is a α-semiderivation on $\mathcal{R}$ with associated function $\hbar$ such that $\mathrm{d}(pq) = \mathrm{d}(p)\mathrm{d}(q)$ for all $p, q \in \mathcal{R}$, then $\mathrm{d} = 0$.*

References

[1] A. Z. Ansari, N. Rehman and F. Shujat, *On Lie ideals with generalized derivations and power values on prime ring*, Palestine J. Math., **12 (3)** (2023), 194-198.

[2] F. Shujat, *On symmetric generalized bi-semiderivations of prime rings*, Bol. Soc. Paran. Mat., (2023), (preprint).

[3] G. Maksa, *A remark on symmetric biadditive functions having non-negative diagonalization*, Glasnik. Mat. 15 **35** (1980), 279-282.

[4] J. Bergen, *Derivations in prime rings,* Canad. Math. Bull., **26** (1983), 267-270.

[5] J. C. Chang, *On semiderivations of prime rings,* Chinese J. Math., **12** (1984), 255-262.

[6] J. H. Mayne, *Centralizing automorphisms of prime rings*, Canad. Math. Bull., **19 (1)** (1976), 113-115.

[7] J. Luh, *A note on commuting automorphisms of rings,* Amer. Math. Monthly., **77** (1970), 61-62.

[8] K. H., Kim, *On α-semiderivations and commutativity of prime rings*, Gulf J. Math., **8 (1)** (2020), 23-31.

[9] N. Rehman and A. Z. Ansari, *On Lie ideals and symmetric bi-additive maps in rings,* Palestine J. Math., **2 (1)** (2013), 14-21.

[10] S. Huang, *On commutativity of prime rings with left generalized derivations,* Palestine J. Math., **9(1)** (2020), 126-131.

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