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A NEW TYPE OF MINIMAL AND MAXIMAL SETS

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Abstract In this paper, we introduce new classes of sets called maximal S_C-open sets, minimal S_C-open sets and S_C-paraopen sets via S_C-open sets in topological spaces. Then, we investigate some of their fundamental properties.

1 Introduction

The study of semi-open sets and their properties were initiated by N. Levine [3] in 1963. Crossley and Hildebrand [11], gave some properties of semi-closure of a set A (denoted by $scl(A)$), they defined $scl(A)$ as intersection of all semi-closed sets containing the set A. Then, some types of semi-open sets introduced such as S_C-open [2, 13] and S_β-open [7, 12] sets and for some related study see [9, 10]. Furthermore, F. Nakaok and N. Oda [4, 5] introduced the notation of maximal open sets and minimal open sets in topological spaces. In 2016, B. M. Ittanagi et al [8], introduced the notation of paraopen and paraclosed sets. Authors of [8], defined as any open subset U of a topological space X is said to be a paraopen set if it is neither minimal open nor maximal open set. The family of all paraopen sets in a topological space X is denoted by $PaO(X)$. Also, any closed subset F of a topological space X is said to be a paraclosed set if and only if its complement $(X\backslash F)$ is paraopen set and the family of all paraclosed sets in a topological space X is denoted by $PaC(X)$. In addition, B. Alias and M. Haji in [1] introduced and investigated some fundamental properties of maximal semi open and minimal semi open sets. Finally, the purpose of the present paper is to introduce the concept of new classes of S_C-open sets called maximal S_C-open sets and minimal S_C-open sets. We also investigate some of their fundamental properties.

2 Preliminaries

Definition 2.1. [2] A subset of semi open set G is said to be S_C-open if for each $x \in G$, there exists a closed set F such that $x \in F \subseteq G$. The complement of S_C-open set is said to be S_C-closed. As the usual sense, the intersection of all S_C-closed sets of X containing A is called the S_C-closure of A. Also the union of all S_C-open sets of X contained A is called the S_C-interior of A.

Definition 2.2. [2] A subset A of a space X is said to be semi-open if $A \subseteq cl(int(A))$. The complement of semi-open set is said to be semi-closed.

Definition 2.3. A proper nonempty open (resp., semi-open) set U of X is said to be a maximal open [5] (resp., maximal semi-open) [1] set if any open set (resp., semi open) which contains U is X or U.

Definition 2.4. [5] A proper nonempty open set U of a space X is said to be a minimal open set if any open set which contained in U is ϕ or U.

Definition 2.5. [4] A proper nonempty closed subset F of a space X is said to be a maximal closed set if any closed set which contains F is X or F.

Definition 2.6. [1] A proper nonempty semi-closed set B of a space X is said to be a minimal semi-closed if any semi-closed set which contained in B is ϕ or B.

3 Maximal and Minimal S_C-Open Sets.

Definition 3.1. A proper nonempty S_C-open subset U of a topological space X is said to be a minimal S_C-open set if any S_C-open set which is contained in U is ϕ or U.

Definition 3.2. A proper nonempty S_C-open subset U of a topological space X is said to be maximal S_C-open set if any S_C-open set which contains U is X or U.

Definition 3.3. A proper nonempty S_C-closed subset F of a topological space X is said to be a minimal S_C-closed set if any S_C-closed set which is contained in F is ϕ or F.

Definition 3.4. A proper nonempty S_C-closed subset F of a topological space X is said to be maximal S_C-closed set if any S_C-closed set which contains F is X or F.

Proposition 3.5. *Let A be a proper nonempty subset of X. Then A is a maximal S_C-open set if $X\backslash A$ is a minimal S_C -closed set.*

Proof. Necessity, let A be a maximal S_C-open set. Then, $A \subseteq X$ or $A \subseteq A$. Hence $\phi \subseteq X\backslash$A or $X\backslash A \subseteq X\backslash A$. Therefore, by Definition 3.1, $X\backslash A$ is a minimal S_C-closed set. Sufficiency, let $X\backslash A$ be a minimal S_C-closed set, Then $\phi \subseteq X\backslash A$ *or* $X\backslash A \subseteq X\backslash A$. Hence $A \subseteq X$ *or* $A \subseteq A$ which implies that A is a maximal S_C-open set. □

The following example shows that the family of maximal S_C-open sets and maximal semi-open sets are independent in general.

Example 3.6. Let (X, τ) be a topological space such that $X = \{a, b, c, d\}$ and $\tau = \{\emptyset, \{a\}, \{b\}, \{a, b\}, \{a, c, d\}, X\}$.
Clearly $SO(X) = \{\phi, X, \{a\}, \{b\}, \{a,b\}, \{a,c\}, \{a,d\}, \{a,b,c\}, \{a,b,d\}, \{a,c,d\}\}$ and $S_CO(X) = \{\phi, X, \{b\}, \{a,c,d\}\}$. Then $\{a,b,d\}$ is a maximal semi-open set which is not maximal S_C-open, and $\{b\}$ is a maximal S_C-open set which is not maximal semi-open.

Theorem 3.7. *The following statements are true for any topological space X.*

(i) Let A be a maximal S_C-open set and B be a S_C-open set, then, $A \cup B = X$ or $B \subseteq A$.

(ii) Let A and B be maximal S_C-open sets, then $A \cup B = X$ or $B = A$.

(iii) Let F be a minimal S_C-closed set and G be a S_C-closed set, then $F \cap G = \phi$ or $F \subseteq G$.

(iv) Let F and G be minimal S_C-closed sets, then $F \cap G = \phi$ or $F = G$.

Proof.

(i) Let A be a maximal S_C-open set and B be a S_C-open set. If $A \cup B = X$, then we are done. But if $A \cup B \neq X$, then we have to prove that $B \subseteq A$. Now, $A \cup B \neq X$ means $B \subseteq A \cup B$ and $A \subseteq A \cup B$. Therefore, we have $A \subseteq A \cup B$ and A is maximal S_C -open, then by definition $A \cup B = X$ *or* $A \cup B = A$ but $A \cup B \neq X$, thus $A \cup B = A$ which implies $B \subseteq A$.

(ii) Let A and B be maximal S_C-open sets. If $A \cup B = X$, then we are done. But if $A \cup B \neq X$, then we have to prove that $B = A$. Now, $A \cup B \neq X$ means $A \subseteq A \cup B$ and $B \subseteq A \cup B$. Now, $A \subseteq A \cup B$ and A is a maximal S_C-open set, then by definition $A \cup B = X$ or $A \cup B = A$ but $A \cup B \neq X$. Therefore, $A \cup B = A$ which it implies $B \subseteq A$. Similarly if $B \subseteq A \cup B$, we obtain $A \subseteq B$. Therefore, $A = B$.

(iii) Let F be a minimal S_C-closed set and G be a S_C-closed set. If $F \cap G = \phi$, then there is nothing to prove. But if $F \cap G \neq \phi$, then we have to prove that $F \subseteq G$. Now, if $F \cap G \neq \phi$, then $F \cap G \subseteq F$ and $F \cap G \subseteq G$. Since $F \cap G \subset F$ and given that F is minimal S_C-closed, then by definition $F \cap G = F$ or $F \cap G = \phi$, but $F \cap G \neq \phi$, then $F \cap G = F$ which implies $F \subseteq G$.

(iv) Let F and G be two minimal S_C-closed sets. If $F \cap G = \phi$, then there is nothing to prove. But if $F \cap G \neq \phi$, then we have to prove that $F = G$. Now, if $F \cap G \neq \phi$, then $F \cap G \subseteq F$ and $F \cap G \subseteq G$. Since $F \cap G \subseteq F$ and given that F is minimal S_C-closed, then by definition $F \cap G = F$ *or* $F \cap G = \phi$. But $F \cap G \neq \phi$, then $F \cap G = F$ which implies $F \subseteq G$. Similarly if $F \cap G \subseteq G$ and given that G is minimal S_C-closed, then by definition $F \cap G = G$ *or* $F \cap G = \phi$. But $F \cap G \neq \phi$ then $F \cap G = G$ which implies $G \subseteq F$. Thus, $F = G$.

□

Theorem 3.8.

(i) Let A be a maximal S_C-open set and x be an element of $X \backslash A$, then $X \backslash A \subset B$ for any S_C-open set B containing x.

(ii) Let A be a maximal S_C-open set, then either of the following (a) or (b) holds:

a. For each $x \in X \backslash A$ and each S_C-open set B containing x, $B = X$.

b. There exists a S_C-open set B such that $X \backslash A \subseteq B$ and $B \subseteq X$.

(iii) Let A be a maximal S_C-open set, then either of the following (a) and (b) holds:

a. For each $x \in X \backslash A$ and each S_C-open set B containing x, we have $X \backslash A \subseteq B$.

b. There exists a S_C-open set B such that $X \backslash A = B \neq X$.

Proof.

(i) Since $x \in X \backslash A$, we have $B \nsubseteq A$ for any S_C-open set B containing x. Then $A \cup B = X$ by Theorem 3.7. Therefore, $X \backslash A \subseteq B$.

(ii) If (1) does not hold, then there exists an element x of $X \backslash A$ and a S_C-open set B containing x such that $B \subseteq X$. By (i), we have $X \backslash A \subseteq B$.

(iii) If (2) does not hold, then by (i), we have $X \backslash A \subseteq B$ for each $x \in X \backslash A$ and each S_C-open set B containing x. Hence, we have $X \backslash A \subseteq B$.

□

Theorem 3.9. *Let A, B, C be maximal S_C-open sets such that $A \neq B$. If $A \cap B \subseteq C$, then either $A = C$ or $B = C$.*

Proof. Given that $A \cap B \subseteq C$. If $A = C$, then there is nothing to prove. But if $A \neq C$, then we have to prove $B = C$. Using Theorem 3.7, we have $B \cap C = B \cap [C \cap X] = B \cap [C \cap (A \cup B)] = B \cap [(C \cap A) \cup (C \cap B)] = (B \cap C \cap A) \cup (B \cap C \cap B) = (A \cap B) \cup (C \cap B)$ since $A \cap B \subseteq C = (A \cup C) \cap B = X \cap B = B$, since $A \cup C = X$. This implies $B \subseteq C$ also from the definition of maximal S_C-open set it follows that $B = C$. □

Theorem 3.10. *Let A, B, C be maximal S_C-open sets which are different from each other. Then $(A \cap B) \nsubseteq (A \cap C)$.*

Proof. Let $(A \cap B) \subseteq (A \cap C)$. Then, $(A \cap B) \cup (C \cap B) \subseteq (A \cap C) \cup (C \cap B)$. Hence $(A \cup C) \cap B \subseteq C \cap (A \cup B)$. Since by Theorem 3.1, $A \cup C = X$. We have $X \cap B \subseteq C \cap X$ which implies $B \subseteq C$. From the definition of maximal S_C-open set it follows that $B = C$. Contradiction to the fact that A, B and C are different from each other. Therefore, $(A \cap B) \nsubseteq (A \cap C)$. □

Theorem 3.11.

(i) Let F be a minimal S_C-closed set of X. If $x \in F$, then $F \subseteq G$ for any S_C -closed set G containing x.

(ii) Let F be a minimal S_C-closed set of X. Then $F = \cap\{G \mid x \in G \in S_C C(X)\}$ for any element x of F.

Proof.

(i) Let F be a minimal S_C-closed and $G \in S_CC(X)$ such that $F \nsubseteq G$. This implies that $F \cap G \subsetneq F$ and $F \cap G \neq \phi$. But since F is minimal S_C-closed, by Definition 3.3 $F \cap G = F$ which contradicts the relation $F \cap G \subsetneq F$. Therefore, $F \subseteq G$.

(ii) From fact that F is S_C-closed containing x, we have $F \subseteq \cap \{G \mid G \in S_CC(X, x)\} \subseteq F$. Therefore, we have the result.

□

Theorem 3.12.

(i) Let F and F_λ $(\lambda \in \Delta)$ be minimal S_C-closed sets. If $F \subseteq \bigcup_{\lambda\in\Delta} F_\lambda$ then there exists $\lambda \in \Delta$ such that $F = F_\lambda$.

(ii) Let F and $(F_\lambda\ ; \lambda \in \Delta)$ be minimal S_C-closed sets. If $F \neq F_\lambda$ for any $\lambda \in \Delta$, then $\bigcup_{\lambda\in\Delta} F_\lambda \cap F = \phi$.

Proof.

(i) Let F and $(F_\lambda\ ; \lambda \in \Delta)$ be minimal S_C-closed sets with $F \subset \bigcup_{\lambda\in\Delta} F_\lambda$. We have to prove that $F_\lambda \cap F \neq \phi$. If $F_\lambda \cap F = \phi$, then $F_\lambda \subseteq X \backslash F$ and hence $F \subseteq \bigcup_{\lambda\in\Delta} F_\lambda \subseteq X \backslash F$ which is a contradiction. Now as $F_\lambda \cap F \neq \phi$. then $F_\lambda \cap F \subseteq$ F and $F_\lambda \cap F \subseteq F_\lambda$. Since $F_\lambda \cap F \subseteq F$ and given that F is minimal S_C-closed, then by definition $F_\lambda \cap F$ = F or $F_\lambda \cap F = \phi$. But $F_\lambda \cap F \neq \phi$. then $F_\lambda \cap F = F$. which implies $F \subseteq F_\lambda$. Similarly if $F_\lambda \cap F = F_\lambda$, and given that is F_λ minimal S_C-closed, then by definition $F_\lambda \cap F = F_\lambda$ or$F_\lambda \cap F = \phi$. But $F_\lambda \cap F \neq \phi$. Then $F_\lambda \cap F = F_\lambda$ which implies $F \subseteq F_\lambda$. Then $F = F_\lambda$.

(ii) Suppose that $\bigcup_{\lambda\in\Delta} F_\lambda \cap F \neq \phi$ then there exists $\lambda \in \Lambda$ such that $F_\lambda \cap F \neq \phi$. By Theorem 3.7, we have $F = F_\lambda$. Which is a contradiction to the fact $F \neq F_\lambda$. Hence $\bigcup_{\lambda\in\Delta} F_\lambda \cap F = \phi$.

□

Theorem 3.13. *Let A be a maximal S_C-open set. Then either $S_CCl(A) = X$ or $S_CCl(A) = A$.*

Proof. Since A is a maximal S_C-open set, then by Theorem 3.7, we have the following cases:

(i) For each $x \in X \backslash A$ and each S_C-open set H of x, let x be any element of $X\backslash A$ and H be any S_C-open set of x. Since $X\backslash A \neq H$, we have $H \cap A \neq \emptyset$, for any S_C-open set H of x. Hence $X\backslash A \subseteq S_CCl(A)$. Since $X = X \cup (X\backslash A) \subseteq A \cup S_CCl(A) = S_CCl(A) \subseteq X$, we have $S_CCl = X$.

(ii) There exists a S_C-open set H such that $X\backslash A = H \neq X$. Since $X\backslash A = H$ is a S_C-open set, then A is S_C-closed set. Therefore, $A = S_CClA$.

□

Theorem 3.14. *Let A be a maximal S_C-open set, and S be a nonempty subset of $X\backslash A$. Then, $S_CCl(S) = X\backslash A$.*

Proof. Since $\phi \neq S \subseteq X\backslash A$, we have $W \cap S \neq \phi$, for any element x of $X\backslash A$ and S_C-open set W of x by Theorem 3.13. Then $X\backslash A \subseteq S_CCl(A)$. Since $X\backslash A$ is S_C-closed set and $S \subseteq X\backslash A$, we see that $S_CCl(S) \subseteq S_CCl(X\backslash A) = X\backslash A$. Therefore, $X\backslash A = S_CCl(S)$. □

Proposition 3.15. *Let A be a maximal S_C-open set and M be a nonempty subset of X with $A \subseteq M$. Then, $S_CCl(M) = X$.*

Proof. Since $A \subseteq M \subseteq X$, there exists a nonempty subset S of $X\backslash A$ such that $M = A \cup S$. Hence, we have $S_CCL(M) = S_CCl(A \cup S) = S_CCl(A) \cup S_CCl(S) \supseteq (X\backslash A) \cup A = X$ by Theorem 3.14. Therefore, $S_CCl(M) = X$. □

Theorem 3.16. *Let A be a maximal S_C-open set and assume that the subset $X\backslash A$ has at least two elements. Then, $S_CCl(X\backslash \{a\}) = X$, for any element a of $X\backslash A$.*

Proof. Since $A \subseteq (X\backslash \{a\})$ by our assumption, we have the result by Proposition 3.15. □

Theorem 3.17. *Let A be a maximal S_C-open set, and N be a proper subset of X with $A \subseteq N$. Then $S_C Int(N) = A$.*

Proof. If $N = A$, then $S_C Int(N) = S_C Int(A) = A$. Otherwise, $N \neq A$, and hence $A \subseteq N$. It follows $that\ A \subseteq S_C Int(N)$. Since A is maximal S_C-open set, we have also $S_C Int(N) \subseteq A$. Therefore, $S_C Int(N) = A$. □

4 S_C-Paraopen Sets.

Definition 4.1. Any open subset U of a topological space X is said to be a S_C-paraopen set if it is neither minimal S_C- open nor maximal S_C- open set. The family of all S_C-paraopen sets in a topological space X is denoted by $S_C PaO(X)$. Any S_C-closed subset F of a topological space X is said to be a S_C-paraclosed set if and only if its complement $x \in X \backslash F$ is S_C-paraopen set. The family of all S_C-paraclosed sets in a topological space X is denoted by $S_C PaC(X)$.

Note that every S_C-paraopen set is an S_C-open set and every S_C-paraclosed set is a S_C-closed set, but not conversely, which is shown by the following example.

Example 4.2. Consider $X = \{a,\ b,\ c,\ d\}$ with
$\tau = \{\phi,\ X,\ \{a\}, \{b\}, \{c\}, \{a,b\}, \{a,c\}, \{b,c\}, \{a,b,c\}, \{a,b,d\}$
$and\ S_C C\,(X) = \{\phi,\ X,\ \{c\}, \{a,d\}, \{b,d\}, \{a,c,d\}, \{b,c,d\}, \{a,b,d\}$.
Then S_C-$MiO(X) = \{\{c\}\}$,S_C-$MaO(X) = \{\{a,b,\ d\}, \{a,c,d\}, \{b,c,d\}\}$, S_C-$MiC(X) = \{\{a\}, \{b\}\{d\}\}$,$S_C MaC(X) = \{\{b,c,\ d\}, \{a,c,\ d\}, \{a,b,\ d\}\}$, S_C-$PaO(X) = \{\phi,\ \{a,d\}, \{b,d\}, X\}$, $S_C PaC(X) = \{X,\ \{b,c\},\ \{a,c\}, \phi\}$. Here $\{c\}$ is an S_C- open set but not an S_C-paraopen set and $\{a\}, \{b\}\{d\}$ are S_C-closed sets but not S_C-paraclosed set.

Remark 4.3. Union and intersection of S_C-paraopen (resp., S_C-paraclosed) sets need not be a S_C-paraopen (resp. S_C-paraclosed) set.

Example 4.4. In Example 4.2, we have $\{a,d\},\ \{b,d\}$ are S_C- paraopen sets but $\{a,\ d\} \cup \{b,d\} = \{a,b,\ d\}$ and $\{a,\ d\} \cap \{b,d\} = \{d\}$ which are not S_C-paraopen sets. (resp., $\{a,c\},\ \{b,c\}$ are S_C-paraclosed sets but $\{a,c\} \cup \{b,c\} = \{a,b,c\}\ and$
$\{a,c\} \cap \{b,c\} = \{c\}$ which are not S_C-paraclosed sets.

Theorem 4.5. *Let X be a topological space and U be a nonempty S_C-paraopen subset of X. Then there exists a minimal S_C-open set N such that $N \subseteq U$.*

Proof. By definition of minimal S_C-open set, it is obvious that $N \subseteq U$. □

Theorem 4.6. *Let X be a topological space and U be a proper S_C-paraopen subset of X then there exists a maximal S_C-open set M such that $U \subseteq M$.*

Proof. By definition of maximal S_C-open set, it is obvious that $U \subseteq M$. □

Theorem 4.7. *Let X be a topological space.*

(i) Let U be a S_C-paraopen and N be a minimal S_C-open set then $U \cap N = \phi$ or $N \subseteq U$.

(ii) Let U be a S_C-paraopen and M be a maximal S_C-open set then $U \cup M = X$ or $U \subseteq M$.

(iii) Intersection of S_C-paraopen sets is either S_C-paraopen or minimal S_C-open set.

Proof.

(i) Let U be a S_C-paraopen and N be a minimal S_C-open set in X. Then $U \cap N = \phi\ or\ U \cap N \neq \phi.\ If\ U \cap N = \phi$, then there is nothing to prove. Suppose $U \cap N \neq \phi$. Now we have $U \cap N$ is an S_C-open set and $U \cap N \subseteq N$. Hence $N \subseteq U$.

(ii) Let U be a S_C-paraopen and M be a maximal S_C-open set in X. Then $U \cup M = X\ or\ U \cup M \neq X.\ If\ U \cup M = X$, then there is nothing to prove. Suppose $U \cup M \neq X$. Now we have $U \cup M$ is an S_C-open set and $M \subseteq U \cup M$. Since M is maximal S_C-open set, $U \cup M = M$ which implies $U \subseteq M$.

(iii) Let U and V be S_C-paraopen sets in X. If $U \cap V$ is a S_C-paraopen set then there is nothing to prove. Suppose $U \cap V$ is not a S_C-paraopen set. Then by definition, $U \cap V$ is a minimal S_C-open or maximal S_C-open set. If $U \cap V$ is a minimal S_C-open set, then there is nothing to prove. Suppose $U \cap V$ is a maximal S_C-open set. Now $U \cap V \subseteq U$ and $U \cap V \subseteq V$ which contradicts the fact that U and V are S_C-paraopen sets. Therefore, $U \cap V$ is not a maximal S_C-open set. That is $U \cap V$ must be a minimal S_C-open set.

□

Theorem 4.8. *Let X be a topological space. A subset F of X is S_C-paraclosed if and only if it is neither maximal S_C-closed nor minimal S_C-closed set.*

Proof. The proof follows from the definition and fact that the complement of minimal S_C-open set is maximal S_C-closed set and the complement of maximal S_C-open set is minimal S_C-closed set. □

Theorem 4.9. *Let X be a topological space and F be a nonempty S_C-paraclosed subset of, then there exists a minimal S_C-closed set N such that $N \subseteq F$.*

Proof. By definition of minimal S_C-closed set, it is obvious that $N \subseteq F$. □

Theorem 4.10. *Let X be a topological space and F be a proper S_C-paraclosed subset of X, then there exists a maximal S_C-closed set M such that $F \subseteq M$.*

Proof. By definition of maximal S_C-closed set, it is obvious that $F \subseteq M$. □

Theorem 4.11. *Let X be a topological space.*

(i) Let F be paraclosed and N be a minimal S_C-closed sets then $F \cap N = \phi$ or $N \subseteq F$.

(ii) Let F be S_C-paraclosed and M be a maximal S_C-closed sets then $F \cup M = X$ or $F \subseteq M$.

(iii) Intersection of paraclosed sets is either S_C-paraclosed or minimal S_C-closed set.

Proof.

(i) Let F be a S_C-paraclosed and N be a minimal S_C-closed sets in X. Then $(X \setminus F)$ is S_C- paraopen and $(X \setminus N)$ is maximal S_C-open sets in X. Then by Theorem 4.7 we have $(X \setminus F) \cup (X \setminus N) = X$ *or* $(X \setminus F) \subseteq (X \setminus N)$ which implies $X \setminus (F \cap N) = X$ or $N \subseteq F$. Therefore, $F \cap N = \phi$ *or* $N \subseteq F$.

(ii) Let F be a S_C- paraclosed and M be a maximal S_C-closed sets in X. Then $(X \setminus F)$ is S_C-paraopen and $(X \setminus M)$ is minimal S_C- open sets in X. Then by Theorem 4.7 we have $(X \setminus F) \cap (X \setminus M) = \phi$ *or* $(X \setminus M) \subseteq (X \setminus F)$ which implies $X \setminus (F \cup M) = \phi$ *or* $F \subseteq M$. Therefore, $F \cup M = X$ or $F \subseteq M$

(iii) Let U and V be S_C-paraclosed sets in X. If $U \cap V$ is a S_C-paraclosed set then there is nothing to prove. Suppose $U \cap V$ is not a S_C-paraclosed set. Then by definition, $U \cap V$ is a minimal S_C-closed or maximal S_C-closed set. If $U \cap V$ is a minimal S_C-closed set, then there is nothing to prove. Suppose $U \cap V$ is a maximal S_C-closed set. Now $U \subseteq U \cap V$ and $V \subseteq U \cap V$ which contradicts the fact that U and V are S_C- paraclosed sets. Therefore, $U \cap V$ is not a maximal S_C-closed set. That is $U \cap V$ must be a minimal S_C-closed set.

□

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