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DIVISOR EQUITABLE EDGE DOMINATION IN FUZZY GRAPHS

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Abstract *Recent research has revealed that resource allocation issues in parallel computing systems can be understood as edge domination issues in graphs. Network routing issues and encoding theory are two other uses of edge domination. In this article, we introduced the divisor edge domination in fuzzy graphs. We studied the properties of minimal divisor edge dominating set. Also, we obtained the equivalent condition for a set to be minimal divisor equitable edge domination set. Further, we investigated characteristics of divisor equitable independent edge dominating sets and connected divisor equitable edge dominating sets.*

1 Introduction

Different mathematicians have made numerous rediscoveries while tackling issues in their respective fields. These issues gave rise to other branches of graph theory, including graph decomposition, graph labelling, graph domination and graph colouring etc. The notion of domination dates back to the 1850s, when some enthusiastic thinkers speculated about the bare minimum number of queens that should be put on an 8×8 *chessboard in order for every square to be either attacked by a queen or occupied by a queen. Berge [1] and Ore [2] formalised the theory of domination in 1958 and 1962, respectively. The first comprehensive title "Fundamentals of Domination in Graphs" by Haynes et al. [3] contains the noteworthy discussion on domination and subset related problems such as independence, covering, matching, etc. Numerous areas of social sciences, engineering and mathematics are closely related to the theory of dominance. In facility location issues, if the number of facilities (such as hospitals and fire stations) is limited and one tries to reduce the distance a person must travel to reach the closest facility, dominance occurs. Problems including identifying sets of representatives, electrical networks or monitoring communication and land surveying (e.g., reducing the number of locations a surveyor must stand in order to obtain accurate height measurements for an entire region) all involve concepts from domination.*

In 1965, Zadeh [4] first developed the idea of fuzzy sets. Rosenfeld [5] introduced fuzzy graphs in 1975 and basic concept of fuzzy graph was first presented by Kauffmann [6] in 1973. In addition, Kauffmann [6] created various fuzzy analogous graph theoretic notions, such as the bridge, tree and cut vertex. Fuzzy graphs have many uses, including modeling real-time systems where the degree of information present changes with varying degrees of accuracy. In 1987, S. R. Jayaram [7] discussed the edge domination number in graph. Anwar Alwardi and N. D. Soner [8] in 2013 discussed the notion of equitable edge domination in graphs. The idea of domination utilising effective edges in fuzzy graphs was first forth by A. Somasundaram and S. Somasundaram [9]. Domination in fuzzy graphs using strong arc was studied by A. Nagoorgani and V. T. Chandrasekaran, [10]. Dharmalingam and Rani [11] created the idea of equitable domination in fuzzy graphs. Following [7] Equitable edge domination in fuzzy graphs was first proposed by C. Gurubaran and A. Prasanna [12]. G. B. Priyanka et.al [13] developed the new *idea in domination as divisor equitable domination in fuzzy graphs.*

In this article, we introduced the concept of divisor equitable edge domination(dEED) and divisor equitable independent edge domination(dEIED) in fuzzy graphs. Also connected divisor equitable edge domination(CdEED) in fuzzy graphs are developed. We studied the properties of divisor equitable edge dominating set(dEED-set), divisor equitable independent edge dominating set(dEIED-set), connected divisor equitable edge dominating set(CdEED-set) of fuzzy graphs.

2 Preliminaries

Definition 2.1. [14] A fuzzy graph $\mathscr{Q} = (\psi, \varrho)$ on a graph $\mathscr{Q}^* = (\mathscr{V}, \mathscr{E})$ is a pair of functions $\psi : \mathscr{V} \to [0,1]$ and $\varrho : \mathscr{V} \times \mathscr{V} \to [0,1]$, where ψ is a fuzzy subset of $\mathscr{V}(\neq \emptyset)$ and ϱ is a symmetric relation on ψ such that $\forall\ m, r$ in $\mathscr{V}$ the relation $\varrho(m,r) \leqslant \psi(m) \wedge \psi(r)$ is satisfied.

Definition 2.2. [15] The order q and size p of $\mathscr{Q} = (\psi, \varrho)$ are described as $q = \sum\limits_{mr \in \mathscr{E}} \varrho(m,r)$ and $p = \sum\limits_{m \in \mathscr{V}} \psi(m)$.

Definition 2.3. [12] The neighbourhood degree of a vertex s is described as the sum of the weights of the edges adjacent to s and is indicated by $d_{\mathbb{N}}(s)$, the minimum neighbourhood degree of $\mathscr{Q}$ is $\delta_{\mathbb{N}}(\mathscr{Q}) = min\{d_{\mathbb{N}}(s) : s \in \mathscr{V}\}$ and the maximum neighbourhood degree of $\mathscr{Q}$ is $\Delta_{\mathbb{N}}(\mathscr{Q}) = max\{d_{\mathbb{N}}(s) : s \in \mathscr{V}\}$.

Definition 2.4. [16] A path $\mathbb{P}$ of length n is a sequence of distinct vertices $f_0, f_1, \ldots, f_n$ such that $\varrho(f_{i-1}, f_i) > 0, i = 1, 2, \ldots, n$ and the degree of membership of a weakest arc is defined as its strength.

Definition 2.5. [11] The strength of the connectedness between two vertices f, r in $\mathscr{Q}$ is $\varrho^{\infty}(f,r) = sup\,\{\varrho^l(f,r) : l = 1, 2, 3...\}$ where $\varrho^l(f,r) = sup\,\{\varrho(f,f_1) \wedge \varrho(f_1,f_2) \wedge \varrho(f_2,f_3) \wedge ...\varrho(f_{l-1},r)\}$.

Definition 2.6. [17] An arc (m,r) in $\mathscr{Q} = (\psi, \varrho)$ is defined as strong if $\varrho^{\infty}(m,r) = \varrho(m,r)$. Then m, r are described as strong neighbours.

Definition 2.7. [17] The strong neighbourhood of the node m is characterised as $\mathbb{N}_S(m) = \{w \in \mathscr{V} \mid (m,w)$ is a strong arc $\}$.

Definition 2.8. [9] For a fuzzy graph $\mathscr{Q} = (\psi, \varrho)$ on a graph $\mathscr{Q}^* = (\mathscr{V}, \mathscr{E})$, a subset B of $\mathscr{V}$ is known as fuzzy dominating set(fD-set) in $\mathscr{Q}$ if $\forall$ vertex r in $\mathscr{V} \backslash B$, $\exists$ m $\in B$ such that m dominates r. The domination number of the minimum cardinality taken over all D-sets in $\mathscr{Q}$ and is indicated by $\gamma(\mathscr{Q})$ or simply γ. A fuzzy dominating set(fD-set) B of $\mathscr{Q}$ is termed as minimal fD-set of $\mathscr{Q}$, if for every node w $\in B$, $B \backslash \{w\}$ is not a fD-set.

Definition 2.9. [11] Let $\mathscr{Q} = (\psi, \varrho)$ be a fuzzy graph on a graph $\mathscr{Q}^* = (\mathscr{V}, \mathscr{E})$. A subset B of $\mathscr{V}$ is said to be fuzzy equitable dominating set(fED-set) if $\forall\ r \in \mathscr{V} \backslash B$ $\exists$ a vertex m $\in B$ $\ni$ mr $\in \mathscr{E}(\mathscr{Q})$ and $|d_{\mathscr{Q}}(m) - d_{\mathscr{Q}}(r)| \leq 1$. The minimum cardinality of such a D-set is indicated by γ_{fe} and is termed as the fuzzy equitable domination number of $\mathscr{Q}$.

Definition 2.10. [12] Let $\mathscr{Q} = (\psi, \varrho)$ be a fuzzy graph on a graph $\mathscr{Q}^* = (\mathscr{V}, \mathscr{E})$. Then the degree of $e \in \mathscr{E}$ is termed as $d_{\mathscr{Q}}(e) = d_{\mathscr{Q}}(l) + d_{\mathscr{Q}}(s) - 2\varrho(l,s)$ for each edge $e(= ls) \in \mathscr{E}$, since $(l,s) > 0$ for $ls \in \mathscr{E}$, $\varrho(l,s) = 0$ for $ls \notin \mathscr{E}$.

Definition 2.11. [12] A set $B \subseteq \mathscr{E}$ of strong edges is called equitable edge dominating set(EED-set) of $\mathscr{Q}$ if for all strong edge z not in B is adjacent to atleast one edge $z' \in B \ni |d_{\mathscr{Q}}(z) - d_{\mathscr{Q}}(z')| \leq 1$. The minimum cardinality of such EED-set is indicated by $\gamma'_{fe}(\mathscr{Q})$ and is known as equitable edge dominating number of $\mathscr{Q}$. EED is also known as fuzzy equitable edge domination. B is minimal EED-set if for any edge $z \in B$, $B \backslash \{z\}$ is not an EED-set of $\mathscr{Q}$.

Definition 2.12. [12] Let $m \in \mathscr{E}$. The fuzzy equitable edge neighbourhood of m indicated by $\mathbb{N}_{fee}(m)$ and described as $\mathbb{N}_{fee}(m) = \{w \in \mathscr{E} : w$ is a strong arc adjacent to $m, |d_{\mathscr{Q}}(w) - d_{\mathscr{Q}}(m)| \leq 1\}$. The cardinality of $\mathbb{N}_{fee}(m)$ is termed as fuzzy equitable edge degree of m and denoted by $d^{fee}_{\mathscr{Q}}(m)$. The maximum equitable degree of edge in $\mathscr{Q}$ are $\Delta'_{fee}(\mathscr{Q}) = \max\limits_{h \in \mathscr{E}} |d^{fee}_{\mathscr{Q}}(h)|$ and the minimum equitable degree of edge in $\mathscr{Q}$ are $\delta'_{fee}(\mathscr{Q}) = \min\limits_{h \in \mathscr{E}} |d^{fee}_{\mathscr{Q}}(h)|$.

Definition 2.13. [18] If a vertex $m \in \mathscr{V}$ be such that $|d_{\mathscr{Q}}(m) - d_{\mathscr{Q}}(r)| \geq 2 \; \forall \; r \in \mathbb{N}(m)$, then m is in every fED-set and the points are said to be fuzzy equitable isolates(fEI). The collection of all fEI is identified as I_{fe}.

Definition 2.14. [12] If a edge $e_1 \in \mathscr{E}$ be such that $|d_{\mathscr{Q}}(m) - d_{\mathscr{Q}}(r)| \geq 2 \; \forall$ edge $r \in \mathbb{N}(e_1)$, then e_1 is in every fEED-set and the edges are said to be fuzzy equitable edge isolates. The collection of all fuzzy equitable edge isolates is identified as I_{fee}.

Definition 2.15. [12] An ED-set B is described as an equitable independent edge dominating set(EIED-set) if no two strong edges in B are equitable adjacent.

Definition 2.16. [12] An EED-set B of $\mathscr{Q}$ is connected equitable edge dominating set(CEED-set) if the induced subgraph $\langle B \rangle$ is connected. The minimum cardinality of such CEED-set is indicated by γ'_{cfee} and is known as connected equitable edge dominating number $\mathscr{Q}$.

Definition 2.17. [13] Let $\mathscr{Q} = (\psi, \varrho)$ be a fuzzy graph on a graph $\mathscr{Q}^* = (\mathscr{V}, \mathscr{E})$. A subset B of $\mathscr{V}$ is known as divisor equitable dominating set(dED-set) if $\forall \; w \in \mathscr{V} \backslash B \; \exists$ a vertex $m \in B \ni mw \in \mathscr{E}(\mathscr{Q})$ and $gcd(d_{\mathscr{Q}}(m), d_{\mathscr{Q}}(w)) \leq 1$. γ_{de} is to determine the minimum cardinality of such a D-set and is termed as the divisor equitable domination number of $\mathscr{Q}$.

3 Divisor equitable edge domination

Definition 3.1. A set $B(\subseteq \mathscr{E})$ of strong edges is called divisor equitable edge dominating set (dEED-set) of $\mathscr{Q}$ if for every strong edge z not in B is adjacent to atleast one edge $z' \in B \ni gcd(d_{\mathscr{Q}}(z), d_{\mathscr{Q}}(z')) \leq 1$. $\gamma'_{dee}(\mathscr{Q})$ is to determine the minimum cardinality of such a dEED-set and is known a dEED number $\mathscr{Q}$.

Example 3.2. Let $\mathscr{Q} = (\psi, \varrho)$ be a fuzzy graph and described as follows.

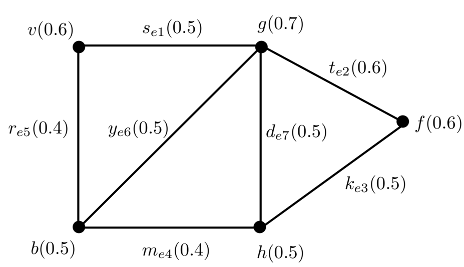

Figure 1. Example of fEED-set and dEED-set

Here $B = \{r_{e5}, d_{e7}\}$ is both fEED-set and dEED-set.

Example 3.3. Let $\mathscr{Q} = (\psi, \varrho)$ be a fuzzy graph and described as follows.

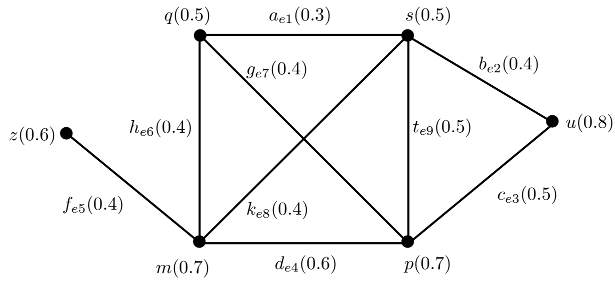

Figure 2. Example of fEED-set which is not dEED-set

Here $B = \{h_{e6}, t_{e9}\}$ is a fEED-set, but not dEED-set as $gcd(d_{\mathscr{Q}}(h_{e6}), d_{\mathscr{Q}}(a_{e1})) = 2.1 > 1$.

Example 3.4. Let $\mathscr{Q} = (\psi, \varrho)$ be a fuzzy graph and described as follows.

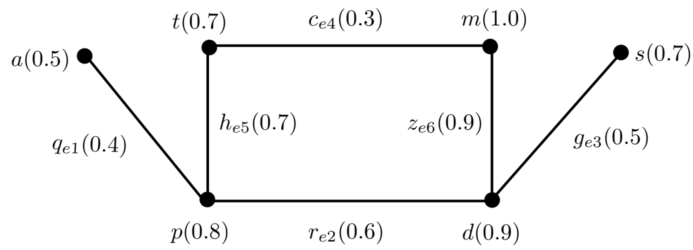

Figure 3. Example of dEED-set which is not fEED-set

Here $B = \{r_{e2}, c_{e4}\}$ is a dEED-set which is not fEED-set as $|d_{\mathscr{Q}}(r_{e2}) - d_{\mathscr{Q}}(q_{e1})| = 1.2 > 1$.
□

Definition 3.5. Let $f \in \mathscr{E}$. The divisor equitable edge neighbourhood of f indicated by $\mathbb{N}_{dee}(f)$ and described as $\mathbb{N}_{dee}(f) = \{r \in \mathscr{E} : r \text{ is a strong arc adjacent to } f, |d_{\mathscr{Q}}(r) - d_{\mathscr{Q}}(f)| \leq 1\}$. The cardinality of $\mathbb{N}_{dee}(f)$ is termed as divisor equitable edge degree of f and denoted by $d^{dee}_{\mathscr{Q}}(f)$.

The maximum divisor equitable degree of edge in $\mathscr{Q}$ is $\Delta'_{dee}(\mathscr{Q}) = \max_{f \in \mathscr{E}} |d^{dee}_{\mathscr{Q}}(f)|$ and the minimum divisor equitable degree of edge in $\mathscr{Q}$ is $\delta'_{dee}(\mathscr{Q}) = \min_{f \in \mathscr{E}} |d^{dee}_{\mathscr{Q}}(f)|$.

Theorem 3.6. *A dEED-set B is minimal if and only if for each edge $m \in B$ one of the below statements holds:*

(i) $\mathbb{N}_{dee}(m) \cap B = \emptyset$.

(ii) there exists a strong edge $h \in \mathscr{E} \backslash B \ni \mathbb{N}_{dee}(h) \cap B = \{m\}$.

Proof. Assume that B is a minimal dEED-set. Take that (i) and (ii) do not hold. Then for some $m \in B \; \exists$ a strong edge $h \in \mathbb{N}_{dee}(m) \cap B$ and for all strong edge $e \in \mathscr{E} \backslash B$, $\mathbb{N}_{dee}(e) \cap B \neq \{m\}$. So $B \backslash \{m\}$ is a dEED-set, a contradiction to the minimality of B. Hence (i) and (ii) holds.

Conversely, take for all edges in $\mathscr{Q}$ be strong edges $m \in B$ one of the statements holds. Assume B is not minimal, then $\exists \; m \in B$ such that $B \backslash \{m\}$ is a dEED-set. Then there exists a strong edge $h \in B \backslash \{m\} \ni h \in \mathbb{N}_{dee}(m)$. So m does not satisfy (i). Hence m must satisfy (ii), for that since $B \backslash \{m\}$ is a dEED-set $\exists$ a strong edge $m' \in B \backslash \{m\} \ni m'$ is divisor equitable adjacent to h. Thus $m' \in \mathbb{N}_{dee}(h) \cap B$ and $m' \neq m$, a contradiction to $\mathbb{N}_{dee}(h) \cap B = \{m\}$. Therefore B is minimal dEED-set. □

Theorem 3.7. *Let $\mathscr{Q} = (\psi, \varrho)$ be a fuzzy graph, then $\gamma'(\mathscr{Q}) \leq \gamma'_{dee}(\mathscr{Q})$.*

Proof. It is trivial. □

Definition 3.8. Let an edge $e_2 \in \mathscr{E}$ be such that $gcd(d_{\mathscr{Q}}(e_2), d_{\mathscr{Q}}(r)) \geq 2$ for all $r \in \mathbb{N}_{dee}(e_2)$. Then e_2 is in every divisor equitable dominating set and the edges are said to be divisor equitable isolate edges. The collection of all divisor equitable isolates is identified as I_{dee}.

Theorem 3.9. *Let $\mathscr{Q}$ be a fuzzy graph without any equitable isolated edges and B be a minimal dEED-set of $\mathscr{Q}$. Then $\mathscr{E}\backslash B$ is dEED-set.*

Proof. Assume B be minimal dEED-set of $\mathscr{Q}$ and $\mathscr{E}\backslash B$ is not an dEED-set. Then $\exists$ an edge $w \ni w \in B$ is not dE adjacent to any strong edge in $\mathscr{E}\backslash B$. Since $\mathscr{Q}$ has no dE isolated edges, we have w is dE dominated by at least one edge in $B\backslash\{w\}$. So $B\backslash\{w\}$ dEED-set, a contradiction to the minimality of B. Hence $\mathscr{E}\backslash B$ is dEED-set. □

Theorem 3.10. *If $\mathscr{Q} = (\psi, \varrho)$ is a bi- regular or regular with $d_{\mathscr{Q}}(s) \leq 1$ for all $s \in \mathscr{E}$, then $\gamma'(\mathscr{Q}) = \gamma'_{dee}(\mathscr{Q})$.*

Proof. Assume $\mathscr{Q}$ is a regular, it has all edges in $\mathscr{Q}$ are strong edges, with the same degree say r and B be a minimal ED-set of $\mathscr{Q}$. So $\gamma'(\mathscr{Q}) = |B|$, take $a \in \mathscr{E}\backslash B$. As B is an ED-set, there is a strong edge $a' \in B$ and aa' are adjacent. Also $d_{\mathscr{Q}}(a) = d_{\mathscr{Q}}(a') = r$. Hence $gcd(d_{\mathscr{Q}}(a), d_{\mathscr{Q}}(a')) = r < 1$ and B is a dEED-set of $\mathscr{Q}$, So $\gamma'_{dee}(\mathscr{Q}) \leq |B| = \gamma'(\mathscr{Q})$. But $\gamma'(\mathscr{Q}) \leq \gamma'_{dee}(\mathscr{Q})$. Thus $\gamma'(\mathscr{Q}) = \gamma'_{dee}(\mathscr{Q})$. □

Theorem 3.11. *For a fuzzy graph $\mathscr{Q} = (\psi, \varrho)$, we have $\gamma'_{dee}(\mathscr{Q}) \leq q - \Delta'_{dee}(\mathscr{Q})$.*

Proof. Assume w be a strong edge in $\mathscr{Q}$ of divisor equitable degree $\Delta'_{dee}(\mathscr{Q})$. Clearly $\mathscr{E}(\mathscr{Q})\backslash\mathbb{N}_{dee}(w)$ is a dEED-set. Thus $\gamma'_{dee}(\mathscr{Q}) \leq q - \Delta'_{dee}(\mathscr{Q})$. □

Corollary 3.12. *For a fuzzy graph $\mathscr{Q} = (\psi, \varrho)$, we have $\gamma'_{dee}(\mathscr{Q}) \leq q - \delta'_{dee}(\mathscr{Q})$.*

Theorem 3.13. *Let $\mathscr{Q}$ be a fuzzy star graph. Then $\gamma'_{dee}(\mathscr{Q}) = \min\{\varrho(s_i); s_i \in \mathscr{E}\}$.*

Proof. Consider $\mathscr{Q}$ be fuzzy star graph and every edges will be a strong edges and all edges will incident to a vertex k, as all edge will dominate remaining edges of a fuzzy star $\mathscr{Q}$. So $\gamma'_{dee}(\mathscr{Q}) = \min\{\varrho(s_i); s_i \in \mathscr{E}\}$. □

4 Divisor equitable independent edge domination

Definition 4.1. An edge dominating set B is known as divisor equitable independent edge dominating set(dEIED-set) if no two strong edges in B are divisor equitable(dE) adjacent. The dEIED-number $\gamma_{dei}(\mathscr{Q})$ is the minimum cardinality taken over all dEIED-set of $\mathscr{Q}$.

Definition 4.2. The divisor edge independence(dEI) number $\beta'_{dei}(\mathscr{Q})$ is defined to be the number of edges in a maximum dEI-set of edges of $\mathscr{Q}$.

Example 4.3. Let $\mathscr{Q} = (\psi, \varrho)$ be a fuzzy graph and described as follows.

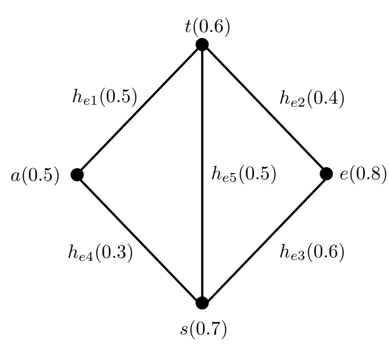

Figure 4. Example of dEIED-set

Here $B = \{h_{e1}, h_{e3}\}$ is a dEIED-set, $\gamma'_{dei}(\mathscr{Q}) = 1.1$.

Theorem 4.4. *An dEI-set B is maximal dEI-set iff B is dEIE-set and dEED-set of $\mathscr{Q}$.*

Proof. Consider a dEI-set B is maximal. Then for every edge $h \in \mathscr{E}\backslash B$, the set $B \cup \{h\}$ is not dEI-set, that is $\forall$ edge $h \in \mathscr{E}\backslash B$, there is an strong edge $s \in B \ni h$ is dE adjacent to s. So B is dEED-set. Thus B is both dEIE-set and dEED-set of $\mathscr{Q}$.

Conversely, consider B is both dEIE-set and dEED-set of B and B is not maximal dEI-set. Then $\exists$ an edge $h \in \mathscr{E}\backslash B \ni B \cup \{h\}$ is dEI which implies there is no strong edge in B, dE adjacent to h. So B is not dEED-set, a contradiction. Hence B is maximal dEI-set. □

Theorem 4.5. *For any $\gamma'_{dee}-$ set B of a fuzzy graph $\mathscr{Q} = (\psi, \varrho)$, $| \mathscr{E}\backslash B | \leq \sum_{(h\in B)} d_{\mathscr{Q}}(h)$ and the equality conditions holds iff*

(i) B is dEI,

(ii) for all edge $h \in \mathscr{E}\backslash B$ $\exists$ only one strong edge $s \in B \ni \mathbb{N}_{dee}(h) \cap B = \{s\}$.

Proof. If every edge in $\mathscr{E}\backslash B$ is dE adjacent to at least one edge of B. Thus each edge in $\mathscr{E}\backslash B$ contributes at least one to the sum of the divisor equitable degrees of the edges of B. Thus

$$| \mathscr{E}\backslash B | \leq \sum_{(h\in B)} d_{\mathscr{Q}}(h).$$

Let $|\mathscr{E}\backslash B| = \sum_{(h\in B)} d_{\mathscr{Q}}(h)$ and B is not dEI. Then each edge $\mathscr{E}\backslash B$ is counted in $\sum_{(h\in B)} d_{\mathscr{Q}}(h)$. If h_1 and h_2 are dE adjacent, then h_1 is counted in $d_{\mathscr{Q}}(h_1)$ and vice versa for any $h_1, h_2 \in B$. So the sum exceeds $|\mathscr{E}\backslash B|$ be at least two contrary to the assumption. Thus B must be dEI.

Consider (ii) is not true. So $|\mathbb{N}_{dee}(h) \cap B| \geq 2$ for some edge $h \in \mathscr{E}\backslash B$. Consider h_1 and h_2 belong to $\mathbb{N}_{dee}(h) \cap B$, so $\sum_{(h\in B)} d_{\mathscr{Q}}(h)$ exceeds $\mathscr{E}\backslash B$ by atleast one, as h counted twice once in $d_{\mathscr{Q}}(h_1)$ and once in $d_{\mathscr{Q}}(h_2)$. So, if there is equality, then both (i) and (ii) must be true. Evidently, the reverse part is true. □

Theorem 4.6. *Let $\mathscr{Q}$ be a fuzzy graph without dE isolated edges. Then $\mathscr{E}\backslash B$ is dEED- set for all minimal dEED- set B of $\mathscr{Q}$.*

Proof. Assume $\mathscr{E}\backslash B$ is not dEED-set and B be minimal dEED-set of $\mathscr{Q}$, $\exists$ an strong edge $h \in B$ $\ni h$ is not dE adjacent to any edge in $\mathscr{E}\backslash B$. Since, $\mathscr{Q}$ does not have any dE isolated edges, h is dE adjacent to at least one strong edge in $B\backslash\{h\}$ and hence $B\backslash\{h\}$ is dEED-set, a contradiction to the minimal dEED-set B of $\mathscr{Q}$. Hence $\mathscr{E}\backslash B$ is dEED-set of $\mathscr{Q}$. □

5 Connected divisor equitable edge domination

Definition 5.1. A dEED-set B of $\mathscr{Q}$ is connected divisor equitable edge dominating set(CdEED-set) if the induced subgraph $\langle B \rangle$ is connected. The CdEED-number γ'_{cdee} of $\mathscr{Q}$ is the minimum cardinality of a CdEED-set.

Example 5.2. Let $\mathscr{Q} = (\psi, \varrho)$ be a fuzzy graph and described as follows.

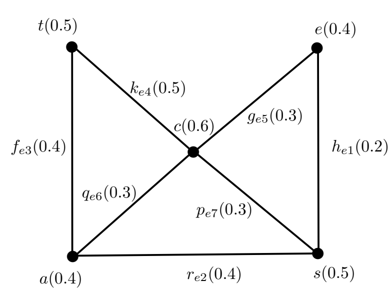

Figure 5. Example of CdEED-set

Here $B = \{r_{e2}, f_{e3}, p_{e7}\}$ is a CdEED-set with $\gamma'_{cdee}(\mathscr{Q}) = 1.1$. □

Remark 5.3. Every CdEED-set is dEED-set.

Theorem 5.4. *For any $\mathscr{Q} = (\psi, \varrho)$,we have $\gamma'(\mathscr{Q}) \leq \gamma'_{dee}(\mathscr{Q}) \leq \gamma'_{cdee}(\mathscr{Q})$.*

Proof. This is obviously true. □

Theorem 5.5. *For any connected fuzzy graph $\mathscr{Q}$ of order p, we have $\gamma'_{cdee}(\mathscr{Q}) \leq p - \Delta'_{dee}(\mathscr{Q})$.*

Proof. Consider F be a spanning tree of $\mathscr{Q}$ and t is an end vertex of F. So its strong edges are incident with t form a CdEED-set of $\mathscr{Q}$. Thus $\gamma'_{cdee}(\mathscr{Q}) \leq p - \Delta'_{dee}(\mathscr{Q})$. □

Corollary 5.6. *For any connected fuzzy graph $\mathscr{Q}$ of order p, we have $\gamma'_{cdee}(\mathscr{Q}) \leq p - \delta'_{dee}(\mathscr{Q})$.*

Theorem 5.7. *A CdEED-set B of $\mathscr{Q}$ is minimal iff for each edge $m \in B$ one of the below statements holds :*

(i) $\mathbb{N}_{dee}(m) \cap B = \emptyset$,

(ii) $\exists$ *a strong edge* $h \in \mathscr{E} \backslash B \ni \mathbb{N}_{dee}(h) \cap B = \{m\}$.

Proof. Let B be a minimal CdEED-set. Then for $m \in B$, $B_m = B \backslash \{m\}$ is not a CdEED-set and so $\exists\, h \in \mathscr{E} \backslash B_m \ni h$ is not dominated by any strong edges of B, if $h = m$ then $\mathbb{N}_{dee}(m) \cap B = \emptyset$ and if $h \neq m$ as there exists an strong edge $h \in \mathscr{E} \backslash B \ni \mathbb{N}_{dee}(h) \cap B = \{m\}$ and h is a strong edge. Obviously, the reverse part is true. □

Theorem 5.8. *Let $\mathscr{Q}$ be without divisor isolated edges with p be order and q be the size of $\mathscr{Q}$, then $\frac{q}{\Delta'_{dee}(\mathscr{Q})+1} \leq \gamma'_{cdee}(\mathscr{Q})$.*

Proof. Let F be a CdEED-set of $\mathscr{Q}$. Since $|F|\ \Delta'_{dee}(\mathscr{Q}) \leq \sum\limits_{r \in F} d_{\mathscr{Q}}(r) = \sum\limits_{r \in F} |\mathbb{N}_{dee}(r)\ |$
$\leq |\ \bigcup\limits_{(r \in F)} \mathbb{N}_{dee}(r)|$
$\leq |\mathscr{E} \backslash F|$
$\leq q - |F|$, which implies $|F|\Delta'_{dee}(\mathscr{Q}) + |F| \leq q$. Hence $\frac{q}{\Delta'_{dee}(\mathscr{Q})+1} \leq \gamma'_{cdee}(\mathscr{Q})$. □

Theorem 5.9. *For any $\mathscr{Q} = (\psi, \varrho)$, we have $\gamma'_{cdee}(\mathscr{Q}) \leq q - \Delta'_{dee}(\mathscr{Q})$.*

6 Conclusion remarks

In recent years, fuzzy graph theory has found widespread usage in modern science and technology. The equitable dominating sets in fuzzy graphs are natural models for facility location problems in operational research. In this article, we have studied the notion of divisor equitable edge domination in fuzzy graphs. We have obtained some properties of connected divisor equitable edge domination and divisor equitable independent edge domination with illustrations. Communication and electrical network issues can be tracked using the characteristics of divisor equitable edge domination that are described in this article. These findings can be applied to intuitionistic fuzzy graphs and pythagorean fuzzy graphs using the techniques discussed here .

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