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Blow up of solutions for a logarithmic Petrovsky equation with variable exponent

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Abstract: In this article we consider the following a logarithmic Petrovsky equations with variable exponents:

$$u_{tt} + \Delta^2 u + |u_t|^{p(x)-2} u_t = |u|^{q(x)-2} u \ln u.$$

We proved that under suitable conditions on the initial data, a finite-time blow up result for solutions with negative initial energy.

1 Introduction

Let be Ω a bounded domain in R^n with a smooth boundary $\partial\Omega$. We study the following boundary value problem:

$$\begin{cases} u_{tt} + \Delta^2 u + |u_t|^{p(x)-2} u_t = |u|^{q(x)-2} u \ln u, & \Omega \times (0,T), \\ u(x,t) = \frac{\partial}{\partial v} u(x,t) = 0, & \partial\Omega \times (0,T), \\ u(x,0) = u_0(x), \ u_t(x,0) = u_1(x), & x \in \Omega, \end{cases} \tag{1.1}$$

here $p(\cdot)$ and $q(\cdot)$ are measurable functions on Ω satisfying

$$\begin{cases} 2 \le p_1 \le p(x) \le p_2 \le p_* \\ 2 \le q_1 \le q(x) \le q_2 \le q_* \end{cases} \tag{1.2}$$

here

$$\begin{cases} p_1 = ess \ \inf_{x\in\Omega} p(x), \ p_2 = ess \sup_{x\in\Omega} \ p(x) \\ q_1 = ess \ \inf_{x\in\Omega} q(x), \ q_2 = ess \sup_{x\in\Omega} \ q(x) \end{cases} \tag{1.3}$$

and

$$\begin{cases} 2 < p_* < \infty & \text{if } n \le 4, \\ 2 < p_* < \frac{2n}{n-4} & \text{if } n > 4, \end{cases} \tag{1.4}$$

also satisfying the log-Hölder continuity condition:

$$|p(x) - p(y)| \le \frac{A}{\ln\left|\frac{1}{x-y}\right|}, \tag{1.5}$$

for all $x, y \in \Omega$ with $|x-y| < \delta$, $0 < \delta < 1$, $A > 0$.

In recent years, a great deal of mathematical effort has been paid to the study of mathematical models of parabolic, elliptic and hyperbolic equaions with variable exponents of nonlinearity. Technological advancements brought many new real-world problems such as flows of electro-rheological fluids, fluids with temperature dependent viscocity, filtration processes through a porous media, image processing and thermorheological fluids and others, which required modeling with non-standard [6, 20].

Messaoudi et al. [11] studied the following wave equation

$$u_{tt} - \Delta u + a |u_t|^{m(\cdot)-2} u_t = b |u|^{p(\cdot)-2},$$

they proved the local existence and the global nonexistence.

Tebba et al. [21] investigated a nonlinear damped wave equation given by:

$$u_{tt} - \Delta u - \Delta u_{tt} + a |u_t|^{m(x)-2} u_t = b |u|^{p(x)-2} u,$$

under appropriate assumptions on the variable exponents, they demonstrated the existence of a unique weak solution using the Faedo-Galerkin method. They also proved the finite time blow-up of solutions.

Antontsev et al. [1] worked the nonlinear Petrovsky equation as follows:

$$u_{tt} + \Delta^2 u - \Delta u_t + |u_t|^{p(x)-2} u_t = |u|^{q(x)-2} u,$$

under suitable assumptions on the variable exponents and inital data, they obtain local weak solutions and obtained global nonexistence.

In the study of Ouaoua and Boughamsa [12] they looked into the following equation

$$u_{tt} + \Delta^2 u - \Delta u + |u_t|^{m(x)-2} u_t = |u|^{r(x)-2} u,$$

they showed the local existence and also proved that the local solution is global.

Hamadouche [9] worked the following nonlinear Petrovsky equation with variable exponents

$$u_{tt} + \Delta^2 u + a |u_t|^{m(\cdot)-2} u_t = b |u|^{p(\cdot)-2},$$

by utilizing the Faedo-Galerkin method, the author established the existence of a unique weak solution for variable exponents m and p under suitable assumptions, and also obtained the blow-up result with negative initial energy.

Rahmoune [18] studied the following wave equation

$$u_{tt} - \Delta u + |u_t|^{m(x)-2} u_t = |u|^{p(x)-2} u \ln u,$$

they proved the local existence and the global nonexistence.

The existence, blow up and decay of solutions was studied by many authors for the equation, see for instance [2, 3, 4, 5, 8, 10, 13, 14, 15, 16, 17, 19, 22, 23, 24]

This work is divided into three sections, apart from the introduction. In Part 2, we present preliminary details about variable exponents, Lebesgue spaces, and Sobolev spaces. Moreover, we introduce important lemmas and assumptions. In Part 3, we demonstrate the occurrence of solution blow-up with negative initial energy.

2 Preliminaries

In this section, we present some Lemmas and corollary for the proof of our result.

Lemma 2.1. *[6, 7]. If $p : \Omega \to [1, \infty]$ is a measurable function u on Ω and*

$$2 < p_1 \leq p(x) \leq p_2 < \frac{2n}{n-4}, \; n \geq 5. \tag{2.1}$$

Then, the embedding $H_0^2(\Omega) \hookrightarrow L^{p(\cdot)}(\Omega)$ is continuous and compact.

So, we give the sufficient conditions for $p(x)$ and $q(x)$

$$2 < p_1 \leq p(x) \leq p_2 < q_1 \leq q(x) < q_2 < \frac{2n}{n-4} \tag{2.2}$$

holds, where

$$E(t) = \frac{1}{2}\|u_t\|^2 + \frac{1}{2}\|\Delta u\|^2 - \int_\Omega \frac{1}{q(x)} |u|^{q(x)} \ln|u|\, dx + \int_\Omega \frac{1}{q^2(x)} |u|^{q(x)}\, dx. \tag{2.3}$$

Lemma 2.2. *The energy associated with the problem (1.1) given by (1.2) satisfies the*

$$E'(t) = -\int_\Omega |u_t|^{p(x)}\, dx \leq 0 \tag{2.4}$$

and the inequality $E(t) \leq E(0)$ holds, where

$$\begin{aligned} E(0) &= \frac{1}{2}\|u_1\|^2 + \frac{1}{2}\|\Delta u_0\|^2 - \int_\Omega \frac{1}{q(x)} |u_0|^{q(x)} \ln|u_0|\, dx \\ &\quad + \int_\Omega \frac{1}{q^2(x)} |u_0|^{q(x)}\, dx. \end{aligned} \tag{2.5}$$

Proof. We multiply the equation of (1.1) by u_t, and integrating over Ω using integrating by parts, we get

$$E'(t) = -\int_\Omega |u_t|^{p(x)}\, dx \leq 0.$$

□

Let

$$\mathcal{H}(t) = -E(t) \text{ for } t \geq 0, \tag{2.6}$$

since $E(t)$ is absolutely continuous, hence $\mathcal{H}'(t) \geq 0$ and

$$0 < \mathcal{H}(0) \leq \mathcal{H}(t) \leq \int_\Omega \frac{1}{q(x)} |u|^{q(x)} \ln |u| \, dx.$$

Lemma 2.3. *[18]. Let the conditions of (2.1) be fulfilled and let u be the solution of (1.1). Then,*

$$\int_\Omega |u|^{q(x)} dx \geq \int_{\Omega_2} |u|^{q_1} dx = \|u\|_{q_1,\Omega_2}^{q_1} \tag{2.7}$$

where $\Omega_2 = \{x \in \Omega : |u(x,t)| \geq 1\}$.

Lemma 2.4. *[18]. Under the assumptions stated in (2.2), the function $\mathcal{H}(t)$ provided above gives the following estimated:*

$$0 < \mathcal{H}(0) \leq \mathcal{H}(t) \leq \frac{|\Omega|}{q_1 e} + \frac{\mathcal{B}_s}{(s-q_2)\, q_1 e} \|\nabla u\|_2^s, \ t \geq 0,$$

where s is chosen sufficiently small such that

$$\begin{cases} q_1 \leq q_2 < s < \infty, \text{ for } n = 1,2, \\ q_1 \leq q_2 < s \leq \frac{2n}{n-2}, \text{ for } n \geq 3 \end{cases}$$

and $\mathcal{B}_s$ is a positive constant of embedding $H_0^2(\Omega)$ in $L^s(\Omega)$ such that

$$\|u\|_s \leq \mathcal{B}_s \|\Delta u\|_2, \ \forall u \in H_0^2(\Omega). \tag{2.8}$$

From above lemma and by using Sobolev Embedding theorem, we have the following corollary:

Corollary 2.5. *Under the assumptions of (2.2), the function $\mathcal{H}(t)$ presented above yields the following estimates:*

$$0 < \mathcal{H}(0) \leq \mathcal{H}(t) \leq \frac{|\Omega|}{q_1 e} + \frac{\mathcal{B}_s}{(s-q_2)\, q_1 e} \|\Delta u\|_2^s, \ t \geq 0, \tag{2.9}$$

where s is chosen sufficiently small such that

$$\begin{cases} q_1 \leq q_2 < s < \infty, \text{ for } n = 1,2,3,4, \\ q_1 \leq q_2 < s \leq \frac{2n}{n-4}, \text{ for } n \geq 5. \end{cases} \tag{2.10}$$

3 Blow up

In this part, we state and prove our main result.

Theorem 3.1. *Assume that (2.2) hold, and $E(0) < 0$. Then any solution of problem (1.1) blows up infinite time.*

Proof. Let

$$\Phi(t) = \mathcal{H}^{1-\sigma}(t) + \varepsilon \int_\Omega u u_t dx, \tag{3.1}$$

with $\sigma > 0$ is small enough to be chosen later and such that

$$0 < \sigma \leq \min\left\{ \frac{q_1 - 2}{2q_1}, \frac{q_1 - p_2}{q_1(p_2 - 1)}, \frac{2(q_1 - p_1)}{s(p_1 - 1) q_1}, \frac{2(q_1 - p_1)}{s(p_2 - 1) q_1} \right\}. \tag{3.2}$$

Utilizing Equation (1.1) a direct derivation of (3.1) yields

$$\begin{aligned} \Phi'(t) &= (1-\sigma)\mathcal{H}^{-\sigma}(t)\mathcal{H}'(t) + \varepsilon \|u_t\|^2 - \varepsilon \|\Delta u\|^2 \\ &\quad -\varepsilon \int_\Omega u |u_t|^{p(x)-1} u_t dx + \varepsilon \int_\Omega |u_t|^{q(x)} \ln u dx. \end{aligned} \tag{3.3}$$

By applying the addition and subtraction of $\varepsilon(1-\eta) q_1 \mathcal{H}(t)$ with $0 < \eta < \frac{q_1-2}{q_1}$ on the right hand side of (3.3), we get

$$\begin{aligned} \Phi'(t) &\geq (1-\sigma)\mathcal{H}^{-\sigma}(t)\mathcal{H}'(t) + \varepsilon(1-\eta) q_1 \mathcal{H}(t) + \varepsilon\left(1 + \frac{(1-\eta) q_1}{2}\right)\|u_t\|^2 \\ &\quad +\varepsilon\left(\frac{(1-\eta) q_1}{2} - 1\right)\|\Delta u\|^2 + \varepsilon\eta \int_\Omega |u_t|^{q(x)} \ln u dx \\ &\quad +\varepsilon(1-\eta)\frac{q_1}{q_2^2} \int_\Omega |u|^{q(x)} dx - \varepsilon \int_\Omega u |u_t|^{p(x)-1} dx. \end{aligned} \tag{3.4}$$

Due to the fact that (2.7), taking into account

$$\frac{1}{q_2^2}\int_\Omega |u|^{q(x)}\,dx < \frac{1}{q_1}\int_\Omega |u_t|^{q(x)}\ln u dx,$$

(3.4) result in

$$\begin{aligned}\Phi'(t) &\geq (1-\sigma)\mathcal{H}^{-\sigma}(t)\mathcal{H}'(t)+\varepsilon\beta\left[\mathcal{H}(t)+\|u_t\|^2+\|\Delta u\|^2+\int_\Omega |u|^{q(x)}\,dx\right]\\ &\quad -\varepsilon\int_\Omega uu_t|u_t|^{p(x)-2}\,dx\\ &\geq (1-\sigma)\mathcal{H}^{-\sigma}(t)\mathcal{H}'(t)+\varepsilon\beta\left[\mathcal{H}(t)+\|u_t\|^2+\|\Delta u\|^2+\|u\|_{q_1,\Omega_2}^{q_1}\right]\\ &\quad -\varepsilon\int_\Omega uu_t|u_t|^{p(x)-2}\,dx, \end{aligned}\tag{3.5}$$

here

$$\beta=\min\left\{(1-\eta)q_1,\left(1+\frac{(1-\eta)q_1}{2}\right),\left(\frac{(1-\eta)q_1}{2}-1\right),\frac{q_1}{q_2^2}\right\}.$$

Now, by applying Young's inequality, we can make an estimate for the final expression in equation (3.3) as demonstrated below

$$\begin{aligned}\int_\Omega uu_t|u_t|^{p(x)-2}\,dx &\leq \frac{1}{p_1}\int_\Omega \gamma^{p(x)}|u|^{p(x)}\,dx\\ &\quad +\frac{p_2-1}{p_2}\int_\Omega \gamma^{-\frac{p(x)}{p(x)-1}}|u_t|^{p(x)}\,dx,\ \ (\forall\gamma>0). \end{aligned}\tag{3.6}$$

As a result, by taking γ such that

$$\gamma^{-\frac{p(x)}{p(x)-1}}=k\mathcal{H}^{-\sigma}(t),\ k>0,$$

substituting the statement into equation (3.6) with a sufficiently large k to be specified later, we derive the following inequality:

$$\begin{aligned}\int_\Omega u|u_t|^{p(x)-1}\,dx &\leq \frac{1}{p_1}\int_\Omega k^{1-p(x)}\mathcal{H}^{\sigma(p(x)-1)}(t)|u|^{p(x)}\,dx\\ &\quad +\frac{p_2-1}{p_2}k\mathcal{H}'(t)^{-\sigma}(t),\ \forall\gamma>0. \end{aligned}\tag{3.7}$$

The result of joining (3.5) with (3.7)

$$\begin{aligned}\Phi'(t) &\geq \left[(1-\sigma)-\varepsilon\frac{p_2-1}{p_2}k\right]\mathcal{H}^{-\sigma}(t)\mathcal{H}'(t)\\ &\quad +\varepsilon\beta\left[\mathcal{H}(t)+\|u_t\|^2+\|\Delta u\|^2+\|u\|_{q_1,\Omega_2}^{q_1}\right]\\ &\quad -\varepsilon\frac{k^{1-p_1}}{p_1}\mathcal{H}^{\sigma(p_2-1)}(t)\int_\Omega |u|^{p(x)}\,dx. \end{aligned}\tag{3.8}$$

Using Corollary 5, we obtain

$$\begin{aligned}&\mathcal{H}^{\sigma(p_2-1)}(t)\int_\Omega |u|^{p(x)}\,dx\\ &\leq 2^{\sigma(p_2-1)-1}C\left(\frac{|\Omega|}{q_1e}\right)^{\sigma(p_2-1)}\left(\left(\|u\|_{q_1,\Omega_2}^{q_1}\right)^{\frac{p_1}{q_1}}+\left(\|u\|_{q_1,\Omega_2}^{q_1}\right)^{\frac{p_1}{q_1}}\right)\\ &\quad +2^{\sigma(p_2-1)-1}C\frac{(\mathcal{B}_s)^{s\sigma(p_2-1)}}{(s-q_2)eq_1}\|\Delta u\|_2^{s\sigma(p_2-1)}\left(\|u\|_{q_1,\Omega_2}^{p_1}+\|u\|_{q_1,\Omega_2}^{p_1}\right). \end{aligned}\tag{3.9}$$

We will estimate the terms to the right of (3.9) using Young's inequality, we get

$$\begin{aligned}\|\Delta u\|_2^{s\sigma(p_2-1)}\|u\|_{q_1,\Omega_2}^{p_1} &\leq \frac{p_1}{q_1}\|u\|_{q_1,\Omega_2}^{q_1}+C\frac{q_1-p_1}{q_1}\|\Delta u\|_2^{\frac{s\sigma(p_2-1)q_1}{q_1-p_1}}\\ &= \frac{p_1}{q_1}\|u\|_{q_1,\Omega_2}^{q_1}+C\frac{q_1-p_1}{q_1}\left(\|\Delta u\|^2\right)^{\frac{s\sigma(p_2-1)q_1}{2(q_1-p_1)}}, \end{aligned}$$

similarly

$$\|\Delta u\|_2^{s\sigma(p_2-1)} \|u\|_{q_1,\Omega_2}^{p_2} \leq \frac{p_2}{q_1} \|u\|_{q_1,\Omega_2}^{q_1} + C\frac{q_1-p_2}{q_1} \|\Delta u\|_2^{\frac{s\sigma(p_2-1)q_1}{q_1-p_1}}.$$

Using the following inequality

$$a^\theta \leq a+1 \leq \left(1+\frac{1}{b}\right)(a+b) \ \forall a \geq 0,\ 0<\theta<1, b\geq 0, \tag{3.10}$$

and condition (2.2) with $a = \|u\|_{q_1,\Omega_2}^{q_1}$, $c_1 = 1+\frac{1}{\mathcal{H}(0)}$, $b = \mathcal{H}(0)$ and $\theta = \frac{p_1}{q_1}(\theta = \frac{p_2}{q_1})$, we get

$$\begin{aligned}\left(\|u\|_{q_1,\Omega_2}^{q_1}\right)^{\frac{p_1}{q_1}} + \left(\|u\|_{q_1,\Omega_2}^{q_1}\right)^{\frac{p_2}{q_1}} &\leq 2c_1\left(\|u\|_{q_1,\Omega_2}^{q_1} + \mathcal{H}(0)\right)\\ &\leq 2c_1\left(\|u\|_{q_1,\Omega_2}^{q_1} + \mathcal{H}(t)\right)\end{aligned}$$

and condition (3.2) with $a = \|\Delta u\|_2^2$, $c_2 = 1+\frac{1}{H(0)}$, $b = \mathcal{H}(0)$ and $\theta = \frac{s\sigma(p_2-1)q_1}{2(q_1-p_1)}$, we get

$$\begin{aligned}\left(\|\Delta u\|_2^2\right)^{\frac{s\sigma(p_2-1)q_1}{2(q_1-p_1)}} &\leq c_2\left(\|\Delta u\|^2 + \mathcal{H}(0)\right)\\ &\leq c_2\left(\|\Delta u\|^2 + \mathcal{H}(t)\right)\end{aligned}$$

also, $a = \|\Delta u\|_2^2$, $c_3 = 1+\frac{1}{\mathcal{H}(0)}$, $b = \mathcal{H}(0)$ and $\theta = \frac{s\sigma(p_2-1)q_1}{2(q_1-p_2)}$, we get

$$\left(\|\Delta u\|_2^2\right)^{\frac{s\sigma(p_2-1)q_1}{2(q_1-p_2)}} \leq c_3\left(\|\Delta u\|^2 + \mathcal{H}(t)\right)$$

and so, (3.9)

$$\mathcal{H}^{\sigma(p_2-1)}(t)\int_\Omega |u|^{p(x)}dx \leq C\left(\|u\|_{q_1,\Omega_2}^{q_1} + \mathcal{H}(t) + \|\Delta u\|^2\right),\ \forall t\in[0,T], \tag{3.11}$$

where C is the positive constant that depends only on Ω, e, a $p_{1,2}$, $q_{1,2}$. Combining (3.8) and (3.11), we get

$$\begin{aligned}\Phi'(t) \geq\ & \left[(1-\sigma) - \varepsilon\frac{p_2-1}{p_2}k\right]\mathcal{H}^{-\sigma}(t)\mathcal{H}'(t)\\ &+\varepsilon\left[\beta - \frac{k^{p_2-1}}{p_2}C\right]\left[\mathcal{H}(t) + \|u_t\|^2 + \|\Delta u\|^2 + \|u\|_{q_1,\Omega_2}^{q_1}\right].\end{aligned} \tag{3.12}$$

At this point we pick $\gamma = \beta - \frac{k^{p_2-1}}{p_2}C \geq 0$, (it is the case when $k > \left(\frac{\beta p_1}{C}\right)^{\frac{1}{1-p_1}}$. Once k is fixed we pick $\varepsilon > 0$ sufficient small so that

$$(1-\sigma) - \varepsilon\frac{p_2-1}{p_2}k \geq 0$$

and

$$\Phi(0) = \mathcal{H}^{1-\sigma}(0) + \varepsilon\int_\Omega u_0(x)\,u_1(x)\,dx > 0.$$

Hence (3.12) takes the form

$$\Phi'(t) \geq \gamma\left[\mathcal{H}(t) + \|u_t\|^2 + \|\Delta u\|^2 + \|u\|_{q_1,\Omega_2}^{q_1}\right]. \tag{3.13}$$

Therefore, we have

$$\Phi(t) \geq \Phi(0) > 0, \text{ for all } t\geq 0$$

On the other hand from (3.1),

$$\Phi^{\frac{1}{1-\sigma}}(t) \leq 2^{\frac{1}{(1-\sigma)}}\left(\mathcal{H}(t) + \left|\int_\Omega uu_t dx\right|^{\frac{1}{(1-\sigma)}}\right) \tag{3.14}$$

by utilizing Holder's inequality, it becomes

$$\begin{aligned}\left|\int_\Omega uu_t dx\right|^{\frac{1}{(1-\sigma)}} &\leq C\|u\|_{q_1}\|u_t\|_2\\ &\leq C\|u\|_{q_1,\Omega}\|u_t\|_2.\end{aligned}$$

Again, algebraic inequality (3.10), with $a = \|u\|_{q_1,\Omega_2}^{q_1}$, $c = 1 + \frac{1}{\mathcal{H}(0)}$, $b = \mathcal{H}(0)$ and $0 < \theta = \frac{2p_1}{(1-2\theta)q_1} \leq 1$ (see 3.2), gives

$$\left(\|u\|_{q_1,\Omega_2}^{q_1}\right)^{\frac{2}{(1-2\sigma)q_1}} \leq C\left(\|u\|_{q_1,\Omega_2}^{q_1} + \mathcal{H}(t)\right).$$

Thus, Young's inequality gives

$$\begin{aligned}
\left|\int_\Omega u u_t dx\right|^{\frac{1}{(1-\sigma)}} &\leq C\left[\|u\|_{q_1,\Omega_2}^{\frac{2(1-\sigma)}{1-2\sigma}} + \|u_t\|_2^{2(1-\sigma)}\right]^{\frac{1}{(1-\sigma)}}, \\
&\leq C\left[\left(\|u\|_{q_1,\Omega_2}^{q_1}\right)^{\frac{2}{(1-2\sigma)q_1}} + \|u_t\|_2^2\right], \\
&\leq C\left[\|u\|_{q_1,\Omega_2}^{q_1} + \mathcal{H}(t) + \|u_t\|_2^2\right], \text{ for all } t \geq 0,
\end{aligned}$$

joining it with (3.13) and (3.14) yields

$$\Phi'(t) \geq \zeta \Phi^{\frac{1}{1-\sigma}}(t) \tag{3.15}$$

where ζ is a positive constant according as (ε, γ, C). With a simple integration of (3.15) over $(0, t)$ we infer that

$$\Phi^{\frac{\sigma}{1-\sigma}}(t) \geq \frac{1}{\Phi^{\frac{\sigma}{1-\sigma}}(0) - \frac{\sigma}{1-\sigma}\zeta t}. \tag{3.16}$$

Consequently, $\Phi(t)$ blows up in a finite time T^*

$$T^* \leq \frac{1-\sigma}{\zeta\sigma\Phi^{\frac{\sigma}{1-\sigma}}(0)}.$$

□

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