Palestine Journal of Mathematics

Vol 13(Special Issue I)(2024) , 22–35

ASSESSMENT OF MATHEMATICAL PROBLEM SOLVING SKILLS IN THE COLLEGE CYCLE IN MOROCCO: A TRIPARTITE EXPLORATION ACROSS VARIOUS MATHEMATICAL DISCIPLINES

Asmae Bahbah and Mohamed Erradi

MSC 2010 Classifications: Primary 97D10, 97C70; Secondary 97D50, 97C30.

Keywords and phrases: Problem solving, mathematics, skills, assessment, teaching, strategy.

Abstract Problem solving is one of the key skills of the 21st century and an essential part of learning mathematics. It is through this complex act that learners acquire, practice, and improve the various cognitive and metacognitive skills essential to success in many areas of life. However, traditional classroom teaching methods do not always enable these skills to be developed effectively. Consequently, these skills need to be assessed methodically to ensure that learners develop the necessary competencies, and this is the aim of this work. The results of our study show the low level of the 66 participating students in the assessment of mathematical problem-solving skills in the different problem situations $\bar{x}$ = 0.55 for the algebra problem, $\bar{x}$ = 0.42 for the geometry problem, and $\bar{x}$ = 0.27 for the statistics problem), particularly metacognitive strategies. There was also a significant difference in problem-solving ability between the different mathematical domains ($\bar{x}$ = 0.12311 with σ = 0.3334 and t = 2.990 with Sig = 0. 004 < 0.05 for Pair1, $\bar{x}$ = 0.2408 with σ = 0.2894 and t = 6.760 with Sig = 0.000 < 0.05 for Pair2, and $\bar{x}$ = 0.1177 with σ = 0.2458 and t =3.898 with Sig = 0.000 < 0.05 for Pair3). In addition, this study confirmed the existence of a significant positive low-intensity correlation in Pair2 and Pair3 (r = 0.334 with sig = 0.006 for Pair2 and r = 0.492 with sig = 0.000 for Pair2).

1 Introduction

The teaching of mathematics, which is one of the main disciplines in the school life of a learner, represents a major challenge for societies given the considerable and striking development of science and technology today [7]. Studying mathematics means acquiring and practicing problem solving [22]. Problem solving is a very important factor in achieving the objectives of learning mathematics by building new knowledge, solving mathematical problems present in the curriculum both as an object or tool for learning mathematics and they even occupy a central place [17], being able to apply and adapt various appropriate strategies to solve a problem and reflecting on the process of solving mathematical problems [20]. Thus, mathematical problem solving is at the heart of mathematics teaching and learning [8].

Problem solving requires mastery of specific knowledge, cognitive and metacognitive procedures [3], as well as emotional and motivational skills [4]. All while preparing students to face the challenges of everyday life. To be competent in mathematics, it is essential to develop a "mathematical disposition" [6] that encompasses different categories of skills [9].

Educational systems are increasingly emphasizing problem solving, and organizations such as the Organization for Economic Cooperation and Development (OECD) and the Programme for International Student Assessment (PISA) have also placed great emphasis on this skill [1]. As a result, several educational systems emphasize the integration of problem solving as an object of instruction or a tool for deep learning and skill improvement [19]. However, its effective implementation in the classroom and not as a subject in its own right is not a trivial process and several national and international assessment test results show the low level of students' problem solving skills, which poses a real problem for them in school and in everyday life.

The objective of this study is to evaluate the students' level of problem solving in the different areas of mathematics (Algebra, Geometry and Statistics). So, what is the level of students in problem solving? Do they have the essential skills for problem solving? Do they have the ability to solve all the problems in the different areas?

The results showed that the students participating in this study use superficial solving approaches and have a low level of problem solving skills, in addition to the existence of a significant difference in the ability to solve the problem in different mathematical areas.

This study also confirmed the results found previously [2], which show that problem solving is an interactive navigation between three domains: the domain of the problem situation, the domain of declarative knowledge stored in long-term memory and the domain of transformations that takes place at the level of working memory, and consequently the model found in the screen.

We have organized the contents of this paper as follows. Section 2 outlines the main points of the literature review. In Section 3, we present the methodological framework, where we justify the choice of students, the situations choice and the technical choice of analysis. Then, we present the results and discussion in Section 4. Section 5 presents a conclusion and future works.

2 Literature Review

When faced with a problem situation, which often causes distress or difficulty to a learner and requires some form of intervention or treatment [21], many cognitive and metacognitive processes are implemented by learners to create new memory traces. These traces allow the brain to store and retrieve information in short-term or long-term memory.

Solving these problems calls upon the learner's abilities to memorize, perceive, reason, conceptualize and appropriate the language [26], but also upon his emotions, motivation and self-confidence.

Teaching problem solving strategies to everyone, in the same way and at the same time, distorts the reality of a strategy [13]. In this regard, there are students who seem to master problem solving strategies but have great difficulty applying them because of a lack of the relative skills to implement an expert and reflective approach [12]. This problem has the potential to disrupt classroom dynamics [24], as effective instruction must teach students both procedural fluency and procedural mastery [14].

Assessment of these mathematical problem-solving skills is important both for measuring student achievement and for identifying ways to improve these skills. It can provide feedback to the teacher to adjust instructional strategies and to understand the cognitive difficulties encountered by students.

However, the assessment of mathematical problem solving skills, particularly metacognitive strategies, is complex because of the very definition of the concept and its different characteristics. There are various tools in the literature to assess these skills, depending on the research objectives. However, the problem-solving test remains the most commonly used tool to assess students' skills in this area [10] using the think-aloud technique and the semi-directed interview [5].

The mathematical competences of the secondary college education that are evaluated in this study they have extracted from the document of orientations and pedagogical choices Moroccan [18].

3 Methodological Framework

This study is based on a qualitative and quantitative approach focusing on the mastery of mathematical problem solving skills in the 3rd year of secondary school in Morocco.

Table 1 present the skills and their descriptions.

Table 1. College high school math skills 1.

Skills	Description
The acquisition of concepts, knowledge; operations, tools and procedures	The ability to : - know situations related to calculus and to perform techniques - know situations related to calculus and to perform operations; - know the concepts and conventional terms of calculation; - use mathematical tools and tools of measurement and construction. - know and recognize situations where concepts are used; - classify; represent; formulate; symbolize.
Development of skills , and enrichment of abilities in the areas of inquiry, observation, abstraction and reasoning	The ability to : - model situations, present a proof, clarify a strategy, or solve a problem orally and in writing or using drawings and graphs or algebraic methods - practice mathematical discovery; - recognize and apply inductive and deductive reasoning - use different methods of proof, to understand and apply methods of reasoning; - formulate conjectures, establish proofs and evaluate them - Accuracy of thinking - check the validity of ideas, to give examples and counter examples;
The acquisition of the methodology of thought (development of the levels of reflection) and that of work and organization	The ability to : - formulate and clarify representations about mathematical ideas and situations and to use them; - discuss mathematical ideas: problem solving strategies, algorithm... - view mathematics as an integrated unit.
Development of precision and clarity of expression; communication through language, symbols, geometric figures and graphics	The ability to : - perceive mathematical ideas well - use listening, writing, and reviewing skills to interpret and evaluate mathematical ideas - appreciate the value and role of mathematical symbolism; - search through problems and describe results using mathematical representations or models.
The use of mathematical notions and their investment in other school disciplines or in the surrounding reality.	The acquisition of - basic knowledge and skills in various mathematics branches; - sufficient mathematical knowledge and skills to continue studies or to enter professional life - mathematical knowledge and skills to understand and assimilate the content of other disciplinary subjects (scientific and technological)
The development of analysis, synthesis and estimation skills	The ability to: - analyze when determining relationships between variables in mathematical situations - identify a property that implies other propositions; - to recognize a property as a unified one among several different situations; - find structural relationships between several statements - use a mathematical idea to assimilate other mathematical ideas.

The acquisition of the methodology of mathematization of situations and treatment of problems, the presentation of justifications to prove, deny or verify, and to state conjectures	The ability to: - formulate problems from mathematical or real-life situations and express them in mathematical models; - formulate hypotheses and conjectures and convincing proofs - Possess a variety of strategies for solving problems and applying them; - verify and interpret results with reference to the original; problem - generalize solutions and strategies to new problems.

3.1 Student Choice :

The 66 students participating in this study are enrolled in the 3rd year of middle school this year 2022 -2023, they are varied in terms of gender; level in mathematics; motivation; and level of language and verbal communication. They were informed that this assignment would not be graded to avoid the stress of getting a grade.

3.2 Choice of situations :

The situations chosen are within the students' reach and stimulating, requiring the mobilization and articulation of precise knowledge or know-how to determine the level of acquisition of skills [11], referring to a limited number of adequate criteria to make this evaluation fairer [23] and to avoid the risk of interdependence between them for a better explanation and interpretation.

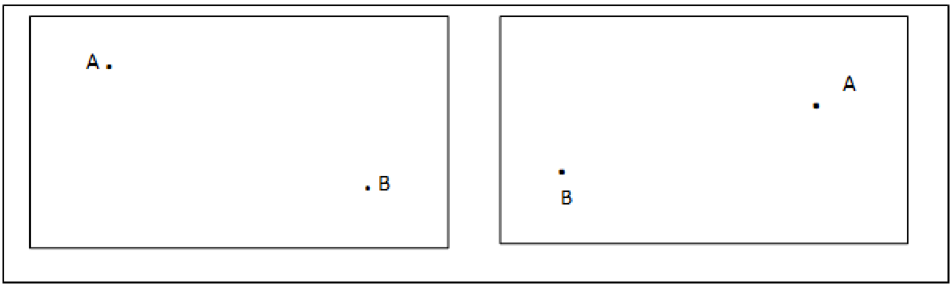
Three situations have been chosen for this evaluation that differ in nature, in the number of problem data and also in the level of the criteria to be evaluated.

Evaluation situation 1 :

A school has organized a visit to the Rabat Zoo. Each student participating in this outing, must contribute a sum of 600 dhs. The day before the trip, 18 students withdrew, so each of the other participants must add 300 dhs to cover the total cost of the visit. What is the cost of the visit?

Evaluation situation 2 [27] :

Each of the following figures consists of two points A and B.



Each of these must be completed by a point J that satisfies the following conditions :

- I, C, and D are three points such that I is the midpoint of segments$[AC]$ and $[BD]$;
- E is the point such that $\overrightarrow{BD} + \overrightarrow{BE} = 2\overrightarrow{BA}$;
- J is the middle of the segment $[CE]$

What can be said about the point J? Justify your answer.

Evaluation situation [27]: Third year college students were asked how many sports they play. The results were plotted in the following diagram given in Figure 1:

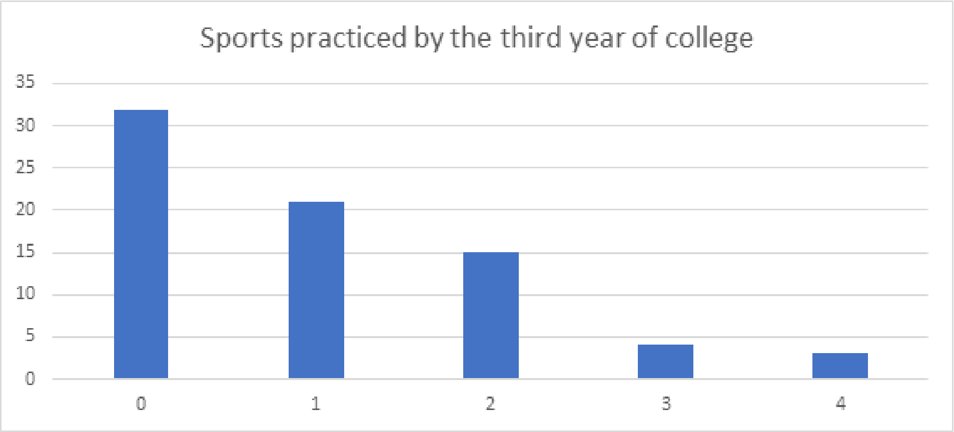


Figure 1. Sports practiced by the third year of college

Is the following statement true or false? Justify your answer. Of the students who play at least one sport, half play more than one sport. The description of the evaluation situations is presented in Table 2.

3.3 Technical choice of assessment :

In our work we will combine several assessment techniques to capture more data and information. We will use the written traces of each student and the resolution aloud which consists in asking the students to verbalize their thoughts while they solve the problem. Facing each assessment situation, we interviewed and recorded audios. We also used a questionnaire at the end of the resolution which acts as a tool to confirm and/or justify the answers or to verbalize the mental representations and is not intended to guide the subjects.

3.4 Technical choice of analysis :

Although different cognitive processes have been developed to evaluate mathematical problem solving such as Montague, Warger, and Morgan who proposed a seven-step process: reading, paraphrasing, visualizing, hypothesizing, estimating, calculating, and verifying [16], Montague has been proposed another cognitive process that includes the following steps: understanding, translation and transformation, observing the relationships between the elements of the problem, formulating a plan, predicting the outcome, regulating the solution, and detecting and correcting errors [15]; Polya's first theoretical and heuristic framework of problem solving remains the origin of all these models [25].

Polya's approach to problem solving is a widely used model in teaching and assessing problem solving in mathematics. And several systems have incorporated this model into their assessments of mathematical thinking. The model that guides this assessment study is Polya's cognitive process model, which consists of four main steps. First, there is the problem comprehension phase (See), where the student analyzes and fully understands the mathematical problem presented to them. This stage involves reading the problem carefully, identifying relevant information, and clarifying the goals to be achieved.

Next, there is the design phase (Plan), where the student thinks about the different possible strategies to solve the problem. This may include finding patterns, translating and transforming information, notations, finding links and relationships between elements of the problem, predicting how the situation will evolve, using mathematical models, or setting up a series of logical steps to follow.

The third step is the execution of the plan (Do), where the student implements the chosen strategy to solve the problem. This involves performing the necessary calculations, manipulating numbers or mathematical objects according to the established plan.

Finally, there is the look back (Check) stage, where the student evaluates the validity of his or her solution and checks whether it meets the requirements of the problem. This may involve

Table 2. Description of the evaluation situations

Situations Problems	**Domaine**	**Number data**	**Skills to be mobilized**
Situation Problem 1	Algebra	3	-Search for and identify the main unknown - Translate and rephrase the sentences - Break down the problem into two sub-problems -Model -Produce a literal expression - Develop and solve an equation or system - Calculate - Reasoning - Verify the solution
Situation Problem 2	Geometry	12	- Self-confidence and taking the initiation -Representing, building -Reasoning: inductive and deductive reasoning deductive reasoning - Observe, research, analyze, model and compare - Communicate evidence Imagining, specifying -Confidence and taking the lead -Conjecture
Situation Problem 3	Statistics	6	- - Mastery of language - Comprehension, rephrasing of the sentence and expressing it in mathematical models - Reading graphs (extracting results), and interpreting results - Recognize and use statistical properties - Linking the meaning of the sentence and chart data - Find structural relationships between given variables - Research, describe, model and discuss - Communicate, reason and argue - Verify and interpret results

checking the calculations, reviewing the steps taken, or ensuring that the answer is consistent and logical.

4 Results and Discutions

The results of the analysis of the answers given by the students to the semi free test using Polya's problem solving indicators, show that the average of 66 students participating in this interview in the three situations is 0.77 in the competence of understanding the problem, 0.5 for the competence of designing a plan, 0.28 for the competence of executing the plan and 0.19 for the competence of validating and evaluating the solution.

Table 3 summarizes the results obtained in the three situations calculated by the SPSS software.

According to this table, the average of students who successfully understood problem situation 1 which is an algebra situation is $\bar{x} = 0.98$ with a standard deviation of $\sigma = 0.057$, $\bar{x} = 0.64$ with a standard deviation of $\sigma = 0.294$ for the geometry problem situation is $\bar{x} = 0.66$ with a standard deviation of $\sigma = 0.233$ for problem situation 3 which enters into graphing and statis-

Table 3. Descriptive statistics results for the means of Polya's four gait skills in the three problems

	Problem 1				Problem 2				Problem 3				Mean
	Min	Max	Mean	Std. Deviation	Min	Max	Mean	Std. Deviation	Min	Max	Mean	Std. Deviation	
Understanding	0,67	1	0,98	0,057	0	1	0,64	0,294	0,33	1	0,66	0,233	0,77
Devising a plan	0	1	0,62	0,414	0	1	0,65	0,364	0	1	0,22	0,308	0,5
Carrying out the plan	0	1	0,35	0,45	0	1	0,27	0,403	0	1	0,23	0,32	0,28
Examination	0	1	0,26	0,42	0	1	0,17	0,28	0	0,5	0,14	0,22	0,19

tical activities. For the second step which is designing a plan, the averages are $\bar{x} = 0.62$ for problem situation 1, $\bar{x} = 0.65$ for problem situation 2 and $\bar{x} = 0.308$ for problem situation 3.

The averages of the students who passed to the resolution and regulation during the execution had a big drop in the problem situation 1 $\bar{x} = 0.35$ with $\sigma = 0.45$) and in the problem situation 2 ($\bar{x}$=0.27 with $\sigma = 0.403$), a slight increase for the problem situation 3 ($\bar{x} = 0.23$ with $\sigma = 0.32$). Regarding the averages of verification and interpretation skills were very low: $\bar{x} = 0.26$ with σ = 0.42 for problem situation 1, $\bar{x} = 0.17$ with σ = 0.28 for problem situation 2 and $\bar{x} = 0.14$ with $\sigma = 0.22$ for problem situation 3.

Regarding the averages of verification and interpretation skills were very low: $\bar{x}$=0.26 with σ = 0.42 for problem situation 1, $\bar{x}$=0.17 with σ = 0.28 for problem situation 2 and $\bar{x}$=0.14 with σ = 0.22 for problem situation 3.

These results show that the means of students' mathematical problem solving skills in all situations and at all stages of problem solving tend towards a low level of skills especially the metacognitive processes (analysis, reflection, verification, planning...) which are generally lower than 0.5.

The difference in average problem solving skills between the three problem situations can be seen in Figure 2.

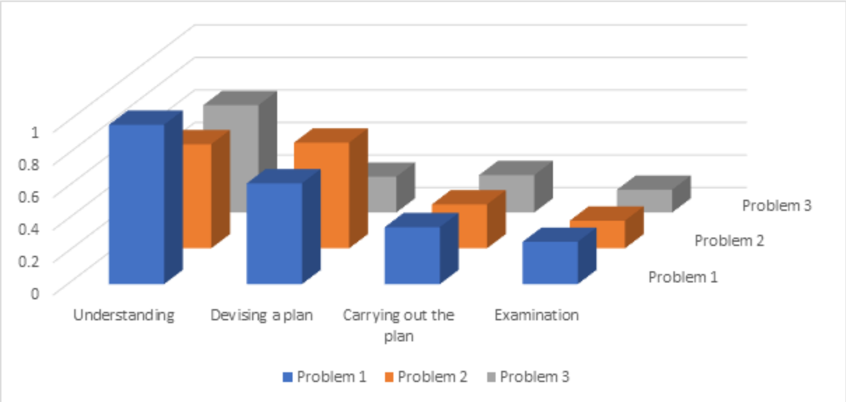


Figure 2. Difference in average problem solving skills between the three problem situations.

The details of the averages of the indicators of each problem-solving skill and the difference between these different averages in the three situations are represented successively in Table 4 and Figure 3.

Table 4. Descriptive statistics results for the means of each indicator of Polya's problem-solving skills

	Problem 1		Problem 2		Problem 3	
	Mean	*Std. Deviation*	*Mean*	*Std. Deviation*	*Mean*	*Std. Deviation*
Identification of data	1	0.000	0.39	0.492	0.71	0.456
Identification of unknowns	0.97	0.173	0.59	0.495	0.30	0.463
The goal to be achieved.	1	0.000	0.94	0.240	0.98	0.123
Translate and transform information	0.73	0.449	0.77	0.422	0.18	0.389
Find the links and relationships between the elements of the problem	0.70	0.463	0.76	0.432	0.41	0.495
Formulating a resolution plan	0.62	0.489	0.48	0.504	0.18	0.389
Choosing the right PR strategies	0.45	0.502	0.58	0.498	0.11	0.310
Mathematical model resolution	0.41	0.495	0.30	0.463	0.39	0.492
Regulation of the path during execution	0.29	0.456	0.24	0.432	0.08	0.267
Verification and reflection	0.11	0.310	0.05	0.210	0.00	0.00
Interpretation of results or making connections	0.41	0.495	0.29	0.456	0.27	0.449

The first indicator to identify the problem data was passed by all students in the first problem, by 39% in the second problem and by 71% in the third problem. This great difference between the different problems is due to the fact that the majority of the students consider that the data of a problem are numerical data and clear relations, on the other hand the verbal sentences and the verbal information are not the main data but they are given for a better understanding of the problem.

In the step of identifying the unknowns 41% of the students did not find the unknown of the second problem saying that in geometry there are no unknowns but 94% succeeded in knowing the goal and what to look for. On the other hand, in problem 1, 97% identified the unknown and 100% of the students knew the goal. And in problem 3, 30% identified the unknown and 98% of the students knew the goal to be reached.

Figure 3 clearly shows the gap between the first, second and third problem situations in the three indicators of the first comprehension skill.

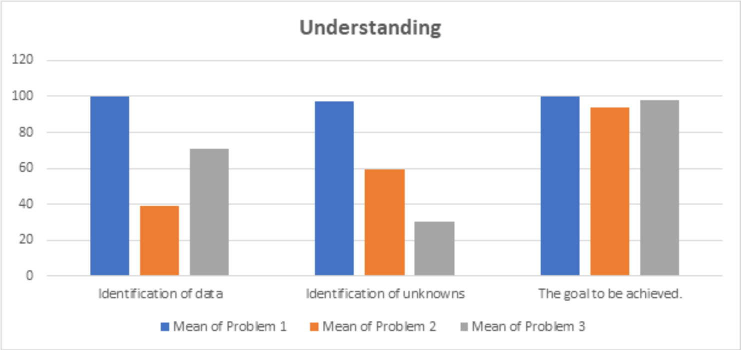


Figure 3. Percentage difference in the three indicators of Understanding between the three problem situations.

During the translation and transformation stage, several students found difficulties in translating information into mathematical notations, algorithms, and equations such as the sentence "At the last moment, 18 people of them did not participate in this outing" in problem situation 1, the sentence " E is the point such that $\overrightarrow{BD}+\overrightarrow{BE}=2\overrightarrow{BA}$ in problem situation 2 and the words "at least, several" in problem situation 3. Thus, the percentages of students who were able to pass this step are 73% for problem situation 1, 77% for problem situation 2 and just 18% for problem situation 3.

Then, they looked for relationships and connections between the elements of the problem by mentioning that to find the cost of the field trip, they must multiply the number of participating students by 600 DH the amount of the participation but 30% were unable to find the relationship between the cost of the field trip and the number of students after the withdrawal of 18 students and to notice that the cost does not change after the withdrawal and consequently to conceive a first degree equation with only one unknown which is the number of participating students or a system of two first degree equations with two unknowns : The cost of the field trip and the number of participating students. This influenced the percentage of the indicator of formulating a plan which was 62%. As a result, only 45% of the students were able to choose the right strategies.

In the second situation, 76%were able to find the links between the different information given, however, only 48% were able to formulate a plan for the solution due to emotional factors such as self-efficacy and self-confidence or due to the inability to find structural relationships between several statements, to identify a property that implies other propositions, to formulate hypotheses and conjectures and convincing proofs and to apply inductive (to prove what is established) and deductive reasoning to show that J is the middle of [AB] and to see that the positions of the first points (A, B and C) do not influence the position of the point J sought, which always remains the middle of the segment [AB]. So they have to move from particular cases to a general case.

Concerning the third situation, the four indicators of the competence to design a plan were passed by only a minority of the students because of the mistranslation of the words "at least" and "several" and the sentence "Among the students who practice at least one sport" as a result the formulation of the plan and the model of the calculation were not correct. Figure 4 shows the difference between the percentages of students in each indicator of the design skill in the three situations.

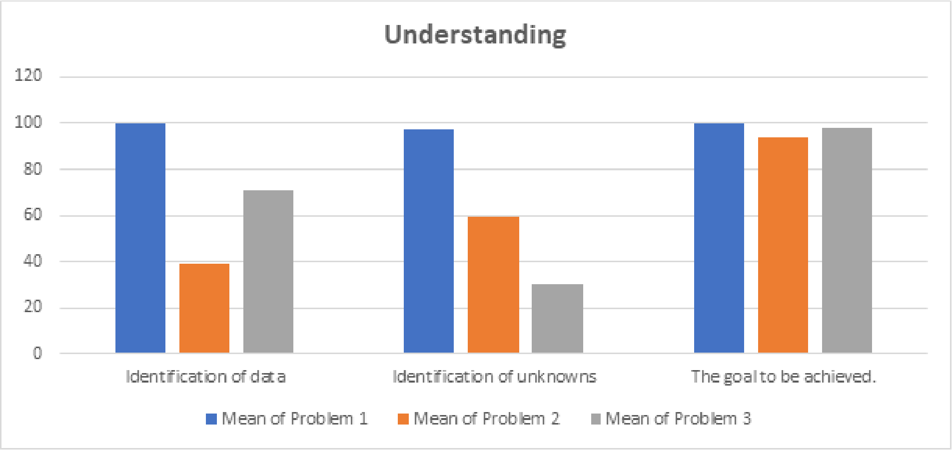


Figure 4. Percentage difference in the three indicators of Devising a plan between the three problem situations.

In addition, the phase of solving the mathematical model is fulfilled by only 41% for the problem situation 1 of which 70.73% succeeded after a better regulation of the course during the execution and the detection of errors (number of friends is negative or is a fraction), 30% for the problem situation 2 of which 80% needed a regulation in the demonstration to be able to convince the teacher while 79.5% were able to solve the problem situation 3 and to make the necessary calculations perfectly without any error and any regulation.

The differences between these percentages in the different problem situations are shown in Figure 5.

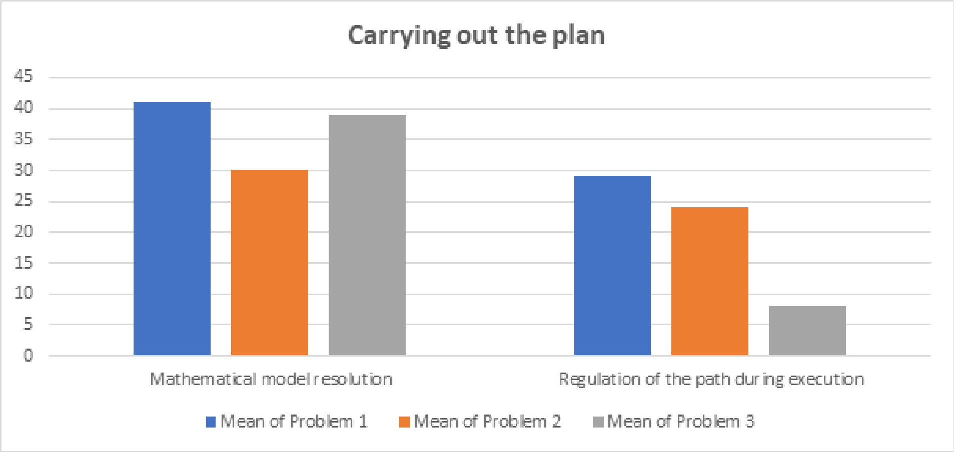


Figure 5. Percentage difference in the three indicators of Devising a plan between the three problem situations.

Regarding the verification indicator, it was only done by 11% of the students in problem situation 1, by 5% of the students in problem situation 2 and was not done by any students in problem situation 3. The majority of students also did not check the correctness of the solutions in the different situations and did not interpret the final answer by only seeing the final results obtained. This last indicator is achieved by only 41% in problem situation 1, 29% in problem situation 2 and 27% in problem situation 3.

We note that there is a large gap in the three situations as presented in Figure 6.

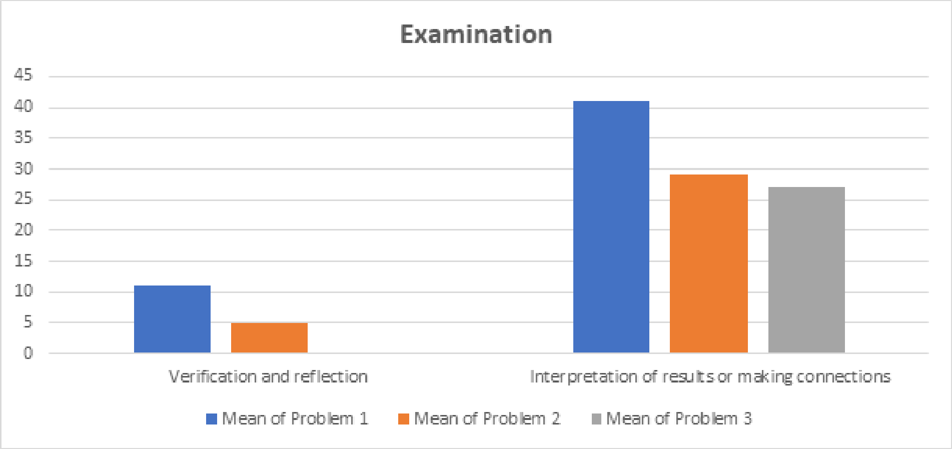


Figure 6. Percentage difference in the three indicators of Examination between the three problem situations.

The T-test of significance for the difference between two paired sample means has been verified in the following tables:

Table 5. Descriptive statistics results for the means of Polya's four gait skills in the three problems

		Mean	N	Std. Deviation	Std. Error Mean
Pair 1	Probem 1	,5552	66	,27782	,03420
	Probem 2	,4321	66	,26383	,03248
Pair 2	Probem 1	,5552	66	,27782	,03420
	Probem 3	,3144	66	,21588	,02657
Pair 3	Probem 2	,4321	66	,26383	,03248
	Probem 3	,3144	66	,21588	,02657

According to this table, there is a difference between the averages of each peer assessed and that these averages are very low which shows the low level of the students in mathematical problem solving.

Table 6. The correlation coefficient between each pair

		N	Correlation	Sig.
Pair 1	Problem 1 & Problem 2	66	,238	,054
Pair 2	Problem 1 & Problem 3	66	,334	,006
Pair 3	Problem 2 & Problem 3	66	,492	,000

The correlation coefficient between Problem 1 and Problem 2 is 0.238. This coefficient is relatively low, which is confirmed by the value sig= 0.054 which is higher than 5% . This coefficient is not significant since it does not differ significantly from 0. The correlation coefficient between Problem 1 and Problem 3 is 0.334 with sig= 0.006 which is lower than 5% . And between Problem 2 and Problem 3 the coefficient is 0.492 with sig= 0.000 which is lower than 5%. The correlation coefficients are between 0.3 and 0.5, which indicates a significant positive low intensity correlation. We can deduce that there is always a perfectly positive linear correlation, this justifies that the value of one of the variables tends to increase at the same time as the other variable.

Table 7. Results of the Paired Samples T-Test on the means of the skills of each problem

		Paired differences					t	ddl	Sig. (2-tailed)
		Mean	Std. Deviation	Std. Error Mean	95% confidence Interval of the Difference				
					Lower	Upper			
Pair 1	Problem 1-Problem 2	,12311	,33447	,04117	,04088	,20533	2,990	65	,004
Pair 2	Problem 1-Problem 3	,24085	,28944	,03563	,16969	,31200	6,760	65	,000
Pair 3	Problem 2 - Problem 3	,11774	,24538	,03020	,05742	,17806	3,898	65	,000

From this table:

The difference in means between each pair ranges from 0.1177 to 0.240: $\overline{x}$=0.12311 with σ = 0.3334 for Pair 1 (Problem 1- Problem 2), $\overline{x}$=0.2408 with σ = 0.2894 for Pair 2 (Problem 1- Problem 3), and $\overline{x}$=0.1177 with σ = 0.2458 for Pair 3 (Problem 2- Problem 3).

Each confidence interval does not contain the value 0, we must reject the null hypothesis of equality of skill means. Thus, there is a significant difference between each pair, and the ability to solve an analysis problem is not the same as the ability to solve an algebra or statistics problem and vice versa. This difference is not negligible, it is considerable.

Looking at the value of T: t = 2.990 with Sig = 0.004 < 0.05 for Pair 1 (problem 1- problem 2), t = 6.760 with Sig = 0.000 < 0.05 for Pair 2 (problem 1- problem 3) and t =3.898 with Sig = 0.000 < 0.05 for Pair 3 (problem 2- problem 3). Thus it confirms the result stated by the 95% confidence interval. Therefore, we must reject the null hypothesis of equality of the skill means. Then the ability of the students participating in this study to solve a problem depends on the nature of the problem.

All these results show that in general the students try to solve a problem using the four steps of Polya but in a random way and that they find some difficulty especially in the phase of translation and transformation of mathematical information into mathematical notations, algorithms and equations, the phase where they try to find the links and relations between the elements of the problem and the phase of choosing the right strategies. The indicator of interpretation and verification, in all the solved problems, is the most rare indicator used especially the verification of the obtained result (the students who found the results were happy to find what is asked and forget to verify them). All these steps play a main role in solving and use high level skills such as analysis, reflection, planning, interpretation and self-evaluation. This explains why the level of skills of these students is very low, especially the metacognitive processes, so when the level of these skills increases the averages decrease. These results also reveal a significant difference in the students' ability to solve a problem between the different mathematical domains (Algebra, Geometry and graphic activities, statistics and numerical functions). And that students' ability to solve an analysis problem is higher compared to the other domains.

The TIMSS 2019 report on Moroccan students also pointed out that there is a difference between the various domains, showing that Moroccan middle school students have a significant mastery of the domain of "geometry" by 413 points against a weakness in the domain of "algebra" by 370 points and that of "data and probability" by 372 points. Even if here the percentages of the students who succeeded in the situation of algebra is higher than that of the situation of geometry. These poor mathematical problem solving skills are due to conventional learning using traditional methods of mathematics teaching, despite the various new reforms, which only allows students to perform algorithms and encourages them to memorize formulas by heart without doing reasoning. As a result, they will not be able to remember mathematical concepts or apply them in real-life problem situations.

Computer tutoring systems for mathematics offer a potential solution by providing personalized and adaptive learning experiences that can help students improve their skills and confidence in mathematics. These systems use various techniques, such as machine learning, data mining, and artificial intelligence, to provide personalized and adaptive tutoring experiences. They can adapt to the individual needs and abilities of each student and provide immediate feedback and guidance. For this reason, we have already worked in a previous paper on modeling mathematical problem solving processes to design a model that will form the basis for designing the Expert System model to rescue this situation and assist in the development of mathematics problem solving strategies in our target learner.

In a previously published work we gave a methodological and analytical approach to verbalize and visualize the networks of mathematical problem solving and presented them schematically and then we gave a general model of these networks with the aim of predicting a model that will constitute the basis for the design of an Expert system model to help the development of problem solving strategies for students.

The analysis of the data we now have confirms the existence of specific processes during the problem solving activity. And that problem solving relies on a complex set of cognitive and metacognitive processes that starts as soon as the student becomes aware of the first information related to the problem (context, numerical and linguistic data) and continues until he stops thinking about the problem by processing the data at the level of working memory and getting out the declarative knowledge stored in long-term memory. In addition the different interactions between these domains that generate the resolution networks. According to the analysis of the different networks of resolution of the three problem situations in the different mathematical domains of this study we can confirm the result that we found before [2], that the model (Figure 7) is composed of the movement of exchange of information between the specific data of the problem, the knowledge stored in long term memory and the data that are processed at the level of working memory [2].

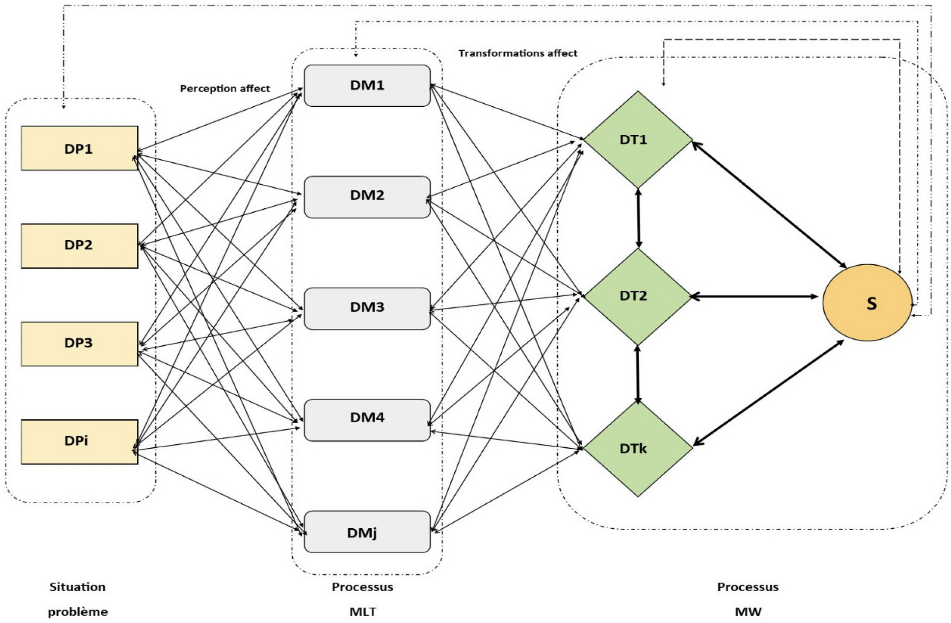


Figure 7. Modeling the network for solving a mathematics problem

5 Conclusion

Mathematics is a fundamental science, that plays a very important role in scientific and technological development. It is also considered a challenging and difficult subject. The results of the present study help us to realize the current level of mathematical problem solving skills of the students participating in this research. This study has shown that the majority of the participating students have difficulties in translating and transforming mathematical information into mathematical notation, in formulating their understanding of the problem, in choosing the right strategies for solving it, in checking the results obtained and in interpreting them, which influences the competence in solving mathematical problems. And that there is a significant difference in the ability of these students between the various mathematical areas. To overcome these problems and improve mathematical problem solving skills, teachers should use new teaching strategies based on linking mathematical learning to real-life problem situations to develop knowledge and improve problem solving skills. It is also necessary to diversify mathematics teaching-learning strategies to develop a meaningful learning process that will enable students to learn interesting problem-solving skills. The mathematics curriculum and assessment system also need to be reformed to focus on problem solving in all areas of mathematics. Several studies have shown that computer tutoring systems can significantly improve students' performance and engagement in mathematics. Hence the idea of designing a model of an expert system that will be able to account for the problem solving processes of our target learners. This system must be personalized and adaptive to improve students' skills and confidence in mathematics.

Our next work is to design this model and subsequently the tutoring system and measure its impact on the development of students' mathematical problem solving strategies.

References

[1] E. M. Albay, Towards a 21st Century Mathematics Classroom: Investigating the Effects of the Problem-Solving Approach Among Tertiary Education Students. *Asia-Pacific Social Science Review* **20** (2020).

[2] A. Bahbah, M., Khaldi, and M. Erradi, Mapping of Mathematical Problem Solving Processes in Middle School Students: A Methodological and Analytical Approach. *RA Journal Of Applied Research*, **9**, 115-121 (2023).

[3] M. A. A. Bakar, N. Ismail, Metacognitive learning strategies in mathematics classroom intervention: a review of implementation and operational design aspect. *International Electronic Journal of Mathematics Education*, **15** , em0555 (2019).

[4] D. Caprioara, Problem solving-purpose and means of learning mathematics in school. *Procedia-Social and Behavioral Sciences*, **191**, 1859-1864 (2015).

[5] M. Chanudet, Problem solving in mathematics in secondary I in French-speaking Switzerland: What evaluation practices? *Actes du seminaire de didactique des mathematiques de l'ARDM-annee 2020*, 146-170 (2021).

[6] M. Crahay, L. Verschaffel, E. De Corte, J. Gregoire, Teaching and learning mathematics: What does psycho-pedagogical research say? *De Boeck*, Brussels, Belgium, .153-176 (2005).

[7] P. J. Fensham, Approaches to the teaching of STS in science education. *International journal of science education*, **10**, 346–356, (1988).

[8] A. Feyfant, Effects of teaching practices on learning: Annie Feyfant. *Dossier d'actualite veille et analyses*, **65**, 1-14 (2011).

[9] V. Freiman, A. Savard, Problem solving in mathematics. *Education and Francophonie*, **42**, 1-6 (2014).

[10] M. Gill, and S. Gill, An Investigation Into Job Satisfaction and Problems Faced By Teachers Of Various Secondary Schools In Creater Mumbai. *Ilkogretim Online*, **19**, 1298-1304 (2020).

[11] F. M. Gerard, C. Lannoye, J. M. De Ketele, Assessing competencies: A practical guide. *Brussels: De Boeck.* (2009).

[12] V. Hanin, C. Van Nieuwenhoven, Evaluation of a pedagogical device aimed at developing cognitive and metacognitive strategies in problem solving in secondary school. *e-JIREF*, **2**, 53-88 (2016).

[13] J. Laflamme, Reading in mathematical problem solving situations. *Bulletin AMQ*, **49**, 46-64 (2009).

[14] L. Lysenko, P. C. Abrami, A. Wade, E. Kiforo, and R. Iminza, Learning Mathematics with Interactive Technology in Kenya Grade-one Classes. *International Journal of Innovation in Science and Mathematics Education*, **30** (2022).

[15] M. Montague, Mathematical problem solving instruction: Components, procedures, and materials. After-school extensions: Including students with disabilities in afterschool programs. *Reston, Va.: Exceptional Innovations.* (2002).

[16] M. Montague, C. Warger, T. H. Morgan, Solve it! Strategy instruction to improve mathematical problem solving. *Learning Disabilities Research and Practice*, **15**, 110-116 (2000).

[17] H. Murray, A. Olivier, P. Human, Learning through Problem Solving. (1998).

[18] Moroccan Minister of National Education Les orientations pedagogiques aout. Ministere de l'education nationale du maroc. (2009).

[19] V. Mudaly, Constructing mental diagrams during problem solving in mathematics. Pythagoras Journal of the Association for Mathematics Education of South Africa. (2021).

[20] NCTM Principles and Standards for School Mathematics Reston Virginia: NCTM. (2000)

[21] G. Polya, How to solve it. A New Aspect of Mathematical Method Collections. *Priceton Science Library.* (1973).

[22] M. Priolet, J. C. Regnier, Mathematics problems in official elementary school instructions and programs, from 1833 to the present. *Grand N*, **90**, 69-87 (2012).

[23] X. Roegiers, School and evaluation: situations to evaluate students' competences. *De Boeck.* (2004).

[24] J. A. Russo, J. Bobis, A. Downton, S. Hughes, S. Livy, M. McCormick, P. Sullivan, Students who surprise teachers when learning mathematics through problem solving in the early primary years. *International Journal of Innovation in Science and Mathematics Education*, **28** (2020).

[25] A. H. Schoenfeld, Problem solving in the United States, 1970–2008: research and theory, practice and politics. *ZDM*, **39**, 537-551 (2007).

[26] Q. F. Velasquez, D. C. Bueno, Metacognitive Skills in Problem-Solving among Senior High School STEM Strand Students. *Institutional Multidisciplinary Research and Development*, **2** (June), 124 - 129 (2019).

[27] https://eduscol.education.fr/document/13132/download

Author information

Asmae Bahbah and Mohamed Erradi, Research team in Computer Science and University Pedagogical Engineering (S2IPU), Higher Normal School, Abdelmalek Essaadi University, Martil 93150, Morocco.
E-mail: `asmae.bahbah@etu.uae.ac.ma`