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Arithmetic Progressions in the Values of Quadratic Polynomials and Some Figurate Numbers

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Abstract In this paper, we investigate arithmetic progressions in integer-valued quadratic polynomials. We prove two results for integer-valued quadratic polynomials. First, we prove that there is no three-term arithmetic progression with a perfect square as a common difference for a class of integer-valued quadratic polynomials. We then find a class of numbers depending on integer-valued quadratic polynomials, which cannot work as a common difference for three-term arithmetic progression. Further, we deduce some results for arithmetic progressions in the sequence of polygonal and centered polygonal numbers.

1 Introduction

The study of arithmetic progression in the sequence of figurate numbers, specially polygonal numbers, is an interesting problem and has attracted mathematicians for a long time. Historically, Euler [7] proved that there is no four-term arithmetic progression of square numbers. In 1965, Sierpiński [11] proved that there exist infinitely many three-term arithmetic progressions of triangular numbers with a triangular number as a common difference.

In recent years, Brown, Dunn, and Harrington [1] extended the result of Euler for any s-gonal numbers, where $s > 2$ is an integer. Moreover, they proved that there exist infinitely many three-term arithmetic progressions of s-gonal numbers for any fixed value of s. In 2013, Ide and Jones [5] proved that there exist no arithmetic progression of triangular numbers with square number as a common difference, but there are infinitely many arithmetic progressions of square numbers with a triangular number as a common difference. In 2017, Jones and Phillips [6] proved that given any family of polygonal numbers, there exist no three-term arithmetic progression with square number as a common difference (see Theorem 1 of [6]). They also proved several results related to arithmetic progressions in polygonal numbers with common difference a polygonal number. In [9], Pongsriiam and Subwattanachai extended the results of Brown *et al.* in [1] to the case of quadratic polynomials.

In the view of the work done in [9] for arithmetic progressions in integer values of quadratic polynomials and the result proved in Theorem 1 of [6], a natural question arises:

Problem 1. Given any integer-valued quadratic polynomial, does there exist a three-term arithmetic progression with square as a common difference in the values of quadratic polynomial?

In this paper, we study this problem and investigate arithmetic progressions in the values of quadratic polynomials with some special class of numbers as a common difference. Consequently, we derive some results for polygonal numbers. Further, we deduce some results related to arithmetic progressions in another class of plane figurate numbers called centered polygonal numbers.

2 Preliminaries

We start this section with the definition of *Pell's and Generalized Pell's equation* followed by a result on their solutions.

Definition 2.1. Let $d > 1$ be a square-free positive integer and N be a non-zero integer. Then the *generalized Pell's equation* is the Diophantine equation of the form

$$x^2 - dy^2 = N. \tag{2.1}$$

For an arbitrary N, the solutions of equation (2.1) are closely related with the solutions of

$$x^2 - dy^2 = 1. \tag{2.2}$$

The relation between the solutions of equation (2.1) and equation (2.2) are indicated in the following result.

Proposition 2.2. *[8] Let N be a non-zero integer and $d > 1$ be a square-free positive integer.*

1 . *There are infinitely many solutions (x_n, y_n) of the equation (2.2), and they are given by*

$$(x_n + y_n\sqrt{d}) = (x_1 + y_1\sqrt{d})^n, \; n \in \mathbb{N},$$

where (x_1, y_1) is the fundamental solution of equation (2.2).

2 . *Let (x_1, y_1) be the fundamental solution of equation (2.2) and let (α, β) be a solution of equation (2.1).Then for each $n \geq 1$, the ordered pair (γ_n, δ_n) is a solution of equation (2.1), where*

$$\gamma_n + \delta_n\sqrt{d} = (x_1 + y_1\sqrt{d})^n(\alpha + \beta\sqrt{d}).$$

We state two results on arithmetic progressions in integer-valued quadratic polynomial proved in [9].

Proposition 2.3. *[9, Theorem 2] Let a, b, c be three real numbers such that $f(n) = an^2 + bn + c$ for all positive integer n. Suppose $f(n)$ is a non-negative integer for every positive integer n. Then $2a, 2b$ and c are integers, and the sequence $(f(n))_{n\geq 1}$ contains no four-term arithmetic progression.*

Proposition 2.4. *[9, Theorem 3] Suppose that $f(n)$ is a polynomial satisfying the same assumptions as in Proposition 2.3. Then the sequence $(f(n))_{n\geq 1}$ contains infinitely many three-term arithmetic progression. Further, for each positive integer $n > -b/(2a)$, we obtain three-term arithmetic progression with $f(n)$ as first term and positive common difference.*

We also need the following propositions to prove our main results:

Proposition 2.5. *[2] Consider the Pythagorean equation*

$$x^2 + y^2 = z^2.$$

Then all solutions of the above equation satisfying conditions

$$\gcd(x, y, z) = 1, \;\; 2|y, \;\; x > 0, y > 0, z > 0$$

are given by

$$x = m^2 - n^2, \; y = 2mn, \; z = m^2 + n^2,$$

where m,n are positive integers such that $m > n > 0$, $\gcd(m, n) = 1$ and $2 \nmid (m - n)$.

Proposition 2.6. *[2] The Diophantine equation*

$$x^4 + y^4 = z^2$$

has no integral solution such that $x > 0, y > 0, z > 0$.

3 On arithmetic progressions in values of quadratic polynomials

In this section, we prove two results for integer-valued quadratic polynomials. We also discuss its consequences for the case of polygonal numbers. In the first theorem, we prove that there is no three-term arithmetic progression with a perfect square as a common difference for a class of integer-valued quadratic polynomials. In the second theorem, we find a class of numbers depending on integer-valued quadratic polynomials, which can not work as a common difference for three-term arithmetic progression.

Theorem 3.1. *Let $a, b, c \in \mathbb{R}$ such that $f(n) = an^2 + bn + c$ for all $n \in \mathbb{N}$. Suppose $f(n)$ is a non-negative integer for every $n \in \mathbb{N}$ such that $2a, 2b$ and c are integers. Let $2a = \lambda^2$, where λ is some positive integer and t be any positive integer. Then there exist no three-term arithmetic progression in the quadratic sequence $(f(n))_{n\geq 1}$ with common difference t^2.*

Proof. We will prove this result by method of contradiction. Let $a, b, c \in \mathbb{R}$ such that $f(n) = an^2 + bn + c$ for all $n \in \mathbb{N}$. Suppose $f(n)$ is a non-negative integer for every $n \in \mathbb{N}$. Then by Proposition 2.3, $2a, 2b$ and c are integers. Multiplying $4a$ in numerator and denominator of polynomial $f(n)$ and rearranging, we get

$$f(n) = \frac{N^2 - (b^2 - 4ac)}{4a}, \text{ where } N = 2an + b. \tag{3.1}$$

Let $f(x), f(y)$ and $f(z)$ be a three-term arithmetic progression in $(f(n))_{n\geq 1}$ with common difference t^2, where $x < y < z$ and t are positive integers. Using equation (3.1), we can write

$$f(x) = \frac{X^2 - (b^2 - 4ac)}{4a},$$
$$f(y) = \frac{Y^2 - (b^2 - 4ac)}{4a},$$
$$f(z) = \frac{Z^2 - (b^2 - 4ac)}{4a},$$

where $X = 2ax + b, Y = 2ay + b$ and $Z = 2az + b$.

Let $2a = \lambda^2$, where λ is some positive integer and consider $t^2 = f(y) - f(x)$, on simplifying we have

$$X^2 = Y^2 - 2\lambda^2 t^2. \tag{3.2}$$

Similarly, for $t^2 = f(z) - f(y)$, we have

$$Z^2 = Y^2 + 2\lambda^2 t^2. \tag{3.3}$$

Multiplying equation (3.2) and (3.3), we get

$$(XZ)^2 = Y^4 - (2\lambda^2 t^2)^2,$$

i.e.,

$$(XZ)^2 + (2\lambda^2 t^2)^2 = (Y^2)^2. \tag{3.4}$$

Without loss of generality, we take

$$\gcd(X, t) = \gcd(Y, t) = \gcd(Z, t) = 1, \tag{3.5}$$

for, if $\gcd(X, t) = d > 1$, then $\gcd(Y, t) = \gcd(Z, t) = d$ and we can produce a smaller arithmetic progression with common difference $(\frac{t}{d})^2$ satisfying the assumed gcd property in equation (3.5). Therefore, we have $\gcd(XZ, 2\lambda^2 t^2, Y^2) = 1$. Hence, $(XZ, 2\lambda^2 t^2, Y^2)$ is a solution of the Pythagorean equation satisfying conditions of Proposition 2.5. Using Proposition 2.5, there exist positive integers m, n such that $m > n > 0$, $\gcd(m, n) = 1$ and $2 \nmid (m - n)$ satisfying $XZ = m^2 - n^2$, $2(\lambda t)^2 = 2mn$, $Y^2 = m^2 + n^2$.

Now $2(\lambda t)^2 = 2mn$ and $\gcd(m, n) = 1$ implies that $m = \alpha^2$ and $n = \beta^2$, for some positive integers α, β. Thus, we can write $Y^2 = \alpha^4 + \beta^4$. This implies that (α, β, Y) is non trivial solution of the Diophantine equation $x^4 + y^4 = z^2$, which contradicts the result of Proposition 2.6. This completes the proof of Theorem 3.1. □

Remark 3.2. In the view of Problem 1 posed in Section 1, we obtain a class of polynomials for which there exist no three-term arithmetic progression with a square number as a common difference in Theorem 3.1. However, in general, it is not true that given any integer-valued quadratic polynomial, there exist no three-term arithmetic progression with square as a common difference (see Example 4.3 in Section 4).

Next, we obtain some results for polygonal numbers as a consequence of Theorem 3.1.

Proposition 3.3. *Let $s > 2$ be a positive integer such that $(s-2)$ is a perfect square, and for any positive integer t, there exist no three-term arithmetic progression of s-gonal numbers with common difference t^2.*

Proof. For each integer $s > 2$, n^{th} s-gonal number is given by

$$P_s(n) = \frac{(s-2)n(n-1)}{2} + n = \frac{(s-2)n^2}{2} - \frac{(s-4)n}{2}.$$

The proof follows immediately from Theorem 3.1 by substituting $a = \dfrac{(s-2)}{2}$, $b = -\dfrac{(s-4)}{2}$, and $c = 0$. □

The result of Ide and Jones [5, Theorem 3.1] for triangular numbers can be seen as a corollary of Proposition 3.3 by substituting $s = 3$.

Corollary 3.4. *There exist no three-term arithmetic progression in the sequence of triangular numbers with a square number as a common difference.*

Corollary 3.5. *There exist no three-term arithmetic progression in the sequence of hexagonal numbers with a square number as a common difference.*

Proof. The proof follows directly by substituting $s = 6$ in Proposition 3.3. □

The following theorem gives a class of numbers depending on the integer-valued quadratic polynomial which can not work as a common difference for any three-term arithmetic progression.

Theorem 3.6. *Suppose that $f(n) = an^2 + bn + c$ is a polynomial satisfying the same assumptions as in Theorem 3.1. Let t be a positive integer such that $\gcd(2a, t) = 1$. Then there exist no three-term arithmetic progression in sequence $(f(n))_{n\geq 1}$ with common difference $2at^2$.*

Proof. Let $f(x), f(y)$ and $f(z)$ be a three-term arithmetic progression in sequence $(f(n))_{n\geq 1}$ with common difference $2at^2$, where $x < y < z$ and t are positive integers such that $\gcd(2a, t) = 1$. As discussed earlier, $f(n)$ can be written as

$$f(n) = \frac{N^2 - (b^2 - 4ac)}{4a} \text{ , where } N = 2an + b. \tag{3.6}$$

Using equation (3.6), we can write

$$f(x) = \frac{X^2 - (b^2 - 4ac)}{4a} \text{ ,}$$

$$f(y) = \frac{Y^2 - (b^2 - 4ac)}{4a} \text{ ,}$$

$$f(z) = \frac{Z^2 - (b^2 - 4ac)}{4a} \text{ ,}$$

where $X = 2ax + b, Y = 2ay + b$ and $Z = 2az + b$.
Since $2at^2$ is common difference, therefore

$$2at^2 = f(y) - f(x) = f(z) - f(y).$$

On further simplification, we have

$$X^2 = Y^2 - 8a^2t^2 \tag{3.7}$$

and

$$Z^2 = Y^2 + 8a^2t^2. \tag{3.8}$$

Multiplying equation (3.7) and (3.8), we get

$$(XZ)^2 = Y^4 - (8a^2t^2)^2,$$

i.e.,

$$(XZ)^2 + (8a^2t^2)^2 = (Y^2)^2. \tag{3.9}$$

Without loss of generality, we take

$$\gcd(X, t) = \gcd(Y, t) = \gcd(Z, t) = 1, \tag{3.10}$$

for, if $\gcd(X, t) = d > 1$, then $\gcd(Y, t) = \gcd(Z, t) = d$. Since $\gcd(2a, t) = 1$, we can produce a smaller arithmetic progression with common difference $2a(\frac{t}{d})^2$ satisfying the assumed gcd property in equation (3.10). Therefore, we have $\gcd(XZ, 8a^2t^2, Y^2) = 1$. Hence, $(XZ, 8a^2t^2, Y^2)$ is a solution of the Pythagorean equation satisfying conditions of Proposition 2.5. Therefore, there exist positive integers m, n such that $m > n > 0$, $\gcd(m, n) = 1$ and $2 \nmid (m - n)$ satisfying $XZ = m^2 - n^2,\ 8(at)^2 = 2mn,\ Y^2 = m^2 + n^2$.

Since, $8(at)^2 = 2mn$ and $\gcd(m, n) = 1$ implies that $m = \alpha^2$ and $n = \beta^2$, for some positive integers α, β. Thus, we can write $Y^2 = \alpha^4 + \beta^4$. This implies that (α, β, Y) is non trivial solution of the Diophantine equation $x^4 + y^4 = z^2$, which contradicts the result of Proposition 2.6. This completes the proof of Theorem 3.6. □

Proposition 3.7. *Let $s > 2$ be any positive integer and t be any positive integer such that* $\gcd((s - 2), t) = 1$*, then there exist no three-term arithmetic progression of s-gonal numbers with common difference* $(s - 2)t^2$.

Proof. Proof is obvious using definition of polygonal numbers and substituting $a = \dfrac{(s-2)}{2}$, $b = -\dfrac{(s-4)}{2}$, and $c = 0$ in Theorem 3.6. □

Corollary 3.8. *Let t be an odd positive integer, then there exist no three-term arithmetic progression of square numbers with common* $2t^2$.

Proof. The proof follows directly from Proposition 3.7 by substituting $s = 4$. □

4 Arithmetic progressions in centered polygonal numbers

In this section, we investigate arithmetic progressions in the sequence of centered polygonal numbers. The centered polygonal numbers form a class of plane figurate numbers, in which layers of polygons are drawn centered about a point [3, pages 48-49]. Algebraically, we can define a centered polygonal number as follows:

Let $s > 2$ be a fixed positive integer and n be an arbitrary positive integer. Then n^{th} centered s-gonal number is obtained as the sum of first n terms of the sequence $1, s, 2s, 3s, \ldots$, and is given by

$$CP_s(n) = \frac{sn(n-1)}{2} + 1 = \frac{sn^2}{2} - \frac{sn}{2} + 1.$$

We first introduce some results for centered polygonal numbers, which are particular case of results derived in [9]. With the help of these results, we construct three-term arithmetic progressions in the sequence of centered polygonal numbers and conclude that the properties of centered polygonal numbers are different from polygonal numbers. Further, we also establish some results as a consequence of Theorem 3.1 and Theorem 3.6.

Proposition 4.1. *Let s be a fixed positive integer with $s > 2$. Then there cannot be four centered s-gonal numbers in an arithmetic progression with integer $d \neq 0$ as a common difference.*

Proof. For a fixed integer $s > 2$, n^{th} centered s-gonal number $CP_s(n)$ is a quadratic polynomial of form $f(n) = an^2 + bn + c$, where $a = \frac{s}{2}$, $b = -\frac{s}{2}$, $c = 1$ in $\mathbb{R}$. Clearly for each $n \in \mathbb{N}$, $CP_s(n)$ is a sequence of non-negative integer such that $2a = s$, $2b = -s$, $c = 1$ are integers. Therefore, $CP_s(n)$ satisfies all assumptions of Proposition 2.3. Hence, by Proposition 2.3, there cannot be four centered s-gonal numbers in an arithmetic progression with integer $d \neq 0$ as a common difference. □

Proposition 4.2. *Let s be a fixed positive integer with $s > 2$ and x be an arbitrary positive integer. Then there exist infinitely many integer $d > 0$ such that there is a three-term arithmetic progression with a common difference d in the centered s-gonal numbers beginning with $CP_s(x)$.*

Proof. As discussed in proof of Proposition 4.1, $CP_s(n)$ satisfies all assumptions of Proposition 2.3. Therefore, by Proposition 2.4, for a fixed integer $s > 2$, the sequence $(CP_s(n))_{n\geq 1}$ contains infinitely many three-term arithmetic progression. Since $b = -\frac{s}{2}$ and $a = \frac{s}{2}$, we have $n > -\frac{b}{2a} = -(\frac{-s}{2s}) = \frac{1}{2}$, which holds for each positive integer n. Therefore, again using Proposition 2.4, for any positive integer x, there exist infinitely many integers $d > 0$ such that there is a three-term arithmetic progression with common difference d in the centered s-gonal numbers beginning with $CP_s(x)$. This completes the proof. □

As illustrations of Proposition 4.2, we discuss the case for $s = 3$ and $s = 4$ as examples and find three-term arithmetic progressions starting with arbitrary term of sequence of centered s-gonal number in either case using similar method as discussed in [9].

Example 4.3 (Centered triangular number). We discuss the case for $s = 3$. Let

$$CP_3(x) = \frac{3x^2 - 3x + 2}{2}$$

be an arbitrary centered triangular number. The integers A and B satisfying equation

$$A^2 - 2B^2 = -1,\ A \equiv 1(mod\ 2) \text{ and } B \equiv 1(mod\ 2) \tag{4.1}$$

with $A > B > 1$ gives the value of

$$y = \frac{XB + s}{2s} = xB + \frac{1 - B}{2},$$

and

$$z = \frac{XA + s}{2s} = xA + \frac{1 - A}{2}$$

in positive integers such that $x < y < z$ and $CP_3(x), CP_3(y), CP_3(z)$ are in arithmetic progression. By second part of Proposition 2.2, the solution (A, B) of equation (4.1) satisfies

$$A + B\sqrt{2} = (3 + 2\sqrt{2})^n(1 + \sqrt{2}),\ n \geq 1 \text{ is an integer.}$$

Also for $n \geq 1$, $A > B > 1$. Therefore, for each n, we can construct a three-term arithmetic progression starting with $CP_3(x)$. As an example, we consider the case $n = 1$. Then $A+B\sqrt{2} = (3 + 2\sqrt{2})(1 + \sqrt{2}) = 7 + 5\sqrt{2}$. Therefore, we get $A = 7$ and $B = 5$. On substituting these values, we get

$$y = xB + \frac{1 - B}{2} = 5x - 2,$$

and

$$z = xA + \frac{1 - A}{2} = 7x - 3.$$

Therefore, $CP_3(x), CP_3(5x - 2), CP_3(7x - 3)$ are in arithmetic progression.
For $x = 1$, we get $CP_3(1) = 1, CP_3(3) = 10, CP_3(4) = 19$ are in arithmetic progression with common difference $d = 9$. For $x = 2$, we get $CP_3(2) = 4,\ CP_3(8) = 85,\ CP_3(11) = 166$ are in arithmetic progression with common difference $d = 81$. We can similarly obtain arithmetic progressions for each value of n and x.

Example 4.4 (Centered square number). Consider the case for $s = 4$. Let x be an arbitrary positive integer such that

$$CP_4(x) = 2x^2 - 2x + 1$$

is a centered square number. As discussed in Example 4.3, we obtain $A = 7$ and $B = 5$ as solution of equation (4.1) for $n = 1$. On substituting these values, we get

$$y = xB + \frac{1-B}{2} = 5x - 2,$$

and

$$z = xA + \frac{1-A}{2} = 7x - 3.$$

Therefore, $CP_4(x)$, $CP_4(5x-2)$, $CP_4(7x-3)$ are in arithmetic progression.
For $x = 1$, we get $CP_4(1) = 1, CP_4(3) = 13, CP_4(4) = 25$ are in arithmetic progression with common difference $d = 12$. For $x = 2$, we get $CP_4(2) = 5, CP_4(8) = 113, CP_4(11) = 221$ are in arithmetic progression with common difference $d = 108$. Similarly, we can proceed for other values of n and x.

Remark 4.5. In [5], Ide and Jones proved that there exist no three-term arithmetic progression of triangular numbers with a square number as a common difference. However, Example 4.3 shows that there exist three-term arithmetic progressions of centered polygonal numbers with a square number as a common difference. Example 4.3 also provides a class of integer-valued quadratic polynomials for which three-term arithmetic progression with square common difference can be constructed.

In the view of above remark natural question arises:

Problem 2. Are there finitely many or infinitely many arithmetic progressions of centered triangular numbers with a square number as a common difference?

The following theorem gives description of arithmetic progression having common difference a square for the special class of centered polygonal numbers.

Theorem 4.6. *For any perfect square positive integer $s > 2$ and for any positive integer t, no three-term arithmetic progression of centered s-gonal numbers with common difference t^2 exist.*

Proof. The proof follows directly from the definition of centered polygonal numbers. Substituting $a = \frac{s}{2}$, $b = -\frac{s}{2}$, $c = 1$, the result follows from Theorem 3.1. □

We establish the following results as corollaries of Theorem 4.6:

Corollary 4.7. *For any perfect square positive integer $s > 2$ and any positive integer t, there exist no three-term arithmetic progressions of centered s-gonal numbers with centered octagonal number $CP_8(t)$ as common difference.*

Proof. Using the definition of centered s-gonal number, we have $CP_8(t) = (2t-1)^2$, where $t > 0$. Therefore, the result follows from Theorem 4.6. □

Corollary 4.8. *For any perfect square positive integer $s > 2$ and any positive integer t, there exist no three-term arithmetic progressions of centered s-gonal numbers with a square number $P_4(t) = t^2$ as common difference.*

Proof. The t^{th} s-gonal number [4] is given by

$$P_s(t) = \frac{(s-2)t(t-1)}{2} + t,$$

where $s > 2$ and t are positive integers. Therefore, we have $P_4(t) = t^2$. Hence, by Theorem 4.6 we conclude the result. □

Remark 4.9. In Theorem 4.6, the condition "perfect square"is not superfluous. For example, if we take $s = 3$, we obtain arithmetic progressions of centered triangular number with square as a common difference (see Example 4.3).

The following theorem provides a class of numbers that can not work as a common difference for three-term arithmetic progression in the given sequence of centered polygonal numbers.

Theorem 4.10. *Let $s > 2$ be a fixed positive integer and $t > 0$ be any integer such that $\gcd(s, t) = 1$. Then there exist no three-term arithmetic progression in centered s-gonal numbers with common difference st^2.*

Proof. For a fixed integer $s > 2$, n^{th} centered s-gonal number $CP_s(n)$ is a quadratic polynomial of form $f(n) = an^2 + bn + c$, where $a = \frac{s}{2}$, $b = -\frac{s}{2}$, $c = 1$ in $\mathbb{R}$. The result follows directly from Theorem 3.6. □

Remark 4.11. In Theorem 4.10, we proved that for any fixed positive integer $s > 2$ if we take $d = st^2$,where t is any integer such that $\gcd(s, t) = 1$ then there exist no three-term arithmetic progression of centered s-gonal numbers. However, the above result is not true if we consider $d \neq st^2$ for any positive integer t. For example, consider $s = 4$ and $d = 12 = 4.3$. Clearly $d \neq 4t^2$ for any integer t, but we have $CP_4(1) = 1, CP_4(3) = 13, CP_4(4) = 25$ in arithmetic progression with common difference $d = 12$ (see Example 4.4).

5 Concluding Remarks

We conclude this paper with few comments on Problem 1. In Theorem 3.1, we provided a subclass of integer valued quadratic polynomials for which there exist no three-term arithmetic progression with square common difference. In Example 4.3, we constructed three-term arithmetic progression for a class of integer valued quadratic polynomial. Therefore, the result of Theorem 1 of [6] cannot be extended to the case of integer valued quadratic polynomials. However, Theorem 3.6 provides a class of numbers which cannot be taken as common difference for three-term arithmetic progressions in integer valued quadratic polynomials.

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