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The C-prime Fuzzy Graph of a Nearring with respect to a Level Ideal

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Abstract In this paper, we introduce a c-prime fuzzy graph of a nearring with respect to a level ideal of a fuzzy ideal. We find a relation between properties of the fuzzy ideal and properties of the fuzzy graph. We introduce ideal symmetry of the fuzzy graph and obtain conditions under which the graph is ideal symmetric. We find a relation between nearring homomorphisms and graph homomorphisms. We investigate conditions required for the homomorphic image of a c-prime fuzzy ideal to be a c-prime fuzzy ideal.

1 Introduction

After the introduction of fuzzy set theory researchers applied it in different fields. Rosenfeld [27] generalized algebraic group structure by introducing fuzzy groups. Davvaz [8] introduced fuzzy ideals of rings and nearrings. Kedukodi, Kuncham and Bhavanari [20, 16, 19] studied different fuzzy ideals of nearrings. Davvaz [7] introduced interval valued L-fuzzy ideals of nearrings. Jagadeesha, Kedukodi, Kuncham [13, 14, 21] studied properties of different interval valued L-fuzzy ideals of nearrings. Beck [5] related a ring with a graph by introducing the zero divisor graph of a ring. Anderson, Livingston [4] generalized the definition of a zero divisor graph. Wu [30], Wu and Lu [31] further studied zero divisor graphs. Bhavanari, Kuncham, Kedukodi [6], Kedukodi, Kunchham, Jagadeesha and Juglal [17] studied graphs of nearrings with respect to different ideals. Mordeson, Nair [24] studied different graphs and fuzzy graphs. Kedukodi, Kuncham and Bhavanari [18] introduced a fuzzy graph of a nearing and studied its properties. Samanta, Akram and Pal [28] introduced m-step fuzzy competition graphs, mm-step neighbourhood graphs, fuzzy economic competition graphs and fuzzy mm-step economic competition graphs and studied some of their properties. Nagoorgani, Akram and Anupriya [25] obtained some properties of double domination of intuitionistic fuzzy graphs and found some applications. Shahzadi, Akram [29] introduced intuitionistic fuzzy soft graphs, complete intuitionistic fuzzy soft graphs, strong intuitionistic fuzzy soft graphs and the self complement of intuitionistic fuzzy soft graphs, gave procedures to constrct these graphs and found applications of these graphs in communication networks and decision-making. Akram, Waseem [2] studied equitable domination, k-domination and restrained domination in bipolar fuzzy graphs and gave application of bipolar fuzzy graphs to decision-making problems. Fathalian, Borzooei, Hamidi [9] proved that every connected graph is a fuzzy labelling graph and gave some applications of these graphs. Akram, Habib [1] studied the notion of q-rung picture fuzzy graphs and obtained regularity of these graphs.

In this paper, we define a c-prime fuzzy graph of a nearring N with respect to a level set ν_t of a fuzzy ideal ν denoted by (N, ν, ρ, t). We prove that, if ν is a c-prime fuzzy ideal of N then ν_t is a strong vertex cut of (N, ν, ρ, t). We obtain a relation between the properties of fuzzy ideal and properties of the fuzzy graph. We introduce ideal symmetry of the fuzzy graph and find conditions under which the graph is ideal symmetric. We investigate conditions required to obtain the fuzzy clique of this fuzzy graph. We find conditions for the fuzzy ideal under which the level set ν_t is a vertex cover of the fuzzy graph. We prove that, if ν is a c-prime ideal of a simple nearring or ν is fuzzy ideal of an integral nearring, then (N, ν, ρ, t) is a complete graph or a star graph. If N_1, N_2 are nearrings and $f : N_1 \to N_2$ is a nearring homomorphism then we prove that f is a graph homomorphism between (N, ν, ρ, t) and $(N, f(\nu), f(rho), t)$. We prove that if x is vertex of (N, ν, ρ, t) such that x is connected to all other vertices of (N, ν, ρ, t) then $f(x)$ is connected to all vertices of $(N, f(\nu), f(\rho), t)$ and vice versa. We find conditions under which the homomorphic image of c-prime fuzzy ideal to be c-prime fuzzy ideal using properties of fuzzy graph.

2 Preliminaries

In this paper N, N_1 and N_2 represent right nearrings. We refer to Pilz [26] for definitions, concepts related to nearrings, Anderson and Fuller [3] for rings, Harary [12] for graphs.
(Harary [12]) A *graph* $G = (V, E)$ consists of a set of objects $V = \{v_1, v_2, ...\}$ called *vertices* (or points) and another set $E = \{e_1, e_2, ..\}$ whose elements are called *edges* such that each edge e is identified with an unordered pair (v_i, v_j) of vertices. Let $G = (V, E)$ be a graph. The degree of a vertex $v \in V$ is the number of edges having end point v denoted by $deg(v)$. A graph $G = (V, E)$ is said to be complete if $(u, v) \in E$ for all $u, v \in V, u \neq v$. A *vertex cover* of graph G is a subset K of V such that if (u, v) is an edge of G, then $u \in K$ or $v \in K$ or both $u \in K$ and $v \in K$. A *clique* of (V, E) is a complete subgraph with maximum number of vertices.

Definition 2.1. *(Hell [22]) Let* $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ be two graphs. A *graph homomorphism* from G_1 to G_2 is a mapping $f : V_1 \to V_2$ such that $(f(u), f(v)) \in E_2$ whenever $(u, v) \in E_1$.

Definition 2.2. [24] A fuzzy graph $H = (\nu, \rho)$ of (V, E) is defined by a fuzzy subset ν of V and a fuzzy subset ρ of E such that $\rho(x, y) \leq \nu(x) \wedge \nu(y) \ \forall \ x, y \in V$.

Let $t \in [0, 1]$. Then
$$\nu_t = \{x \in V \mid \nu(x) \geq t\};$$
$$\rho_t = \{(x, y) \in E \mid \rho(x, y) \geq t\}.$$
Then ν_t is called level set of ν and ρ_t is called level set of ρ. Then (ν_t, ρ_t) is a graph with vertex set ν_t and edge set ρ_t. Let (ν, ρ) be a fuzzy graph. Then (ν, ρ) said to be *complete* if $\rho(u, v) = \nu(u) \wedge \nu(v) \, \forall \, u, v \in V$.
A complete fuzzy subgraph with maximum number of vertices is called a *fuzzy clique* of (ν, ρ).
Let I, J, K be ideals of N. Then ideal I is called a *prime ideal* if $JK \subseteq I$ implies $J \subseteq I$ or $K \subseteq I$.

Definition 2.3. (Groenewald [10, 11]) Let I be an ideal of N. Then I is called a *c-prime ideal* of N if $xy \in I$ implies $x \in I$ or $y \in I$ for all $x, y \in N$.

Definition 2.4. (Groenewald [10, 11]) An ideal I of N is called *c-semiprime* if $x \in N$ and $x^2 \in I$ implies $x \in I$.

Definition 2.5. (Pilz [26]) Nearring N is called *integral* if it has no zero divisors.

Definition 2.6. (Pilz [26]) Nearring N is called *simple* if its only ideals are $\{0\}$ and N.

Definition 2.7. (Davvaz [8]) Let $\alpha, \beta \in [0, 1]$ and
$\alpha < \beta$. Let ν be a fuzzy subset of N. Then ν is called a fuzzy ideal with thresholds α, β if for all $x, y, i \in N$,
(i) $\alpha \vee \nu(x+y) \geq \beta \wedge \nu(x) \wedge \nu(y)$, (ii) $\alpha \vee \nu(-x) \geq \beta \wedge \nu(x)$, (iii) $\alpha \vee \nu(y+x-y) \geq \beta \wedge \nu(x)$,
(iv) $\alpha \vee \nu(xy) \geq \beta \wedge \nu(x)$, (v) $\alpha \vee \nu(x(y+i) - xy) \geq \beta \wedge \nu(i)$.
Here α is called the lower threshold of ν and β is the upper threshold of ν. In this paper fuzzy ideal ν means fuzzy ideal with lower threshold α and upper threshold β and and $\nu(0) \geq \beta$.

Theorem 2.8. *(Davvaz [8]) Let ν be a fuzzy subset of N. Then ν is a fuzzy ideal of N if and only if for every $t \in (\alpha, \beta]$ the level subset ν_t is an ideal of N.*

Definition 2.9. (Kedukodi, Kuncham and Bhavanari[20]) A fuzzy ideal ν of N is called *a c-prime fuzzy ideal* if for all $a, b \in N$, $\alpha \vee \nu(a) \vee \nu(b) \geq \beta \wedge \nu(ab)$.

Definition 2.10. A fuzzy ideal μ is called a *c-semiprime fuzzy ideal* if for all $a \in N$, $\alpha \vee \mu(a) \geq \beta \wedge \mu(a^2)$.

Theorem 2.11. *(Kedukodi, Kuncham and Bhavanari[20]) Let ν be a fuzzy ideal of N. Then ν is a c-prime (resp. c-semiprime) fuzzy ideal of N if and only if for every $t \in (\alpha, \beta]$, the level subset ν_t is a c-prime (resp. c-semiprime) ideal of N.*

Definition 2.12. (Davvaz [7])

Let $f : N_1 \to N_2$ be a mapping. Let μ be a fuzzy subset of N_1. Then the image of μ under f is given by $f(\hat{\mu})(y) = \begin{cases} \vee_{x\in f^{-1}(y)}\hat{\mu}(x) & \text{if} \quad f^{-1}(y) \neq \emptyset \\ 0 & \text{if} \quad f^{-1}(y) = \emptyset \end{cases}$ for all $y \in N_2$.

Remark 2.13. In the next part of this paper by fuzzy (resp. fuzzy c-prime, fuzzy c-semiprime) ideal we mean fuzzy (resp. fuzzy c-prime, fuzzy c-semiprime) ideal with lower threshold α and upper threshold β.

3 C-prime Fuzzy Graph of a Nearring with respect to a Level Ideal

Definition 3.1. Let $\nu : N \to (0, 1]$ be a fuzzy ideal of N. Let $t \in (\alpha, \beta]$ be fixed. Define $\rho : N \times N \to [0, 1]$ as follows:

$$\rho(x, y) = \begin{cases} \nu(x) \wedge \nu(y) & x \neq y \text{ and } (xy \in \nu_t \text{ or } yx \in \nu_t) \\ \\ 0 & \text{Otherwise} \end{cases}$$

Then the fuzzy graph with respect to ν_t denoted by (N, ν, ρ, t) is called the *c-prime fuzzy graph of N with respect to the level ideal ν_t.*

Now we provide some examples of cprime fuzzy graphs of nearrings.

Example 3.2. Let $N = \{0, a, b, c\}$ be a set with binary operations $+$ and $\cdot$ defined as in Table 1. Then $(N, +, \cdot)$ is a nearring.

$+$	0	a	b	c
0	0	a	b	c
a	a	0	c	b
b	b	c	0	a
c	c	b	a	0

$\cdot$	0	a	b	c
0	0	0	0	0
a	0	a	0	a
b	b	b	b	b
c	b	c	b	c

Table 1. Nearring for Example 3.2

We define $\nu : N \to (0, 1]$ by

$$\nu(x) = \begin{cases} 0.9 & \text{if} \quad x = 0 \\ 0.6 & \text{if} \quad x = a \\ 0.3 & \text{if} \quad x \in \{b, c\} \end{cases}$$

If we take thresholds $\alpha = 0.3$ and $\beta = 0.6$ then ν is a fuzzy ideal of N. Let $t = \beta$. Then $\nu_t = \{0, a\}$. Then the values of ρ are as in Table 2.

The graph is given in Figure 1.

If we take thresholds $\alpha = 0.6$ and $\beta = 0.9$ then ν is a fuzzy ideal of N. Let $t = \beta$. Then $\nu_t = \{0\}$. Then the values of ρ are as in Table 3.

The graph is given in Figure 2.

$\rho(x,y)$	$y=0$	$y=a$	$y=b$	$y=c$
$x=0$	0	0.6	0.3	0.3
$x=a$	0.6	0	0.3	0.3
$x=b$	0	0	0	0
$x=c$	0	0	0	0

Table 2. Values of $\rho(x,y)$ for Example 3.2 when $\nu_t = \{0, a\}$
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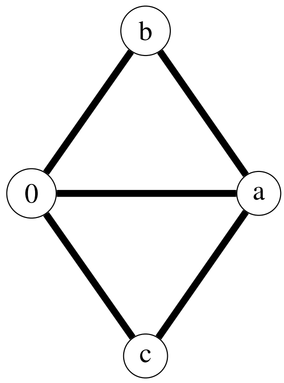

Figure 1. Graph when $\nu_t = \{0, a\}$

Example 3.3. Let $N = Z_6 = \{0, 1, 2, ..., 5\}$ be the ring of integers modulo 6. Let $a, b \in (0.1, 0.9)$ and $a \neq b$. Define a fuzzy subset $\nu : Z_6 \to (0, 1]$ by

$$\nu(x) = \begin{cases} 0.9 & \text{if } x = 0 \\ a & \text{if } x = 3 \\ b & \text{if } x \in \{2, 4\} \\ 0.1 & \text{otherwise} \end{cases}$$

If we take thresholds $\alpha = a \wedge b$ and $\beta = a \vee b$ then ν is a fuzzy ideal of Z_6. Let $t = \beta$. Let $a = 0.8$ and $b = 0.5$. Then $\alpha = 0.5$, $\beta = 0.8$, $t = 0.8$ and $\nu_t = \{0, 3\}$. Then the values ρ are as in Table 4.

The graph is given in Figure 3.

Let $a = 0.5$ and $b = 0.8$. Then $\alpha = 0.5$, $\beta = 0.8$, $t = 0.8$ and $\nu_t = \{0, 2, 4\}$. The table for ρ is in Table 5. The graph is given in Figure 4.

Remark 3.4. A vertex v of (N, ν, ρ, t) is connected to all other vertices if and only if $\rho(v, v_1) > 0 \;\forall\; v_1 \neq v;\; v_1 \in N$.

Proposition 3.5. *Let ν be a fuzzy ideal of N and $t \in (\alpha, \beta]$.*
(i) Let ν be a c-prime fuzzy ideal of N. Then ν_t is a strong vertex cut of (N, ν, ρ, t).
(ii) Let ν be a c-semiprime fuzzy ideal of N and ν_t a strong vertex cut of (N, ν, ρ, t).
Then ν is a c-prime fuzzy ideal of N.

Proof. To prove (i), let ν be a c-prime fuzzy ideal of N. Suppose $\nu_t = N$. Then ν_t is a strong vertex cut of (N, ν, ρ, t). Let $\nu_t \subset N$. Let $x \in N \setminus \nu_t$ and $y \in N \setminus \nu_t$ such that there exists an edge between x and y in (N, ν, ρ, t). Then $xy \in \nu_t$ or $yx \in \nu_t$. Without loss of generality assume $xy \in \nu_t$. As ν is a c-prime fuzzy ideal of N, we get ν_t is a c-prime ideal of N. Then $x \in \nu_t$ or $y \in \nu_t$. A contradiction to the fact that $x \in N \setminus \nu_t$ and $y \in N \setminus \nu_t$. Hence ν_t is strong vertex cut of (N, ν, ρ, t).
To prove (ii), let ν be a c-semiprime fuzzy ideal of N and ν_t, a strong vertex cut of (N, ν, ρ, t). Let $xy \in \nu_t$. Suppose $x = y$. Then $x \in \nu_t$ (ν_t is a c-semiprime.) Let $x \neq y$. Suppose $x \in N \setminus \nu_t$ and $y \in N \setminus \nu_t$. As ν_t is a strong vertex cut of (N, ν, ρ, t) there is no edge between x and y in

$\rho(x,y)$	$y=0$	$y=a$	$y=b$	$y=c$
$x=0$	0	0.6	0.3	0.3
$x=a$	0.6	0	0.3	0
$x=b$	0	0	0	0
$x=c$	0	0	0	0

Table 3. Values of $\rho(x,y)$ for Example 3.2 when $\nu_t = \{0\}$

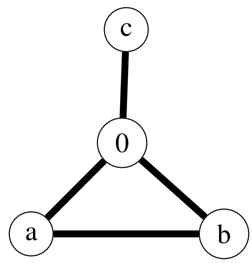

Figure 2. Graph when $\nu_t = \{0\}$

(N,ν,ρ,t). Then $xy \notin \nu_t$ and $yx \notin \nu_t$, a contradiction since $xy \in \nu_t$. Hence ν_t is a c-prime ideal of N. Therefore ν is a c-prime fuzzy ideal of N. □

Remark 3.6. In Proposition 3.5(i), if ν is not a c-prime fuzzy ideal of N then ν_t is not a strong vertex cut of N. We provide an example.
In Example 3.2, note that $\nu_t = \{0\}$ is not a c-prime fuzzy ideal of N. Since $\alpha \vee \nu(a) \vee \nu(b) = 0.6 \vee 0.6 \vee 0.3 = 0.6 \ngeq 0.9 = 0.9 \wedge 0.9 = \beta \wedge \nu(ab)$. Observe that $\nu_t = \{0\}$ is not a strong vertex cut of (N,ν,ρ,t).
In Proposition 3.5(ii) if ν_t is not a c-semiprime fuzzy ideal of N then even if ν_t is a strong vertex cut of (N,ν,ρ,t) it is not a c-prime fuzzy ideal of N. We provide Example 3.7.

Example 3.7. Let $N = Z_4$ be the ring of integers modulo 4. We define

$$\nu : N \to [0,1] \text{ by } \nu(x) = \begin{cases} 0.8 & \text{if } \ x = 0 \\ 0.5 & \text{if } \ x = 2 \\ 0.2 & \text{if } \ x \in \{1,3\} \end{cases}$$

Take thresholds $\alpha = 0.5$ and $\beta = 0.7$. Then ν is a fuzzy ideal of N. Let $t = \beta$. Then $\nu_t = \{0\}$. Then values of ρ are as in Table 6.

The graph is given in Figure 5.

Note that ν is not a c-semiprime fuzzy ideal of N. Since $\alpha \vee \nu(2) \vee \nu(2) = 0.5 \vee 0.5 \vee 0.5 = 0.5 \ngeq 0.7 = 0.7 \wedge 0.9 = \beta \wedge \nu(ab)$. Observe that ν is not a c-prime fuzzy ideal of N.

Proposition 3.8. *Let ν be a c-prime fuzzy ideal of N and $t \in (\alpha,\beta]$.*
(i) Let x be a vertex in (N,ν,ρ,t). If $\rho(x,z) > 0 \ \forall \ z \neq x;\ z \in N$ then $x \in \nu_t$.
(ii) Let x be a vertex in (N,ν,ρ,t). If $x \in \nu_t$ then $\rho(x,z) > 0 \ \forall \ z \neq x;\ z \in N$.
(iii) ν is c-prime fuzzy ideal of N if and only if every element $x \in \nu_t$ is connected to all other elements of N in (N,ν,ρ,t).
(iv) If ν is c-prime fuzzy ideal N then (ν_t,ν,ρ,t) is complete subgraph of (N,ν,ρ,t).

Proof. To prove (i), let x be a vertex in (N,ν,ρ,t) such that $\rho(x,z) > 0 \ \forall \ z \neq x;\ z \in N$. Then $xy \in \nu_t$ or $yx \in \nu_t$ for all $y \in N$ with $y \neq x$. Without loss of generality, assume $xy \in \nu_t$. Suppose $\nu_t = N$. Then $x \in \nu_t$. Let $\nu_t \subset N$. Choose $y \in N \setminus \nu_t$. Then $x \in \nu_t$. (Since ν is a c-prime fuzzy ideal of N, by Theorem 2.11 we get ν_t is a c-prime ideal of N. Then $xy \in \nu_t$ and $y \notin \nu_t$ implies $x \in \nu_t$.)
To prove (ii), let $t \in (\alpha,\beta]$ and $x \in \nu_t$. Suppose $x = 0$. Then $xy = 0 \in \nu_t$ and $yx = 0 \in \nu_t$ for all $y \in N$. (Since ν is a fuzzy ideal of N, ν_t is an ideal of N. Then $0 \in \nu_t$.) Then $\rho(x,y) > 0$.

$\rho(x,y)$	$y=0$	$y=1$	$y=2$	$y=3$	$y=4$	$y=5$
$x=0$	0	0.1	0.5	0.8	0.5	0.1
$x=1$	0.1	0	0	0.1	0	0
$x=2$	0.5	0	0	0.5	0	0
$x=3$	0.8	0.1	0.5	0	0.5	0.1
$x=4$	0.5	0	0	0.5	0	0
$x=5$	0.1	0	0	0.1	0	0

Table 4. Values of $\rho(x,y)$ for Example 3.3 when $\nu_t = \{0,3\}$

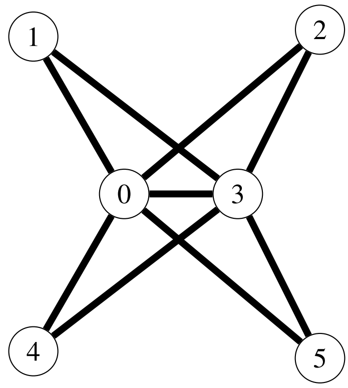

Figure 3. Graph when $\nu_t = \{0,3\}$ for Example 3.3

Hence the result is true. Let $x \neq 0$. Now, suppose there exists a vertex $y \in N$ such that y is not connected to x in (N,ν,ρ,t). This implies $xy \notin \nu_t$ and $yx \notin \nu_t$. Now as $x \in \nu_t$ and ν_t is an ideal of N, we get $xy \in \nu_t$, a contradiction. Hence x is connected to all other elements of N in (N,ν,ρ,t). The result (iii) follows from (i) and (ii). The result (iv) follows from (iii). □

Remark 3.9. (i) In Example 3.2, when $\alpha = 0.3, \beta = 0.6$, ν is a c-prime fuzzy ideal of N. Note that $\rho(b,z) \ngtr 0$ for all $z \in N$. Observe that $b \notin \nu_t$.
(ii) In Example 3.2, when $\alpha = 0.3, \beta = 0.6$, ν is a c-prime fuzzy ideal of N. Note that $c \notin \nu_t$ and observe that $\rho(c,b) = 0$.
(iii) In Example 3.3 when $\alpha = 0.5, \beta = 0.8$, ν is a c-prime fuzzy ideal of N. For $t = \beta$ we get (ν_t,ν,ρ,t) as in Figure 6. Observe that (ν_t,ν,ρ,t) is a complete subgraph of (N,ν,ρ,t).

Definition 3.10. Let (N,ν,ρ,t) be a c-prime fuzzy graph. Then (N,ν,ρ,t) is said to be *ideal symmetric* for every pair of vertices a,b in (N,ν,ρ,t) with an edge between them, if either $[\rho(a,c) > 0 \;\; \forall \;\; c \neq a;\; c \in N]$ or $[\rho(b,c) > 0 \;\; \forall \;\; c \neq b;\; c \in N]$.

Proposition 3.11. *Let ν be a fuzzy ideal of N and $t \in (\alpha,\beta]$.*
(a) If ν is a c-prime fuzzy ideal of N then (N,ν,ρ,t) is ideal symmetric.
(b) Suppose (i) (N,ν,ρ,t) is ideal symmetric ;
(ii) ν is a c-semiprime fuzzy ideal of N;
(iii) for every $x \in N$ and $\rho(x,z) > 0 \;\; \forall \;\; z \neq x;\; z \in N$ implies $x \in \nu_t$.
Then ν is c-prime fuzzy ideal of N.

Proof. Let $t \in (\alpha,\beta]$. To prove (a), let $m,n \in N$ be such that there is an edge between m and n in (N,ν,ρ,t). Then $mn \in \nu_t$ or $nm \in \nu_t$. Without loss of generality, assume $mn \in \nu_t$. Then $m \in \nu_t$ or $n \in \nu_t$ (As ν is a c-prime fuzzy ideal of N, ν_t is a c-prime ideal of N.) By Proposition 3.8 (ii) we get $\rho(m,z) > 0 \;\; \forall \;\; z \neq m;\; z \in N$ or $\rho(n,z) > 0 \;\; \forall \;\; z \neq n;\; z \in N$. Hence (N,ν,ρ,t) is ideal symmetric.
To prove (b), let $x,y \in N$ and $xy \in \nu_t$. Suppose $\nu_t = N$. Then $x \in \nu_t$. Let $\nu_t \subset N$. Suppose $x = y$. Then $x \in \nu_t$ (by (ii) ν is c-semiprime fuzzy ideal of N, ν_t is a c-semiprime ideal of N). Let $x \neq y$. Now there exists an edge between x and y in (N,ν,ρ,t). As (N,ν,ρ,t) is ideal

$\rho(x,y)$	$y=0$	$y=1$	$y=2$	$y=3$	$y=4$	$y=5$
$x=0$	0	0.1	0.8	0.5	0.8	0.1
$x=1$	0.1	0	0.1	0	0.1	0
$x=2$	0.8	0.1	0	0.5	0.8	0.1
$x=3$	0.5	0	0.5	0	0.5	0
$x=4$	0.8	0.1	0.8	0.5	0	0.1
$x=5$	0.1	0	0.1	0	0.1	0

Table 5. Values of $\rho(x,y)$ for Example 3.3 when $\nu_t = \{0,2,4\}$

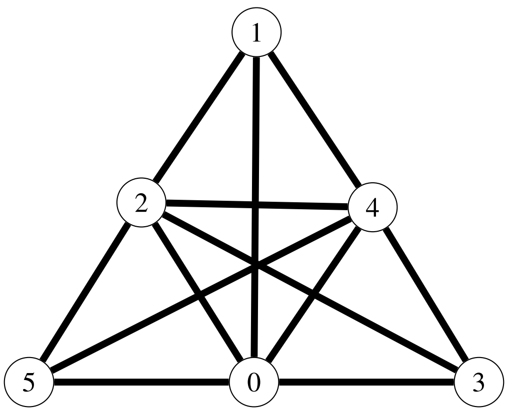

Figure 4. Graph when $\nu_t = \{0,2,4\}$ for Example 3.3

symmetric we get either $\rho(x,z) > 0 \ \forall \ z \neq x;\ z \in N$ or $\rho(y,z) > 0 \ \forall \ z \neq y;\ z \in N$. By Proposition 3.8 (i), we get $x \in \nu_t$ or $y \in \nu_t$. Thus ν is a c-prime fuzzy ideal of N. □

Remark 3.12. (i) In Example 3.2, when we take thresholds $\alpha = 0.6, \beta = 0.9$ note that ν is not a c-prime fuzzy ideal of N, since $\alpha \vee \nu(a) \vee \nu(b) = 0.6 \vee 0.6 \vee 0.3 = 0.6 \not\geq 0.9 = 0.9 \wedge 0.9 = \beta \wedge \nu(ab)$. Observe that (N,ν,ρ,t) is not ideal symmetric.
(ii) In Example 3.7, note that the graph is ideal symmetric however ν_t is not a c-semiprime ideal of N, ($2^2 = 0 \in \nu_t$ however $2 \notin \nu_t$) observe that ν is not a c-prime fuzzy ideal of N, since $\alpha \vee \nu(2) \vee \nu(2) = 0.5 \vee 0.5 \vee 0.5 = 0.5 \not\geq 0.7 = 0.7 \wedge 0.8 = \beta \wedge \nu(2 \cdot 2)$.
(iii) In Example 3.2, when we take thresholds $\alpha = 0.6, \beta = 0.9$ note that $\nu_t = \{0\}$ is a c-semiprime ideal of N, however (N,ν,ρ,t) is not ideal symmetric. Observe that ν is not a c-prime fuzzy ideal of N. Since $\alpha \vee \nu(a) \vee \nu(b) = 0.6 \vee 0.6 \vee 0.3 = 0.6 \not\geq 0.9 = 0.9 \wedge 0.9 = \beta \wedge \nu(ab)$.

Proposition 3.13. *Let ν be a c-prime fuzzy ideal of N and $n \notin \nu_t$. Then $C = \nu_t \cup \{n\}$ is a fuzzy clique of (N,ν,ρ,t).*

Proof. By Proposition 3.8 (iv), we get (ν_t,ν,ρ,t) is complete subgraph of (N,ν,ρ,t). Let $n \notin \nu_t$ and $C = \nu_t \cup \{n\}$. By Proposition 3.8 (iii), we get every element of ν_t is connected to n. This proves that (C,ν,ρ,t) is complete. It remains to prove that C is maximal. First, we show that if $p \in N \setminus \nu_t, q \in N \setminus \nu_t$ and $p \neq q$ then p is not connected to q in (N,ν,ρ,t). If possible suppose p is connected to q. Then $pq \in \nu_t$ or $qp \in \nu_t$. Without loss of generality, assume $pq \in \nu_t$. As ν_t is c-prime ideal of N, we get $p \in \nu_t$ or $q \in \nu_t$. This is a contradiction to the fact that $p \in N \setminus \nu_t, q \in N \setminus \nu_t$. Now, let $C_1 = C \cup \{m\};\ m \notin \nu_t$ and $m \neq n$. Then $\rho(n,m) = 0 \neq \nu(m) \wedge \nu(n)$. This implies (C_1,ν,ρ,t) is not complete. Hence (C_1,ν,ρ,t) cannot be a fuzzy clique. Thus $C = \nu_t \cup \{n\}$ is a fuzzy clique of (N,ν,ρ,t). □

Remark 3.14. (i) In Example 3.3, when $\alpha = 0.5, \beta = 0.8$ and $\nu_t = \{0,3\}$, ν is a c-prime fuzzy ideal of N. Take $n = 2 \notin \nu_t$. Then $C = \nu_t \cup \{n\}$ is a fuzzy clique of (N,ν,ρ,t) as in Figure 7.

In Example 3.3, when $\alpha = 0.5, \beta = 0.8$ and $\nu_t = \{0,2,4\}$, ν is a c-prime fuzzy ideal of N. Take $n = 2 \notin \nu_t$ Then $C = \nu_t \cup \{n\}$ is a fuzzy clique of (N,ν,ρ,t) as is Figure 8.

$\rho(x,y)$	$y=0$	$y=1$	$y=2$	$y=3$
$x=0$	0	0.2	0.5	0.2
$x=1$	0.2	0	0	0
$x=2$	0.5	0	0	0
$x=3$	0.2	0	0	0

Table 6. Values of $\rho(x,y)$ for Example 3.7 when $\nu_t = \{0\}$
.

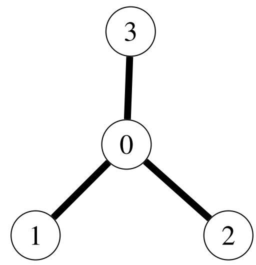

Figure 5. Graph when $\nu_t = \{0\}$

(ii) In Example 3.2, when $\alpha = 0.6, \beta = 0.9$ and $\nu_t = \{0\}$, ν is not a c-prime fuzzy ideal of N. Take $n = c \notin \nu_t$ Then $C = \nu_t \cup \{n\}$ is a complete subgraph of (N, ν, ρ, t) as is Figure 9, but not fuzzy clique (N, ν, ρ, t).

Proposition 3.15. *Let ν be a c-prime fuzzy ideal of N and $t \in (\alpha, \beta]$. Then*
(i) ν_t is a vertex cover of (N, ν, ρ, t).
(ii) $(N \setminus \nu_t, \nu, \rho, t)$ is an empty graph.

Proof. To prove (i), let $x, y \in N$ be such that (x, y) is an edge in (N, ν, ρ, t). Then $xy \in \nu_t$ or $yx \in \nu_t$. Without loss of generality assume $xy \in \nu_t$. As ν is a c-prime fuzzy ideal of N we get ν_t is a c-prime ideal of N. Then $x \in \nu_t$ or $y \in \nu_t$. Hence ν_t is a vertex cover of (N, ν, ρ, t).
To prove (ii), let $x, y \in N \setminus \nu_t$ is such that (x, y) is an edge in $(N \setminus \nu_t, \nu, \rho, t)$. Then $xy \in \nu_t$ or $yx \in \nu_t$. Without loss of generality assume $xy \in \nu_t$. As ν is a c-prime fuzzy ideal of N we get that ν_t is a c-prime ideal of N. Then $x \in \nu_t$ or $y \in \nu_t$, a contradiction. Hence $(N \setminus \nu_t, \nu, \rho, t)$ is an empty graph. □

Remark 3.16. (i) In Example 3.3, when $\alpha = 0.5, \beta = 0.8$ and $\nu_t = \{0, 2, 4\}$, ν is a c-prime fuzzy ideal of N and ν_t is a vertex cover of (N, ν, ρ, t).
(ii) In Example 3.2, when $\alpha = 0.6, \beta = 0.9$ and $\nu_t = \{0\}$, ν is not a c-prime fuzzy ideal of N. Observe that ν_t not a vertex cover of (N, ν, ρ, t). There is a edge between a and b however $a \notin \nu_t$ and $b \notin \nu_t$.
(iii) In Example 3.2, when $\alpha = 0.6, \beta = 0.9$ and $\nu_t = \{0\}$, ν is not a c-prime fuzzy ideal of N. Observe that $(N \setminus \nu_t, \nu, \rho, t)$ is not an empty graph. In fact, $(N \setminus \nu_t, \nu, \rho, t)$ as in Figure 10.

Proposition 3.17. *Let N be a simple nearring and ν be a fuzzy ideal of N. Then (N, ν, ρ, t) is a star graph or (N, ν, ρ, t) is a complete graph if any one of the following conditions are satisfied. (i) ν is a c-prime fuzzy ideal of N. (ii) N is integral nearring.*

Proof. To prove (i), let ν be a c-prime fuzzy ideal of N and $t \in (\alpha, \beta]$. Then ν_t is a c-prime ideal of N. Now, suppose $\nu_t = N$. Then (N, ν, ρ, t) is a complete graph. Let $\nu_t \neq N$. Then $\nu_t = \{0\}$. Let $x \neq 0$ and $y \neq 0$ such that (x, y) is an edge of (N, ν, ρ, t). Then $xy \in \nu_t$ or $yx \in \nu_t$. Without loss of generality let us assume $xy \in \nu_t$. Then $x \in \nu_t = \{0\}$ or $y \in \nu_t = \{0\}$ (since ν_t is c-prime.) Then $x = 0$ or $y = 0$ a contradiction to the fact that $x \neq 0$ and $y \neq 0$. Hence (N, ν, ρ, t) is a star graph.
To prove (ii), let ν is a fuzzy ideal of a integral nearring N and and $t \in (\alpha, \beta]$. Then ν_t is an ideal of N. Suppose $\nu_t = N$. Then (N, ν, ρ, t) is a complete graph. Now, suppose $\nu_t \neq N$. Then

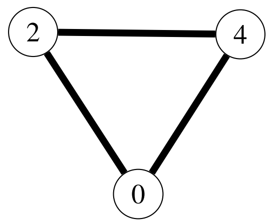

Figure 6. Graph of (ν_t, ν, ρ, t)

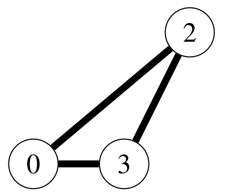

Figure 7. Fuzzy clique when $\nu_t = \{0, 3\}$

$\nu_t = \{0\}$. Let $x \neq 0$ and $y \neq 0$ such that (x, y) is an edge of (N, ν, ρ, t). Then $xy \in \nu_t$ or $yx \in \nu_t$. Without loss of generality let us assume $xy \in \nu_t$. Then $xy = 0$. As N is an integral nearring we get $x = 0$ and $y = 0$, a contradiction to the fact that $x \neq 0$ and $y \neq 0$. Hence (N, ν, ρ, t) is a star graph. □

Remark 3.18. In Example 3.2, note that the nearring is not a simple nearring and ideal $\nu_t = \{0\}$ is not a c-prime fuzzy ideal. Observe that the graph is not a star graph.

4 Graph Homomorphism and Ring Homomorphism

Proposition 4.1. *Let $f : N_1 \to N_2$ be an one to one and onto nearring homomorphism. Let ν be a fuzzy ideal of N_1 and $t \in (\alpha, \beta]$. Then f is one to one and onto graph homomorphism from (N_1, ν, ρ, t)*
to $(N_2, f(\nu), f(\rho), t)$.

Proof. Let ν be a fuzzy ideal of N_1 and $t \in (\alpha, \beta]$. Then by Proposition 3.25 in [20] we get $f(\nu)$ is a fuzzy ideal of N_2 with the same thresholds as that of ν. Let $x, y \in N_1$ such that $x \neq y$ and (x, y) is an edge of (N_1, ν, ρ, t). Then $xy \in \nu_t$ or $yx \in \nu_t$. Without loss of generality assume $xy \in \nu_t$. Then $f(xy) \in f(\nu_t) \subseteq f(\nu)_t$ (by Remark 3.30 in [20]). Then $f(xy) \in f(\nu)_t$. As f is a nearring homomorphism we get that $f(xy) = f(x)f(y) \in f(\nu)_t$. As f is one to one we get $f(x) \neq f(y)$. Also $f(N_1) = N_2$ (f is onto). Hence $(f(x), f(y))$ is an edge in $(N_2, f(\nu), f(\rho), t)$. Therefore f is a graph homomorphism from (N_1, ν, ρ, t) to $(N_2, f(\nu), f(\rho), t)$. Let $x_1, x_2 \in N_1$ such that $x_1 \neq x_2$. Then $f(x_1) \neq f(x_2)$(since f is one to one). Hence there is a one to one correspondence between the vertex set of (N_1, ν, ρ, t) and $(N_2, f(\nu), f(\rho), t)$. Let (x_1, y_1) and (x_2, y_2) be edges of (N_1, ν, ρ, t) such that $(x_1, y_1) \neq (x_2, y_2)$. Then $x_1y_1 \in \nu_t$ or $y_1x_1 \in \nu_t$ and $x_2y_2 \in \nu_t$ or $y_2x_2 \in \nu_t$. Without loss of generality assume $x_1y_1 \in \nu_t$ and $x_2y_2 \in \nu_t$. Then $f(x_1y_1) = f(x_1)f(y_1) \in f(\nu)_t$ and $f(x_2y_2) = f(x_2)f(y_2) \in f(\nu)_t$. As f is one to one, $f(x_1y_1) \neq f(x_2y_2)$. Hence $f(x_1)f(y_1) \neq f(x_2)f(y_2)$. Therefore there is one to one correspondence between the edge set of (N_1, ν, ρ, t) and $(N_2, f(\nu), f(\rho), t)$. □

Example 4.2. Let Z_n be the ring of integers modulo n. Let $N_1 = Z_1 \times Z_4 = \{(0, 0), (0, 1), (0, 2), (0, 3)\}$ and $N_2 = Z_4$. Define $f : N_1 \to N_2$ by $f((x, y)) = y$. Then f is a one to one and onto nearring homomorphism.

We define $\nu : N_1 \to (0, 1]$ by $\nu(x) = \begin{cases} 0.8 & \text{if} \quad x = (0, 0) \\ 0.4 & \text{if} \quad x = (0, 2) \\ 0.2 & \text{if} \quad x \in \{(0, 1), (0, 3)\} \end{cases}$

If we take thresholds $\alpha = 0.2$ and $\beta = 0.4$ then ν is a fuzzy ideal of N_1. Let $t = \beta$, then $\nu_t = \{(0, 0), (0, 2)\}$. Then values of ρ are as in Table 7.

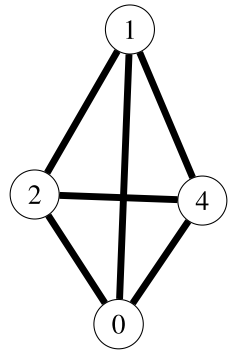

Figure 8. Fuzzy clique when $\nu_t = \{0, 2, 4\}$

Figure 9. Complete subgraph when $\nu_t = \{0\}$

$\rho(x, y)$	$y = (0, 0)$	$y = (0, 1)$	$y = (0, 2)$	$y = (0, 3)$
$x = (0, 0)$	0	0.2	0.4	0.2
$x = (0, 1)$	0.2	0	0.4	0
$x = (0, 2)$	0.4	0.2	0	0.2
$x = (0, 3)$	0.2	0	0.2	0

Table 7. Values of $\rho(x, y)$ for Example 4.2 when $\nu_t = \{(0, 0), (0, 2)\}$

The graph is given in Figure 11.

Then $f(\nu)$ is a fuzzy ideal of N_2 with the same thresholds as that of ν.

Then $f(\nu) : N_2 \to (0, 1]$ by $\nu(x) = \begin{cases} 0.8 & \text{if} \quad x = 0 \\ 0.4 & \text{if} \quad x = 2 \\ 0.2 & \text{if} \quad x \in \{1, 3\} \end{cases}$

Then $f(\nu_t) = f(\{(0, 0), (0, 2)\}) = \{0, 2\}$ is an ideal of N_2.
Then the values of ρ are as in Table 8.

The graph is given in Figure 12.

Observe that f is an one to one graph homomorphism from N_1 to N_2.

Proposition 4.3. *Let $f : N_1 \to N_2$ be an one to one and onto nearring homomrphism. Let ν be a fuzzy ideal of N_1 and $t \in (\alpha, \beta]$. If $x \in \nu_t$ then $\rho(f(x), f(z)) > 0 \ \ \forall \ \ f(z) \neq f(x);\ f(z) \in N_2$.*

Proof. Let ν be a fuzzy ideal of N_1, $t \in (\alpha, \beta]$ and $x \in \nu_t$. Case(i) Suppose $x = 0_1$ where 0_1 is the additive identity of N_1. Then $xy = 0_1 \in \nu_t$ for all $y \in N_1$. As f is a nearring homomorphism we get $f(xy) = f(x)f(y) = f(0_1) = 0_2$ where 0_2 is the additive identity of N_2. Then $\rho(f(x), f(y)) > 0$.
Case(ii) Let $x \neq 0_1$. As $x \in \nu_t$ and ν_t is an ideal we get $xy \in \nu_t$ for all $y \in N_1$. Then $f(xy) \in f(\nu_t) \subseteq f(\nu)_t$ for all $f(y) \in N_2$. As f is a nearring homomoephism we get $f(x)f(y) \in f(\nu)_t$. As f is one to one we get $f(x) \neq f(y)$. Hence $(f(x), f(y)) \in E((N_2, f(\nu), f(\rho), t))$. Therefore $\rho(f(x), f(z)) > 0 \ \ \forall \ \ f(z) \neq f(x);\ f(z) \in N_2$. □

Remark 4.4. In Example 4.2, note that $(0, 1) \notin \nu_t$. Then $f((0, 1)) = 1$ is not connected to all other vertices of N_2. Also $(0, 2) \in \nu_t$. Then $f((0, 2)) = 2$ is connected to all other vertices of N_2.

Figure 10. Graph $(N \setminus \nu_t, \nu, \rho, t)$

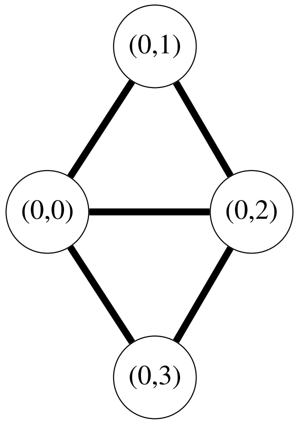

Figure 11. Graph when $\nu_t = \{(0,0),(0,2)\}$

Proposition 4.5. *Let $f : N_1 \to N_2$ be an one to one and onto nearring homomrphism. Let ν be a fuzzy ideal of N_1 and $t \in (\alpha, \beta]$. If $\rho(f(x), f(z)) > 0 \ \forall \ \ f(z) \neq f(x);\ f(z) \in N_2$ then $\rho(x,z) > 0 \ \forall \ \ z \neq x;\ z \in N_1$.*

Proof. Let ν be a fuzzy ideal of N_1 and $x, y \in N_1$. Let $\rho(f(x), f(z)) > 0 \ \ \forall \ \ f(z) \neq f(x);\ f(z) \in N_2$. Suppose $\rho(x,z) \not> 0$ for some $z \neq x;\ z \in N_1$. Then there exists $z \in N_1$ such that $xz \notin \nu_t$ and $zx \notin \nu_t$. Then $f(x)f(z) = f(xz) \notin f(\nu_t) \subseteq f(\nu)_t$ and $f(z)f(x) = f(zx) \notin f(\nu_t) \subseteq f(\nu)_t$. Then $f(x)f(z) \notin f(\nu)_t$ and $f(z)f(x) \notin f(\nu)_t$, a contradiction to the fact that $\rho(f(x), f(z)) > 0 \ \forall \ \ f(z) \neq f(x);\ f(z) \in N_2$. Hence $\rho(x,z) > 0$ for some $z \neq x;\ z \in N_1$. □

Remark 4.6. In Example 4.2, note that $3 = f((0,3))$ is not connected to all other vertices of N_2. Observe that $(0,3)$ is not connected to all other vertices of N_1.

Definition 4.7. Let $f : N_1 \to N_2$ be an onto nearring homomorphism, ν be a fuzzy ideal of N_1 and $t \in (\alpha, \beta]$. Then f is said to preserve the vertex cover of the c-prime fuzzy graph if ν_t is a vertex cover of (N_1, ν, ρ, t) then $f(\nu)_t$ is a vertex cover of $(N_2, f(\nu), f(\rho), t)$.

Remark 4.8. In Example 4.2, note that $\nu_t = \{(0,0),(0,2)\}$ is vertex cover of (N_1, ν, ρ, t) and $f(\nu)_t = \{0,2\}$ is vertex cover of $(N_2, f(\nu), f(\rho), t)$.

Proposition 4.9. *Let $f : N_1 \to N_2$ be a nearring homomorphism, ν be a c-prime fuzzy ideal of N_1 and $t \in (\alpha, \beta]$. If (i) f preserves vertex cover (ii) $f(\nu)_t$ is c-semiprime then $f(\nu)$ is a c-prime fuzzy ideal of N_2.*

Proof. Let ν be a c-prime fuzzy ideal of N_1. Then $f(\nu)$ is a fuzzy ideal of N_2 with the same thresholds as that of ν. Suppose $\nu_t = N_1$. Then $f(\nu_t) = f(\nu)_t = N_2$ is a c-prime ideal of N_2. Then $f(\nu)$ is a c-prime fuzzy ideal of N_2. Now suppose $\nu_t \subset N_1$. As ν is a c-prime fuzzy ideal of N_1 we get ν_t is a c-prime ideal of N_1. By Proposition 3.15(i) we get ν_t is a vertex cover of (N_1, ν, ρ, t). As f preserves vertex cover we get $f(\nu)_t$ is a vertex cover of $(N_2, f(\nu), f(\rho), t)$. Let $(x,y) \in E(N_2, f(\nu), f(\rho), t)$, then $x \in f(\nu)_t$ or $y \in f(\nu)_t$. Then by Proposition 3.8, we get that x is connected to all other vertices of $(N_2, f(\nu), f(\rho), t)$ or y is connected to all other vertices of $(N_2, f(\nu), f(\rho), t)$. Hence $(N_2, f(\nu), f(\rho), t)$ is ideal symmetric.
Let x in N_2 such that x is connected to all other vertices of $(N_2, f(\nu), f(\rho), t)$, then $xy \in f(\nu)_t$ for all y in N_2. Suppose $x = y$. Then $x \in f(\nu)_t$ (since $f(\nu)_t$ is c-semiprime). As f is onto, $x = f(x_1)$ for some $x_1 \in N_1$. Now $f(x_1)$ is connected to all other vertices of $(N_2, f(\nu), f(\rho), t)$. Then by Proposition 4.5, we get that x is connected to all other vertices of (N_1, ν, ρ, t). Then

$\rho(x,y)$	$y=0$	$y=1$	$y=2$	$y=3$
$x=(0,0)$	0	0.2	0.4	0.2
$x=1$	0.2	0	0.4	0
$x=2$	0.4	0.2	0	0.2
$x=3$	0.2	0	0.2	0

Table 8. Values of $\rho(x,y)$ for Example 4.2 when $\nu_t = \{0,2\}$

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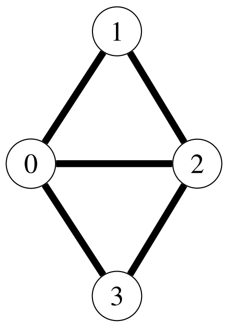

Figure 12. Graph when $\nu_t = \{0,2\}$

$x_1y_1 \in \nu_t$ for all $y_1 \in N_1$. Suppose $\nu_t = N_1$. Then $f(\nu)_t = N_2$ is a c-prime ideal of N_2. Let $\nu_t \subset N_1$. Choose $y_1 \in N_1 \setminus \nu_t$, then $x_1 \in \nu_t$. (As ν be a c-prime fuzzy ideal of N_1, then ν_t is a c-prime ideal of N_1.) Then $f(x_1) \in f(\nu)_t$, then $x \in f(\nu)_t$. Hence $(N_2, f(\nu), f(\rho), t)$ satisfies all conditions of Proposition 3.11. Therefore $f(\nu)$ is a c-prime fuzzy ideal of N_2. □

Conclusions: We have introduced c-prime fuzzy graphs of nearrings with respect to level ideals of fuzzy ideals. We obtained different graph theoretical properties of this graph and related graph theoretical properties with ideal theoretical properties. We introduced ideal symmetry of the graph and obtained a relation between fuzzy ideal properties and fuzzy graph properties. Finally we related graph homomorphisms with nearring homomorphisms.

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Compliance with Ethical Standards

Conflict of Interest:

Author Jagadeesha B declares that he has no conflict of interest. Author Kedukodi Babushri Srinivas declares that he has no conflict of interest. Author Kuncham Syam Prasad declares that he has no conflict of interest.

Ethical approval:

This article does not contain any studies with human participants or animals performed by any of the authors.

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