Palestine Journal of Mathematics

Vol 13(Special Issue III)(2024) , 82–91

On Intuitionistic Fuzzy f-Primary Ideals Of Commutative Γ-Rings

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Communicated by Madeleine Al-Tahan

MSC 2010 Classifications: Primary 03F55, 13A15; Secondary 16Y80.

Keywords and phrases: Γ-ring, Intuitionistic fuzzy f-primary ideal, Intuitionistic fuzzy 2-absorbing f-primary ideal, Intuitionistic fuzzy ideal expansion, Global intuitionistic fuzzy ideal expansion.

Abstract In this paper, we introduce the notion of expansion of intuitionistic fuzzy ideals of a commutative Γ-ring and by using this concept, we develop the notion of intuitionistic fuzzy f-primary ideals (2-absorbing f-primary ideals) which unify the notion of intuitionistic fuzzy prime ideals (2-absorbing ideals) and intuitionistic fuzzy primary ideals (2-absorbing primary ideals) of Γ-ring. A number of important results about intuitionistic fuzzy prime ideals (2-absorbing ideals) and intuitionistic fuzzy primary ideals (2-absorbing primary ideals) are extended into this general frame work.

1 Introduction

Algebraic structures play a prominent role in mathematics with wide ranging applications in many disciplines such as theoretical physics, computer sciences, control engineering, information sciences, coding theory etc. It is well known that the concept of Γ-rings was first introduced and investigated by N. Nobusawa in [1], which is a generalization of the concept of rings. The class of Γ-rings contains not only all rings but also all Hestenes ternary rings. Later W. E. Barnes [2] weakened slightly the conditions in the definition of Γ-ring in the sense of Nobusawa. After these two papers were published, many mathematicians obtained interesting results on Γ-rings in the sense of Barnes and Nobusawa extending and generalizing many classical notions and results of the theory of rings. The structure of Γ-rings was investigated by several authors such as Barnes in [2], Kyuno in [3, 4]. Warsi in [5] studied the decomposition of primary ideal of Γ-rings. Paul in [6] studied various types of ideals of Γ-rings and the corresponding operator rings. The notion of expansion of Γ-ideal in Γ-ring was introduced by Jun et al. in [7]. Elkettani et al. in [8] studied the concept of 2-absorbing δ-primary ideal of Γ-ring.

The idea of intuitionistic fuzzy sets was first published by Atanassaov [9, 10], as a generalization of the notion of fuzzy set given by Zadeh [11]. Kim et al. in [12] considered the intuitionistic fuzzification of ideal of Γ-ring which were further studied by Palaniappan et al. in [13, 14, 15]. The notion of intuitionistic fuzzy prime ideal and semiprime were studied by Palaniappan and Ramachandran in [16]. Authors in [17], [18] and [19] studied intuitionistic fuzzy characteristic ideals; intuitionistic fuzzy primary ideals and intuitionistic fuzzy 2-absorbing primary ideals in Γ-ring respectively.

Intuitionistic fuzzy 2-absorbing ideals and intuitionistic fuzzy 2-absorbing primary ideals are two of the most important structures in ″Intuitionistic Fuzzy Commutative Algebra″. Although different from each other in many aspects, they share quite a number of similar properties as well (see [19, 21, 22]). However, these two structures have been treated rather differently, and all of their properties were proved separately. It is therefore natural to examine whether it is possible to have a unified approach to study these two structures. In this paper we introduce the notion of intuitionistic fuzzy f-primary ideals (2-absorbing f-primary ideals), where f is a mapping that assigns to each intuitionistic fuzzy ideal A, an intuitionistic fuzzy ideal $f(A)$ of the same Γ-ring. Such an intuitionistic fuzzy f-primary ideals (2-absorbing f-primary ideals) unify the intuitionistic fuzzy prime ideals (2-Absorbing ideals) and intuitionistic fuzzy primary ideals (2-Absorbing primary ideals) under one frame. This approach clearly reveals how similar the two structures are and how they are related to each other.

In the first section, we introduce the concept of intuitionistic fuzzy ideal expansion and define intuitionistic fuzzy primary ideals with respect to such an expansion. Besides the familiar expansions, we also have a new expansion $\mathcal{M}$ defined by means of intuitionistic fuzzy maximal ideals. Further, we investigate intuitionistic fuzzy ideal expansions satisfying some additional conditions and prove more properties of the generalized intuitionistic fuzzy primary ideals with respect to such expansions.

In the second section, we introduce the notion of intuitionistic fuzzy 2-Absorbing ideal expansion and define intuitionistic fuzzy 2-absorbing primary ideals with respect to such an expansion. Besides the familiar expansions, we also have a new expansion $\mathcal{M}$ defined by means of intuitionistic fuzzy maximal ideals. Further, we investigate intuitionistic fuzzy 2-absorbing ideal expansions satisfying some additional conditions and prove more properties of the generalized intuitionistic fuzzy 2-absorbing primary ideals with respect to such expansions.

2 Preliminaries

Let us recall some definitions and results, which are necessary for the development of the paper. Throughout this paper unless stated otherwise all Γ-rings are commutative Γ-ring with unity.

Definition 2.1. ([1, 2]) If $(M, +)$ and $(\Gamma, +)$ are additive Abelian groups. Then M is called a Γ-ring (in the sense of Barnes [2]) if there exist mapping $M \times \Gamma \times M \to M$ [image of (x, α, y) is denoted by $x\alpha y$, $x, y \in M, \gamma \in \Gamma$] satisfying the following conditions:
(1) $x\alpha y \in M$.
(2) $(x+y)\alpha z = x\alpha z + y\alpha z$, $x(\alpha+\beta)y = x\alpha y + x\beta y$, $x\alpha(y+z) = x\alpha y + x\alpha z$.
(3) $(x\alpha y)\beta z = x\alpha(y\beta z)$. for all $x, y, z \in M$, and $\gamma \in \Gamma$.

The Γ-ring M is called commutative if $x\gamma y = y\gamma x, \forall x, y \in M, \gamma \in \Gamma$. An element $1 \in M$ is said to be the unity of M if for each $x \in M$ there exists $\gamma \in \Gamma$ such that $x\gamma 1 = 1\gamma x = x$. A subset N of a Γ-ring M is a left (right) ideal of M if N is an additive subgroup of M and $M\Gamma N = \{x\alpha y | x \in M, \alpha \in \Gamma, y \in N\}$, $(N\Gamma M)$ is contained in N. If N is both a left and a right ideal then N is a two-sided ideal, or simply an ideal of M. A mapping $\sigma : M \to M'$ of Γ-rings is called a Γ-homomorphism [2] if $\sigma(x+y) = \sigma(x) + \sigma(y)$ and $\sigma(x\alpha y) = \sigma(x)\alpha\sigma(y)$ for all $x, y \in M, \alpha \in \Gamma$.

An ideal I of a Γ-ring M is called prime (primary) if for any ideal U, V of M $U\Gamma V \subseteq I$ implies $U \subseteq I$ or $V \subseteq I$ ($U \subseteq I$ or $V \subseteq \sqrt{I}$, where $\sqrt{I} = \{x \in M : (x\gamma)^{n-1}x \in I$ for some $n \in \mathbf{N}$ and $\gamma \in \Gamma\}$ is the radical of ideal I, here for $n = 1$, $(x\gamma)^{n-1}x = x$ [5].

Definition 2.2. ([8]) A proper ideal I of a Γ-ring M is called a 2-absorbing ideal of M if whenever $x, y, z \in M$, $\gamma_1, \gamma_2 \in \Gamma$ such that $x\gamma_1 y\gamma_2 z \in I$ imply that $x\gamma_1 y \in I$ or $x\gamma_2 z \in I$ or $y\gamma_2 z \in I$.

Definition 2.3. ([8]) A proper ideal I of a Γ-ring M is called a 2-absorbing primary ideal of M if whenever $x, y, z \in M$, $\gamma_1, \gamma_2 \in \Gamma$ such that $x\gamma_1 y\gamma_2 z \in I$ imply that $x\gamma_1 y \in I$ or $x\gamma_2 z \in \sqrt{I}$ or $y\gamma_2 z \in \sqrt{I}$.

Remark 2.4. ([8]) Every 2-absorbing ideal of Γ-ring M is 2-absorbing primary ideal of M.

Definition 2.5. ([9, 10]) An intuitionistic fuzzy set A in X can be represented as an object of the form $A = \{< x, \mu_A(x), \nu_A(x) >: x \in X\}$, where the functions $\mu_A, \nu_A : X \to [0, 1]$ denote the degree of membership (namely $\mu_A(x)$) and the degree of non-membership (namely $\nu_A(x)$) of each element $x \in X$ to A respectively and $0 \leq \mu_A(x) + \nu_A(x) \leq 1$ for each $x \in X$.

Remark 2.6. ([9, 10]) When $\mu_A(x) + \nu_A(x) = 1, \forall x \in X$. Then A is called a fuzzy set.

If $A, B \in IFS(X)$, then $A \subseteq B$ if and only if $\mu_A(x) \leq \mu_B(x)$ and $\nu_A(x) \geq \nu_B(x), \forall x \in X$ and $A = B \Leftrightarrow A \subseteq B$ and $B \subseteq A$. For any subset Y of X, the intuitionistic fuzzy characteristic function χ_Y is an intuitionistic fuzzy set of X, defined as $\chi_Y(x) = (1,0), \forall x \in Y$ and $\chi_Y(x) = (0,1), \forall x \in X \backslash Y$. Let $\alpha, \beta \in [0,1]$ with $\alpha + \beta \leq 1$. Then the crisp set $A_{(\alpha,\beta)} = \{x \in X : \mu_A(x) \geq \alpha$ and $\nu_A(x) \leq \beta\}$ is called the (α,β)-level cut subset of A.In particular $A_{(1,0)}$ which is denoted by A_*, is defined as $A_* = \{x \in M : \mu_A(x) = \mu_A(0_M)$ and $\nu_A(x) = \nu_A(0_M)\}$. Also the IFS $x_{(\alpha,\beta)}$ of X defined as $x_{(\alpha,\beta)}(y) = (\alpha,\beta)$, if $y = x$, otherwise $(0,1)$ is called the intuitionistic fuzzy point (IFP) in X with support x. By $x_{(\alpha,\beta)} \in A$ we mean $\mu_A(x) \geq \alpha$ and $\nu_A(x) \leq \beta$. Further if $\sigma : X \to Y$ is a mapping and A, B be respectively IFS of X and Y. Then the image $\sigma(A)$ is an IFS of Y is defined as $\mu_{\sigma(A)}(y) = Sup\{\mu_A(x) : \sigma(x) = y\}$, $\nu_{\sigma(A)}(y) = Inf\{\nu_A(x) : \sigma(x) = y\}$, for all $y \in Y$ and the inverse image $\sigma^{-1}(B)$ is an IFS of X is defined as $\mu_{\sigma^{-1}(B)}(x) = \mu_B(\sigma(x))$, $\nu_{\sigma^{-1}(B)}(x) = \nu_B(\sigma(x))$, for all $x \in X$, i.e., $\sigma^{-1}(B)(x) = B(\sigma(x))$, for all $x \in X$. Also the IFS A of X is said to be σ-invariant if for any $x, y \in X$, whenever $\sigma(x) = \sigma(y)$ implies $A(x) = A(y)$.

Definition 2.7. ([12, 14]) Let A and B be two IFSs of a Γ-ring M and $\gamma \in \Gamma$. Then the product $A\Gamma B$ is defined by

$$(\mu_{A\Gamma B}(x), \nu_{A\Gamma B}(x)) = \begin{cases} (\vee_{x=y\gamma z}(\mu_A(y) \wedge \mu_B(z)), \wedge_{x=y\gamma z}(\nu_A(y) \vee \nu_B(z)), & \text{if } x = y\gamma z \\ (0,1), & \text{otherwise} \end{cases}$$

Definition 2.8. ([12, 14]) Let A be an IFS of a Γ-ring M. Then A is called an intuitionistic fuzzy ideal of M if for all $x, y \in M, \gamma \in \Gamma$, the following are satisfied
(i) $\mu_A(x-y) \geq \mu_A(x) \wedge \mu_A(y)$;
(ii) $\mu_A(x\gamma y) \geq \mu_A(x) \vee \mu_A(y)$;
(iii) $\nu_A(x-y) \leq \nu_A(x) \vee \nu_A(y)$;
(iv)$\nu_A(x\gamma y) \leq \nu_A(x) \wedge \nu_A(y)$.

The set of all intuitionistic fuzzy ideals of Γ-ring M is denoted by $IFI(M)$. Note that if $A \in IFI(M)$, then $\mu_A(0_M) \geq \mu_A(x)$ and $\nu_A(0_M) \leq \nu_A(x), \forall x \in M$ (See [11]).

Definition 2.9. ([16, 18]) Let P be an IFI of a Γ-ring M. Then P is said to be prime (primary) if P is non-constant and for any IFIs A, B of M, $A\Gamma B \subseteq P$ implies $A \subseteq P$ or $B \subseteq P$ ($A \subseteq P$ or $B \subseteq \sqrt{P}$, where $\sqrt{P}$ is an IF radical of P defined as $\mu_{\sqrt{P}}(x) = \vee\{\mu_P((x\gamma)^m x) :$ where $m \in \mathbf{N}\}$ and $\nu_{\sqrt{P}}(x) = \wedge\{\nu_P((x\gamma)^m x) :$ where $m \in \mathbf{N}\}$, where $\gamma \in \Gamma$).

Remark 2.10. ([16, 18]) Let $x_{(p,q)}, y_{(t,s)} \in IFP(M)$. Then

$$x_{(p,q)}\Gamma y_{(t,s)} = (x\Gamma y)_{(p\wedge t, q\vee s)}.$$

Proposition 2.11. *([20]) Let M, M' be Γ-rings. If $\sigma : M \to M'$ is a surjective homomorphism, then $\forall x \in M, p, q \in (0,1]$ such that $p + q \leq 1$, we have*

$$\sigma(x_{(p,q)}) = (\sigma(x))_{(p,q)}$$

Theorem 2.12. *([16, 18]) Let M be a commutative Γ-ring and A be an IFI of M. Then following are equivalent*
(i) A is an intuitionistic fuzzy prime (primary) ideal of M.
(ii) For any $x_{(p,q)}, y_{(t,s)} \in IFP(M)$ such that $x_{(p,q)}\Gamma y_{(t,s)} \subseteq A \Rightarrow x_{(p,q)} \subseteq A$ or $y_{(t,s)} \subseteq A$ ($x_{(p,q)} \subseteq A$ or $y_{(t,s)} \subseteq \sqrt{A}$).

Definition 2.13. ([19]) Let Q be a non-constant IFI of a Γ-ring M. Then Q is called an intuitionistic fuzzy 2-absorbing ideal of M if for any $IFPs$ $x_{(p,q)}, y_{(t,s)}, z_{(u,v)}$ of M and $\gamma_1, \gamma_2 \in \Gamma$ such that $x_{(p,q)}\gamma_1 y_{(t,s)}\gamma_2 z_{(u,v)} \subseteq Q$ implies that either $x_{(p,q)}\gamma_1 y_{(t,s)} \subseteq Q$ or $x_{(p,q)}\gamma_2 z_{(u,v)} \subseteq Q$ or $y_{(t,s)}\gamma_2 z_{(u,v)} \subseteq Q$.

Definition 2.14. ([19]) Let Q be a non-constant IFI of a Γ-ring M. Then Q is called an intuitionistic fuzzy 2-absorbing primary ideal of M if for any $IFPs$ $x_{(p,q)}, y_{(t,s)}, z_{(u,v)}$ of M and $\gamma_1, \gamma_2 \in \Gamma$ such that $x_{(p,q)}\gamma_1 y_{(t,s)}\gamma_2 z_{(u,v)} \subseteq Q$ implies that either $x_{(p,q)}\gamma_1 y_{(t,s)} \subseteq Q$ or $x_{(p,q)}\gamma_2 z_{(u,v)} \subseteq \sqrt{Q}$ or $y_{(t,s)}\gamma_2 z_{(u,v)} \subseteq \sqrt{Q}$.

Theorem 2.15. *([19]) Every intuitionistic fuzzy* 2*-absorbing ideal of* Γ*-ring* M *is an intuitionistic fuzzy* 2*-absorbing primary ideal of* M.

Definition 2.16. ([25, 26]) A non-constant intuitionistic fuzzy ideal A of a Γ-ring M is called an intuitionistic fuzzy maximal ideal if for any intuitionistic fuzzy ideal B of M, if $A \subseteq B$, then either $B_* = A_*$ or B_* = M.

3 Expansion Of Intuitionistic Fuzzy Ideals Of Γ-Ring

In this section, we introduce the notion of expansion of intuitionistic fuzzy ideals of a commutative Γ-ring and using this concept, we develop the notion of intuitionistic fuzzy f-primary ideals, where f is a map satisfying additional conditions, and prove more results with respect to such expansions.

Definition 3.1. Let $\mathcal{A}(M)$ denote the set of all IFIs of Γ-ring M. Then we define a map $f : \mathcal{A}(M) \to \mathcal{A}(M)$ which is called an expansion of IFIs of M (or briefly as IFI expansion) if it satisfies the following properties

(i) $A \subseteq f(A), \forall A \in \mathcal{A}(M)$

(ii) $A \subseteq B \Rightarrow f(A) \subseteq f(B), \forall A, B \in \mathcal{A}(M)$.

Example 3.2.

(1) The identity map $i : \mathcal{A}(M) \to \mathcal{A}(M)$ defined by $i(A) = A$ is an expansion of IFIs of M.
Proof: Proof is straight forward.

(2) [18,(Proposition 3.3 and 3.5)] The map $f : \mathcal{A}(M) \to \mathcal{A}(M)$ defined by $f(A) = \sqrt{A}$ is an expansion of IFIs of M.
Proof: $\sqrt{A}$ is an IFI of M.

(i) $A \subseteq \sqrt{A}, \forall A \in \mathcal{A}(M)$

(ii) $A \subseteq B \Rightarrow \sqrt{A} \subseteq \sqrt{B}, \forall A, B \in \mathcal{A}(M)$.

(3) Denote $\mathcal{M}(A) = \bigcap\{Q : Q \supseteq A$ and Q is an IF maximal ideal of $M\}$. Then the map $g : \mathcal{A}(M) \to \mathcal{A}(M)$ defined by $g(A) = \mathcal{M}(A)$ is an expansion of IFIs of M.
Proof: $\mathcal{M}(A)$ is IFI of M.

(i) $A \subseteq \mathcal{M}(A)\ \forall A \in \mathcal{A}(M)$ [By definition of $\mathcal{M}(A)$]

(ii) $A \subseteq B \Rightarrow \mathcal{M}(A) \subseteq \mathcal{M}(B)\ \forall A, B \in \mathcal{A}(M)$.[By definition of $\mathcal{M}(A)$]

(4) The constant map $c : \mathcal{A}(M) \to \mathcal{A}(M)$ defined as $c(A) = \chi_M$ = (1,0) $\forall x \in M$ and (0,1) $\forall x \notin M$ is an expansion of IFIs of M.
Proof: Proof is straight forward.

Definition 3.3. Given an expansion f of IFIs of M. An IFI $A \in \mathcal{A}(M)$ is said to be an intuitionistic fuzzy f-primary if it satisfies

$$x_{(p,q)}\gamma y_{(t,s)} \subseteq A \Rightarrow x_{(p,q)} \subseteq A \text{ or } y_{(t,s)} \subseteq f(A), \forall x_{(p,q)}, y_{(t,s)} \in IFP(M), \gamma \in \Gamma.$$

Example 3.4. (1) If f(A) = i(A), then intuitionistic fuzzy 2-absorbing f-primary ideal is just intuitionistic fuzzy 2-absorbing ideal as defined in definition 2.13.
(2) If f(A) = $\sqrt{A}$, then intuitionistic fuzzy 2-absorbing f-primary ideal is just intuitionistic fuzzy 2-absorbing primary ideal as defined in definition 2.14.

Theorem 3.5. *Let* f, g *be two expansions of IFIs of* Γ*-ring* M*. If* $f(A) \subseteq g(A), \forall A \in \mathcal{A}(M)$*, then every intuitionistic fuzzy* f*-primary ideal is also an intuitionistic fuzzy* g*-primary ideal.*

Proof. Let $A \in \mathcal{A}(M)$ be an intuitionistic fuzzy f-primary ideal of Γ-ring M. Let $x_{(p,q)}, y_{(t,s)} \in IFP(M), \gamma \in \Gamma$ such that $x_{(p,q)}\gamma y_{(t,s)} \subseteq A, x_{(p,q)} \nsubseteq A$ implies that $y_{(t,s)} \subseteq f(A) \subseteq g(A)$, by assertion. Hence A is an intuitionistic fuzzy g-primary ideal of M. □

Theorem 3.6. *Let* f_1, f_2 *be two expansions of IFIs of* Γ*-ring* M*. Let* $f : \mathcal{A}(M) \to \mathcal{A}(M)$ *defined by* $f(A) = f_1(A) \cap f_2(A)$*,* $\forall A \in \mathcal{A}(M)$*. Then* f *is an IFIs expansion of* M*.*

Proof. For every $A \in \mathcal{A}(M)$, we have by definition $A \subseteq f_1(A)$ and $A \subseteq f_2(A)$ and so $A \subseteq f_1(A) \cap f_2(A) = f(A)$. Thus $A \subseteq f(A)$. Further let $B, C \in \mathcal{A}(M)$ such that $B \subseteq C$. Then $f_1(B) \subseteq f_1(C)$ and $f_2(B) \subseteq f_2(C)$ and so $f(B) = f_1(B) \cap f_2(B) \subseteq f_1(C) \cap f_2(C) = f(C)$, i.e., $f(B) \subseteq f(C)$. Hence f is an IFI expansion of Γ-ring M. □

Theorem 3.7. *Let f be an expansion of IFIs of Γ-ring M. For any subset S of M. Denote $\mathcal{A}_f(S) = \bigcap\{B : B$ is an IF f-primary ideal of M such that $\chi_S \subseteq B\}$. Then the map $h : \mathcal{A}(M) \to \mathcal{A}(M)$ defined by $h(A) = \mathcal{A}_f(A_*)$, $\forall A \in \mathcal{A}(M)$ is an expansion of IFIs of M.*

Proof. Obviously $A \subseteq \mathcal{A}_f(A_*) = h(A), \forall A \in \mathcal{A}(M)$.
Let $A^{'}, A^{''} \in \mathcal{A}(M)$ such that $A^{'} \subseteq A^{''}$. Then

$$\begin{aligned} h(A^{'}) &= \mathcal{A}_f(A^{'}_*) = \bigcap\{B : B \in \mathcal{A}(M) \text{ such that } \chi_{A^{'}_*} \subseteq B \text{ and } B \text{ is an IF } f\text{-primary }\} \\ &\subseteq \bigcap\{B : B \in \mathcal{A}(M) \text{ such that } \chi_{A^{''}_*} \subseteq B \text{ and } B \text{ is an IF } f\text{-primary }\} \\ &= \mathcal{A}_f(A^{''}_*) \\ &= h(A^{''}). \end{aligned}$$

Hence h is an expansion of IFIs of M. □

Theorem 3.8. *Let f be an expansion of IFIs of Γ-ring M. If $\{A_i : i \in \Lambda\}$ is a directed collection of IF f-primary ideals of M, where Λ is an index set, then $A = \bigcup_{i\in\Lambda} A_i$ is an IF f-primary ideal of M.*

Proof. Let $x_{(p,q)}, y_{(t,s)} \in IFP(M), \gamma \in \Gamma$ be such that $x_{(p,q)}\gamma y_{(t,s)} \subseteq A$ and $x_{(p,q)} \nsubseteq A = \bigcup_{i\in\Lambda} A_i$. Then there exists A_i such that $x_{(p,q)}\gamma y_{(t,s)} \subseteq A_i$ and $x_{(p,q)} \nsubseteq A_i$. Since each A_i is an IF f-primary ideal and $A_i \subseteq A$. It follows that $y_{(t,s)} \subseteq f(A_i) \subseteq f(A)$. Hence A is an IF f-primary ideal of M. □

Theorem 3.9. *Let f be an expansion of IFIs of Γ-ring M. If Q is an IF f-primary ideal of M, then for every $A, B \in \mathcal{A}(M)$ such that $A\Gamma B \subseteq Q$ and $A \nsubseteq Q$ implies that $B \subseteq f(Q)$.*

Proof. Assume that Q is an IF f-primary ideal of M and let $A, B \in \mathcal{A}(M)$ such that $A\Gamma B \subseteq Q$ and $A \nsubseteq Q$. Suppose that $B \nsubseteq f(Q)$. Then there exists $x, y \in M$ such that $\mu_A(x) > \mu_Q(x), \nu_A(x) < \nu_Q(x)$ and $\mu_B(y) > \mu_{f(Q)}(y), \nu_B(y) < \nu_{f(Q)}(y)$. Let $\mu_A(x) = p, \nu_A(x) = q$ and $\mu_B(x) = t, \nu_B(x) = s$. Then $\mu_Q(x) < p, \nu_Q(x) > q$ and $\mu_{f(Q)}(y) < t, \nu_{f(Q)}(y) > s$. This implies that $x_{(p,q)} \subseteq A$ and $y_{(t,s)} \subseteq B$, but $x_{(p,q)} \nsubseteq Q$ and $y_{(t,s)} \nsubseteq f(Q)$. Now

$\mu_Q(x\gamma y) \geq \mu_{A\Gamma B}(x\gamma y) \geq \{\mu_A(x) \wedge \mu_B(y)\} = p \wedge t = \mu_{x_{(p,q)}\gamma y_{(t,s)}}(x\gamma y)$ and $\nu_Q(x\gamma y) \leq \nu_{A\Gamma B}(x\gamma y) \leq \{\nu_A(x) \vee \nu_B(y)\} = q \vee s = \nu_{x_{(p,q)}\gamma y_{(t,s)}}(x\gamma y)$. Hence $x_{(p,q)}\gamma y_{(t,s)} \subseteq Q$. But $x_{(p,q)} \nsubseteq Q$ and $y_{(t,s)} \nsubseteq f(Q)$. This contradict the assumption that Q is IF f-primary ideal of M. Consequently the result is valid. □

Remark 3.10. In the definition of IF f-primary ideals, the statement $^{''}A\Gamma B \subseteq Q^{''}$ and $A \nsubseteq Q$ implies that $B \subseteq f(Q)$ in Theorem (3.9) can be replaced by $^{''}A\Gamma B \subseteq Q^{''}$ and $A \nsubseteq f(Q)$ implies that $B \subseteq Q$.

For any IFI A of a Γ-ring M and for any IFS B of M, the IF residual quotient of A by B is denoted by $(A : B) = \bigcup\{x_{(p,q)} \in IFP(M) : x_{(p,q)}\Gamma B \subseteq A\}$. It is easy to see that $(A : B)$ is an IFI of M such that $A \subseteq (A : B)$.

Theorem 3.11. *Let f be an expansion of IFIs of Γ-ring M. Then*
(i) If Q is an IF f-primary ideal and A be an IFI of M such that $A \nsubseteq f(Q)$, then $(Q : A) = Q$.
(ii) For any IF f-primary ideal Q and for any subset N of M, $(Q : \chi_N)$ is also an IF f-primary ideal.

Proof. **(i)** Since $Q \supseteq A \cap Q \supseteq A\Gamma Q$, i.e., $A\Gamma Q \subseteq Q$, so $Q \subseteq (Q : A)$. Also by definition we have $A\Gamma(Q : A) \subseteq Q$. Since $A \nsubseteq f(Q)$ we have $(Q : A) \subseteq Q$. Therefore $(Q : A) = Q$.

(ii) Let $x_{(p,q)}\Gamma y_{(t,s)} \subseteq (Q : \chi_N)$ and $x_{(p,q)} \nsubseteq (Q : \chi_N)$. Then $x_{(p,q)}\Gamma\chi_N \nsubseteq Q$. Therefore $\exists$, $n \in N, \gamma_1 \in \Gamma$ such that $\mu_{x_{(p,q)}\Gamma\chi_N}(x\gamma_1 n) > \mu_Q(x\gamma_1 n)$ and $\nu_{x_{(p,q)}\Gamma\chi_N}(x\gamma_1 n) < \nu_Q(x\gamma_1 n)$, i.e., $p > \mu_Q(x\gamma_1 n)$ and $q < \nu_Q(x\gamma_1 n)$ and so $(x\gamma_1 n)_{(p,q)} \nsubseteq Q$, i.e., $x_{(p,q)}\gamma_1 n_{(p,q)} \nsubseteq Q$. But $x_{(p,q)}\gamma_1 n_{(p,q)}\gamma_2 y_{(t,s)} = (x\gamma_1 n\gamma_2 y)_{(p\wedge t,q\vee s)} = (x\gamma_3 y)_{(p\wedge t,q\vee s)} \subseteq Q$, where $\gamma_3 = \gamma_1 n\gamma_2$. As Q is an IF f-primary ideal so $y_{(t,s)} \subseteq f(Q) \subseteq f((Q : \chi_N))$. Hence $(Q : \chi_N)$ is an IF f-primary ideal. □

Definition 3.12. Let f be an expansion of IFIs of Γ-ring M. Then f is said to be intersection preserving if it satisfies $f(A \cap B) = f(A) \cap f(B)$, for every $A, B \in \mathcal{A}(M)$.

Also, f is said to be global if for each Γ-homomorphism $\sigma : M \to M'$ of Γ-rings, the following hold:

$$f(\sigma^{-1}(A)) = \sigma^{-1}(f(A)) \text{ for every } A \in \mathcal{A}(M').$$

Note that an expansion i of IFIs of Γ-ring M in example (3.2)(i) is both intersection preserving as well as global.

Theorem 3.13. *For each $A \in \mathcal{A}(M)$, let $\mathcal{P}(A) := \bigcap\{B : B \supseteq A$ and B is IFPI of $M\}$. Then the map $f : \mathcal{A}(M) \to \mathcal{A}(M)$ given by $f(A) = \mathcal{P}(A)$ is an intersection preserving expansion of IFIs of Γ-ring M.*

Proof. Obviously, f is an expansion of IFIs of Γ-ring M. For every $A, B \in \mathcal{A}(M)$, let us denote $\mathcal{P}_1 := \{P : P \supseteq A \cap B,\ P \text{ is IFPI of } M\}$; $\mathcal{P}_2 := \{P : P \supseteq A \text{ or } P \supseteq B, P \text{ is IFPI of } M\}$.
Then $\bigcap \mathcal{P}_1 = \mathcal{P}(A \cap B)$ and $\bigcap \mathcal{P}_2 = \mathcal{P}(A) \cap \mathcal{P}(B)$. Obviously $\mathcal{P}_2 \subseteq \mathcal{P}_1$. If $P \in \mathcal{P}_1$ then $A\Gamma B \subseteq A \cap B \subseteq P$. As P is IFPI, so $A \subseteq P$ or $B \subseteq P$. i.e., $P \in \mathcal{P}_2$ and so $\mathcal{P}_1 \subseteq \mathcal{P}_2$, then $\mathcal{P}_1 = \mathcal{P}_2$. Thus $f(A \cap B) = \mathcal{P}(A \cap B) = \bigcap \mathcal{P}_1 = \bigcap \mathcal{P}_2 = \mathcal{P}(A) \cap \mathcal{P}(B) = f(A) \cap f(B)$. This complete the proof. □

Theorem 3.14. *Let f be an expansion of IFIs of Γ-ring M which is intersection preserving. If $A_1, A_2, ...A_n$ are IF f-primary ideals of M and $B = f(A_k)$ for all $k = 1, 2, ...n$, then $A := \bigcap_{k=1}^n A_k$ is an IF f-primary ideal of M.*

Proof. Obviously, $A := \bigcap_{k=1}^n A_k$ is an IFI of M. Let C, D are IFIs of M such that $C\Gamma D \subseteq A$ and $C \nsubseteq A$. Then $C \nsubseteq A_k$ for some A_k, where $k \in \{1, 2, ..., n\}$. But $C\Gamma D \subseteq A \subseteq A_k$ and A_k is IF f-primary ideal of M, which imply that $D \subseteq f(A_k)$. Since f is intersecting preserving, so

$$f(A) = f(\textstyle\bigcap_{k=1}^n A_k) = \bigcap_{k=1}^n f(A_k) = B = f(A_k)$$

and so $D \subseteq f(A)$. Therefore A is an IF f-primary ideal of M. □

Let $\sigma : M \to M'$ be a Γ-homomorphism of Γ-rings. Note that if B be an IFI of M', then $\sigma^{-1}(B)$ is an IFI of M, and that if σ is surjective and A is an IFI of M, then $\sigma(A)$ is an IFI of M'.

Theorem 3.15. *Let f be an expansion of IFIs which is global and let $\sigma : M \to M'$ be a Γ-homomorphism of Γ-rings. If B is an IF f-primary ideal of M'. then $\sigma^{-1}(B)$ is an IF f-primary ideal of M.*

Proof. Let C, D be two IFIs of M such that $C\Gamma D \subseteq \sigma^{-1}(B)$ and $C \nsubseteq \sigma^{-1}(B)$. Then $\sigma(C)\Gamma\sigma(D) = \sigma(C\Gamma D) \subseteq B$ and $\sigma(C) \nsubseteq B$, which implies that $\sigma(D) \subseteq f(B)$. Since f is global, it follows that $D \subseteq \sigma^{-1}(f(B)) = f(\sigma^{-1}(B))$. Hence $\sigma^{-1}(B)$ is an IF f-primary ideal of M. □

It can be easily verified that if $\sigma : M \to M'$ is a Γ-homomorphism of Γ-rings, then $\sigma^{-1}(\sigma(A)) = A$ for every $A \in \mathcal{A}(M)$ that contains $Ker(\sigma)$.

Theorem 3.16. *Let $\sigma : M \to M'$ be a surjective Γ-homomorphism of Γ-rings and let A be an IFI of M that contains $Ker(\sigma)$. Then A is an IF f-primary ideal of M if and only if $\sigma(A)$ is an IF f-primary ideal of M', where f is a global IFI expansion.*

Proof. If $\sigma(A)$ is an IF f-primary ideal of $M^{'}$, then A is an IF f-primary ideal of M, by Theorem (3.13) and $A = \sigma^{-1}(\sigma(A))$. Suppose that A is an IF f-primary ideal of M and let B, C be IFIs of $M^{'}$ such that $B\Gamma C \subseteq \sigma(A)$ and $B \nsubseteq \sigma(A)$. Since σ is surjective we have $\sigma(D) = B$ and $\sigma(E) = C$ for some IFIs D and E in M. Then $\sigma(D\Gamma E) = \sigma(D)\Gamma\sigma(E) = B\Gamma C \subseteq \sigma(A)$ and $\sigma(D) = B \nsubseteq \sigma(A)$, which imply that $D\Gamma E \subseteq \sigma^{-1}(\sigma(A)) = A$ and $D \nsubseteq \sigma^{-1}(\sigma(A)) = A$. Since A is an IF f-primary ideal of M, it follows that $E \subseteq f(A)$ so that $C = \sigma(E) \subseteq \sigma(f(A))$. Using the fact that f is global, we have

$$f(A) = f(\sigma^{-1}(\sigma(A))) = \sigma^{-1}(f(\sigma(A))$$

and so $\sigma(f(A)) = \sigma(\sigma^{-1}(f(\sigma(A)))) = f(\sigma(A))$. Since σ is surjective. Therefore $C \subseteq f(\sigma(A))$ and so $\sigma(A)$ is an IF f-primary ideal of $M^{'}$. This complete the proof. □

4 Intuitionistic Fuzzy 2-Absorbing f-primary Ideals Of Γ-Ring

In this section we investigate intuitionistic fuzzy 2-Absorbing f-primary ideals of Γ-ring, where f is an expansion of IFIs of Γ-ring M.

Definition 4.1. Given an expansion f of IFIs of M. An IFI $A \in \mathcal{A}(M)$ is said to be intuitionistic fuzzy 2-absorbing f-primary if for any $IFPs$ $x_{(p,q)}, y_{(t,s)}, z_{(u,v)}$ of M and $\gamma_1, \gamma_2 \in \Gamma$ such that

$$x_{(p,q)}\gamma_1 y_{(t,s)}\gamma_2 z_{(u,v)} \subseteq A \Rightarrow x_{(p,q)}\gamma_1 y_{(t,s)} \subseteq A \text{ or } x_{(p,q)}\gamma_2 z_{(u,v)} \subseteq f(A) \text{ or } y_{(t,s)}\gamma_2 z_{(u,v)} \subseteq f(A).$$

Example 4.2.
(1) The identity map $i : \mathcal{A}(M) \to \mathcal{A}(M)$ defined by $i(A) = A$, intuitionistic fuzzy 2-absorbing f-primary ideal is just intuitionistic fuzzy 2-absorbing ideal as defined in definition (2.13).
(2) The map $f : \mathcal{A}(M) \to \mathcal{A}(M)$ defined by $f(A) = \sqrt{A}$, intuitionistic fuzzy 2-absorbing f-primary ideal is just intuitionistic fuzzy 2-absorbing primary ideal as defined in definition (2.14).

In the following, we will give a list of results which are an extension of some results in [19].

Theorem 4.3. *Let f, g be two expansions of IFIs of Γ-ring M. If $f(A) \subseteq g(A), \forall A \in \mathcal{A}(M)$, then every intuitionistic fuzzy 2-absorbing f-primary ideal is also an intuitionistic fuzzy 2-absorbing g-primary ideal.*

Proof. Let $A \in \mathcal{A}(M)$ be intuitionistic fuzzy 2-absorbing f-primary ideal of Γ-ring M. Let $x_{(p,q)}, y_{(t,s)}, z_{(u,v)}$ of M and $\gamma_1, \gamma_2 \in \Gamma$ such that $x_{(p,q)}\gamma_1 y_{(t,s)}\gamma_2 z_{(u,v)} \subseteq A \Rightarrow x_{(p,q)}\gamma_1 y_{(t,s)} \subseteq A$ or $x_{(p,q)}\gamma_2 z_{(u,v)} \subseteq f(A) \subseteq g(A)$ or $y_{(t,s)}\gamma_2 z_{(u,v)} \subseteq f(A) \subseteq g(A)$, by assertion. Hence A is intuitionistic fuzzy 2-absorbing g-primary ideal of M. □

Theorem 4.4. *Let f be an expansion of IFIs of Γ-ring M. For any subset S of M. Denote $\mathcal{A}_f(S) = \bigcap\{B : B$ is an IF 2-absorbing f-primary ideal of M such that $\chi_S \subseteq B\}$. Then the map $h : \mathcal{A}(M) \to \mathcal{A}(M)$ defined by $h(A) = \mathcal{A}_f(A_*)$, $\forall A \in \mathcal{A}(M)$ is an expansion of IFIs of M.*

Proof. Obviously $A \subseteq \mathcal{A}_f(A_*) = h(A), \forall A \in \mathcal{A}(M)$.
Let $A^{'}, A^{''} \in \mathcal{A}(M)$ such that $A^{'} \subseteq A^{''}$. Then

$$\begin{aligned} h(A^{'}) &= \mathcal{A}_f(A^{'}_*) = \bigcap\{B : B \in \mathcal{A}(M) \text{ such that } \chi_{A^{'}_*} \subseteq B \text{ and } B \text{ is IF 2-absorbing } f\text{-primary }\} \\ &\subseteq \bigcap\{B : B \in \mathcal{A}(M) \text{ such that } \chi_{A^{''}_*} \subseteq B \text{ and } B \text{ is IF 2-absorbing } f\text{-primary }\} \\ &= \mathcal{A}_f(A^{''}_*) \\ &= h(A^{''}). \end{aligned}$$

Hence h is an expansion of IFIs of M. □

Theorem 4.5. *Let f be an expansion of IFIs of Γ-ring M. If $\{A_i : i \in \Lambda\}$ is a directed collection of IF 2-absorbing f-primary ideals of M, where Λ is an index set, then $A = \bigcup_{i\in\Lambda} A_i$ is IF 2-absorbing f-primary ideal of M.*

Proof. Let $x_{(p,q)}, y_{(t,s)}, z_{(u,v)}$ of M and $\gamma_1, \gamma_2 \in \Gamma$ such that $x_{(p,q)}\gamma_1 y_{(t,s)}\gamma_2 z_{(u,v)} \subseteq A$. Then there exists $i \in \Lambda$ such that $x_{(p,q)}\gamma_1 y_{(t,s)}\gamma_2 z_{(u,v)} \subseteq A_i$. Since each A_i is IF 2-absorbing f-primary ideal and $A_i \subseteq A$. It follows that $x_{(p,q)}\gamma_1 y_{(t,s)} \subseteq A_i$ or $x_{(p,q)}\gamma_2 z_{(u,v)} \subseteq f(A_i)$ or $y_{(t,s)}\gamma_2 z_{(u,v)} \subseteq f(A_i)$. Since $A_i \subseteq f(A_i) \subseteq f(A)$, $x_{(p,q)}\gamma_1 y_{(t,s)} \subseteq A$ or $x_{(p,q)}\gamma_2 z_{(u,v)} \subseteq f(A)$ or $y_{(t,s)}\gamma_2 z_{(u,v)} \subseteq f(A)$, so that A is IF 2-absorbing f-primary ideal of M. □

Theorem 4.6. *Let f be an expansion of IFIs of Γ-ring M which is intersection preserving. If $A_1, A_2, ...A_n$ are IF 2-absorbing f-primary ideals of M and $B = f(A_k)$ for all $k = 1, 2, ...n$, then $A := \bigcap_{k=1}^{n} A_k$ is an IF 2-absorbing f-primary ideal of M.*

Proof. Obviously, $A := \bigcap_{k=1}^{n} A_k$ is an IFI of M. Let $x_{(p,q)}, y_{(t,s)}, z_{(u,v)} \in IFP(M)$ and $\gamma_1, \gamma_2 \in \Gamma$ such that $x_{(p,q)}\gamma_1 y_{(t,s)}\gamma_2 z_{(u,v)} \subseteq A$ and $x_{(p,q)}\gamma_1 y_{(t,s)} \nsubseteq A$. Then $x_{(p,q)}\gamma_1 y_{(t,s)} \nsubseteq A_k$ for some $k \in \{1, 2, ..., n\}$. But $x_{(p,q)}\gamma_1 y_{(t,s)}\gamma_2 z_{(u,v)} \subseteq A \subseteq A_k$ and A_k is an IF 2-absorbing f-primary ideal of M, which imply that $x_{(p,q)}\gamma_2 z_{(u,v)} \subseteq f(A_k)$ or $y_{(t,s)}\gamma_2 z_{(u,v)} \subseteq f(A_k)$. Since f is intersecting preserving, so

$$f(A) = f(\bigcap_{k=1}^{n} A_k) = \bigcap_{k=1}^{n} f(A_k) = B = f(A_k)$$

and so $x_{(p,q)}\gamma_2 z_{(u,v)} \subseteq f(A)$ or $y_{(t,s)}\gamma_2 z_{(u,v)} \subseteq f(A)$. Therefore A is an IF 2-absorbing f-primary ideal of M. □

Theorem 4.7. *Let f be an expansion of IFIs which is global and let $\sigma : M \to M'$ be a Γ-homomorphism of Γ-rings. If B is an IF 2-absorbing f-primary ideal of M', then $\sigma^{-1}(B)$ is an IF 2-absorbing f-primary ideal of M.*

Proof. Let $x_{(p,q)}, y_{(t,s)}, z_{(u,v)} \in IFP(M)$ and $\gamma_1, \gamma_2 \in \Gamma$ such that $x_{(p,q)}\gamma_1 y_{(t,s)}\gamma_2 z_{(u,v)} \subseteq \sigma^{-1}(B)$. Then $\sigma(x_{(p,q)})\gamma_1\sigma(y_{(t,s)})\gamma_2\sigma(z_{(u,v)}) \subseteq B$, i.e., $(\sigma(x))_{(p,q)}\gamma_1(\sigma(y))_{(t,s)}\gamma_2(\sigma(z))_{(u,v)} \subseteq B$, which imply that $(\sigma(x))_{(p,q)}\gamma_1(\sigma(y))_{(t,s)} \subseteq B$ or $(\sigma(x))_{(p,q)}\gamma_2(\sigma(z))_{(u,v)} \subseteq f(B)$ or $(\sigma(y))_{(t,s)}\gamma_2(\sigma(z))_{(u,v)} \subseteq f(B)$. Since f is global, it follows that $x_{(p,q)}\gamma_1 y_{(t,s)} \subseteq \sigma^{-1}(B)$ or $x_{(p,q)}y_{(t,s)}\gamma_2 z_{(u,v)} \subseteq \sigma^{-1}(f(B)) = f(\sigma^{-1}(B))$ or $y_{(t,s)}\gamma_2 z_{(u,v)} \subseteq \sigma^{-1}(f(B)) = f(\sigma^{-1}(B))$. Hence $\sigma^{-1}(B)$ is an IF 2-absorbing f-primary ideal of M. □

It can be easily verified that if $\sigma : M \to M'$ is a Γ-homomorphism of Γ-rings, then $\sigma^{-1}(\sigma(A)) = A$ for every $A \in \mathcal{A}(M)$ that contains $Ker(\sigma)$.

Theorem 4.8. *Let $\sigma : M \to M'$ be a surjective Γ-homomorphism of Γ-rings and let A be an IFI of M that contains $Ker(\sigma)$. Then A is an IF 2-absorbing f-primary ideal of M if and only if $\sigma(A)$ is an IF 2-absorbing f-primary ideal of M', where f is a global IFI expansion.*

Proof. If $\sigma(A)$ is an IF 2-absorbing f-primary ideal of M', then A is an IF 2-absorbing f-primary ideal of M, by Theorem (3.13) and $A = \sigma^{-1}(\sigma(A))$. Suppose that A is an IF 2-absorbing f-primary ideal of M. Let $x_{(p,q)}, y_{(t,s)}, z_{(u,v)} \in IFP(M')$ and $\gamma_1, \gamma_2 \in \Gamma$ such that $x_{(p,q)}\gamma_1 y_{(t,s)}\gamma_2 z_{(u,v)} \subseteq \sigma(A)$. Since σ is surjective we have $\sigma(a) = x, \sigma(b) = y, \sigma(c) = z$, for some $a, b, c \in M$. Then $\sigma(a_{(p,q)}\gamma_1 b_{(t,s)}\gamma_2 c_{(u,v)}) = (\sigma(a))_{(p,q)}\gamma_1(\sigma(b))_{(t,s)}\gamma_2(\sigma(c))_{(u,v)} = x_{(p,q)}\gamma_1 y_{(t,s)}\gamma_2 z_{(u,v)} \subseteq \sigma(A)$, which imply that $a_{(p,q)}\gamma_1 b_{(t,s)}\gamma_2 c_{(u,v)} \subseteq \sigma^{-1}(\sigma(A)) = A$. Since A is an IF 2-absorbing f-primary ideal of M, it follows that $a_{(p,q)}\gamma_1 b_{(t,s)} \subseteq A$ or $a_{(p,q)}\gamma_2 c_{(u,v)} \subseteq f(A)$ or $b_{(t,s)}\gamma_2 c_{(u,v)} \subseteq f(A)$, i.e., $x_{(p,q)}\gamma_1 y_{(t,s)} \subseteq \sigma(A)$ or $x_{(p,q)}\gamma_2 z_{(u,v)} \subseteq \sigma(f(A))$ or $y_{(t,s)}\gamma_2 z_{(u,v)} \subseteq \sigma(f(A))$. Using the fact that f is global, we have

$$f(A) = f(\sigma^{-1}(\sigma(A))) = \sigma^{-1}(f(\sigma(A))$$

and so $\sigma(f(A)) = \sigma(\sigma^{-1}(f(\sigma(A)))) = f(\sigma(A))$. Since σ is surjective. Therefore $\sigma(A)$ is an IF 2-absorbing f-primary ideal of M'. This complete the proof. □

Conclusion

In this paper, we have developed the notion of intuitionistic fuzzy f-primary ideals (2-absorbing f-primary ideals) which unifies the notion of intuitionistic fuzzy prime ideals (2-absorbing ideals) and intuitionistic fuzzy primary ideals (2-absorbing primary ideals) of a Γ-ring. The study of these notions will open a new door toward the foundation of the study of decomposition property for intuitionistic fuzzy f-primary ideal (2-absorbing f-primary ideal).

Acknowledgements

The second author would like to thank Lovely Professional University, Phagwara for providing the opportunity to do the research work.

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