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ON SOME CLASSES OF n-BINORMAL OPERATORS IN MINKOWSKI SPACE

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Abstract In this article, we defined n-binormal operator, n-quasibinormal operator and skew n-binormal operators to Minkowski space $\mathcal{M}$ from Hilbert space. And we stickout some conditions for algebraic properties of these operators in Minkowski space $\mathcal{M}$. Also we proved two unitary equivalent binormal operators may belongs to the same class of operator in Minkowski Space.

1 Introduction

In this article, we introduced n-binormal operator and also exteded the concept of n-quasibinormal operator from n-quasinormal operator [4], as well as skew n-binormal operator from skew n-normal operator in Minkowski Space.

Definition 1.1. An Operator P is said to be Binormal in $\mathcal{M}$, if $P^{\sim}PPP^{\sim} = PP^{\sim}P^{\sim}P$.

Definition 1.2. An Operator P is called n-binormal in $\mathcal{M}$, if $P^{\sim}P^nP^nP^{\sim} = P^nP^{\sim}P^{\sim}P^n$.

Definition 1.3. An Operator P is called skew n-binormal in $\mathcal{M}$, if $[P^{\sim}P^nP^nP^{\sim}]P = P[P^nP^{\sim}P^{\sim}P^n]$.

Definition 1.4. An Operator P is called Quasi n-binormal in $\mathcal{M}$, if $P[P^{\sim}P^nP^nP^{\sim}] = [P^{\sim}P^nP^nP^{\sim}]P$.

Definition 1.5. An operator P is called n-normal in $\mathcal{M}$ if P^n is a normal operator in $\mathcal{M}$. This is equivalent to $P^{\sim}P^n = P^nP^{\sim}$.

2 n-Binormal in Minkowski space

In this section we have investigated some basic properties of n-binormal operator in $\mathcal{M}$.

Theorem 2.1. *If P is n-binormal in $\mathcal{M}$ and if P is unitarily equivalent to Q, Then Q is also n-binormal in $\mathcal{M}$.*

Proof: $Q = UPU^{\sim} \Rightarrow Q^{\sim} = UP^{\sim}U^{\sim}$.
And $Q^n = UP^nU^{\sim}$
$P^{\sim}P^nP^nP^{\sim} = P^nP^{\sim}P^{\sim}P^n$
$Q^{\sim}Q^nQ^nQ^{\sim} = UP^{\sim}U^{\sim}UP^nU^{\sim}UP^nU^{\sim}UP^{\sim}U^{\sim}$
$= UP^{\sim}P^nP^nP^{\sim}U^{\sim}$
$Q^nQ^{\sim}Q^{\sim}Q^n = UP^nU^{\sim}UP^{\sim}U^{\sim}UP^{\sim}U^{\sim}UP^nU^{\sim}$
$= UP^nP^{\sim}P^{\sim}P^nU^{\sim}$
$= UP^{\sim}P^nP^nP^{\sim}U^{\sim}$
$\Rightarrow Q^{\sim}Q^nQ^nQ^{\sim} = Q^nQ^{\sim}Q^{\sim}Q^n$.

Theorem 2.2. *If P, Q are n-binormal in $\mathcal{M}$ and if P and Q are doubly commuting then PQ is n-binormal in $\mathcal{M}$.*

Proof: P is n-binormal $\Rightarrow P^{\sim}P^nP^nP^{\sim} = P^nP^{\sim}P^{\sim}P^n$
Q is n-binormal $\Rightarrow Q^{\sim}Q^nQ^nQ^{\sim} = Q^nQ^{\sim}Q^{\sim}Q^n$
Now, $(PQ)^n(PQ)^{\sim}(PQ)^{\sim}(PQ)^n = Q^nP^nQ^{\sim}P^{\sim}Q^{\sim}P^{\sim}Q^nP^n$
$= Q^nQ^{\sim}P^nQ^{\sim}P^{\sim}Q^nP^{\sim}P^n$
$= Q^nQ^{\sim}Q^{\sim}P^nQ^nP^{\sim}P^{\sim}P^n$
$= Q^nQ^{\sim}Q^{\sim}Q^nP^nP^{\sim}P^{\sim}P^n$
$= Q^{\sim}Q^nQ^nQ^{\sim}P^{\sim}P^nP^nP^{\sim}$
$= Q^{\sim}Q^nQ^nP^{\sim}Q^{\sim}P^nP^nP^{\sim}$
$= Q^{\sim}Q^nP^{\sim}Q^nP^nQ^{\sim}P^nP^{\sim}$
$= Q^{\sim}P^{\sim}Q^nP^nQ^nP^nQ^{\sim}P^{\sim}$
$= (PQ)^{\sim}(PQ)^n(PQ)^n(PQ)^{\sim}$.
$\Rightarrow (PQ)$ is n-binormal in $\mathcal{M}$

Theorem 2.3. *Let P be decomposed as $P = U + iV \in \mathcal{M}$. Then P is binormal if and only if (i)$UV^3 + V^3U = U^3V + VU^3$ and (ii)$U^2VU + UVU^2 = V^2UV + VUV^2$.*

Proof: Since P is binormal then $P^{\sim}PPP^{\sim} = PP^{\sim}P^{\sim}P$.
$P^{\sim}PPP^{\sim} \Leftrightarrow (U - iV)(U + iV)(U + iV)(U - iV)$
$\Leftrightarrow (U^2 + iUV - iVU + V^2)(U^2 - iUV + iVU + V^2)$
$\Leftrightarrow U^4 - iU^3V + iU^2VU + U^2V^2 + iUVU^2$
$+UVUV - UVVU + iUV^3 - iVU^3 - VUUV + VUVU - iVUV^2$
$+V^2U^2 - iV^2UV + iV^3U + V^4$
$PP^{\sim}P^{\sim}P \Leftrightarrow (U + iV)(U - iV)(U - iV)(U + iV)$
$\Leftrightarrow (U^2 - iUV + iVU + V^2)(U^2 + iUV - iVU + V^2)$
$\Leftrightarrow U^4 + iU^3V - iU^2VU + U^2V^2 - iUVU^2$
$+UVUV - UVVU - iUV^3 + iVU^3$
$-VUUV + VUVU + iVUV^2 + V^2U^2 + iV^2UV - iV^3U + V^4$.
It is easy to observe that P is binormal in $\mathcal{M}$

Theorem 2.4. *Let P and Q be commuting n-binormal operator in $\mathcal{M}$ such that $(P+Q)^n$ commute with $\sum\limits_{k=0}^{n} nC_kP^{n-k}Q^k$. Then $(P+Q)$ is n-binormal in $\mathcal{M}$.*

Proof: $(P+Q)^{\sim}(P+Q)^n(P+Q)^n(P+Q)^{\sim}$
$= \left((P+Q)^{\sim}\sum\limits_{k=0}^{n} nC_kP^{n-k}Q^k\right)\left(\sum\limits_{k=0}^{n} nC_kP^{n-k}Q^k(P+Q)^{\sim}\right)$
$=\left((P+Q)^{\sim}P^n + (P+Q)^{\sim}\sum\limits_{k=1}^{n-1} nC_kP^{n-k}Q^k + (P+Q)^{\sim}Q^n\right)$
$\left(P^n(P+Q)^{\sim} + \sum\limits_{k=1}^{n-1} nC_kP^{n-k}Q^k(P+Q)^{\sim} + Q^n(P+Q)^{\sim}\right)$
$= \Big((P^{\sim} + Q^{\sim})P^n + (P+Q)^{\sim}\sum\limits_{k=1}^{n-1} nC_kP^{n-k}Q^k$
$+(P^{\sim} + Q^{\sim})Q^n(P^n(P^{\sim} + Q^{\sim}))$
$+\sum\limits_{k=1}^{n-1} nC_kP^{n-k}Q^k(P+Q)^{\sim} + Q^n(P^{\sim} + Q^{\sim})\Big)$
$= \Big(P^{\sim}P^n + Q^{\sim}P^n + (P+Q)^{\sim}\sum\limits_{k=1}^{n-1} nC_kP^{n-k}Q^k$
$+P^{\sim}Q^n + Q^{\sim}Q^n\Big)\Big(P^nP^{\sim} + P^nQ^{\sim}$
$+\sum\limits_{k=1}^{n-1} nC_kP^{n-k}Q^k(P+Q)^{\sim} + Q^nP^{\sim} + Q^nQ^{\sim}\Big)$
Since P and Q are commuting n-binormal operator in $\mathcal{M}$ such that $(P+Q)^{\sim}$ commutes with $\sum\limits_{k=1}^{n-1} nC_kP^{n-k}Q^k$

$= \left(P^n P^\sim + P^n Q^\sim + \sum_{k=1}^{n-1} nC_k P^{n-k} Q^k (P+Q)^\sim + Q^n P^\sim + Q^n Q^\sim\right)$
$\left(P^\sim P^n + Q^\sim P^n + (P+Q)^\sim \sum_{k=1}^{n-1} nC_k P^{n-k} Q^k + P^\sim Q^n + Q^\sim Q^n\right)$
$=\Big(P^n(P^\sim + Q^\sim) + \sum_{k=1}^{n-1} nC_k P^{n-k} Q^k (P+Q)^\sim$
$+Q^n(P^\sim + Q^\sim)\Big)$
$\left((P^\sim + Q^\sim)P^n + (P+Q)^\sim \sum_{k=1}^{n-1} nC_k P^{n-k} Q^k + (P^\sim + Q^\sim)Q^n\right)$
$= \left(\left(P^n + \sum_{k=1}^{n-1} nC_k P^{n-k} Q^k + Q^n\right)(P+Q)^\sim\right)$
$\left((P+Q)^\sim \left(P^n + \sum_{k=1}^{n-1} nC_k P^{n-k} Q^k + Q^n\right)\right)$
$= \sum_{k=0}^{n} nC_k P^{n-k} Q^k (P+Q)^\sim (P+Q)^\sim \sum_{k=0}^{n} nC_k P^{n-k} Q^k$
$= (P+Q)^n (P+Q)^\sim (P+Q)^\sim (P+Q)^n.$
Hence $(P+Q)$ is n-binormal in $\mathcal{M}$.

Theorem 2.5. *If P is n-normal in $\mathcal{M}$, then P is n-binormal in $\mathcal{M}$.*

Proof: P is a n-normal operator in $\mathcal{M} \Rightarrow P^\sim P^n = P^n P^\sim$
Postmultiply by $P^n P^\sim$ on both sides, we get
$P^n P^\sim P^\sim P^n = P^n P^\sim P^n P^\sim$
$= P^\sim P^n P^n P^\sim$
Hence P is n-binomal in $\mathcal{M}$.

3 Quasi n-Binormal in Minkowski space

In this section we have investigated some basic properties of quasi n-binormal operator in $\mathcal{M}$.

Theorem 3.1. *If $P$is quasi n-binormal in $\mathcal{M}$ and if $Q = UPU^\sim$, Then Q is also quasi n-binormal in $\mathcal{M}$.*

Proof: $Q = UPU^\sim \Rightarrow Q^\sim = UP^\sim U^\sim$
And $Q^n = UP^n U^\sim$
$[P^\sim P^n P^n P^\sim]P = P[P^\sim P^n P^n P^\sim]$
$[Q^\sim Q^n Q^n Q^\sim]Q = [UP^\sim U^\sim UP^n U^\sim UP^n U^\sim UP^\sim U^\sim]UPU^\sim$
$= [UP^\sim P^n P^n P^\sim U^\sim]UPU^\sim$
$= U[P^\sim P^n P^n P^\sim P]U^\sim$
$Q[Q^\sim Q^n Q^n Q^\sim] = UPU^\sim[UP^\sim U^\sim UP^n U^\sim UP^n U^\sim UP^\sim U^\sim]$
$= UP[P^\sim P^n P^n P^\sim]U^\sim$
$= U[P^\sim P^n P^n P^\sim]PU^\sim$
$\Rightarrow Q[Q^\sim Q^n Q^n Q^\sim] = [Q^\sim Q^n Q^n Q^\sim]Q$
Hence, Q is quasi n-binormal in $\mathcal{M}$.

Theorem 3.2. *If P, Q are quasi n-binormal in $\mathcal{M}$ and if P and Q are doubly commuting then PQ is quasi n-binormal in $\mathcal{M}$.*

Proof: P is Quasi n-binormal $\Rightarrow P[P^\sim P^n P^n P^\sim] = [P^\sim P^n P^n P^\sim]P$
Q is Quasi n-binormal $\Rightarrow Q[Q^\sim Q^n Q^n Q^\sim] = [Q^\sim Q^n Q^n Q^\sim]Q$
Now,
$(PQ)[(PQ)^\sim (PQ)^n (PQ)^n (PQ)^\sim] = PQ[Q^\sim P^\sim Q^n P^n Q^n P^n Q^\sim P^\sim]$
$= PQP^\sim Q^\sim P^n Q^n P^n Q^n P^\sim Q^\sim$
$= PP^\sim QP^n Q^\sim P^n Q^n P^\sim Q^n Q^\sim$
$= PP^\sim P^n P^n P^\sim Q^\sim Q^n Q^n Q^\sim$

$= PP^{\sim}P^nP^nP^{\sim}QQ^{\sim}Q^nQ^nQ^{\sim}$
$= P[P^{\sim}P^nP^nP^{\sim}]Q[Q^{\sim}Q^nQ^nQ^{\sim}]$
$= [P^{\sim}P^nP^nP^{\sim}]P[Q^{\sim}Q^nQ^nQ^{\sim}]Q$
$= P^{\sim}P^nP^nP^{\sim}Q^{\sim}PQ^nQ^nQ^{\sim}Q$
$= P^{\sim}P^nP^nQ^{\sim}P^{\sim}Q^nPQ^nQ^{\sim}Q$
$= P^{\sim}P^nQ^{\sim}P^nQ^nP^{\sim}Q^nPQ^{\sim}Q$
$= P^{\sim}Q^{\sim}P^nQ^nP^nQ^nP^{\sim}Q^{\sim}PQ$
$= Q^{\sim}P^{\sim}Q^nP^nQ^nP^nQ^{\sim}P^{\sim}QP$
$= [(PQ)^{\sim}(PQ)^n(PQ)^n(PQ)^{\sim}](PQ).$

Theorem 3.3. *Let P and Q be commuting Quasi n-binormal operator in $\mathcal{M}$ such that $(P+Q)^n$ commute with $(\sum_{k=0}^{n} nC_kP^{n-k}Q^k)$. And if $(P+Q)$ is quasi n-normal, then $(P+Q)$ is Quasi n-binormal in $\mathcal{M}$.*

Proof: $(P+Q)(P+Q)^{\sim}(P+Q)^n(P+Q)^n(P+Q)^{\sim}$
$= (P+Q)\Big((P+Q)^{\sim}(\sum_{k=0}^{n} nC_kP^{n-k}Q^k)\Big)$
$\Big(\sum_{k=0}^{n} nC_kP^{n-k}Q^k(P+Q)^{\sim}\Big)$
$= (P+Q)\Big((P+Q)^{\sim}P^n + (P+Q)^{\sim}(\sum_{k=1}^{n-1} nC_kP^{n-k}Q^k)$
$+ (P+Q)^{\sim}Q^n\Big)\Big(P^n(P+Q)^{\sim}$
$+ \sum_{k=1}^{n-1} nC_kP^{n-k}Q^k(P+Q)^{\sim} + Q^n(P+Q)^{\sim}\Big)$
$= (P+Q)\Big((P^{\sim}+Q^{\sim})P^n + (P+Q)^{\sim}(\sum_{k=1}^{n-1} nC_kP^{n-k}Q^k)$
$+(P^{\sim}+Q^{\sim})Q^n\Big)\Big(P^n(P^{\sim}+Q^{\sim}) + \sum_{k=1}^{n-1} nC_kP^{n-k}Q^k(P+Q)^{\sim}$
$+Q^n(P^{\sim}+Q^{\sim})\Big)$
$= (P+Q)\Big(P^{\sim}P^n + Q^{\sim}P^n + (P+Q)^{\sim}(\sum_{k=1}^{n-1} nC_kP^{n-k}Q^k)$
$+P^{\sim}Q^n + Q^{\sim}Q^n\Big)\Big(P^nP^{\sim} + P^nQ^{\sim} + \sum_{k=1}^{n-1} nC_kP^{n-k}Q^k(P+Q)^{\sim}$
$+Q^nP^{\sim} + Q^nQ^{\sim}\Big)$

As P and Q are commuting Quasi n-binormal operators such that $(P+Q)^{\sim}$ commute with $\sum_{k=1}^{n-1} nC_kP^{n-k}Q^k$
$= (P+Q)\Big(P^nP^{\sim} + P^nQ^{\sim} + (\sum_{k=1}^{n-1} nC_kP^{n-k}Q^k)(P+Q)^{\sim} + Q^nP^{\sim} + Q^nQ^{\sim}\Big)$
$\Big(P^{\sim}P^n + Q^{\sim}P^n + (P+Q)^{\sim}\sum_{k=1}^{n-1} nC_kP^{n-k}Q^k + P^{\sim}Q^n + Q^{\sim}Q^n\Big)$
$= P\Big(P^nP^{\sim} + P^nQ^{\sim} + (\sum_{k=1}^{n-1} nC_kP^{n-k}Q^k)(P+Q)^{\sim} + Q^nP^{\sim} + Q^nQ^{\sim}\Big)$
$+Q\Big(P^nP^{\sim} + P^nQ^{\sim} + (\sum_{k=1}^{n-1} nC_kP^{n-k}Q^k)(P+Q)^{\sim} + Q^nP^{\sim} + Q^nQ^{\sim}\Big)$
$\Big(P^{\sim}P^n + Q^{\sim}P^n + (P+Q)^{\sim}\sum_{k=1}^{n-1} nC_kP^{n-k}Q^k + P^{\sim}Q^n + Q^{\sim}Q^n\Big)$
$= \Big(P^nP^{\sim} + P^nQ^{\sim} + (\sum_{k=1}^{n-1} nC_kP^{n-k}Q^k)(P+Q)^{\sim} + Q^nP^{\sim} + Q^nQ^{\sim}\Big)$

$$(P+Q)\Big(P^{\sim}P^n + Q^{\sim}P^n + (P+Q)^{\sim}\sum_{k=1}^{n-1} nC_k P^{n-k}Q^k + P^{\sim}Q^n + Q^{\sim}Q^n\Big)$$
$$= \Big(P^n(P^{\sim}+Q^{\sim}) + (\sum_{k=1}^{n-1} nC_k P^{n-k}Q^k)(P+Q)^{\sim} + Q^n(P^{\sim}+Q^{\sim})\Big)$$
$$(P+Q)\Big((P^{\sim}+Q^{\sim})P^n + (P+Q)^{\sim}\sum_{k=1}^{n-1} nC_k P^{n-k}Q^k + (P^{\sim}+Q^{\sim})Q^n\Big)$$
$$= \Big(P^n(P+Q)^{\sim} + (\sum_{k=1}^{n-1} nC_k P^{n-k}Q^k)(P+Q)^{\sim} + Q^n(P+Q)^{\sim}\Big)$$
$$(P+Q)\Big((P+Q)^{\sim})P^n + (P+Q)^{\sim}\sum_{k=1}^{n-1} nC_k P^{n-k}Q^k + ((P+Q)^{\sim})Q^n\Big)$$
$$= \Big(P^n + (\sum_{k=1}^{n-1} nC_k P^{n-k}Q^k) + Q^n\Big)(P+Q)^{\sim}(P+Q)$$
$$(P+Q)^{\sim}\Big(P^n + \sum_{k=1}^{n-1} nC_k P^{n-k}Q^k + Q^n\Big)$$
$$= \Big(P^n + (\sum_{k=1}^{n-1} nC_k P^{n-k}Q^k) + Q^n\Big)(P+Q)^{\sim}$$
$$(P+Q)^{\sim}\Big(P^n + \sum_{k=1}^{n-1} nC_k P^{n-k}Q^k + Q^n\Big)(P+Q)$$
$$= \Big(\sum_{k=0}^{n} nC_k P^{n-k}Q^k\Big)(P+Q)^{\sim}(P+Q)^{\sim}\Big(\sum_{k=0}^{n} nC_k P^{n-k}Q^k\Big)(P+Q)$$
$$= (P+Q)^n(P+Q)^{\sim}(P+Q)^{\sim}(P+Q)^n(P+Q).$$

Hence $(P+Q)$ is quasi n-binormal operator in $\mathcal{M}$.

Theorem 3.4. *Let P be decomposed as $P = U + iV$ in $\mathcal{M}$. Then P is quasi binormal in $\mathcal{M}$ if and only if $UV^3 + V^3U = U^3V + VU^3$ and $U^2VU + UVU^2 = V^2UV + VUV^2$.*

Proof: $P(P^{\sim}PPP^{\sim}) \Leftrightarrow (U+iV)(U-iV)(U+iV)(U+iV)(U-iV)$
$\Leftrightarrow (U+iV)(U^2+V^2)(U^2+V^2)$
$\Leftrightarrow (U+iV)(U^2+V^2)^2$
$\Leftrightarrow (U+iV)(U^4+V^4+U^2V^2+V^2U^2)$
$\Leftrightarrow U^5+V^2U^3+iU^4V+iV^3U^2+U^3V^2+U^3V^2+V^4U+iU^2V^3+iV^5$
$(P^{\sim}PPP^{\sim})P = (U-iV)(U+iV)(U+iV)(U-iV)(U+iV)$
$\Leftrightarrow (U^2+V^2)(U^2+V^2)(U+iV)$
$\Leftrightarrow (U^2+V^2)^2(U+iV)$
$\Leftrightarrow (U^4+V^4+U^2V^2+V^2U^2)(U+iV)$
$\Leftrightarrow U^5+U^3V^2+iU^4V+iU^2V^3+V^2U^3+UV^4+iV^2U^3+iV^5$
$P(P^{\sim}PPP^{\sim}) = (P^{\sim}PPP^{\sim})P$ if and only if
$UV^3+V^3U = U^3V+VU^3$
and $U^2VU+UVU^2 = V^2UV+VUV^2$ is true.

4 Skew n-Binormal in Minkowski space

In this section we have investigated some basic properties of skew n-binormal operator in $\mathcal{M}$.

Theorem 4.1. *If P is skew n-binormal in $\mathcal{M}$ and if P is unitarily equivalent to Q, Then Q is also skew n-binormal in $\mathcal{M}$.*

Proof: $Q = UPU^{\sim} \Rightarrow Q^{\sim} = UP^{\sim}U^{\sim}$.
And $Q^n = UP^nU^{\sim}$
$[P^{\sim}P^nP^nP^{\sim}]P = P[P^nP^{\sim}P^{\sim}P^n]$
$[Q^{\sim}Q^nQ^nQ^{\sim}]Q = [UP^{\sim}U^{\sim}UP^nU^{\sim}UP^nU^{\sim}UP^{\sim}U^{\sim}]UPU^{\sim}$
$= [UP^{\sim}P^nP^nP^{\sim}U^{\sim}]UPU^{\sim}$
$= UP^{\sim}P^nP^nP^{\sim}PU^{\sim}$
$Q[Q^nQ^{\sim}Q^{\sim}Q^n] = UPU^{\sim}[UP^nU^{\sim}UP^{\sim}U^{\sim}UP^{\sim}U^{\sim}UP^nU^{\sim}]$
$= UPU^{\sim}[UP^nP^{\sim}P^{\sim}P^nU^{\sim}]$
$= UP[P^nP^{\sim}P^{\sim}P^n]U^{\sim}$

$= U[P^{\sim}P^nP^nP^{\sim}]PU^{\sim}$
$\Rightarrow Q^{\sim}Q^nQ^nQ^{\sim} = Q^nQ^{\sim}Q^{\sim}Q^n$
Hence, Q is skew n-binormal in $\mathcal{M}$.

Theorem 4.2. *If P, Q are skew n-binormal in $\mathcal{M}$ and if P and Q are doubly commuting then PQ is skew n-binormal in $\mathcal{M}$.*

Proof: P is skew n-binormal $\Rightarrow [P^{\sim}P^nP^nP^{\sim}]P = P[P^nP^{\sim}P^{\sim}P^n]$
Q is skew n-binormal $\Rightarrow [Q^{\sim}Q^nQ^nQ^{\sim}]Q = Q[Q^nQ^{\sim}Q^{\sim}Q^n]$
Now,
$[(PQ)^{\sim}(PQ)^n(PQ)^n(PQ)^{\sim}](PQ) = [Q^{\sim}P^{\sim}Q^nP^nQ^nP^nQ^{\sim}P^{\sim}]QP$
$= Q^{\sim}Q^nP^{\sim}Q^nP^nQ^{\sim}P^nQP^{\sim}P$
$= Q^{\sim}Q^nQ^nP^{\sim}Q^{\sim}P^nQP^nP^{\sim}P$
$= Q^{\sim}Q^nQ^nQ^{\sim}P^{\sim}QP^nP^nP^{\sim}P$
$= Q^{\sim}Q^nQ^nQ^{\sim}QP^{\sim}P^nP^nP^{\sim}P$
$= [Q^{\sim}Q^nQ^nQ^{\sim}]Q[P^{\sim}P^nP^nP^{\sim}]P$
$= Q[Q^nQ^{\sim}Q^{\sim}Q^n]P[P^nP^{\sim}P^{\sim}P^n]$
$\Rightarrow (PQ)$ is skew n-binormal in $\mathcal{M}$.

Theorem 4.3. *Every normal operator in $\mathcal{M}$ is skew n-binormal operator in $\mathcal{M}$.*

Proof: Since P is normal operator in $\mathcal{M} \Rightarrow PP^{\sim} = P^{\sim}P$
And it is easy to prove that P is also an n-normal operator in $\mathcal{M} \Rightarrow P^nP^{\sim} = P^{\sim}P^n$
$[P^{\sim}P^nP^nP^{\sim}]P = P^nP^{\sim}P^{\sim}P^nP$
$= P^nP^{\sim}P^{\sim}PP^n$
$= P^nP^{\sim}PP^{\sim}P^n$
$= P^nPP^{\sim}P^{\sim}P^n$
$= P[P^nP^{\sim}P^{\sim}P^n]$
Hence, P is skew n-binormal operator in $\mathcal{M}$.

Theorem 4.4. *If the operator P is n-normal and quasi n-normal operator in $\mathcal{M}$, then P is skew n-binormal operator in $\mathcal{M}$.*

Proof: $[P^{\sim}P^nP^nP^{\sim}]P = P^{\sim}P^nP^nP^{\sim}P$
$= P^{\sim}P^nP^{\sim}P^nP$
$= P^{\sim}P^nPP^{\sim}P^n$
$= PP^{\sim}P^nP^{\sim}P^n$
$= P[P^nP^{\sim}P^{\sim}P^n]$
Hence, P is skew n-binormal in $\mathcal{M}$.

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