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Evolution of viscous fluid string universe in Bianchi type-III metric with Λ term

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Abstract Here we investigate the accelerating behaviour of the evolution of viscous fluid string universe in a higher dimensional Bianchi type-III metric with variable cosmological term Λ. The proposed model assign with the exact solution of Einstein's field equations by considering the physical conditions such as shear scalar (σ) proportional to expansion factor (θ), this show to $\gamma = \tau^n$, where n is the arbitrary constant and coefficient of bulk viscosity (ξ) is inversely proportional to the expansion θ in the model. And also discussed about behaviour of the cosmological parameters like spatial volume (V), scale factor (R), scalar expansion (θ), Hubble parameter (H), deceleration parameter (q), anisotropy parameter (Δ) and shear scalar (σ^2) for the different conditions. Marginal variations have been noticed in the behaviours of the physical parameters. Interestingly we found the cosmological parameter Λ is an appealing candidate for dark energy and the physical and geometrical significance of the model discussed in detail.

1 Introduction

Gravitational theory is considered as a perfect explanation for the accelerating expansion of the universe by Einstein. Many researchers have suggested that the expansion of the universe goes through an anisotropic phase before reaching an isotropic phase. From our earlier studies and from different literatures we come to know that an anisotropic universe can be described by a Bianchi type cosmological model. The passage of time can also reveal the isotropic nature of this type of universe. On the other hand, the problem of cosmological constant Λ is still transient issued in cosmology and particle physics. A cosmological constant issue can be framed as a discrepancy between a negligible value for Λ for the present universe since its value is very small at the current epoch because our universe is very ancient. After the big bang, the universe's temperature will have lowered, leading the phase transition. During the phase conversion, as a result of spontaneous decay, the symmetry of the universe is broken. It can lead to topologically stable defects, vacuum domain walls, string and monopoles. Out of these three topological features, only string can shows to very interesting cosmological significance [1]. On a large scale, the universe appears homogeneous and isotropic, which is in conformity with the cosmological principle. A recent epoch of rapid expansion has also occurred. In spite of the fact that it is not definitely known why this cosmic acceleration occurred, the prevailing opinion that dark energy is to blame for the events.

A variety of factors have led to modern cosmologists and particle physicists focusing considerable attention on the study of higher dimensional cosmological models in Einstein's field equation. But the study on the evolution of viscous fluid within the frame work of cosmic strings and time varying cosmological constant seems to be very less in higher dimensional space time. In addition to possessing stress-energy, these strings are also coupled to gravitational fields, it may be fascinating to study the gravitational reaction that arise from string. There was a general relativistic approach to string developed by Letelier [2] and Stachel [3]. It has been used by Letelier as a provenance for cosmologies of Bianchi type-I and Kantowski-Sachs. Afterwards, Krori et al. [4] and Wang [5, 6] have consider the solutions of Bianchi type- II, VI, VII and IX for a cloud string. Tikekar Patel [7] and Chakraboraty and Chakraborty [8] have analysis the exact solutions of Bianchi type-III and spherical symmetric cosmology respectively for a cloud string.

The large-scale distribution of galaxies in our universe also allows a perfect fluid to describe the matter distribution satisfactorily. The ideal fluid distribution is not the only consideration to be considered in a realistic treatment of this problem. As the universe evolved, it was well known that matter behaved like a viscous fluid when neutrinos decoupled . Literature on four-dimensional space and time discusses viscous fluid cosmological models in the early universe [9, 10, 11]. Recently Bali and Dave [12] have presented Bianchi type-III string cosmological model with bulk viscosity under four dimensional space time, in this case, bulk viscosity is considered as a constant coefficient. According to this theory, the bulk viscosity coefficient reduce as the universe expands, rather than being constant. Also Wang [13] study the behaviour Bianchi type-III string cosmological universe with bulk viscosity. Bali and Pradhan [14] create a formalism for study the new combining of massive string in Bianchi type-III spacetime in existence of variable bulk viscosity. Adhav et al. [15] using cosmic strings and domain walls in general relativity, bianchi type-III cosmological models can be designed. Mohanty and Samanta [16] studied five dimensional LRS Bianchi type-I string cosmological models with bulk viscosity in general relativity and Reddy et al. [17] discussed Nambu-Takabayasi string cosmological models in five dimensional space-time within the framework of the scalar-tensor theory of gravitation. As a follow up of the recent works of the author Baro and Singh [18] which investigated about the the higher dimensional bulk viscous cosmological model with string in Bianchi type-III space-time considering constant Singh. K.P. et al. [19] presented Einstein's field equations for cloud string interactions with electromagnetic fields. Mollah and Singh [20] studied a five dimensional Bianchi type-III geometry containing strings stacked with particles and get a cosmological model shows with quadratic equation of state in Lyra's Manifold. Singh et al. [21] investigate the five dimensional string cosmological models with particles connect to them in Bianchi type-I space-time in general relativity we examined the proposed work too as of late Trivedi and Bhabor [22] constructed a five dimensional Bianchi-III model with two fluid structure in Brans-Dicke theory. Furthermore, numerous authors [23, 24, 25, 26, 27, 28] have also analyzed the behaviour of string cosmological model in modified gravity.

The cosmological constant Λ and the gravitational constant G are the two parameters display in Einstein's field equations. The newtonian constant of gravitation G, plays the part of coupling constant between geometry and matter in Einstein field equations. Dirac [29] was the first researcher, to suggest the idea of a variable G on certain physical grounds. The gravitational 'constant' G has been modified frequently in recent decades in order to account for time variations in general relativity. The expansion of Einstein theory with time subordinate G has also been proposed by [30, 31] for conceivable unification of gravitation and elementary particle physics or for the incorporation of Mach's principle into general relativity . The cosmological constant problem is also exceptionally curiously subject now-a-day.

Recent observations indicate that $\Lambda \sim 10^{55}cm^2$, in spite of the fact that the particle physics prediction for Λ is greater than this value by a factor of 10^{120}. There is a discrepancy between these two estimates, which is called the cosmological constant defect. An expansion of the universe leads to a small value for a decaying cosmological term observed today, which decayed from a huge value at its inception to the small value estimated today [32, 33, 34, 35]. Several phenomenological models have been proposed by considering Λ as a function of time [36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48]. Also Bali and Tinker [49]. Analyzed the bulk viscous barotropic fluid cosmological model of Bianchi type-III by incorporating G and Λ variables. Anisotropic bulk viscous cosmological models with variable G and Λ discussed by Singh and Kale [50]. An expanding universe with varying cosmological and gravitational constants in the presence of bulk viscous fluid according to the Bianchi type-III cosmological model [51, 52, 53]. Tiwari et al. [54] discussed LRS Bianchi type-II cosmological universe with a Decaying Lambda Term. Singh and Beesham [55] studied higher dimensional FRW type cosmological models with time-varying gravitational constant G and decaying vacuum energy Λ. Pradhan et al. [56] discussed models of cosmology of Bianchi type-I with variable gravitational and cosmological constants and time-dependent deceleration parameter. Soni and Shrimali [57] analysis the behaviour of Bianchi type-III string cosmological models with bulk viscosity and cosmological term Λ. Alfedeel and Abebe [58] studied using recent observational data of $H(z)$, SNeIa, and BAO, we reduce the system of differential equations (DEs) of Cosmology of type-I Bianchi Einstein equations time-varying models G and Λ in terms of constrained parameters Ω_m, Ω_r, and Ω_Λ. A detailed discussion of the viability of the Bianchi type-V cosmological model, which assumes a bulk viscous universe along with cosmological parameters with time-dependent characteristics (Λ) and Newtonian gravitational parameters (G) was investigated by Abebe et al. [59]. Ivashchuk et al. [60] analysis the SO(6) Yang–Mills field, the Gauss–Bonnet term, and the Λ term are included in this 10-dimensional gravitational model. Alfedeel, A.H.A. [61] investigate With G and Λ as variables, FLRW's dynamic behaviour in a higher-dimensional spacetime is described as well as the matter creation process.

Our motivation for writing this article comes from discussing the articles mentioned above. In this article we have investigate optimal dynamics of the evolution of viscous fluid string universes in the presence of a variable Λ cosmological term in a higher dimensional Bianchi type-III metric. It assume that a shear scalar equals a expansion scalar factor in order to analyse an explicit solution, which leads to the relation $\gamma = \tau^n$ and coefficient of the bulk viscosity is inversely proportional to expansion scalar θ, which leads to $\xi\theta = K$. The model also described in terms of its geometric and physical characteristics. We present the current work in the following manner: The metric and field equations are presented in sec. 2. In sec. 3 we deal with the physical explanation of the solutions. Last part sect. 4 is the conclusion.

2 Field equations and solution

As a part of the analysis, we consider a five-dimension cosmological model universe for the Bianchi type-III metric [62, 63]

$$ds^2 = \tau^2\left(dx^2 + e^{-2\alpha x}dy^2 + dz^2\right) + \gamma^2 dm^2 - dt^2, \tag{2.1}$$

where $\tau(t)$ *and* $\gamma(t)$ are the scale factors, $\alpha \neq 0$ is an arbitrary constant, $'m'$ represents a fifth space-like coordinate, with zero spatial curvature.

As the string dust cloud moves through a viscous liquid, the energy-momentum tensor is as follows:

$$T^i_j = \rho u_i u^j - \lambda x_i x^j - \xi u^i_{;l}(g^j_i + u_i u^j), \tag{2.2}$$

here u_i *and* x_i satisfy the condition

$$u^i u_j = -x^i x_j = -1, \quad u^i x_i = 0. \tag{2.3}$$

In equation (2.2) shows that ρ is the proper energy density for a cloud string containing particles them, ξ is the coefficient of bulk viscosity, λ is the string tension density of particles, u^i is the cloud four-velocity vector and x^i is a unit space-like vector that represents the string's direction. If the particle density of the presentation is denoted by ρ_p, then we have

$$\rho = \rho_p + \lambda. \tag{2.4}$$

Einstein's field equations in general theory of relativity with $8\pi G = 1$ and variable cosmological term Λ in suitable units are

$$R_{ij} - \frac{1}{2}g_{ij}R = T_{ij} - \Lambda(t)g. \tag{2.5}$$

From equation (2.1) to (2.5) we have the following system of equations:

$$\frac{\ddot{\tau}}{2\tau} + \frac{2\dot{\tau}\dot{\gamma}}{\tau\gamma} + \frac{\ddot{\gamma}}{\gamma} + \frac{5\dot{\tau}^2}{2\tau^2} + \frac{\dot{\gamma}^2}{\gamma^2} + \frac{\alpha^2}{\tau^2} = -\xi\theta - \Lambda, \tag{2.6}$$

$$-\frac{3\ddot{\tau}}{2\tau} + \frac{2\dot{\tau}\dot{\gamma}}{\tau\gamma} + \frac{\ddot{\gamma}}{\gamma} + \frac{\dot{\tau}^2}{2\tau^2} + \frac{\dot{\gamma}^2}{\gamma^2} - \frac{3\alpha^2}{\tau^2} = -\xi\theta - \Lambda, \tag{2.7}$$

$$\frac{\ddot{\tau}}{2\tau} + \frac{2\dot{\tau}\dot{\gamma}}{\tau\gamma} + \frac{\ddot{\gamma}}{\gamma} + \frac{5\dot{\tau}^2}{2\tau^2} + \frac{\dot{\gamma}^2}{\gamma^2} = -\xi\theta - \Lambda, \tag{2.8}$$

$$\frac{3\dot{\gamma}}{\gamma} - \frac{\ddot{\tau}}{2\tau} + \frac{5\dot{\tau}^2}{2\tau^2} = -\lambda - \xi\theta - \Lambda \tag{2.9}$$

and

$$-\frac{\ddot{\tau}}{2\tau} + \frac{3\dot{\tau}\dot{\gamma}}{\tau\gamma} - \frac{\dot{\tau}^2}{2\tau^2} + \frac{\dot{\gamma}^2}{\gamma^2} = -\rho - \Lambda. \tag{2.10}$$

Using overhead dots to represent differentiation over time 't'.

Since there are 5(five) highly nonlinear independent Eqns. (2.6) - (2.10) involving 7(seven) undetermined variables viz. $\tau,\ \gamma,\ \xi,\ \theta,\ \Lambda,\ \lambda$ *and* ρ therefore to get determinate solutions of the above system of equations. It is necessary to assume two physical plausible conditions in order to obtain the 2(two) extra equations.

(i) Initially, we consider shear scalar σ proportional to expansion factor θ, resulting in [64, 65, 66, 27]

$$\gamma = \tau^n, \tag{2.11}$$

where $n \neq 0$ is arbitrary constant.

(ii) Our deterministic model assumes the coefficient of bulk viscosity to be inversely proportional to θ which gives [14, 57]

$$\xi\theta = K, \tag{2.12}$$

where K is a constant of proportionality.

Using equation (2.11), equations (2.7) and (2.8) becomes

$$\tau = (c + bt - \frac{3}{4}\alpha^2 t^2)^{\frac{1}{2}}, \tag{2.13}$$

$$\gamma = (c + bt - \frac{3}{4}\alpha^2 t^2)^{\frac{n}{2}}, \tag{2.14}$$

where b and c are arbitrary constants.

Using equation (2.13) and (2.14), equation (2.8), (2.9) and (2.10) becomes

$$\lambda = \frac{12c\alpha^2(-1+2n) - 4b^2(1-6n+n^2) + 12\alpha^2 b(n-4)nt - 9\alpha^4(n-4)nt^2}{(4c+4bt-3\alpha^2t^2)^2}, \tag{2.15}$$

$$\rho = \frac{\begin{aligned}&16c^2K + 32cbKt - 12b\alpha^2 t(3-2n+n^2+2Kt^2) + 9\alpha^4 t^2(3-2n+n^2+Kt^2) - 12c\alpha^2(1+n \\ &\qquad + 2Kt^2) + 4b^2(2-3n+n^2+4Kt^2)\end{aligned}}{(4c+4bt-3\alpha^2t^2)^2}. \tag{2.16}$$

Using equations (2.15) and (2.16), equation (2.4) becomes

$$\rho_p = \frac{\begin{aligned}&16c^2K + 9\alpha^4t^2(3-6n+2n^2+Kt^2) - 12c\alpha^2(3n+2Kt^2) + 4b^2(3-9n+2n^2+4Kt^2) \\ &\qquad - 4bt(-8cK + 3\alpha^2(3-6n+2n^2+2Kt^2)\end{aligned}}{(4c+4bt-3\alpha^2t^2)^2}. \tag{2.17}$$

Using equation (2.12) - (2.14), equation (2.7) becomes

$$\Lambda = -\frac{\begin{aligned}&32c^2K + 64bcKt + 16b^2(1+n^2+2Kt^2) + 9\alpha^4t^2(9+2n+4n^2+2Kt^2) \\ &\qquad - 12c\alpha^2(5+2n+4Kt^2) - 12\alpha^2bt(9+2n+4n^2+4Kt^2)\end{aligned}}{2(4c+4bt-3\alpha^2t^2)^2}. \tag{2.18}$$

As a result, we have the following expressions for the parameters.

Spatial volume:

$$V = \left(c + bt - \frac{3}{4}\alpha^2 t^2\right)^{\frac{3+n}{4}}. \tag{2.19}$$

Scale factor:

$$a = \left(c + bt - \frac{3}{4}\alpha^2 t^2\right)^{\frac{3+n}{16}}. \tag{2.20}$$

Scalar expansion:

$$\theta = \frac{(n+3)\left(2b - 3\alpha^2 t\right)}{8c + 8bt - 6\alpha^2 t^2}. \tag{2.21}$$

Hubble parameter:

$$H = \frac{(n+3)\left(2b - 3\alpha^2 t\right)}{32c + 32bt - 24\alpha^2 t^2}. \tag{2.22}$$

Deceleration parameter:

$$q = \frac{12\alpha^2(8c + b(n-5)t) - 4b^2(n-13) - 9\alpha^4(n-5)t^2}{(n+3)\left(2b - 3\alpha^2 t\right)^2}. \tag{2.23}$$

Shear Scalar:

$$\sigma^2 = -\frac{3(n(5n-2)+13)\left(2b - 3\alpha^2 t\right)^2}{32\left(t\left(4b - 3\alpha^2 t\right) + 4c\right)^2}. \tag{2.24}$$

Anisotropy parameter:

$$\Delta = \frac{n(13n-18)+21}{(n+3)^2}. \tag{2.25}$$

From equation (2.12) and (2.21) we have

$$\xi = \frac{K\left(8bt + 8c - 6\alpha^2 t^2\right)}{(n+3)\left(2b - 3\alpha^2 t\right)}. \tag{2.26}$$

Once more, the redshift z is described in terms of the average scale factor $a(t)$ by $z = -1 + \frac{a_0}{a(t)}$, where a_0 is the current value of $a(t)$ can be expressed as

$$z = -1 + \left(c + bt - \frac{1}{4}3\alpha^2 t^2\right)^{\frac{-n-3}{16}}. \tag{2.27}$$

Table 1. Behaviour of cosmographic parameters for $b = 0.13,\ K = 1,\ c = 0.056,\ \alpha = 0.0132$ *and* $n = 30$

t	λ	ρ	ρ_p	Λ	V	a	θ	H	q	σ	ξ	z
0	-971.235	1094.901	2066.1360	-2428.68	4.7049×10^{-11}	0.00262	19.1518	0.4788	-0.5147	562.438	0.0522	380.8231
0.1	-639.484	721.263	1361.3677	-1599.48	2.6329×10^{-10}	0.00401	15.5406	0.4678	-0.5146	355.736	0.0643	247.2508
0.2	-452.637	510.822	964.3389	-1132.46	1.0933×10^{-9}	0.00568	13.0748	0.4574	-0.5145	241.675	0.0765	172.9066
0.3	-337.127	380.726	718.8359	-843.742	3.6794×10^{-9}	0.00759	11.2841	0.4474	-0.5144	172.614	0.0886	127.3973
0.4	-260.774	294.730	556.5191	-652.894	1.0594×10^{-8}	0.00975	9.9245	0.4378	-0.5143	127.924	0.1008	97.5682
0.5	-207.692	234.943	443.6451	-520.209	2.704×10^{-8}	0.01213	8.8571	0.4286	-0.5142	97.520	0.1129	76.9825
0.6	-169.302	191.704	361.9946	-424.249	6.2713×10^{-8}	0.01471	7.9969	0.4198	-0.5141	76.014	0.1250	62.1918
0.7	-140.644	159.426	301.0286	-352.614	1.3453×10^{-7}	0.01749	7.2888	0.4113	-0.5140	60.319	0.1372	51.2154
0.8	-118.687	134.695	254.3067	-297.728	2.7047×10^{-7}	0.02044	6.6958	0.4032	-0.5139	48.569	0.1493	42.8501
0.9	-101.493	115.330	217.7133	-254.75	5.1486×10^{-7}	0.02355	6.1920	0.3954	-0.5139	39.584	0.1615	36.3317

3 Physical explanation of the solution

In this paper table 1 and 2 show the rally of tension density(λ), rest energy density(ρ), particle density(ρ_p), cosmological constant(Λ), spatial volume(V), scale factor(a), scalar expansion(θ), Hubble parameter(H), deceleration parameter(q), shear scalar(σ^2), coefficient of bulk viscosity(ξ) and redshift(z) for specific values of arbitrary constant viz. $b = 0.13,\ K = 1,\ c = 0.056,\ \alpha = 0.0132$ *and* $n = 30$.

From Eqns. (2.15) and fig. 1 shows that string tension density is negative. Letelier [67] as saying that λ can be either less than 0 or more than 0. When $\lambda < 0$, the phase of the string

Table 2. Behaviour of cosmographic parameters for $b = 0.13$, $K = 1$, $c = 0.056$, $\alpha = 0.0132$ *and* $n = 30$

t	λ	ρ	ρ_p	Λ	V	a	θ	H	q	σ	ξ	z
1	-87.7796	99.884	188.5186	-220.469	9.3533×10^{-7}	0.02680	5.7586	0.3879	-0.5138	32.590	0.1737	31.1558
2	-30.3373	35.182	66.1122	-76.8698	7.3534×10^{-5}	0.06399	3.3859	0.3259	-0.5128	6.336	0.2953	9.7989
3	-15.1927	18.122	33.7616	-39.0051	1.2518×10^{-3}	0.10059	2.3965	0.2809	-0.5118	1.404	0.4173	4.3163
4	-9.08701	11.245	20.6880	-23.7374	1.0244×10^{-2}	0.12645	1.8537	0.2467	-0.5108	0.205	0.5394	2.1433
5	-6.03416	7.806	14.1358	-16.1026	0.0545	0.13541	1.5108	0.2200	-0.5098	0.000	0.6618	1.0701
6	-4.29312	5.845	10.3903	-11.7479	0.2178	0.12548	1.2746	0.1984	-0.5088	0.109	0.7845	0.4639
7	-3.20765	4.623	8.0497	-9.03256	0.7116	0.09888	1.1019	0.1806	-0.5077	0.318	0.9074	0.0888
8	-2.48585	3.810	6.4896	-7.2267	1.9998	0.06197	0.9702	0.1657	-0.5067	0.552	1.0306	-0.1591
9	-1.98182	3.243	5.3977	-5.96554	5.0001	0.02509	0.8664	0.1531	-0.5057	0.782	1.1540	-0.3313
10	-1.61612	2.832	4.6036	-5.05036	11.388	0.00201	0.7826	0.1423	-0.5047	0.999	1.2777	-0.4556

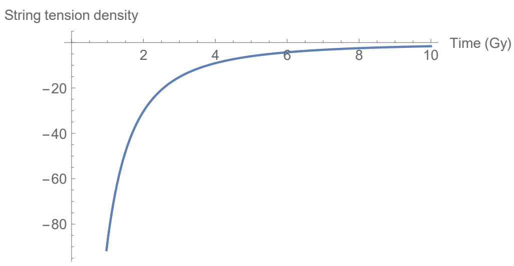

Figure 1. Variation of λ vs. t, whenever $b = 0.13$, $K = 1$, $c = 0.056$, $\alpha = 0.0132$ *and* $n = 30$

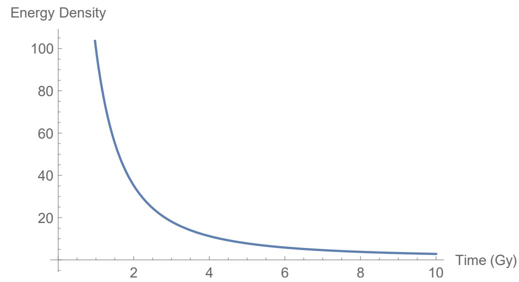

Figure 2. Variation of ρ vs. t, whenever $b = 0.13$, $K = 1$, $c = 0.056$, $\alpha = 0.0132$ *and* $n = 30$

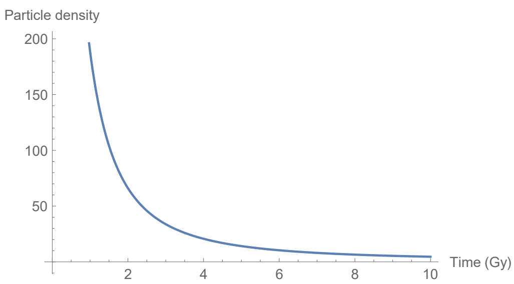

Figure 3. Variation of ρ_p vs. t, whenever $b = 0.13$, $K = 1$, $c = 0.056$, $\alpha = 0.0132$ *and* $n = 30$

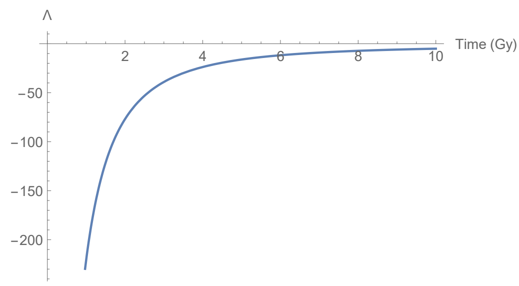

Figure 4. Variation of Λ vs. t, whenever $b = 0.13$, $K = 1$, $c = 0.056$, $\alpha = 0.0132$ *and* $n = 30$

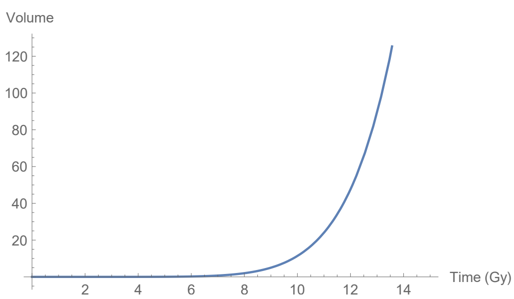

Figure 5. Variation of V vs. t, whenever $b = 0.13$, $c = 0.056$, $\alpha = 0.0132$ *and* $n = 30$

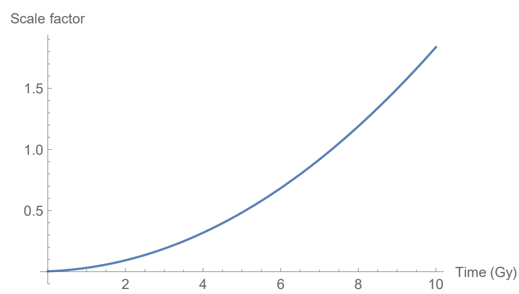

Figure 6. Variation of a vs. t, whenever $b = 0.13$, $c = 0.056$, $\alpha = 0.0132$ *and* $n = 30$

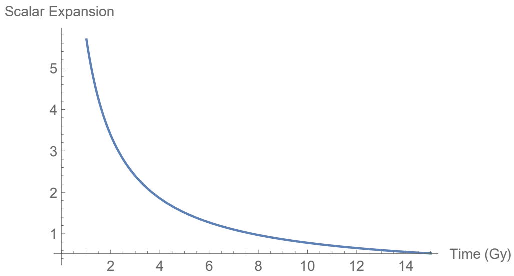

Figure 7. Variation of θ vs. t, whenever $b = 0.13,\ c = 0.056,\ \alpha = 0.0132$ *and* $n = 30$

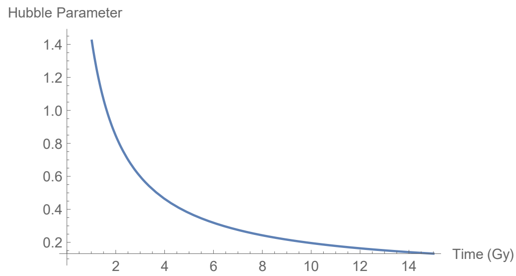

Figure 8. Variation of H vs. t, whenever $b = 0.13,\ c = 0.056,\ \alpha = 0.0132$ *and* $n = 30$

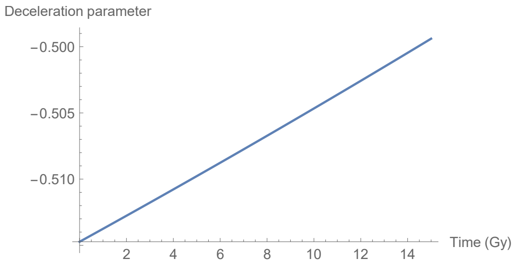

Figure 9. Variation of q vs. t, whenever $b = 0.13,\ c = 0.056,\ \alpha = 0.0132$ *and* $n = 30$

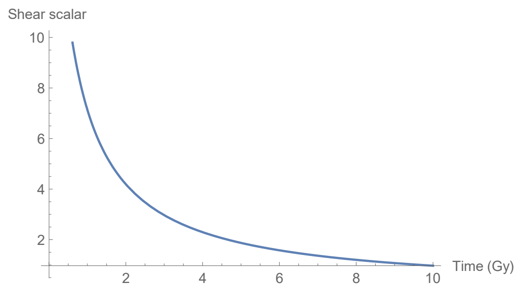

Figure 10. Variation of σ^2 vs. t, whenever $b = 0.13,\ c = 0.056,\ \alpha = 0.0132$ *and* $n = 30$

vanishes. The model in the late time universe satisfies the strong energy condition $\rho \geq 0,\ \lambda < 0$ as stated by Hawking and Ellis [68]. From Eqns. (2.16) and (2.17) shows that energy density and particle density are constant when $t \to 0$ and 0 when $t \to \infty$. The variation of energy density and particle density vs. time are presented in fig. 2 and fig. 3 respectively. Given that $\rho_p > 0$ for every time t, we show that $\frac{\rho_p}{|\lambda|} > 1$, which demonstrates that λ decreases more quickly than ρ_p. This demonstrates the particle dominance in the late universe. As a result, the reality criterion is met.

The behaviour of the cosmological term Λ vs time t for eqn. (2.18) is shown in reference figure 4. We note that Λ is negative when $t = 0$ and increases as time increases and finally become zero at late time. As a result, we say that dark energy has dominated the universe since the late-time period. Although in the past, it was possible for it to have negative values, it is merely interesting to note that. As such, the cosmological parameter Λ is an appealing candidate for dark energy, as dark energy dominates in this model while other fluids may have dominated earlier in cosmological history.

The spatial volume, scale factor, scalar expansion and Hubble parameter are constants at $t = 0$, but as time t increases spatial volume, scale factor are increase and scalar expansion,

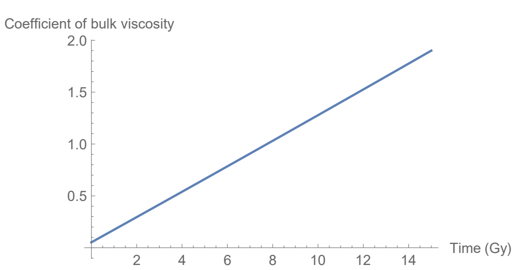

Figure 11. Variation of ξ vs. t, whenever $b = 0.13,\ c = 0.056,\ \alpha = 0.0132$ *and* $n = 30$

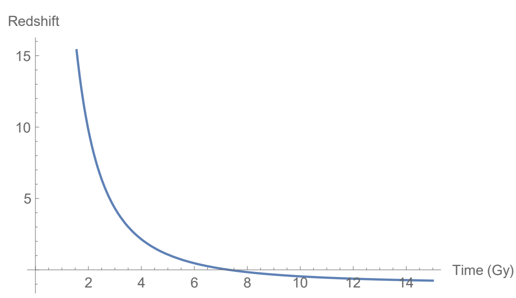

Figure 12. Variation of redshift z vs. t, whenever $b = 0.13,\ c = 0.056,\ \alpha = 0.0132$ *and* $n = 30$

Hubble parameter are decrease when time increases as indicated by eqn. (2.19) to (2.22) and clearly shown by fig. 5 to fig. 8 respectively. As a result, the universe begins to expand at an exponential rate and evolves with constant volume. So there is no initial singularity in the model. It also shown in the model that the universe is expanding over time; however, the rate of expansion slows as time increases, and this expansion stops at $t \to \infty$.

From eqn. (2.23) the value of deceleration parameter q is negative when $t = 0$ and $t \to \infty$ is shown in fig. 9. It indicates that the universe is accelerating at the present time by a negative value for the deceleration parameter. The universe is also accelerating, according to recent observations from SNeIa and similarly, Ade et al. [69] result indicates that the universe accelerates and the deceleration parameter lies somewhere between $-1 < q < 0$. Hence, accelerating models of the universe can be constructed in this case. Our inflationary model is also characterized by a negative deceleration parameter. At the beginning of the model, the fifth dimension's metric coefficient is constant, but as time passes, it contracts until it's unobservable.

In equation (2.24) and figure 10, it can be seen that initially the epoch can reached where the shear scalar is constant, but it decreases as time progresses. From equation (2.25), the mean anisotropy parameter Δ remains constant and non-zero for all time "t". Additionally, the value of $\lim_{t\to\infty} \frac{\sigma}{\theta} \neq 0$ indicates that the model does not become isotropic for large values of t. The result of our study are supported by [28, 70, 71], and can suggest that our proposed model universe retains anisotropic characteristics for a significant period as it evolves. However, this does not contradict the observation that our present universe is isotropic, because the initial anisotropy of the Bianchi type-III universe quickly dissipates and evolves into FRW as suggested by [72]. Also, Bianchi type-III universes have three spatial directions with different scale factors, which introduces anisotropy into the system. With the overall geometry of space, Bianchi model describes deviations from isotropy that preserve homogeneity. However, Plank temperature and polarization data from CMB can be used to test isotropy more generally, Saadesh et al. [73] strongly distaste for anisotropic expansion of the universe. But still some share of anisotropic is there, therefore, we need to conduct further mathematical and observational research in this area. In addition, when we set $n = 1$, our model universe approaches an isotropic one, supporting the recent observational findings.

From eqn. (2.26) coefficient of bulk viscosity is constant at $t = 0$ and $\xi \to \infty$ as $t \to \infty$ is shown in fig. 11. From this condition and graphical representation we come to know that besides affecting background expansion, coefficient of viscosity also affects the growth rate of structures. It also executed in this model of the accelerating universe.

Based on eqn. (2.27) representing redshift vs. time and fig. 12 representing the changes of redshift z with modify in cosmic time, it is perceive that z is negative for all time increases. In view of this result, our model also corresponds to the expanding universe.

4 Conclusion

We present a five-dimensional string cosmological model consisting of bulk viscosity, based on Bianchi type-III string cosmology with Λ term. We have extended each phase, the geometrical and kinematical properties of the different parameters are described in detail. It has been clarified the nature of the singularities within the models, and explicit scale factors have been determined for each case. As a result, the universe expands exponentially and evolves with a constant volume. The Cosmological term is found to be negative and approaches to zero at the late time as the results of type Ia supernovae explosion (SN Ia). It is found that the models reduce to the string model with bulk viscosity in the absence of the cosmological term Λ investigated by Bali and Pradhan [14] The bulk viscosity coefficient plays a significant role in cosmological consequences. A negative tension density indicates the existence of particle-dominated universe in the present day. Based on the studies of [74, 75, 76, 77], the current value of the deceleration parameter is approximately -0.55. In our proposed model also the value of q is ≈ -0.5 at $t \approx 14.4333 Gyr$. Therefore, we conclude that the present age of the model universe is 14.4333 Gyr such value is nearly for the finding of [74, 77]. Consequently, our model universe is consistent with current observations in the present epoch.

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