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AN APPROACH TO VAGUE IDEALS IN GAMMA NEAR ALGEBRA

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Abstract In this paper, we study the notion of vague ideals, the sum of vague ideals, and the vague co-set of a vague ideal in a Γ-near algebra. Additionally, we investigate the vague quotient Γ-near algebra and explore several properties and characterizations of these concepts.

1 Introduction

In 1964, N. Nabusawa [18] studied the concept of Γ-ring. Satyanarayana [19] presented the notion of a Γ-near-ring, which serves as an extension of both near-ring and Γ-ring. Later, Tamizh et al. [23, 24, 25] initiated the concepts of bi-ideals, minimal bi-ideals in near-ring, and bi-ideals in Γ-near-ring.

In 1965, L.A. Zadeh [27], a distinguished professor in electrical engineering and computer science at the University of California, Berkley, put forth the concept of the fuzzy set. Fuzzy logic, in a narrower context, denotes a logical system that generalizes classical two-valued logic to enable reasoning in the presence of uncertainty. Prof. Zadeh believed that, all real world problems could be solved more efficiently, and analytically using fuzzy set concept. This branch of mathematics initially got significant attention in Japan. As a result, close collaboration and technology transfer between universities , and industries was established in 1988. The Japanese government launched a careful feasibility study about setting national research projection fuzzy logic involving both universities, and industries.

The concept of fuzzy ideals and their characteristics have been utilized in different fields such as ring [21], near ring [22], and Γ- near ring [11, 26]. Brown [7] and Irish [10] have studied research on the composition of Near algebras. The notion of Γ-near algebra, introduced by Srinivas [20], serves as an extension to both near algebra and Γ-near-ring. The fundamental work on fuzzy ideals in near algebra and Γ-near algebra which was elaborated in [8, 14, 16, 17].

In 1983, the concept of the intuitionistic fuzzy set (IFS) was examined by Atanassov[1]. The notion of vague set (VS) was discussed by Gau and Buehrer [9]. Both VS and IFS serve as generalizations of a fuzzy set. Instead of using point based membership as in fuzzy Set, interval based membership is used in a VS. This interval-based membership provides a more effective means of capturing the ambiguous nature of data. Bustince and Burillo [6] showed that IFS and VS are identical, in this sense an IFS is isomorphic to a VS.

In several areas of study, various authors have dedicated their research to the investigation of vague ideals. These areas include semirings [12], near rings [3, 13], Γ-near rings [2], and the study of vague algebra was initiated by Biswas [5]. Furthermore, Bhaskar et al. [4, 15] have contributed to the discourse on vague ideals in near algebra and vague gamma near algebra.

This paper aims to present a new concept called vague left (or, right) ideals, the summation of vague ideals, and the vague co-set of a vague ideal in a Γ-near algebra. Additionally, we explore the notion of a vague quotient Γ-near-algebra. Finally, we delve into several properties and theorems that are closely related to the vague ideals of a Γ-near algebra.

2 Preliminaries

Definition 2.1. [9] A vague set A in the universe of discourse ψ is characterized by two membership functions given by

(i) A true membership function $t_A : \psi \to$[0,1] and
(ii) A false membership function $f_A : \psi \to$[0,1],
where $t_A(p)$ is a lower bound of the grade of membership of p derived from the evidence for p and $f_A(p)$ is a lower bound on the negation of p derived from the evidence against p with $t_A(p) + f_A(p) \leq 1$.

Definition 2.2. [20] Let S be a linear space over a field Z and Γ- be a non-empty set. Then S is said to be a Γ-near-algebra (GNA) over a field Z if there exist a mapping $S \times \Gamma \times S \to S$ (the image of (p,α,q) is denoted by pαq) must meet the following three criteria:
(i) $(p\alpha q)\beta r = p\alpha(q\beta r)$,
(ii) $(p+q)\alpha r = p\alpha r + q\alpha r$,
(iii) $(\lambda_1 p)\alpha q = \lambda_1(p\alpha q)$, for all $p, q, r \in S$, $\alpha, \beta \in \Gamma$ and $\lambda_1 \in Z$.

Definition 2.3. [20] Let S be a Γ-near-algebra over a field Z. Then S is said to be a zero symmetric Γ-near-algebra if $p\alpha 0 = 0$ for every $p \in S$ and $\alpha \in \Gamma$, where 0 is the additive identity in S.

Definition 2.4. [4] Let Z be a field and S be a Γ-near-algebra over a field Z. Let F be a vague field of Z and M be a vague set of S. Then M is a *vague* Γ*-near-algebra* (VGNA) of S over a vague field F of Z if the four conditions follow:
(i) $V_M(p+q) \geq \min(V_M(p), V_M(q))$,
(ii) $V_M(\lambda p) \geq \min(V_F(\lambda), V_M(p))$,
(iii) $V_M(p\alpha q) \geq \min(V_M(p), V_M(q))$,
(iv) $V_F(1) \geq V_M(p)$ for every $p, q \in S, \lambda \in Z, \alpha \in \Gamma$.

Note: Throughout this article Γ*-near-algebra* is denoted by *"GNA"*
and *vague* Γ*-near-algebra* is denoted by *"VGNA"*.

3 Vague Ideals of Γ-near algebra

Definition 3.1. Let M be a *VGNA* of a *GNA* S over a vague field F of Z. Then M is called a vague ideal of S, if $V_M(p\alpha q) \geq V_M(p)$ and $V_M(q\alpha(p+i) - q\alpha p) \geq V_M(i)$ for every $p, q, i \in S, \alpha \in \Gamma$.
M is a vague right ideal of S if $V_M(p\alpha q) \geq V_M(p)$ for every $p, q \in S, \alpha \in \Gamma$.
M is a vague left ideal of S if $V_M(q\alpha(p+i) - q\alpha p) \geq V_M(i)$ for every $p, q, i \in S, \alpha \in \Gamma$.

In alternate way, a vague subset M of a *GNA* S over a vague field F of Z is said to be *vague ideal* of S if the four conditions follows:
(i) $V_M(p+q) \geq \min(V_M(p), V_M(q))$,
(ii) $V_M(\lambda p) \geq \min(V_F(\lambda), V_M(p))$,
(iii) $V_F(1) \geq V_M(p)$,
(iv) $V_M(p\alpha q) \geq V_M(p)$,
(v) $V_M(q\alpha(p+i) - q\alpha p) \geq V_M(i)($ or equivalently $V_M(q\alpha r - q\alpha p) \geq V_M(r-p))$ for every $p, q, r, i \in S$ and 1 is the unity in Z.
If M satisfies (i), (ii), (iii) and (iv), then M is called a *vague right ideal* of S.
If M satisfies (i), (ii), (iii) and (v), then M is called a *vague left ideal* of S.

Example 3.2. Consider the set $S = \{0, p, q, r\}$ equipped with the binary operation "$+$" as:

$+$	0	p	q	r
0	0	p	q	r
p	p	0	r	q
q	q	r	0	p
r	r	q	p	0

Define a scalar multiplication on S by $0 \cdot p = 0$ and $1 \cdot p = p$ for all $p \in S$ and $0, 1 \in Z$.

From the table, it is clear that $(S, +)$ is an abelian group. Let $0, 1 \in Z, p \in S$. Then $0 \cdot p = 0 \in S$ and $1 \cdot p = p \in S$. Therefore $\lambda p \in S$ for every $\lambda \in Z$ and $p \in S$. Let $0, 1 \in Z$ and $p, q \in S$. Then particularly for $p = a, q = b$, we have $0(a+b) = 0c = 0$, $0a + 0b = 0 + 0 = 0$,

$1(a+b) = 1c = c$ and $1a+1b = a+b = c$. Therefore, in general $\lambda(p+q) = \lambda p+\lambda q$, for every $\lambda \in Z$ and $p, q \in S$. Similarly, we can easily verify that $(\lambda + \mu)p = \lambda p + \mu p$, $(\lambda\mu)p = \lambda(\mu p)$ and $1p = p$, for every $\lambda, \mu \in Z, p \in S$, where 1 is the unity in Z. Thus, S is a linear space over the field Z.
Let $\Gamma = \{\alpha, \beta\}$ be a non-empty set. Define a mapping $S \times \Gamma \times S \to S$ by the following tables:

α	0	p	q	r
0	0	0	0	0
p	0	0	0	0
q	0	0	0	0
r	0	0	0	0

β	0	p	q	r
0	0	0	0	0
p	0	p	p	p
q	0	q	q	q
r	0	r	r	r

For any $p, q, r \in S$, $\lambda \in Z$ and $\alpha, \beta \in \Gamma$, we can obtain the following:
$(p\alpha q)\beta r = p\alpha(q\beta r)$, $(p+q)\alpha r = p\alpha r + q\alpha r$, $(\lambda p)\alpha q = \lambda(p\alpha q)$.
Thus, S is a *GNA* over a field Z.
Let F be a vague subset of Z defined by

$$t_F(p) = \begin{cases} 0.9 & \text{if } p = 0, \\ 0.8 & otherwise. \end{cases}$$

and

$$f_F(p) = \begin{cases} 0.1 & \text{if } p = 0, \\ 0.2 & otherwise. \end{cases}$$

For every $p, q \in Z$, we get $V_F(p - q) \geq \min(V_F(p), V_F(q))$ and particularly for $q \neq 0$, $V_F(pq^{-1}) \geq \min(V_F(p), V_F(q))$. Thus F is a vague field of Z. Let M be a vague subset of S defined by

$$t_M(p) = \begin{cases} 0.6 & \text{if } p = 0, \\ 0.4 & \text{otherwise.} \end{cases}$$

and

$$f_M(p) = \begin{cases} 0.4 & \text{if } p = 0, \\ 0.6 & \text{otherwise.} \end{cases}$$

For any $\lambda, \mu \in Z, \alpha \in \Gamma$ and $p, q, i \in S$, we have $p\alpha q, \lambda p, \mu q, \lambda p + \mu q, q\alpha(p+i) - q\alpha p \in S$.
(i) $V_M(\lambda p + \mu q) \geq \min(\min(V_F(\lambda), V_M(p)), \min(V_F(\mu), V_A(q)))$,
(ii) $V_M(p\alpha q) \geq V_M(p)$,
(iii) $V_F(1) \geq V_M(p)$ (where 1 is the unity in φ),
(iv) $V_M(q\alpha(p+i) - q\alpha p) \geq V_M(i)$.
Then, M is a vague ideal of the *GNA* S over the vague field F of Z.

Theorem 3.3. *Let M be a vague ideal of a GNA S over a vague field F of Z. Then each level subset $M_t = \{p \in S : V_M(p) \geq t, t \in [0, 1], t \leq V_M(0)\}$ is an ideal of S, where $V_F(\lambda) \geq t$ for any $\lambda \in Z$.*

Proof. Let $p, q \in M_t, \alpha \in \Gamma$ and $\lambda \in Z$. Then $p, q \in S$ and $V_M(p) \geq t, V_M(q) \geq t$. Since M is a vague ideal, we get $V_M(p - q) \geq \min\{V_M(p), V_M(q)\} \geq \min(t, t) = t$. Therefore $p - q \in M_t$. Now $V_M(\lambda p) \geq \min(V_F(\lambda), V_M(p)) \geq \min(t, t) = t$. Therefore $\lambda p \in M_t$ (since S is a vector space, then $\lambda p \in S$). Thus M_t is a linear subspace of S.
Let $p, q \in S$, $\alpha \in \Gamma$ and $i \in M_t$. Then $q\alpha(p+i)-q\alpha p \in S$, and so $V_M(q\alpha(p+i)-q\alpha p) \geq V_M(i) \geq t$. Therefore $q\alpha(p+i) - q\alpha p \in M_t$. Thus M_t is a left ideal of S. Now $V_M(i\alpha p) \geq V_M(i) \geq t$. Therefore $i\alpha p \in M_t$. Thus M_t is a right ideal of S. Hence M_t is an ideal of S. □

Theorem 3.4. *Let M be a VGNA of S over a vague field F of Z. If M is a vague ideal of S, then $V_M(0) \geq V_M(p)$ for every $p \in S$.*

Proof. We have

$$\begin{aligned} V_M(0) &= V_M(p-p) \\ &= V_M(1p-1p) \\ &\geq \min(\min(V_F(1), V_M(p)), \min(V_F(-1), V_M(p))) \\ &\geq \min(\min(V_M(p), V_M(p)), \min(V_F(1), V_M(p))) \\ &\geq \min(V_M(p), \min(V_M(p), V_M(p)) \\ &= V_M(p). \end{aligned}$$

□

Theorem 3.5. *Let M be a vague ideal of a zero symmetric GNA S over a vague field F of Z. Let $V_F(\lambda) \geq V_M(0)$ for all $\lambda \in Z$. Then the set $M_* = \{p \in S : V_M(p) = V_M(0)\}$ is an ideal of S.*

Proof. First we verify that M_* is a linear subspace of S. Let $p, q \in M_*$. Then $V_M(p) = V_M(0) = V_M(q)$. Since M is a vague ideal in S, then $V_M(p-q) \geq \min(V_M(p), V_M(q)) = \min(V_M(0), V_M(0)) = V_M(0)$. But $V_M(0) \geq V_M(p-q)$. Therefore $V_M(p-q) = V_M(0)$. Thus $p - q \in M_*$. For every $\lambda \in Z$, we have $V_M(\lambda p) \geq \min(V_F(\lambda), V_M(p)) \geq (V_F(\lambda), V_M(0)) = V_M(0)$. But $V_M(0) \geq V_M(\lambda p)$ for every $\lambda \in Z, p \in S$. Therefore $V_M(\lambda p) = V_M(0)$ and $\lambda p \in S$. Thus $\lambda p \in M_*$. Hence M_* is a linear subspace of S.

Let $p, q \in S, \alpha \in \Gamma, i \in M_*$. Then $V_M(i) = V_M(0)$. Now $V_M(q\alpha(p+i) - q\alpha p) \geq V_M(i) = V_M(0)$. But $V_M(0) \geq V_M(q\alpha(p+i) - q\alpha p)$ for every $p, q \in S, \alpha \in \Gamma, i \in M_*$. Therefore $V_M(q\alpha(p+i) - q\alpha p) = V_M(0)$. Thus $q\alpha(p+i) - q\alpha p \in M_*$. Now $V_M(i\alpha p) \geq V_M(i) = V_M(0)$. On the other hand $V_M(0) \geq V_M(i\alpha p)$. Therefore $V_M(i\alpha p) = V_M(0)$. Thus $i\alpha p \in M_*$. Hence M_* is an ideal of S. □

Proposition 3.6. *If M_1 and M_2 are two vague ideals of a zero symmetric GNA S over a vague field F of Z, then $M_1 + M_2$ is also a vague ideal of S.*

Proof. Let M_1 and M_2 be two vague ideals of a zero symmetric *GNA* S.
Now (i) Let $p = p_1 + p_2, q = q_1 + q_2; p, q, p_1, q_1, p_2, q_2 \in S$.
Then $p + q = p_1 + p_2 + q_1 + q_2 = p_1 + q_1 - q_1 + p_2 + q_1 + q_2$. This implies that

$$\begin{aligned} V_{(M_1+M_2)}(p+q) &= \sup\{\min(V_{M_1}(p_1+q_1), V_{M_2}(-q_1+p_2+q_1+q_2))\} \\ &\geq \sup\{\min(V_{M_1}(p_1), V_{M_1}(q_1)), \min(V_{M_2}(-q_1+p_2+q_1), V_{M_2}(q_2))\} \\ &\geq \sup\{\min(V_{M_1}(p_1), V_{M_1}(q_1), min(V_{M_2}(p_2), V_{M_2}(q_2))\} \\ &= \min\{\sup_{p=p_1+p_2} \min(V_{M_1}(p_1), V_{M_2}(p_2)), \sup_{q=q_1+q_2} \min(V_{M_1}(q_1), V_{M_2}(q_2))\} \\ &= \min(V_{(M_1+M_2)}(p), V_{(M_1+M_2)}(q)). \end{aligned}$$

(ii) Let $p = p_1 + p_2, \lambda p = \lambda p_1 + \lambda p_2; p, p_1, p_2 \in S, \lambda \in Z$. Then

$$\begin{aligned} V_{(M_1+M_2)}(\lambda p) &= \sup\{\min(V_{M_1}(\lambda p_1), V_{M_2}(\lambda p_2))\} \\ &\geq \sup\{\min(V_F(\lambda), V_{M_1}(p_1)), \min(V_F(\lambda), V_{M_2}(p_2))\} \\ &= \min\{V_F(\lambda), \sup(V_{M_1}(p_1), V_{M_2}(p_2))\} \\ &= \min\{V_F(\lambda), V_{(M_1+M_2)}(p)\}. \end{aligned}$$

(iii) Since M_1, M_2 are VGNAs, we get $V_F(1) \geq V_{M_1}(p)$ and $V_F(1) \geq V_{M_2}(p)$.
Let $p = p_1 + p_2; p, p_1, p_2 \in S$. Then
$V_{(M_1+M_2)}(p) = \sup\{\min(V_{M_1}(p_1), V_{M_2}(p_2))\} \leq \min(V_F(1), V_F(1)) = V_F(1)$.

(iv) Let $p = p_1 + p_2; p, p_1, p_2 \in S, \alpha \in \Gamma$. This implies that $p\alpha q = p_1\alpha q + p_2\alpha q; q \in S$. Then

$$\begin{aligned} V_{(M_1+M_2)}(p\alpha q) &= \sup\{\min(V_{M_1}(p_1\alpha q), V_{M_2}(p_2\alpha q))\} \\ &\geq \sup\{\min(V_{M_1}(p_1), V_{M_2}(p_2))\} \\ &= V_{(M_1+M_2)}(p). \end{aligned}$$

(v) Let $r-p=t_1+t_2; p,t_1,t_2\in S$. This implies that $r=t_1+t_2+p$, and so $q\alpha r-q\alpha p= q\alpha(t_1+t_2+p)-q\alpha p=q\alpha(t_1+t_2+p)-q\alpha(t_2+p)+q(t_2+p)-q\alpha p$. Then

$$\begin{aligned} V_{(M_1+M_2)}(q\alpha r-q\alpha p) &= V_{(M_1+M_2)}\{q\alpha(t_1+t_2+p)-q\alpha(t_2+p)+\\ &\qquad\qquad y\alpha(t_2+p)-q\alpha p\}\\ &\geq \sup\{\min(V_{M_1}(q\alpha(t_1+t_2+p)-q\alpha(t_2+p)),\\ &\qquad\qquad V_{M_2}(q\alpha(t_2+p)-q\alpha p))\}\\ &\geq \sup\{\min(V_{M_1}(t_1+t_2+p-(t_2+p)),V_{M_2}(t_2+p-p))\}\\ &= \sup_{r-p=t_1+t_2}(V_{M_1}(t_1),V_{M_2}(t_2))\\ &= V_{(M_1+M_2)}(r-p). \end{aligned}$$

Hence, M_1+M_2 is a vague ideal of S. □

Theorem 3.7. *Intersection of family of vague ideals of a GNA S over a vague field F of Z is a vague ideal of S.*

Proof. Let $\{M_i\}_{i\in\Lambda}$ be the family of vague ideals of a *GNA* S over a vague field F of Z.
Let M be the intersection of $\{M_i\}_{i\in\Lambda}$. Let $V_M(p)=\bigcap_{i\in\Lambda}V_{M_i}(p)=\inf_{i\in\Lambda}V_{M_i}(p)$. For any $p,q\in S,\alpha\in\Gamma$ and $\lambda,\mu\in Z$ we have

$$\begin{aligned} V_M(\lambda p+\mu q) &= \inf_{i\in\Lambda}V_{M_i}(\lambda p+\mu q)\\ &\geq \inf_{i\in\Lambda}[\min(\min(V_F(\lambda),V_{M_i}(p)),\min(V_F(\mu),V_{M_i}(q)))]\\ &\geq \min(\min(V_F(\lambda),\inf_{i\in\Lambda}V_{M_i}(p)),\min(V_F(\mu),\inf_{i\in\Lambda}V_{M_i}(q)))\\ &= \min(\min(V_F(\lambda),V_M(p)),\min(V_F(\mu),V_M(p))). \end{aligned}$$

Since each M_i is a vague ideal, we get $V_F(1)\geq V_{M_i}(p)\geq\inf_{i\in\Lambda}V_{M_i}(p)=V_M(p)$ for every $p\in S$ and $i\in\Lambda$. Now

$$\begin{aligned} V_M(p\alpha q) &= \inf_{i\in\Lambda}V_{M_i}(p\alpha q)\\ &\geq \inf_{i\in\Lambda}(V_{M_i}(p))\\ &= V_M(p). \end{aligned}$$

Thus, M is a vague right ideal of S. Let $p,q,j\in S$. Then

$$\begin{aligned} V_M(q\alpha(p+j)-q\alpha p) &= \inf_{i\in\Lambda}V_{M_i}(q\alpha(p+j)-q\alpha p)\\ &\geq \inf_{i\in\Lambda}(V_{M_i}(j))\\ &= V_M(j). \end{aligned}$$

Thus, M is a vague left ideal of S.
Hence, M is a vague ideal of a *GNA* S. □

Definition 3.8. Let M be a vague ideal of a *GNA* S over a vague field F of Z and $m\in S$. Then the vague subset $m+V_M$ of M is defined by $(m+V_M)(n)=V_M(n-m)$ for every $n\in S$, is called a `vague co-set` of the vague ideal M.

Theorem 3.9. *Let M be a vague ideal of a GNA S over a vague field F of Z. Let $p,q,r\in S,\alpha\in\Gamma,\lambda\in Z$. Then we have the following:*
(i) $p+V_M=q+V_M$ *if and only if* $V_M(p-q)=V_M(0)$,
(ii) $p+V_M=q+V_M$ *implies* $V_M(p)=V_M(q)$,
(iii) $V_M(p+q)=V_M(q+p)$.

Notation: Let M be a vague ideal of a *GNA* S over a vague field F of Z. Then we write $\frac{S}{V_M}$ or $S/V_M=\{m+V_M : m\in S\}$ the set of all vague co-sets of M.

Theorem 3.10. *Let M be a vague ideal of a GNA S over a vague field F of Z. Then the set S/V_M of all vague co-sets of M is a GNA with respect to the operations defined by*

$$\begin{array}{lll}(p+V_M)+(q+V_M) & = & (p+q)+V_M,\\ (p+V_M)\alpha(q+V_M) & = & p\alpha q+V_M,\\ \lambda(p+V_M) & = & \lambda p+V_M\end{array}$$

for every $p, q \in S, \alpha \in \Gamma$ and $\lambda \in Z$.

Definition 3.11. Let M be a vague ideal of a *GNA* S over a vague field F of Z. Then the set S/V_M or $\frac{S}{V_M}$ of all vague co-sets of M is called a *vague quotient* Γ*-near algebra* of S by M with respect to the following operations:

$$\begin{array}{lll}(p+V_M)+(q+V_M) & = & (p+q)+V_M,\\ (p+V_M)\alpha(q+V_M) & = & p\alpha q+V_M,\\ \lambda(p+V_M) & = & \lambda p+V_M\end{array}$$

for every $p, q \in S, \alpha \in \Gamma$ and $\lambda \in Z$.

Theorem 3.12. *Let M be a vague ideal of a GNA S. Define $f_M : S/V_M \to [0,1]$ by $f_M(p+V_M) = V_M(p)$ for every $p \in S$. Then f_M is a vague ideal of S/V_M.*

Notation: Let A and B be two vague ideals of a *GNA* S over a vague field F of Z such that $V_A \subseteq V_B$ and $V_A(0) = V_B(0)$. Then we define a vague subset $f_B : S/V_A \to [0,1]$ by $f_B(p+V_A) = V_B(p)$ for every $p+V_A \in S/V_A$.

Theorem 3.13. *Let A and B be two vague ideals of a GNA S over a vague field F of Z. Then f_B is a vague ideal of S/V_A such that $f_A \subseteq f_B$, where f_B and f_A are given by the above notation. Also $f_A(0) = f_B(0)$.*

Theorem 3.14. *If M is a vague ideal of a GNA S over a vague field F of Z, then the mapping $f : S \to S/V_M$ defined by $f(p) = p+V_M, p \in S$ is a GNA epimorphism with Kernel M_*, where $M_* = \{p \in S : V_M(p) = V_M(0)\}$.*

4 Conclusion

Within this study, we presented and investigated the concept of vague ideals, as well as the summation of these ideals in gamma near-algebra, and discussed vague co-set of a vague ideal in a gamma near algebra. Also, vague quotient gamma near-algebra. Further, we give several properties with suitable examples. we proved that the intersection of family of vague ideals of gamma near algebra is again a vague ideal of gamma near algebra, also studied some theorems related to the vague ideals of gamma near-algebra. Furthermore, we studied challenging problems related to vague ideals of gamma near algebra and their impact on various branches of Mathematics.

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