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On the Sombor index and Sombor energy of m-splitting graph and m-shadow graph of regular graphs

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Abstract I. Gutman recently introduced the Sombor index, a vertex-degree-based topological index of a simple graph G with n vertices. It is defined as $SO(G) = \sum_{uv \in E(G)} \sqrt{d_G(u)^2 + d_G(v)^2}$, where $d_G(u)$ is the degree of the vertex u and $E(G)$ is the edge set of G. In this paper, we find the Sombor index of the m-splitting graph and m-shadow graph of k-regular graph. Additionally, we establish the relationship between the m-shadow graph of the k-regular graph and the energy and Sombor energy of the m-splitting graph.

1 Introduction

Topological graph indices, also known as molecular descriptors, are mathematical formulas applicable to any graph that models a molecular structure. These indices are valuable tools for analyzing mathematical properties and investigating specific physicochemical characteristics of molecules, providing a cost-effective alternative to expensive and time-consuming laboratory experiments. Various topological indices have been extensively studied in the literature [4, 11].

The use of graph representations to study the physical properties of chemicals marked the beginning of research into topological indices. Over time, these indices have evolved beyond their chemical origins, being studied independently in graph theory and developing correlations with chemical properties [2]. The study of topological indices remains an area of active research, with new concepts and indices continuously being proposed by researchers exploring the diverse applications and theoretical underpinnings of these mathematical tools. Recently, Gutman [6] introduced a new vertex-degree-based topological index known as the Sombor index. Since then, the Sombor index has been used in a wide range of contexts, generating more attention and research [3, 14, 16]. This article aims to delve deeper into the properties and applications of the Sombor index, providing a comprehensive overview of its theoretical foundations and practical implications in both graph theory and molecular chemistry.

2 Preliminary

In graph theory, a finite graph $G = (V, E)$ consists of a finite non-empty set V, the elements of which are the vertices of G, and a finite set E of unordered pairs of distinct elements of V called the edges of G. For any vertex u in V, the neighborhood $N(u)$ of the vertex u is the set of vertices adjacent to u, given by $N(u) = \{v \in V : v \text{ is adjacent to } u\}$. The degree of a vertex u, denoted as $d_G(u)$, is the size of its neighborhood, i.e., the number of vertices adjacent to u.

Let $A(G)$ be the adjacency matrix of graph G with vertices $v_1, v_2, ..., v_n$. The adjacency matrix $A(G)$ of the graph G is defined as:

$$A(G) = (a_{ij}) = \begin{cases} 1 & \text{if } v_i \text{ and } v_j \text{ are adjacent} \\ 0 & \text{if } i = j \\ 0 & \text{if } v_i \text{ and } v_j \text{ are not adjacent.} \end{cases}$$

The Sombor index, introduced by Gutman [6], is defined as:

$$SO(G) = \sum_{uv \in E(G)} \sqrt{d_G(u)^2 + d_G(v)^2},$$

where the sum runs over all edges uv in the graph G.

The energy of graphs has many applications in different fields, such as chemistry and computer science. Because of its application, many researchers have shown interest in finding different types of energy of graphs; see [9, 10, 13, 15]. The energy $\varepsilon(G)$ of a simple graph G, introduced by Gutman [7], is defined as the sum of the absolute eigenvalues of the adjacency matrix $A(G)$ of the graph G. That is,

$$\varepsilon(G) = \sum_{i=1}^{n} |\lambda_i| .$$

where, $\lambda_1, \lambda_2, ..., \lambda_n$ are the eigenvalues of the adjacency matrix $A(G)$ of the graph G. Recently, Vaidya et al. [17, 18] obtained some results on the energy of shadow graph and splitting graph. The *shadow graph* $D_2(G)$ of a connected graph G is definitively constructed by taking two copies of G, denoted as G' and G'', and rigorously joining each vertex u' in G' to the neighbors of the corresponding vertex u'' in G''. The m-*shadow* graph $D_m(G)$ of a connected graph G is created by making m copies of the graph G, labeled as $G_1, G_2, ..., G_m$, and then linking each vertex u in G_i to the neighboring vertex v in G_j, where $1 \leq i, j \leq m$. The *splitting graph* $S'(G)$ were introduced by Sampathkumar [12]. The *splitting graph* $S'(G)$ of a graph G is created by adding a new vertex v' to each vertex v in G, such that v' is connected to every vertex adjacent to v in G. The m-*splitting graph* $Spl_m(G)$ of a graph G is created by adding m new vertices, v_{ij}, to each vertex v_i in G, such that v_{ij} is connected to each vertex v_k adjacent to v_i in G. In a recent publication, Gowtham et al. [5] introduced the Sombor matrix for graph G and defined the Sombor energy $SE(G)$ as a new variant of graph energy. The Sombor matrix of a graph G with n vertices is denoted as $S(G) = (s_{ij})_{n \times n}$.

$$s_{ij} = \begin{cases} \sqrt{d_G(u)^2 + d_G(v)^2} & \text{if } uv \in E(G) \\ 0 & \text{otherwise.} \end{cases}$$

The Sombor energy $SE(G)$ of graph G is the sum of the absolute eigenvalues of the Sombor matrix $S(G)$ of graph G.

To obtain the result, we need the *Kronecker product* of matrices. Let $A = (a_{ij}) \in \mathbb{R}^{m \times n}$ and $B \in \mathbb{R}^{p \times q}$. The *Kronecker product* of A and B is defined by

$$A \otimes B = \begin{bmatrix} a_{11}B & a_{12}B & ... & a_{1n}B \\ a_{21}B & a_{22}B & ... & a_{2n}B \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1}B & a_{m2}B & ... & a_{mn}B \end{bmatrix}_{mp \times nq}$$

Proposition 2.1. *[1] Let $A \in M^m$ and $B \in M^n$. Let λ and μ represent the eigenvalues of the matrices A and B, with corresponding eigenvectors x and y, respectively. Then $\lambda\mu$ is an eigenvalue of $A \otimes B$ with corresponding eigenvector $x \otimes y$.*

For the graph-theoretical notions, we refer to [1, 19].

3 Main Result

3.1 Sombor index of m-splitting graph and m-shadow graph

Now, we give the general formula for the Sombor index of the m-shadow graph and m-splitting graph of the k-regular graph.

Sombor index of m-splitting graph

Theorem 3.1. *Let G be a k-regular graph with n vertices, where $n \geq 2$. Then*

(i) $SO(G) = \frac{nk^2}{\sqrt{2}}$,

(ii) $SO(Spl_m(G)) = SO(G)(m\sqrt{2m^2+4m+4} + (m+1))$.

Proof. (i) Proof is trivial.

(ii) Note that, $|V(Spl_m(G))\backslash V(G)| = nm$ and $d_{Spl_m(G)}(v_{ij}) = k$ for all $1 \leq i \leq n$ and $1 \leq j \leq m$. Since, degree $d_G(v_i)$ of v_i for all $1 \leq i \leq n$ increased by mk in splitting graph $Splm(G)$ of G. Therefore, $d_{Spl_m(G)}(v_i) = (m+1)k$.

Since, $N(v_{ij}) = \{v_l : v_l$ is adjacent to $v_i\}$ for all $i \neq l$ and $1 \leq j \leq m$ and $d_{Spl_m(G)}(v_l) = (m+1)k$.

Therefore,

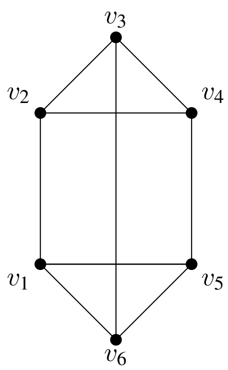
$$\begin{aligned} SO(Spl_m(G)) &= \sum_{v_{ij}v_l \in E(Spl_m(G))} \sqrt{(d_{Spl_m(G)}(v_{ij}))^2 + (d_{Spl_m(G)}(v_l))^2} \\ &\quad + \sum_{v_iv_j \in E(Spl_m(G))} \sqrt{(d_{Spl_m(G)}(v_i))^2 + (d_{Spl_m(G)}(v_j))^2} \\ &= |N(v_{ij})||V(Spl_m(G))\backslash V(G)|\sqrt{k^2 + ((m+1)k)^2} \\ &\quad + |E|\sqrt{((m+1)k)^2 + ((m+1)k)^2} \\ &= knm\sqrt{k^2(m^2+2m+2)} + \frac{n(m+1)k^2}{2}\sqrt{2} \\ &= \frac{nk^2}{\sqrt{2}}[m\sqrt{2m^2+4m+4} + (m+1)] \\ &= SO(G)[m\sqrt{2m^2+4m+4} + (m+1)]. \end{aligned}$$

□

Consider complete graph K_n which is $(n-1)$-regular. Sombor index of $m-$splitting graph of complete graph K_n is:

$$SO(Spl_m(K_n)) = \frac{n(n-1)^2}{\sqrt{2}}[m\sqrt{2m^2+4m+4} + (m+1)]$$

Example 3.2. Consider $3-$regular graph with 6 vertices



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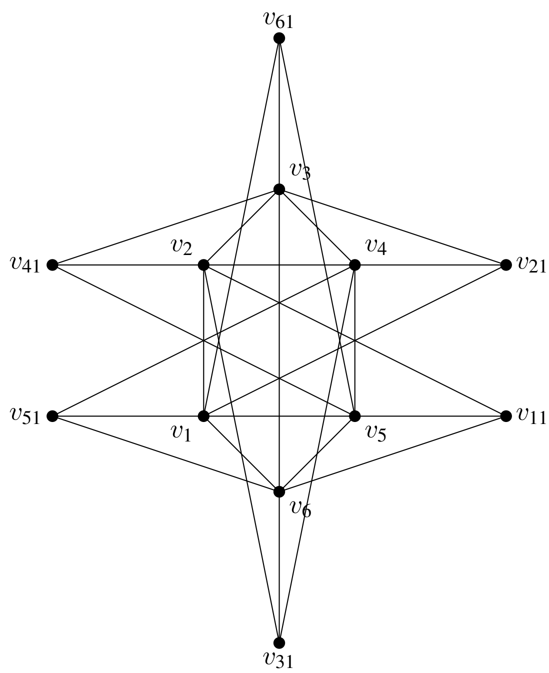


$Spl_m(G)$

Here, $m = 1$, $k = 3$, and $n = 6$.

$$
\begin{aligned}
SO(G) &= \sum_{v_iv_j \in E(G)} \sqrt{d_G(v_i)^2 + d_G(v_j)^2} \\
&= 9\sqrt{3^2 + 3^2} \\
&= 27\sqrt{2}. \\
SO(Spl_m(G)) &= \sum_{v_{ij}v_l \in E(Spl_m(G))} \sqrt{d_{Spl_m(G)}(v_{ij})^2 + d_{Spl_m(G)}(v_l)^2} \\
&\quad + \sum_{v_iv_j \in E(Spl_m(G))} \sqrt{d_{Spl_m(G)}(v_i)^2 + d_{Spl_m(G)}(v_j)^2} \\
&= 18\sqrt{3^2 + 6^2} + 9\sqrt{6^2 + 6^2} \\
&= 54(\sqrt{5} + \sqrt{2}) = SO(G)(\sqrt{10} + 2).
\end{aligned}
$$

Sombor index of m-shadow graph

Proposition 3.3. *If G is a k-regular graph with n vertices, then the m-shadow graph $D_m(G)$ of G is mk-regular.*

Theorem 3.4. *If G is a k-regular graph with n vertices, then $SO(D_m(G)) = SO(G)(m^3 + m^2)$.*

Proof. Since, $D_m(G)$ is constructed from m copies of G and $|V(G)| = n$. Therefore, $|V(D_m(G))| = mn + n = (m+1)n$ and $|E(D_m(G))| = \frac{(nm+n)(mk)}{2}$. The Sombor index of the shadow graph of k-regular graph G is

$$SO(D_m(G)) = \sum_{uv \in E(D_m(G))} \sqrt{\left(d_{D_m(G)}(u)\right)^2 + \left(d_{D_m(G)}(v)\right)^2}$$

From Proposition 3.3, $d_{D_m(G)}(u) = mk$ for all $u \in D_m(G)$. Therefore,

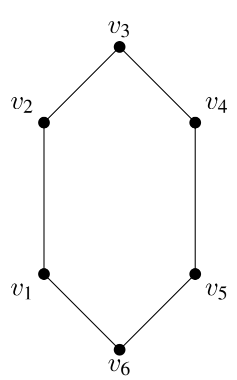
$$\begin{aligned} SO(D_m(G)) &= |E(D_m(G))|\sqrt{(mk)^2+(mk)^2} \\ &= \frac{(nm+n)(mk)}{2}\sqrt{2(mk)^2} \\ &= \frac{(nm+n)(mk)^2}{\sqrt{2}} \\ &= \frac{nk^2}{\sqrt{2}}(m^3+m^2) \\ &= SO(G)(m^3+m^2). \end{aligned}$$

□

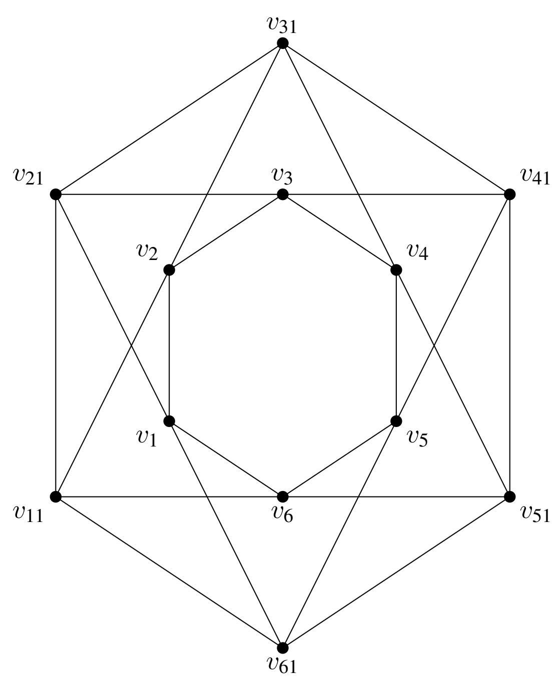
Consider cycle graph C_n, which is 2-regular.

$$SO(D_m(C_n)) = \frac{4n}{\sqrt{2}}(m^3+m^2).$$

Example 3.5. Consider cycle graph C_6. $SO(C_6) = 12\sqrt{2}$
$SO(D_1(C_6)) = 24\sqrt{2} = 2SO(C_6)$



$\mathbf{C_6}$



$\mathbf{D_1(C_6)}$

3.2 Sombor energy of m-splitting and m-shadow graph

Sombor energy of m-splitting graph

Theorem 3.6. *If G is a $k-$regular graph, then $SE(Spl_m(G)) = k(m+1)\sqrt{2}\varepsilon(G)$.*

Proof. Let G be a $k-$regular graph with vertex set $V(G) = \{v_1, v_2, v_3, ..., v_n\}$. Let $Spl_m(G)$ be m-splitting graph of graph G with vertex set $V(Spl_m(G)) = V(G) \cup \{v_{ij} : v_{ij}$ is adjacent to $v_l, v_i \neq v_l\}$. The Sombor matrix of $Spl_m(G)$ is

$$S(Spl_m(G)) = \begin{bmatrix} S_1 & S_2 & ... & S_2 \\ S_2 & 0 & ... & 0 \\ \vdots & \vdots & \ddots & \vdots \\ S_2 & 0 & ... & 0 \end{bmatrix}_{n+mn},$$

where S_1 and S_2 are $n \times n$ symmetric matrices. Entries of matrix S_1 are

$$a_{ij} = \begin{cases} (m+1)k\sqrt{2} & \text{if } v_i \text{ is adjacent to } v_j \\ 0 & \text{Otherwise.} \end{cases}$$

Because $d_{Spl_m}(v_i) = (m+1)k$ and entries of S_2 are

$$b_{ij} = \begin{cases} k\sqrt{m^2+2m+2} & \text{if } v_i \text{ is adjacent to } v_{ij} \\ 0 & \text{Otherwise.} \end{cases}$$

Because $b_{ij} = \sqrt{(d_{Spl_m(G)}(v_{ij}))^2 + (d_{Spl_m(G)}(v_l))^2}$ with $d_{Spl_m(G)}(v_{ij}) = (m+1)k$. Therefore, we can write matrix S_1 and S_2 as $S_1 = ((m+1)k\sqrt{2})A(G)$ and $S_2 = k\sqrt{m^2+2m+2}A(G)$. That is

$$S(Spl_m(G)) = \begin{bmatrix} (m+1)k\sqrt{2}\,A(G) & k\sqrt{m^2+2m+2}\,A(G) & ... & k\sqrt{m^2+2m+2}\,A(G) \\ k\sqrt{m^2+2m+2}\,A(G) & 0A(G) & ... & 0A(G) \\ \vdots & \vdots & \ddots & \vdots \\ k\sqrt{m^2+2m+2}\,A(G) & 0A(G) & ... & 0A(G) \end{bmatrix}_{n+mn}$$

$$= \begin{bmatrix} (m+1)k\sqrt{2} & k\sqrt{m^2+2m+2} & ... & k\sqrt{m^2+2m+2} \\ k\sqrt{m^2+2m+2} & 0 & ... & 0 \\ \vdots & \vdots & \ddots & \vdots \\ k\sqrt{m^2+2m+2} & 0 & ... & 0 \end{bmatrix}_{m+1} \otimes A(G)$$

$$= \mathcal{B} \otimes A(G),$$

$$\text{where, } \mathcal{B} = \begin{bmatrix} (m+1)k\sqrt{2} & k\sqrt{m^2+2m+2} & ... & k\sqrt{m^2+2m+2} \\ k\sqrt{m^2+2m+2} & 0 & ... & 0 \\ \vdots & \vdots & \ddots & \vdots \\ k\sqrt{m^2+2m+2} & 0 & ... & 0 \end{bmatrix}_{m+1}.$$

The matrix $\mathcal{B}$ has eigenvalues $\mu_1, \mu_2, \ldots, \mu_{m+1}$. Because $\mathcal{B}$ is a rank two matrix, it has two non-zero eigenvalues, denoted as μ_1 and μ_2. For convenience, set $a = (m+1)k\sqrt{2}$ and $b =$

$k\sqrt{m^2+2m+2}$. We have $tr(\mathcal{B}) = \sum_{i=1}^{m+1} \mu_i$ and $tr(\mathcal{B}^2) = \sum_{i=1}^{m+1} \mu_i^2$. Therefore,

$$\mu_1 + \mu_2 = a \tag{3.1}$$

and

$$\mu_1^2 + \mu_2^2 = 2mb^2 + a^2. \tag{3.2}$$

Solving (1) and (2), we get

$$\mu^2 - a\mu - mb^2 = 0$$

with roots are μ_1 and μ_2. Therefore, the characteristic equation of matrix $\mathcal{B}$ is

$$Char(\mathcal{B}) = \mu^{m-1}(\mu^2 - k(m+1)\sqrt{2}\mu - mk^2(m^2+2m+2)).$$

Therefore, eigenvalues of matrix $\mathcal{B}$ are $\frac{k(m+1)\sqrt{2} \pm k\sqrt{4m^3+10m^2+12m+2}}{2}$ and 0 with multiplicity $m-1$. The spectrum of matrix $\mathcal{B}$ is

$$spec(\mathcal{B}) = \begin{pmatrix} 0 & \frac{k(m+1)\sqrt{2}+k\sqrt{4m^3+10m^2+12m+2}}{2} & \frac{k(m+1)\sqrt{2}-k\sqrt{4m^3+10m^2+12m+2}}{2} \\ m-1 & 1 & 1 \end{pmatrix}$$

By Proposition (2.1), the spectrum of $S(Spl_m(G))$ is

$$spec(S(Spl_m(G))) = \begin{pmatrix} 0\lambda_1 & ... & 0\lambda_n & ... & \alpha\lambda_1 & ... & \alpha\lambda_n & \beta\lambda_1 & ... & \beta\lambda_n \\ m-1 & ... & m-1 & ... & 1 & ... & 1 & 1 & ... & 1 \end{pmatrix},$$

where $\alpha = \frac{k(m+1)\sqrt{2}+k\sqrt{4m^3+10m^2+12m+2}}{2}$ and $\beta = \frac{k(m+1)\sqrt{2}-\sqrt{4m^3+10m^2+12m+2}}{2}$. Therefore,

$$\begin{aligned} SE(Spl_m(G)) &= \sum_{i=1}^{mn+n} \left| \frac{k(m+1)\sqrt{2} \pm k\sqrt{4m^3+10m^2+12m+2}}{2} \lambda_i \right| \\ &= \sum_{i=1}^{n} |\lambda_i| \left[\frac{k(m+1)\sqrt{2}+k\sqrt{4m^3+10m^2+12m+2}}{2} \right. \\ &\quad \left. + \frac{k(m+1)\sqrt{2}-k\sqrt{4m^3+10m^2+12m+2}}{2} \right] \\ &= \sum_{i=1}^{n} |\lambda_i| |k(m+1)\sqrt{2}| \\ &= k(m+1)\sqrt{2}\varepsilon(G). \end{aligned}$$

□

Sombor energy of m-shadow graph

Theorem 3.7. *If G is a $k-$regular graph. Then $SE(D_m(G)) = mk\sqrt{2+8m}\varepsilon(G)$.*

Proof. Let G be $k-$regular graph. Sombor matrix $S(D_m(G))$ of $D_m(G)$ is

$$S(D_m(G)) = \begin{bmatrix} mk\sqrt{2}A(G) & mk\sqrt{2}A(G) & ... & mk\sqrt{2}A(G) \\ mk\sqrt{2}A(G) & 0A(G) & ... & 0A(G) \\ \vdots & \vdots & \ddots & \vdots \\ mk\sqrt{2}A(G) & 0A(G) & ... & 0A(G) \end{bmatrix}_{mn+n}$$

$$= \begin{bmatrix} mk\sqrt{2} & mk\sqrt{2} & ... & mk\sqrt{2} \\ mk\sqrt{2} & 0 & ... & 0 \\ \vdots & \vdots & \ddots & \vdots \\ mk\sqrt{2} & 0 & ... & 0 \end{bmatrix}_{m+1} \otimes A(G)$$

Let $\mathcal{C} = \begin{bmatrix} mk\sqrt{2} & mk\sqrt{2} & ... & mk\sqrt{2} \\ mk\sqrt{2} & 0 & ... & 0 \\ \vdots & \vdots & \ddots & \vdots \\ mk\sqrt{2} & 0 & ... & 0 \end{bmatrix}_{m+1}$ and let eigenvalues of the matrix $\mathcal{C}$ be $\delta_1, \delta_2, ..., \delta_{m+1}$.

Since $\mathcal{C}$ is of rank two, there are two non-zero eigenvalues δ_1 and δ_2 (say). We have, $tr(\mathcal{C}) = \sum_{i=1}^{m+1} \delta_i$ and $tr(\mathcal{C}^2) = \sum_{i=1}^{m+1} \delta_i^2$. Therefore,

$$tr(\mathcal{C}) = \delta_1 + \delta_2$$

That is

$$\delta_1 + \delta_2 = mk\sqrt{2} \tag{3.3}$$

and

$$tr(\mathcal{C}^2) = \delta_1^2 + \delta_2^2$$

That is

$$\delta_1^2 + \delta_2^2 = 4m^3k^2 + 2m^2k^2 \tag{3.4}$$

Solving (3) and (4) we get

$$\delta^2 - mk\sqrt{2}\delta - 2m^3k^2 = 0.$$

whose roots are δ_1 and δ_2. So, the characteristic equation of $\mathcal{C}$ is

$$Char(\mathcal{C}) = \delta^{(m-1)}(\delta^2 - mk\sqrt{2}\delta - 2m^3k^2).$$

Therefore, the eigenvalues of matrix $\mathcal{C}$ are $mk\sqrt{2}(\frac{1 \pm \sqrt{4m+1}}{2})$ and 0 with multiplicity $m-1$. The spectrum of matrix $\mathcal{C}$ is

$$spec(\mathcal{C}) = \begin{pmatrix} 0 & mk\sqrt{2}(\frac{1+\sqrt{4m+1}}{2}) & mk\sqrt{2}(\frac{1-\sqrt{4m+1}}{2}) \\ m-1 & 1 & 1 \end{pmatrix}$$

By proposition 2.1, the spectrum of matrix $S(D_m(G))$

$$spec(S(D_m(G))) = \begin{pmatrix} 0\lambda_1 & ... & 0\lambda_n & p\lambda_1 & ... & p\lambda_n & q\lambda_1 & ... & q\lambda_n \\ m-1 & ... & m-1 & 1 & ... & 1 & 1 & ... & 1 \end{pmatrix},$$

where $p = mk\sqrt{2}(\dfrac{1+\sqrt{4m+1}}{2})$ and $q = mk\sqrt{2}(\dfrac{1-\sqrt{4m+1}}{2})$. Therefore,

$$\begin{aligned} SE(D_m(G)) &= \sum_{i=1}^{n} \left| mk\sqrt{2}(\frac{1 \pm \sqrt{4m+1}}{2})\lambda_i \right| \\ &= mk\sqrt{2}\sum_{i=1}^{n} |\lambda_i| \left[\frac{\sqrt{4m+1}+1}{2} + \frac{\sqrt{4m+1}-1}{2} \right] \\ &= mk\sqrt{8m+2}\sum_{i}^{n} |\lambda_i| \\ &= mk\sqrt{8m+2}\varepsilon(G). \end{aligned}$$

□

Table 1. Sombor index and Sombor energy of m-splitting and m-shadow graph of some specific regular graph.

Graph (G)	$\mathbf{SO(G)}$	$\mathbf{SO(Spl_m(G))}$	$\mathbf{SO(D_m(G))}$	$\mathbf{SE(Spl_m(G))}$	$\mathbf{SE(D_m(G))}$
C_n	$2\sqrt{2}n$	$2\sqrt{2}n(m\sqrt{2m^2+4m+4}+(m+1))$	$2\sqrt{2}n(m^3+m^2)$	$2\sqrt{2}(m+1)\varepsilon(C_n)$	$2m\sqrt{8m+2}\varepsilon(C_n)$
K_n	$\frac{n(n-1)^2}{\sqrt{2}}$	$\frac{n(n-1)^2}{\sqrt{2}}(m\sqrt{2m^2+4m+4}+(m+1))$	$\frac{n(n-1)^2}{2}(m^3+m^2)$	$2\sqrt{2}(n-1)^2(m+1)$	$2(n-1)^2m\sqrt{8m+2}$
Q_n	$2^{(n-\frac{1}{2})}n^2$	$2^{(n-\frac{1}{2})}n^2(m\sqrt{2m^2+4m+4}+(m+1))$	$2^{(n-\frac{1}{2})}n^2(m^3+m^2)$	$2\sqrt{2}n(m+1)\lceil\frac{n}{n}\rceil\binom{n}{\lceil\frac{n}{n}\rceil}$	$2mn\sqrt{8m+2}\lceil\frac{n}{n}\rceil\binom{n}{\lceil\frac{n}{n}\rceil}$
$K_{n,n}$	$\sqrt{2}n^3$	$\sqrt{2}n^3(m\sqrt{2m^2+4m+4}+(m+1))$	$\sqrt{2}n^3(m^3+m^2)$	$2\sqrt{2}n^2(m+1)$	$2mn^2\sqrt{8m+2}$

4 Conclusion.

Recently, I. Gutman introduced a new degree-based topological index called the Sombor index. We have obtained the general formula for the Sombor index and the relation between the Sombor energy and energy of the m-splitting and m-shadow graphs of the k-regular graph.

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