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INTEGRAL FORMULAS RELATED TO THE CAUCHY-FANTAPPIÈ INTEGRAL FORMULA

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Abstract. This article is devoted to representations of the Cauchy-Fantappiè integral formula in some domains of the space $\mathbb{C}^n$, and studied one can be considered as a Bochner-Hua Lo-ken integral formula in matrix domains.

1 Introduction

In classical complex analysis, the significance of Cauchy's integral formula is well-known, which has the following remarkable properties: Firstly, it is true for any domain with a smooth or piecewise smooth boundary and does not depend on the type of domain (property of universality). Secondly, the kernel of this formula is holomorphic with respect to the outer variable.

In multidimensional complex analysis, there are numerous analogues of Cauchy's integral formula, but they do not simultaneously have these two properties. For example, Cauchy's multiple integral formulas for a polydisc, Leray's formula for ball, the Weil formula for polyhedra and the Bochner-Hua Lo-Ken integral formula for classical domains are with holomorphic kernels and are not universal, but the integral formula Martinelli-Bochner is a universal formula with a non-holomorphic kernel (see [1, 2, 3, 4]). In [6] the properties of the functions from H^p class are given in the polydisk, it is given by descriptions of traces for several concrete functional classes on polyballs defined with the help of Bergman metric ball. These results are new even in polydisk. In [8] the eigenfunctions and eigenvalues of the Bochner-Martinelli operator in a half-space are investigated. The work [5] is devoted to the regularity of the Cauchy-Fantappiè integral on strictly convex domains and the monograph [4] is devoted to integral representations of holomorphic several complex variable functions, such as integral formulas of Bochner-Martinelli, Cauchy-Fantappiè, Koppelman and multidimensional logarithmic residue, etc., and their boundary properties. The applications under consideration are problems of analytic continuation of functions from the boundary of a bounded domain in $\mathbb{C}^n$. The Cauchy-Fantappiè integral formula, which contains all the most commonly used integral formulas, depends on an unknown function, associated with the domain, i.e. this formula has an unknown kernel. We present the Cauchy-Fantappiè integral formula (see [1]).

Theorem 1.1. *For any domain $D \subset \mathbb{C}^n$ with piecewise smooth boundary for and any function $f(z) \in \mathcal{A}(D)$[1] holds*

$$f(z) = \frac{(n-1)!}{(2\pi i)^n} \int\limits_{\partial D} f(\zeta) \frac{\delta(\lambda(\zeta)) \wedge d\zeta}{\langle \zeta - z, \lambda(\zeta) \rangle^n}, \tag{1.1}$$

where $\lambda(\zeta)$ is an arbitrary smooth vector function on ∂D such that

[1]The function f belongs to the function space $\mathcal{A}(D)$, if f is holomorphic in D and continuous on the closure of $\overline{D}$, i. e. $f(z) \in \mathcal{O}(D) \cap C(\overline{D})$.

$$\langle \zeta - z, \lambda(\zeta)\rangle \neq 0 \ \textit{for all } z \in D \textit{ and } \zeta \in \partial D,\ d\zeta = d\zeta_1 \wedge d\zeta_2 \wedge \ldots \wedge d\zeta_n,$$

$$\zeta - z = (\zeta_1 - z_1, \zeta_2 - z_2, \ldots, \zeta_n - z_n), \delta(w) = \sum_{\nu=1}^{n} (-1)^{\nu-1} w_\nu dw[\nu]\,,$$

$$dw[\nu] = dw_1 \wedge dw_2 \wedge ... \wedge dw_{\nu-1} \wedge dw_{\nu+1} \wedge ... \wedge dw_n,\ \langle z, w\rangle = \sum_{\nu=1}^{n} z_\nu w_\nu.$$

2 Cauchy-Fantappiè integral formula in some domain

Despite the great generality of the Cauchy-Fantappiè formula, the question of finding integral representations with a holomorphic kernel for specific domains from $\mathbb{C}^n$ is not removed. In this article, the formula (1.1) integral formulas with holomorphic kernels are obtained for some domains from $\mathbb{C}^n$.

onsider the domain $D_1 \subset \mathbb{C}^n$ of the following type:

$$D_1 = \left\{ z \in \mathbb{C}^n : \sum_{\nu=1}^{n} |z_\nu|^2 < 1 \right\}.$$

For D_1 in formula (1.1) we choose the vector function $\lambda(\zeta)$ in the form

$$\lambda(\zeta) = \left(\frac{1}{\zeta_1}, \frac{1}{\zeta_2}, \ldots, \frac{1}{\zeta_n} \right), \zeta \in \partial D_1.$$

Then

$$\langle \zeta - z, \lambda(\zeta)\rangle = \left\langle (\zeta_1 - z_1, \zeta_2 - z_2, \ldots, \zeta_n - z_n), \left(\frac{1}{\zeta_1}, \ldots, \frac{1}{\zeta_n} \right) \right\rangle =$$

$$= n - \sum_{\nu=1}^{n} \frac{1}{\zeta_\nu} z_\nu \neq 0$$

for $z \in D_1$ and $\zeta \in \partial D_1$.

Further,

$$\delta(\lambda(\zeta)) \wedge d\zeta = \sum_{\nu=1}^{n} (-1)^{\nu-1} |\zeta_\nu|^2 d\bar{\zeta}[\nu] \wedge d\zeta.$$

Now formula (1.1) takes the form

$$f(z) = \frac{(n-1)!}{(2\pi i)^n} \int_{\partial D_1} f(\zeta) \frac{\sum_{\nu=1}^{n} (-1)^{\nu-1} |\zeta_\nu|^2 d\bar{\zeta}[\nu] \wedge d\zeta}{\left(n - \sum_{\nu=1}^{n} \frac{1}{\zeta_\nu} z_\nu \right)^n}. \tag{2.1}$$

b) We consider the domain

$$D_2 = \{ z \in \mathbb{C}^n : |z_1 z_2|^2 + |z_2 z_3|^2 + \ldots + |z_n z_1|^2 < 1 \}.$$

In this case, choosing

$$\lambda(\zeta) = \left(\bar{\zeta}_1 |\zeta_2|^2, \ldots, \bar{\zeta}_{n-1} |\zeta_n|^2, \bar{\zeta}_n |\zeta_1|^2 \right), \zeta \in \partial D_2,$$

we have

$$\langle \zeta - z, \lambda(\zeta)\rangle =$$

$$= \left\langle (\zeta_1 - z_1, \zeta_2 - z_2, \ldots \zeta_n - z_n), \left(\bar{\zeta}_1 |\zeta_2|^2, \ldots, \bar{\zeta}_{n-1} |\zeta_n|^2, \bar{\zeta}_n |\zeta_1|^2 \right) \right\rangle =$$

$$= \left(|\zeta_1 \zeta_2|^2 + |\zeta_2 \zeta_3|^2 + \ldots + |\zeta_n \zeta_1|^2 \right) - \left(\bar{\zeta}_1 |\zeta_2|^2 z_1 + \bar{\zeta}_2 |\zeta_3|^2 z_2 + \ldots + \bar{\zeta}_n |\zeta_1|^2 z_n \right) =$$

$$= 1 - \sum_{\nu=1}^{n} \bar{\zeta}_\nu |\zeta_{\nu+1}|^2 z_\nu \neq 0,$$

for $\zeta \in \partial D_2$ and $z \in D_2$ (where $\zeta_{n+1} = \zeta_1$).

Now, let's calculate

$$\delta(\lambda(\zeta)) \wedge d\zeta = \left(\sum_{\nu=1}^{n} (-1)^{\nu-1} \overline{\zeta}_\nu |\zeta_{\nu+1}|^2 \partial_{\overline{\zeta}_1} (\overline{\zeta}_1 |\zeta_2|^2) \wedge \ldots [\nu] \ldots \wedge \partial_{\overline{\zeta_n}} (\overline{\zeta}_n |\zeta_1|^2) \wedge d\zeta \right) =$$

$$= \prod_{k=1}^{n} |\zeta_k|^2 \sum_{\nu=1}^{n} (-1)^{\nu-1} \overline{\zeta}_\nu d\overline{\zeta}[\nu] \wedge d\zeta.$$

Therefore, formula (1.1) in this case has the form

$$f(z) = \frac{(n-1)!}{(2\pi i)^n} \int\limits_{\partial D_2} f(\zeta) \frac{\prod\limits_{k=1}^{n} |\zeta_k|^2 \sum\limits_{\nu=1}^{n} (-1)^{\nu-1} \overline{\zeta}_\nu d\overline{\zeta}[\nu] \wedge d\zeta}{\left(1 - \sum\limits_{\nu=1}^{n} \overline{\zeta}_\nu |\zeta_{\nu+1}|^2 z_\nu \right)^n}. \tag{2.2}$$

Note that for $n = 1$ formulas (2.1) and (2.2) coincide with the Cauchy's integral formula for the unit circle.

c) Let's consider a domain with a more difficult configuration:

$$D_3 = \{z \in \mathbb{C}^n : \alpha_1 |z_1 z_2|^{2\beta_1} + \alpha_2 |z_2 z_3|^{2\beta_2} + \ldots + \alpha_n |z_n z_1|^{2\beta_n} < 1\},$$

$\alpha_\nu > 0, \beta_\nu \geq 1, \nu = 1, \ldots, n.$

For this domain $\lambda(\zeta)$ can be taken as a vector - function of the following form:

$$\lambda(\zeta) = \left(\alpha_1 \zeta_1^{\beta_1 - 1} \overline{\zeta}_1^{\beta_1} |\zeta_2|^{2\beta_1}, \ldots, \alpha_\nu \zeta_\nu^{\beta_\nu - 1} \overline{\zeta}_\nu^{\beta_\nu} |\zeta_{\nu+1}|^{2\beta_\nu}, \ldots, \alpha_n \zeta_n^{\beta_n - 1} \overline{\zeta}_n^{\beta_n} |\zeta_1|^{2\beta_n} \right), \zeta \in \partial D_3.$$

In this case

$$\langle \zeta - z, \lambda(\zeta) \rangle = 1 - \sum_{\nu=1}^{n} \alpha_\nu \zeta_\nu^{\beta_\nu - 1} \overline{\zeta}_\nu^{\beta_\nu} |\zeta_{\nu+1}|^{2\beta_\nu} z_\nu \neq 0,$$

where $\zeta \in \partial D_3$ and $z \in D_3$ $(\zeta_{n+1} = \zeta_1)$.

The expression $\delta(\lambda(\zeta)) \wedge d\zeta$ has the form:

$$\delta(\lambda(\zeta)) \wedge d\zeta =$$

$$= \prod_{k=1}^{n} (\alpha_k \beta_k) |\zeta_1|^{2\beta_1} \cdots |\zeta_n|^{2\beta_n} \sum_{\nu=1}^{n} (-1)^{\nu-1} \frac{1}{\alpha_\nu \beta_\nu} |\zeta_\nu|^{2(\beta_\nu - 1)} \overline{\zeta}_\nu d\overline{\zeta}[\nu] \wedge d\zeta.$$

Therefore, formulas (1.1) gives the following integral representation:

$$f(z) = \frac{(n-1)!}{(2\pi i)^n} \int\limits_{\partial D_3} f(\zeta) \times$$

$$\times \frac{\prod\limits_{k=1}^{n} (\alpha_k \beta_k) |\zeta_1|^{2\beta_1} \cdots |\zeta_n|^{2\beta_n} \sum\limits_{\nu=1}^{n} (-1)^{\nu-1} \frac{1}{\alpha_\nu \beta_\nu} |\zeta_\nu|^{2(\beta_\nu - 1)} \overline{\zeta}_\nu d\overline{\zeta}[\nu] \wedge d\zeta}{\left(1 - \sum\limits_{\nu=1}^{n} \alpha_\nu \zeta_\nu^{\beta_\nu - 1} \overline{\zeta}_\nu^{\beta_\nu} |\zeta_{\nu+1}|^{2\beta_\nu} z_\nu \right)^n}. \tag{2.3}$$

It should be noted that for $\alpha_\nu = \beta_\nu = 1, \nu = 1, \ldots, n$, representation (2.2) follows from formula (2.3).

The Cauchy-Fantappiè representation has proven to be very useful and has many applications in multidimensional complex analysis. For example, receiving integral representations have holomorphic kernels by variable z, which makes it possible to uniformly approximate holomorphic functions in corresponding domains by polynomials.

3 Multiple integral Bochner-Hua Lo-ken formula as a generalized Cauchy-Fantappiè formula in matrix domains

The group of automorphisms can be used to find integral formulas for homogeneous domains. Domains with rich automorphism groups are often realize as matrix domains (see [13], [12]). These domains turned out to be useful in solving various problems in the theory of several complex variable functions.

Complex homogeneous bounded domains represent great interest from different points of view. This is explained by the fact that they are a relatively wide class of domains in $\mathbb{C}^n$, for which a number of meaningful, essentially multidimensional results have been obtained in [9].

In 1935 E.Cartan (see [10]) initiated a systematic study of homogeneous domains and found all bounded homogeneous domains in the space $\mathbb{C}^2$ and $\mathbb{C}^3$. It is shown that in the space $\mathbb{C}^2$ any bounded homogeneous domain can be biholomorphically mapped into the ball

$$\mathbb{B}^2(1) = \left\{ z \in \mathbb{C}^2 : |z_1|^2 + |z_2|^2 < 1 \right\},$$

or bicircle

$$\mathbb{U}^2 = \left\{ z \in \mathbb{C}^2 : |z_1| < 1, |z_2| < 1 \right\}.$$

There are some differences for the space $\mathbb{C}^3$, in this space any bounded homogeneous domain can be mapped biholomorphically into one of the following four domains:

1) the ball

$$\mathbb{B}^3(1) = \left\{ z \in \mathbb{C}^3 : |z_1|^2 + |z_2|^2 + |z_3|^2 < 1 \right\};$$

2) the domain

$$\mathbb{G} = \mathbb{B}^2(1) \times \mathbb{U}^1 = \left\{ z \in \mathbb{C}^3 : |z_1|^2 + |z_2|^2 < 1, |z_3| < 1 \right\};$$

3) the polydisc

$$\mathbb{U}^3 = \left\{ z \in \mathbb{C}^3 : |z_1| < 1, |z_2| < 1, |z_3| < 1 \right\};$$

4) a bounded domain, which is obtained by a biholomorphic mapping from the domain

$$\tau^+(2) = \left\{ z \in \mathbb{C}^3 : (\operatorname{Im} z_3)^2 > (\operatorname{Im} z_1)^2 + (\operatorname{Im} z_2)^2, \operatorname{Im} z_3 > 0 \right\}$$

to the future tube (see[11]).

In multidimensional complex analysis, E. Cartan [10] proposed a classification of all bounded symmetric domains. With respect to biholomorphic mappings, these bounded symmetric domains are divided into equivalence classes. Each such class can be specified by specifying one domain belonging to it. After this, it is obvious that it suffices to consider only irreducible classes, i.e., classes of domains that are inexpressible as products of bounded symmetric domains of lower dimensions. E. Cartan [10] established that there are six types of classes of irreducible bounded symmetric domains. Domains belonging to four of these types are called classical because their automorphism groups are classical semisimple Lie groups. Two of these types are special in the sense that each of them occurs in the space $\mathbb{C}^n$ of only one dimension n, respectively for $n = 16$ and $n = 27$.

Consider the classical domains (according to E. Cartan's classification) (see [9, 10]):

$$\Re_I(m,k) = \left\{ Z \in \mathbb{C}[m \times k] : I^{(m)} - Z\bar{Z}' > 0 \right\},$$

$$\Re_{II}(m) = \left\{ Z \in \mathbb{C}[m \times m] : I^{(m)} - Z\bar{Z} > 0, \forall Z' = Z \right\},$$

$$\Re_{III}(m) = \left\{ Z \in \mathbb{C}[m \times m] : I^{(m)} + Z\bar{Z} > 0, \forall Z' = -Z \right\},$$

$$\Re_{IV}(n) = \left\{ z \in \mathbb{C}^n : |\langle z, \bar{z} \rangle|^2 - 2|z|^2 + 1 > 0, |\langle z, \bar{z} \rangle| < 1 \right\},$$

where $I^{(m)}$ is the identity matrix of order m, $\bar{Z}'-$ is the complex conjugate matrix of the transposed matrix Z'. ($H > 0$ for a Hermitian matrix H means, as usual, that H is positive definite).

All these domains are homogeneous, symmetric, convex, complete, circular domains centered at O (O is zero matrix). All these domains are biholomorphically non-equivalent, so the complex analysis for them is constructed in different ways.

The E.Cartan domains $\Re_V$ and $\Re_{VI}$ in $\mathbb{C}^{16}$ and $\mathbb{C}^{27}$, respectively, are quite essential. The question of an efficient description of these two domains is still open.

In the theory of functions of a single complex variable, we often study the theory of functions in the unit circle $\mathbb{U} = \{z \in \mathbb{C} : |z| < 1\}$, since in the general case all symmetric domains are equivalent to the unit circle, therefore, the above four classes (symmetric classical domains) play an important role in multidimensional complex analysis. By explicitly writing out the transitive automorphism group of the four types of classical domains and matrix balls (see e.g. [14], [15]) associated with classical domains, one can find the Bergman and Cauchy-Szegő kernels for these domains by direct calculation. Then, using the properties of the Poisson kernel, we find a formula that restores the value of the holomorphic function in the domain itself from its values on some boundary sets of uniqueness. In [7] paper presents the functional properties of the Poisson integral on a Lie ball (a classical sphere of the fourth type). In this case, the scheme for finding the Bergman and Cauchy-Szegő kernels from [9] is used. In [18] the volumes of a matrix ball of the third type and a generalized Lie ball are calculated. To find the kernels of integral formulas for these domains (Bergman, Cauchy-Szegő, Poisson kernels, etc.), the total volumes of these domains are needed, and these volumes are also used for the integral representation of holomorphic functions in these domains, in the mean value theorem, and in other important concepts (see for exam. [16], [17]).

Now, let us consider the holomorphic continuation of the classical domain of the first type $\Re_I(m,k)$ and its skeleton $\mathbb{X}_I$. Consider the space $L^2(\mathbb{X}_I, d\mu)$, i.e., the space of square-integrable functions f, with respect to the normalized Lebesgue measure $d\mu$. It is the Haar measure on the skeleton $\mathbb{X}_I$, and hence is invariant under rotations. As is known, the Hardy class $H^2(\Re_I(m,k))$ consists of all functions f, that are holomorphic in the domain $\Re_I(m,k)$ for which

$$\|f\|_{H^2} = \sup_{0<r<1}\left(\int_{\mathbb{X}_I} |f(rZ)|^2 d\mu\right)^{\frac{1}{2}} < \infty.$$

Since $\Re_I(m,k)$ is a bounded complete circular domain, functions f of class $H^2(\Re_I(m,k))$ has the following properties (see [12], [19]):

1^0. The slice functions $f_Z(\lambda) = f(\lambda Z)$ (in measure μ) belong to the space H^2 in the unit circle $\Delta = \left\{\lambda \in \mathbb{C}^1 : |\lambda| < 1\right\}$, for almost all $Z \in \mathbb{X}_I$;

2^0. The function f has radial boundary values

$$\lim_{r\to 1-0} f(rZ) = f^*(Z), Z \in \mathbb{X}_I,$$

and these boundary values f^* belong to the class $L^2(\mathbb{X}_I, d\mu)$;

3^0. The following formula is valid

$$\lim_{r\to 1-0} \int_{\mathbb{X}_I} |f(rZ)|\, d\mu = \int_{\mathbb{X}_I} |f^*(Z)|\, d\mu;$$

4^0. If slice functions $f_Z(\lambda)$ of some function holomorphic in $\Re_I(m,k)$ the function f belong to the Hardy class H^2 in the unit circle for almost all $Z \in \mathbb{X}_I$ and radial boundary values f^* lie down in $L^2(\mathbb{X}_I, d\mu)$, then $f \in H^2(\Re_I(m,k))$;

5^0. Any function $f \in H^2(\Re_I(m,k))$ can be represented by the Bochner-Hua Lo-ken formula as

$$f(Z) = \int_{\mathbb{X}_I} \det{}^{-k}\left(I^{(k)} - \langle Z, U\rangle\right) f(U) d\mu, \tag{3.1}$$

the function $f \in H^2(\Re_I(m,k))$ is restored to $\Re_I(m,k)$ by their radial boundary values f^* .

6^0. If the set $V \subset \mathbb{X}_I$ has positive measure ($\mu(V) > 0$), then V is a set of uniqueness for the Hardy class $H^2(\Re_I(m,k))$;

7^0. The Hardy class $H^2(\Re_I(m,k))$ is invariant under automorphisms of the ball $\Re_I(m,k)$.

4 Conclusion remarks

The Bochner-Hua Lok-en integral formula (3.1) given above is a general case of the Cauchy-Fantappiè formula considered in the previous section. If $m = 1$, then $\mathbb{B}^k(1) = \{z \in \mathbb{C}^k : |z| < 1\}$ – represents the Cauchy-Fantappiè formula for the unit ball, that is, the kernel of formula (3.1) appears as

$$\det{}^k(I - \langle Z, U\rangle) = \{1 - \langle z, u\rangle\}^k.$$

Hence the required integral formula $\mathbb{B}^k(1)$ becomes the Cauchy-Fantappiè formula for the ball:

$$f(Z) = \int_{\mathbb{X}_I} \frac{f(U)}{\det^n(I - \langle Z, U\rangle)} d\mu(U) = \int_{\mathbb{S}^k(1)} \frac{f(u)}{\{1 - \langle z, u\rangle\}^n} d\mu(u) \ ,$$

where $z = (z_1, z_2, ..., z_k) \in \mathbb{B}^k(1)$ and $w = (w_1, w_2, ..., w_k) \in \mathbb{S}^k(1)$.

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