Palestine Journal of Mathematics

Vol 14(1)(2025) , 506–516

TACKLING UNCERTAINTY: DEFUZZIFICATION STRATEGIES FOR FACILITY LOCATION PROBLEMS

G. Kaur*, A. Majumder, P. Pandey and S. Mishra

Communicated by Greeta Arora

MSC 2010 Classifications: Primary 33C20; Secondary 33C65.

Keywords and phrases: Facility location problem, Fuzzy uncertainty, Triangular fuzzy number, Defuzzification.

The authors would like to thank the reviewers and editor for their constructive comments and valuable suggestions that improved the quality of our paper.

Abstract Investigating the unknowns in the dynamic field of global marketing, a unique study presents a novel, fuzzy, two-tiered model for facility placement. This novel model has a single product, one method of transportation, two suppliers, and two locations for manufacturing and warehousing. Because of space limits in factories and stockrooms, the model employs triangular fuzzy numbers to account for demand, total expense, stock level cost, capacity, and lead time, which are unpredictable. To find the best answer, we also look at other defuzzification methods, such as the fuzzy ranking approach, the fuzzy centroid method, and the fuzzy entropy method. To evaluate the effectiveness of any approach, we carefully examine the costs that arise from it. To verify the model's effectiveness, we provide a numerical example and comprehensively evaluate its results.

1 Introduction

One aspect of supply chain management (SCM) that focuses on locating factories, stockrooms, and other facilities in a way that minimizes total expenditures is known as the facility location problem (FLP) [1]. It is possible to classify this complex topic into two broad categories: capacitated and uncapacitated facility location problems. CFLP and UFLP deal with limited capabilities, whereas UFLP discusses unlimited capacities. Reducing inventory expenditures and other supply chain costs within the bounds of capacity, demand, and binary restrictions is the end aim of both kinds. In addition, operations research relies heavily on the capacitated facility location problem (CFLP), especially when formulating strategies for distribution networks and supply chain management as a whole [2]. Supply, demand, and transportation costs are assumed to be known exactly in traditional CFLP models. However, in actual situations, factors like market volatility, lack of complete data, and inaccurate predictions can make these parameters uncertain [3]. Incorporating fuzzy set theory into CFLP models provides individuals with a robust and adaptable strategy for handling uncertainty, enabling them to make more informed location decisions in unpredictable scenarios [4]. Under conditions of fuzzy uncertainty, this integration offers a comprehensive framework for resolving the capacitated facility location issue. It is a potential field for further study and practical application since it provides answers that may be effectively used in real-life scenarios [5, 6].

The various real-world applications of CFLP in areas including capacity, demand, cost, resource allocation, and time restrictions have constructed a rich history in the discipline. Several scholars have researched CFLP under controlled settings to boost efficiency and profitability [7]. However, significant unknowns exist in the contemporary world regarding storage capacity, pricing, and demand. Many people throughout the globe are interested in the CFLP model since specific parameters are widely unclear [8]. Probabilistic or fuzzy parametric uncertainties define the modern global corporate environment. Demand could be unpredictable due to market instability, seasonal changes, and customers' unpredictable preferences. Total costs could be hard to forecast due to fluctuations in transportation expenses, raw material expenses, and operational charges. Changes in production capacities, equipment breakdowns, and personnel availability issues are potential reasons for capacity uncertainty [9, 10, 11]. Unpredictable events, including bad weather or supplier issues, may add uncertainty to lead times and delay shipping, production, or procurement. Assuming randomness in parameters requires a large amount of data. Therefore, it has been more common to include uncertainty in demand, pricing, storage capacity, and decision factors [14].

Although fuzzy uncertainty in CFLP is gaining recognition, research on it is still in its early stages, in contrast to deterministic techniques. One needs an extensive understanding of the theory, methodology, and implications of fuzzy uncertainty-based CFLP models to fill this information gap. This work aspires to contribute to the field by illuminating current advancements, methodologies, and practical applications of the topic by analyzing the literature on the capacitated facility locating problem under fuzzy uncertainty.

Novelty and contribution

Incorporating uncertainty into Capacitated Facility Location Problem (CFLP) models enhances their reliability and trustworthiness, enabling individuals to make more informed judgments in the face of unpredictable and ever-changing circumstances. This study examines the two-echelon CFLP from two distinct perspectives: it uses a fuzzy objective function and fuzzy constraints. An innovative approach to incorporating fuzzy numbers into the components of a two-dimensional CFLP model makes this work stand out. Quantity, total cost, demand, stock levels, expenses, and time to delivery are all examples of these fuzzy numbers. It verifies that the model is flexible enough to respond to market conditions. However, under this model, transportation expenses remain stable. This research expands upon previous work by Majumder and Sarkar by demonstrating how parametric uncertainty may help businesses save costs while improving service allocation to satisfy customers' demands. Emphasis is placed on fuzzy parameters such as fuzzy cost, fuzzy demand, and fuzzy capacity to maximize profit [10]. To tackle the problems associated with CFLP, we use a fuzzy linear programming model that encapsulates the parameters discussed before in triangular fuzzy numbers (TFN). After that, we use some methods for defuzzification, including signed distance, centroid, and fuzzy entropy weighted. Ultimately, we get the most optimal solution by using fuzzy integer ranking to transform a fuzzy model into a precise form.

2 Related work

Facility location problem

Within the realm of FLP, specialists have performed research into a wide variety of innovative approaches to tackle the difficulty of fuzzy parameters, reduce expenses, and boost credibility. There has been much study into the best methods for finding FLP model optimum solutions. To lower the overall cost objective of a two-stage CFLP successfully, Yang et al. suggested a novel strategy that integrates three separate approaches [15]. Wang and Watada also looked at a similar idea that manipulates expenditure and demand [16]. Arana-Jimenez and Blanco created the fuzzy CFLP framework in TFN form to solve minimax multiple objective mixed integer programming problems [17]. In their research, Kratica et al. examined a multi-level UFLP model using mixed-integer programming. Using the CPLEx and Gurobi solvers, they were able to determine the best solution [18]. An FLP problem, including point and area destinations, was investigated by Taghi-Nezhad et al. In their study, they included customer desire as an unknown variable. Utilizing a fuzzy weighted average methodology and a fuzzy critical point method, the researchers then proposed a solution [19]. As an NP-hard problem, the placement of capacitated facilities was tackled by Sadat-Asi A.A. Using a hybrid meta-heuristic approach, the ambiguity around customer demand was eradicated [20].

Fuzzy uncertainty

The mathematical framework of fuzzy set theory, which Lotfi A. Zadeh first proposed during the 1960s, offers a distinctive approach to addressing uncertainty by permitting components to possess varying degrees of membership inside sets [12, 13]. Fuzzy set theory in the context of CFLP enables the depiction of uncertain factors such as demand levels, transportation costs, and facility capacity, effectively reflecting the inherent uncertainty associated with these variables. Fuzzy CFLP models can accommodate uncertainty via the use of fuzzy optimization approaches, hence offering decision-makers solutions that are more robust and flexible. Wen and Iwamura developed a facility location/allocation model in a situation characterized by unpredictable demand, described by the Hurwicz criterion. The solution proposed by this model encompasses three unique methodologies: a simplex algorithm, fuzzy stimulation, and a genetic algorithm [21]. Handfield et al. have shown that demand, lead-time, supplier yield, and penalty cost may represent a triangular fuzzy distribution to manage inventory model risk variables effectively [22]. According to Shavandi and Bozorgi, a genetic algorithm was used to address the issue of fuzzy demand in location-inventory or transportation models [23]. In their study, Phruksarphanrat and Tanthatemee introduced a lot-size order amount to solve uncertain market situations characterized by ambiguous demand and supply [24]. The facility placement model proposed by Wang and Watada integrates risks, fuzzy costs, and demand. This model was solved using a swarm Particle swarm optimization (PSO) optimizer [25].

Defuzzification

There has been consistent demand for numerous publications. Maintaining a consistent demand, on the other hand, is no longer sufficient in the present marketing scene. By including demand uncertainty and travel time into their fuzzy Economic Order Quantity (EOQ) model, Milenkovic and Bojovic improved the rail freight system using triangular fuzzy numbers. They defuzzified the model and determined the optimal total cost using the signed distance technique [26]. However, using triangular fuzzy numbers, Lin L. et al. looked into the issue of optimizing the quickest path. As a result of using the graded mean integration strategy, the algorithmic model was defuzzified. Furthermore, numerous domains and applications of SCM make heavy use of robust ranking, a fundamental defuzzification strategy. Similarly, to tackle the problem of fuzzy transportation and provide a first workable solution, Hunwisai and Kumar combined the Allocation Table Method (ATM) with a robust ranking approach [28]. While a transportation model included the fuzzy cost of transportation, supply, and consumption, Nagstiti et al. investigated the zero-suffix and zero-point techniques. They employed the robust ranking strategy to defuzzify the model. The subject of Singh A.P.'s research was a fuzzy assignment problem involving four people and four different professions. To choose which courses each department will offer the next semester, they employed the robust ranking method [30].

3 Preliminaries

The following definitions and terminologies are essential for defining the model and the defuzzification technique.

Definition 3.1. Fuzzy number [31, 32, 33]
When a fuzzy set with the symbol $\tilde{A}$ satisfies the following requirements, it is considered a *fuzzy number*:

- The fuzzy set $\tilde{A}$ must exhibit *convexity*.
- The fuzzy set $\tilde{A}$ needs to be a normalized fuzzy set.
- The membership function should display piecewise continuity. In this context, a fuzzy number is an extension of $\alpha - cut$ represented in an interval format, i.e., $\tilde{A}_\alpha = [\alpha_l, \alpha_r]$.

Definition 3.2. α**-Level set** [31, 37]
An α-Level set is characterized by the α-cut, denoted as $\tilde{A}_\alpha = \{\alpha | \mu_{\tilde{A}}(\alpha) \geq \alpha\}$, where α ranges arbitrarily within the interval $[0, 1]$.

Definition 3.3. Triangular Fuzzy Number (TFN) [31, 35]
A triangular fuzzy number is represented as $T = (t_1, t_2, t_3)$ through three distinct points. The

expression of its membership function is

$$\mu_{(T)}(x) = \begin{cases} 0, & x < t_1 \\ \frac{x-t_1}{t_2-t_1}, & t_1 \le x \le t_2 \\ \frac{t_3-x}{t_3-t_2}, & t_2 \le x \le t_3 \\ 0, & x > t_3; \textit{otherwise} \end{cases}$$

Definition 3.4. Robust ranking method (RRM) [36]
The Robust ranking method (RRM) transforms a Triangular Fuzzy Number, as per definition 3.3, into a crisp form by rank-ordering the fuzzy number using α-cut. The obtained crisp value through RRM is crucial for decision-making.

$$d(T, 0) = \frac{1}{2}\int_0^1 [T_l(\alpha), T_r(\alpha)]d\alpha.$$

Remark 3.5. If $[T_l(\alpha), T_r(\alpha)]$ is the α-level cut of TFN $T = (t_1, t_2, t_3)$, then by definition 3.4 of the robust ranking method, the crisp value is derived using the following formula [36]:

$$T_{RR} := \frac{1}{4}(t_1 + 2t_2 + t_3)$$

Definition 3.6. Center of area (COA) [34]
Within the centroid method, the crisp output for a TFN is determined as the Center of Area (COA) or centroid of the continuous membership function $\mu_{(T)}(x)$. The COA is computed using the formula:

$$T_{COA} = \frac{\int x * \mu_{(T)}(x)dx}{\int \mu_{(T)}(x)dx}$$

Here, T_{COA} symbolizes the crisp output, while x represents possible output values.

Remark 3.7. Utilizing definition 3.6 for a TFN $T = (t_1, t_2, t_3)$, the centroid of the fuzzy number is as follows [12]:

$$T_{COA} = \frac{t_1+t_2+t_3}{3}$$

Definition 3.8. Fuzzy entropy weighted method (FEWM) [35, 38, 39]
The inventive FEWM method integrates entropy to assign weights to the components of a fuzzy number, capturing the level of uncertainty within the group. The method aims to harmonize a wide range of information in the fuzzy set by maximizing entropy. The crisp value for the TFN $T = (t_1, t_2, t_3)$ is computed as follows:

$$T_{FEWM} = \frac{t_1 * log(t_1) + t_2 * log(t_2) + t_3 * log(t_3)}{log(t_1) + log(t_2) + log(t_3)}$$

4 Problem Definition

This study delves into a distinct logistical problem called the two-echelon capacitated facility location problem (TECFLP) inside a two-dimensional framework. The primary goal is to get products from the plant to the stockroom and the vendor. The structure consists of a delivery plan that is particular to the product, two suppliers, two stockrooms, two production locations, and a two-dimensional organization of the decision factors. Storage expenses, capacity needs, and plant and stockroom delivery schedules are all variables with some unpredictability. The capacity of the facilities is constant and unaffected by factors such as product type, transportation mode, or vendor location. Binary conditions, expressed as binary variables, determine the placement of facilities at specific locations. Despite focusing on only one kind of product, the kind of product affects vendor needs in this case. To find the optimal TECFLP solution, we first display the unknown parameters in TFN format and use several methods to optimize these parameters

Notations:

Table 1 lists the notations employed to create the two-dimensional fuzzy TECFLP model. The variables in this table are continuous and binary, containing two parameters: fuzzy and fixed.

Table 1: Notations used to formulate the TECFLP model

Continuous variables	
x_{vw}	A portion of goods drop-shipped in relation to $\widetilde{D_v}$;
y_{wf}	A portion of goods drop-shipped in relation to $\widetilde{SW_w}$;
s_v	A portion of goods supplied to vendor v from an external distributor in relation to $\widetilde{D_v}$;
IW_w	Stock levels of goods at stockroom w;
JF_f	Stock levels of goods at the factory f;
Binary variables	
WO_w	1, in case stockroom is open w; or else 0;
FO_f	1, in case factory is open f; or else 0;
Fixed parameters	
v	Count of vendors $v \in V$;
w	Count of stockrooms $w \in W$;
f	Count of factory locations $f \in F$;
SC_{vw}	Expense for delivering each item of goods from stockroom w to vendor v;
TCE_v	Expense for delivering each item of goods to vendor v from an external distributor;
PSC_{wf}	Manufacturing and delivering expense of goods per unit from factory f to stockroom w;
LT	The economic value of a lead time frame;
Fuzzy parameters	
$\widetilde{CWO_w}$	Overall expense of stockroom w open;
$\widetilde{CFO_f}$	Overall expense of factory f open;
$\widetilde{UHW_w}$	Stock levels expense of goods at stockroom w;
$\widetilde{UHF_f}$	Stock levels expense of goods at factory f;
$\widetilde{SW_w}$	Capacity of stockroom w;
$\widetilde{SF_f}$	Capacity of factory f;
$\widetilde{D_v}$	Demand of vendor v;
$\widetilde{LWV_{vw}}$	The time range of delivering goods per unit from stockroom w to vendor v;
$\widetilde{LFW_{wf}}$	The time range of delivering goods per unit from factory f to stockroom w;

Assumptions:

The mathematical formulation relies on a set of assumptions, encompassing the following conditions:

(i) Fuzzy parameters effectively handle the vague nature of TECFLP.

(ii) The movement of items within the supply chain is assumed to be uniform.

(iii) Limited-capacity factories and stockrooms necessitate the need for openness.

(iv) Meeting each customer's requirements is essential. Hence, external distributors ensure customer satisfaction. The delivery lead time is a crucial factor in the process.

5 Fuzzy TECFLP model in mathematical terms

The model for fuzzy TECFLP is constructed in two dimensions, aligning with the assumptions mentioned earlier, notations, and definitions. Subject to the model's constraints, the objective function incorporates various fuzzy variable parameters.

Objective function:

$$
\begin{aligned}
MinF \; = \; & \sum_{v\in V}\sum_{w\in W} SC_{vw}\widetilde{D_v}x_{vw} + \sum_{f\in F}\sum_{w\in W} PSC_{wf}\widetilde{SW_w}y_{wf} + \sum_{v\in V}\widetilde{D_v}TCE_v s_v + \sum_{w\in W}\widetilde{UHW_w}IW_w \\
+ & \sum_{f\in F}\widetilde{UHF_f}JF_f + \sum_{w\in W}\widetilde{CWO_w}WO_w + \sum_{f\in F}\widetilde{CFO_f}FO_f + \sum_{v\in V}\sum_{w\in W} LT\widetilde{D_v}\widetilde{LWV_{vw}}x_{vw} \\
+ & \sum_{w\in W}\sum_{f\in F} LT\widetilde{SW_w}\widetilde{LFW_{wf}}y_{wf} \qquad (5.1)
\end{aligned}
$$

Equation (5.1) represents the goal function of the TECFLP model. The first term in this equation denotes the variable cost associated with delivering goods from stockrooms to vendors to satisfy the vendor's demand. The second component of the equation represents the variable cost associated with the manufacturing and delivery of individual units of commodities from the factory to the stockroom. As shown in the third term, an external distributor bears the variable cost of shipping each unit of goods to the vendor. The fourth and fifth terms denote the variable costs related to stock-level expenditures at the stockroom and factories. Conversely, the sixth and seventh terms represent the variable expenses associated with setting up and running stockrooms and factories at a particular site. The ninth and tenth terms explain the variable costs related to the delivery time frame from the stockroom to the vendor and from the manufacturer to the stockroom. Sustaining the following restrictions is required to achieve the goal of this function:

Subject to constraints:

$$\sum_{w\in W} x_{vw} \geq 1 \qquad (5.2)$$

$$\sum_{v\in V} s_v \geq 1 \qquad (5.3)$$

$$\sum_{f\in F} y_{wf} \geq 1 \qquad (5.4)$$

$$\sum_{w\in W} \widetilde{SW_w}y_{wf} + JF_f \leq \widetilde{SF_f}FO_f \qquad (5.5)$$

$$\sum_{v\in V} \widetilde{D_v}x_{vw} + IW_w \leq \widetilde{SW_w}WO_w \qquad (5.6)$$

$$WO_w, FO_f \in \{0,1\}\forall w \in W, f \in F \qquad (5.7)$$

$$0 \leq x_{vw}, y_{wf}, s_v \leq 1 \qquad (5.8)$$

The constraints in the model ensure that each customer's demand, as well as each retailer's demand, is met. These constraints are demand constraints and capacity constraints. The demand constraints (5.2) and (5.3) guarantee the fulfillment of customer and retailer demand, while capacity constraints (5.5) and (5.6) ensure not to exceed the capacity of stockroom and plant. Furthermore, constraint (5.4) provides that at least one plant is open to supply goods to stockrooms, and constraints set (5.7) specify that closed sites do not have facilities. The decision variable is either '1' for an open facility or '0' for a closed one, as defined by constraint (5.8).

6 Solution methodology

The TECFLP model turns fuzzy parameters into a crisp shape to reduce costs. By considering two vendors, two stockrooms, and two factory locations, the model is expanded, denoted as 1,2, to demonstrate its validity.

Objective function:

$$
\begin{aligned}
MinF \;=\; & [SC_{11}\widetilde{D}_1x_{11} + SC_{21}\widetilde{D}_2x_{21} + SC_{12}\widetilde{D}_1x_{12} + SC_{22}\widetilde{D}_2x_{22}] + [PSC_{11}\widetilde{SW}_1y_{11} + PSC_{12}\widetilde{SW}_1y_{12} \\
+ \; & PSC_{21}\widetilde{SW}_2y_{21} + PSC_{22}\widetilde{SW}_2y_{22}] + \widetilde{D}_1TCE_1s_1 + \widetilde{D}_2TCE_2s_2] + [\widetilde{UHW}_1IW_1 + \widetilde{UHW}_2IW_2] \\
+ \; & [\widetilde{UHF}_1JF_1 + \widetilde{UHF}_2JF_2] + [\widetilde{CWO}_1WO_1 + \widetilde{CWO}_2WO_2] + [\widetilde{CFO}_1FO_1 + \widetilde{CFO}_2FO_2] \\
+ \; & [LT\widetilde{D}_1\widetilde{LWV}_{11}x_{11} + LT\widetilde{D}_2\widetilde{LWV}_{21}x_{21} + LT\widetilde{D}_1\widetilde{LWV}_{12}x_{12} + LT\tilde{D}_2\widetilde{LWV}_{22}x_{22}] \\
+ \; & [LT\widetilde{SW}_1\widetilde{LFW}_{11}y_{11} + LT\widetilde{SW}_2\widetilde{LFW}_{21}y_{21} + LT\widetilde{SW}_1\widetilde{LFW}_{12}y_{12} + LT\widetilde{SW}_2\widetilde{LFW}_{22}y_{22}] \qquad (6.1)
\end{aligned}
$$

Subject to constraints:

$$x_{11} + x_{12} \geq 1, x_{12} + x_{22} \geq 1, s_1 + s_2 \geq 1, y_{11} + y_{12} \geq 1, y_{21} + y_{22} \geq 1 \qquad (6.2)$$

$$\widetilde{SW}_1y_{11} + \widetilde{SW}_2y_{21} + JF_1 \leq \widetilde{SF}_1FO_1 \qquad (6.3)$$

$$\widetilde{SW}_1y_{12} + \widetilde{SW}_2y_{22} + JF_2 \leq \widetilde{SF}_2FO_2 \qquad (6.4)$$

$$\widetilde{D}_1x_{11} + \widetilde{D}_2x_{21} + IW_1 \leq \widetilde{SW}_1WO_1 \qquad (6.5)$$

$$\widetilde{D}_1x_{12} + \widetilde{D}_2x_{22} + IW_2 \leq \widetilde{SW}_2WO_2 \qquad (6.6)$$

$$WO_1, WO_2, FO_1, FO_2 \in \{0, 1\} \forall w \in W, f \in F \qquad (6.7)$$

$$0 \leq x_{11}, x_{12}, x_{21}, x_{22}, y_{11}, y_{12}, y_{21}, y_{22}, s_1, s_2 \leq 1 \qquad (6.8)$$

Process of fuzzification

This specific model harbors inherent uncertainty within its parameters. In order to address the imprecise nature of these parameters, the process of fuzzification is implemented. This procedure entails converting precise inputs into a language that the system can effectively comprehend, particularly in relation to degrees of membership within fuzzy sets. Consequently, the parameters of the TECFLP model are conceptualized as fuzzy numbers, referred to as TFN (see Definition (3.3)), to effectively manage uncertainty by utilizing the adaptable membership function they offer. Within this framework, the objective function (6.1) and constraints (6.3), (6.4), (6.5), and (6.6) are also subjected to fuzzification.

Process of defuzzification

To minimize the objective function and determine the optimal values for the decision variables x_{vw}, y_{wf}, s_v, IW_w, JF_f, WO_w, and FO_f, the process of defuzzification is initiated to manage the fuzzy objective function under fuzzy constraints in TFN format. At this stage, we transform the fuzzy model into a definitive output, which is easy to apply in practical scenarios. The fuzzy output establishes the most probable value. Furthermore, the academic literature presents various methodologies for converting fuzzy numbers into precise values. This study employs the RRM, COA, and FEWM techniques for defuzzification, resulting in an efficient resolution to the issue.

7 Numerical experiment and Result discussion

Numerical experiment

Within this research, the input parameters such as overall expenses, stock level expenses, capacity, demand, and delivery timeframes of stockrooms and factories are represented as fuzzy numbers within the model due to their inherent vagueness. The parameter values listed in Table 2 and Table 3 act as the initial data for achieving the optimal solution. This article utilized empirical data to construct a practical and effective service-providing scenario [**?**]. The fuzzy values were initially converted into crisp form using the defuzzification techniques outlined in Section 3 to determine the optimal solution for these parameters. On an Intel Core i3-8145U CPU, This study applied MATLAB 2016a and LINGO 19.0 to optimize the solution. The results were then compared to other defuzzification methods. To validate the results, we compared outcomes from various defuzzification techniques, considering the computational time required by each method to identify the most efficient approach.

Parametric values for the fuzzy parameters

Presented below are the input parametric values of the fuzzy parameters in TFN format. Various defuzzification approaches are employed to convert the TFN values into crisp form, aiding in obtaining the precise solution for the problem.

Table 2: Input Parametric values the in TFN form

$\widetilde{LWV_{11}} = (15, 16, 17)$	$\widetilde{LWV_{12}} = (18, 19, 20)$	$\widetilde{LWV_{21}} = (17, 19, 21)$	$\widetilde{LWV_{22}} = (19, 22, 24)$
$\widetilde{LFW_{11}} = (12, 14, 16)$	$\widetilde{LFW_{12}} = (13, 14, 15)$	$\widetilde{LFW_{21}} = (16, 18, 20)$	$\widetilde{LFW_{22}} = (19, 20, 22)$
$\widetilde{CWO_1} = (65, 67, 69)$	$\widetilde{CWO_2} = (70, 72, 74)$	$\widetilde{CFO_1} = (60, 64, 68)$	$\widetilde{CFO_2} = (67, 70, 75)$
$\widetilde{UHW_1} = (40, 42, 44)$	$\widetilde{UHW_2} = (43, 44, 45)$	$\widetilde{UHF_1} = (45, 47, 59)$	$\widetilde{UHF_2} = (48, 50, 52)$
$\widetilde{D_1} = (35, 40, 45)$	$\widetilde{D_2} = (45, 47, 49)$	$\widetilde{SW_1} = (60, 63, 66)$	$\widetilde{SW_2} = (65, 67, 70)$
$\widetilde{SF_1} = (72, 73, 74)$	$\widetilde{SF_2} = (75, 77, 80)$		

Parametric values for the fixed parameters

The parameters related to transportation costs are treated as fixed in the current problem. Here are the input parametric values of the fixed parameters in the form of TFN.

Table 3: Input Parametric values in fixed form

$SC_{11} = \$90$	$SC_{12} = \$100$	$SC_{21} = \$120$	$SC_{22} = \$80$	$PSC_{11} = \$81$	$PSC_{12} = \$97$
$PSC_{21} = \$110$	$PSC_{22} = \$77$	$TCE_1 = \$82$	$TCE_2 = \$90$	$LT = \$10$	

Result analysis

The RRM method provides the best solution for the objective function and decision variables in a problem, to minimize costs and improve decisions. This unique strategy is known for its ability to decrease computational time compared to other commonly used defuzzification techniques. Interestingly, while the results for decision variables are consistent across these methods, there is a significant difference in the minimal cost of the TECFLP objective function. Table 4 depicts the outcomes related to decision variables in the TECFLP model. To simplify and avoid confusion, we represent different facility and vendor sites with the simple names 1 and 2, signifying the movement of goods between locations such as 1 to 1, 1 to 2, 2 to 1, and 2 to 2. This naming system corresponds to the use of two vendors (V = 1,2), two stockrooms (W = 1,2), and two factory locations (F = 1,2) in this extensive investigation.

Table 4: Optimistic values of the decision variables and objective function:

Continuous variables	**Binary variables**
$x_{11} = 1.000$	$WO_1 = 1.000$
$x_{22} = 1.000$	$WO_2 = 1.000$
$y_{11} = 1.000$	$FO_1 = 1.000$
$y_{22} = 1.000$	$FO_2 = 1.000$
$s_1 = 1.000$	

When items are moved from location 1 to location 1 and from location 2 to position 2, i.e., within the same area, the problem finds its most cost-effective solution, as shown in Table 4. For example, we may contrast the movement of products to transfers between any two states within a country if we use location 1 to represent that nation, which implies a concentrated distribution and movement optimization pattern within a limited geographic region, resulting in efficient and economical logistical operations. Table 5 depicts a comparison of the outcomes obtained from a variety of defuzzification methods. It becomes apparent that the most efficient cost for the TECFLP achieved utilizing the RRM approach is \$59067.50. The amount of computation needed for this specific method is significantly lower when compared to others, standing at a mere 0.10 seconds. After this, the centroid technique generates a minimal cost of \$59086.77, slightly surpassing that obtained through RRM, with a computational time of 0.012 seconds.

Table 5: Optimistic cost for objective function:

Defuzzification technique	Minimal Cost	CPU Time
COA	\$ 59086.77	0.12 seconds
RRM	**\$ 59067.50**	**0.10 seconds**
FEWM	\$ 51941.20	0.15seconds
Total variables = 18		
Total constraints = 30		

Nonetheless, the FEWM defuzzification method requires more CPU time than the others, amounting to 0.15 seconds, while presenting a minimal cost value of \$51941.20 for the model. Allocation methods yield results within four iterations, functioning with 18 decision variables and 30 constraints combined.

8 Conclusion

The conclusive remarks and the future extensions of this study are summarized in the following points:

(i) This article explores the intricate matter of the inherent unpredictability found in different facets of the TECFLP model, including demand, overall expense, stock levels expense, capacity, and lead time. The model encompasses two factory locations, two stockrooms, two suppliers, and a solitary external distributor, all operating under a single transportation mode to handle a singular product.

(ii) The primary objective of this model is to minimize overall costs and strategically optimize the allocation of services provided by an organization to fulfill customer demands at the most economical cost possible. The focus is fuzzy costs, demand, and capacity to maximize profits.

(iii) The approaches such as the ranking method RRM, the centroid method, and the entropy-based FEWM method, using the TFN approach, are used to handle the fuzziness of the model. Next, the expenses associated with each approach are analyzed to assess the efficiency of the defuzzification methods.

(iv) Upon examination of the results in Table 5, it is evident that ranking the fuzzy numbers in a triangular format yields superior outcomes for the objective function, specifically achieving a cost of \$59067.50, surpassing other approaches. Furthermore, the execution time required for this technique within the RRM method is notably shorter than alternative methods.

Future directions

(i) As a direction for future research, there is potential to enhance this model by transforming it into a multi-echelon CFLP configuration while expanding its dimensions up to n, with n representing any positive integer.

(ii) Additionally, a valuable avenue for further exploration would involve incorporating a multi-facility locations-retailers-vendors approach, thus capturing a more comprehensive perspective of the dynamics within the supply chain.

(iii) By incorporating these enhancements, the model elucidated in this study is poised to attain a heightened level of sophistication and relevance, paving the way for more comprehensive insights into supply chain operations and optimization.

References

[1] Martin, X.A., Panadero, J., Peidro, D., Perez-Bernabeu, E. and Juan, A.A., *Solving the time capacitated arc routing problem under fuzzy and stochastic travel and service times*, Networks, **82(4)**, 318-335(2023).

[2] Wang, X., Zhao, J., Cheng, C. and Qi, M., *A multi-objective fuzzy facility location problem with congestion and priority for drone-based emergency deliveries*, Computers & Industrial Engineering, **179**, 109167 (2023).

[3] Hajghani, M., Forghani, M.A., Heidari, A., Khalilzadeh, M. and Kebriyaii, O., *A two-echelon location routing problem considering sustainability and hybrid open and closed routes under uncertainty*, Heliyon, **9(3)**, (2023).

[4] Tai, P.D., Krit J. and Jirachai B., *A fuzzy multi-objective linear programming approach for green multi-echelon distribution network design*, Journal of Modelling in Management, **19(3)**, 1024-1055 (2024).

[5] Rizqi, Z.U., *Capacitated Continuous Review Inventory with Partial Backorder Under Time-Dependent Demand and Fuzzy Supply Bi-Objective Optimization via Simulation Model*, SSRN, **4663662** (2023).

[6] Yadegari, Z., Molana, S.M.H., Kashan, A.H. and Najafi, S.E., *Designing a Fuzzy Mathematical Model for a Two-Echelon Allocation-Routing Problem by Applying Route Conditions A New Interactive Fuzzy Approach*, Decision-Making Applications in Management and Engineering, **7(2)**, 172-196 (2024).

[7] Rezakhanlou, M. and Mirzapour Al-e-Hashem, S.M.J., *A dual-channel multi-objective green supply chain network design considering pricing and transportation mode choice under fuzzy uncertainty*, Environment, Development and Sustainability, **1-32** (2024).

[8] Wang, Y., Zhu, T., Yuan, K. and Li, X., *A fuzzy interval optimization approach for the p-hub median problem under uncertain information*, Plos one, **19(3)**, e0297295 (2024).

[9] Lin, P.C., Watada, J. and Wu, B., *A parametric assessment approach to solving facility-location problems with fuzzy demands*, IEEJ Transactions on Electrical and Electronic Engineering, **9(5)**,484-493 (2014).

[10] Sarkar, B. and Arunava M., *A study on three different dimensional facility location problems*, Economic Modelling, **30**, 879-887 (2013).

[11] Sarkar, B., Muhammad O. and Namhun K., *A cooperative advertising collaboration policy in supply chain management under uncertain conditions*, Applied Soft Computing, **88**, 105948 (2020).

[12] Zadeh, L.A., *Fuzzy sets*, Information and control, **8(3)**, 338-353 (1965).

[13] Tanaka, K. and Werners, B., *An introduction to fuzzy logic for practical applications*, New York: Springer **1-6**, (1997).

[14] Asefi, H., Shahrooz S. and Prem C., *Integrated Municipal Solid Waste Management under uncertainty A tri-echelon city logistics and transportation context*, Sustainable Cities and Society, **50**, 101606 (2019).

[15] Yang, Z., Chen, H., Chu, F. and Wang, N., *a practical hybrid approach to the two-stage capacitated facility location problem*, European Journal of Operational Research, **275(2)**, 467-480 (2019).

[16] Wang, S. and Junzo W., *Capacitated two-stage facility location problem with fuzzy costs and demands*, International Journal of Machine Learning and Cybernetics, **4**, 65-74 (2013).

[17] Arana-Jiménez, M. and Víctor B., *On a fully fuzzy framework for minimax mixed integer linear programming*, Computers & Industrial Engineering, **128**, 170-179 (2019).

[18] Kratica, J., Dugošija, D. and Savić, A., *A new mixed integer linear programming model for the multi-level uncapacitated facility location problem*, Applied Mathematical Modelling, **38(7-8)**, 2118-2129 (2014).

[19] Taghi-Nezhad, N.A., Moradi, S. and Karamali, G., *Fuzzy facility location problem with point and rectangular destinations*, International Journal of Mathematics in Operational Research, **18(1)**, 21-44 (2021).

[20] Sadat Asl, A.A., Fazel Zarandi, M.H., Sotudian, S. and Amini, A., *A fuzzy capacitated facility location-network design model A hybrid firefly and invasive weed optimization (FIWO) solution*, Iranian Journal of Fuzzy Systems, **17(2)**, 79-95 (2020).

[21] Wen, M. and Kakuzo I., *Fuzzy facility location-allocation problem under the Hurwicz criterion*, European journal of operational research, **184(2)**, 627-635 (2008).

[22] Handfield, R., Warsing, D. and Wu, X., *(Q, r) Inventory policies in a fuzzy uncertain supply chain environment*, European Journal of Operational Research, **197(2)**, 609-619 (2009).

[23] Shavandi, H. and Bita B., *Developing a location–inventory model under fuzzy environment*, The International Journal of Advanced Manufacturing Technology, **63**, 191-200 (2012).

[24] Phruksarphanrat, B. and Thipbodee T., *Fuzzy logic approach to inventory lot-sizing problem under uncertain environment*, IAENG Transactions on Engineering Technologies Springer Netherlands, (2013).

[25] Wang, S. and Watada, J., *A hybrid modified PSO approach to VaR-based facility location problems with variable capacity in fuzzy random uncertainty*, Information Sciences, **192**, 3-18 (2012).

[26] Milenkovic, M. and Bojovic, N., *Fuzzy modeling approach to the rail freight car inventory problem*, Transportation Planning and Technology, **37(2)**, 119-137 (2014).

[27] Lin, L., Wu, C. and Ma, L., *A genetic algorithm for the fuzzy shortest path problem in a fuzzy network*, Complex & Intelligent Systems, **7**, 225-234 (2021).

[28] Hunwisai, D. and Kumam, P., *A method for solving a fuzzy transportation problem via Robust ranking technique and ATM*, COGENT mathematics, **4(1)**, 1283730 (2017).

[29] Ngastiti, P.T.B., Surarso, B. and Sutimin, *Zero point and zero suffix methods with robust ranking for solving fully fuzzy transportation problems*, Journal of Physics Conference Series, **1022**, (2018).

[30] Singh, A. P., *Allocation of subjects in an educational institution by robust ranking method*, Arya Bhatta Journal of Mathematics and Informatics, **13(2)**, 247-254 (2021).

[31] Kacher, Y., & Singh, P., *A generalized parametric approach for solving different fuzzy parameter based multi-objective transportation problem*. Soft Computing, **28(4)**, 3187-3206 (2024).

[32] Ahmed, S. A. M., M. A. Talib, N. F. M. Noor, and R. Jani., *A fuzzy data envelopment analysis model for measuring the efficiency of Malaysian public research universities.* Palestine Journal of Mathematics, **11**, 109-124, (2022).

[33] Adak, Amal Kumar, and Dheeraj Kumar., *Some properties of pythagorean fuzzy ideals of γ-near-rings.* Palestine journal of mathematics, **11(4)**, 336-346, (2022).

[34] Yager, R.R., *A procedure for ordering fuzzy subsets of the unit interval*, Information sciences, **24(2)**, 143-161 (1981).

[35] Bandemer, H., *Fuzzy Analysis of Fuzzy Data*, Fuzzy Logic State of the Art. Dordrecht Springer Netherlands, 385-394, (1993).

[36] Kaur, G., Majumder, A. and Pandey, P., *A robust's ranking index to capacitated facility location problem with triangular fuzzy uncertainty in demand, cost, lead time, and capacity*, AIP Conference Proceedings, **2986(1)**, (2024).

[37] Jha, Navnit., *Trigonometry basis approximated fuzzy components and high-resolution scheme for two-point boundary value problems.* Palestine Journal of Mathematics, **12**, 21-33, (2023).

[38] Hamidi, Mohammad., *Graph Based on Linear Inequalities and Uncertain System With Applications.* Palestine Journal of Mathematics, **12(3)**, 395–407 (2023).

[39] Es-saiydy, Mohssine, Mohammed Zarhouni, and Mohamed Zitane., *Oscillation of a Delayed Quaternion-Valued Fuzzy Recurrent Neural Networks on Time Scales.* Palestine Journal of Mathematics, **11(4)**, 99-118, (2022).

Author information

G. Kaur*, Department of Mathematics, Lovely Professional University, Phagwara (Punjab)-144411, India.
E-mail: `gurpreetramgaria12@gmail.com (*Corresponding author)`

A. Majumder, Department of Mathematics, Lovely Professional University, Phagwara (Punjab)-144411, India.
E-mail: `am.arunavamajumder@gmail.com`

P. Pandey, Department of Mathematics, Lovely Professional University, Phagwara (Punjab)-144411, India.
E-mail: `pankaj.anvarat@gmail.com`

S. Mishra, Department of Mathematics, Lovely Professional University, Phagwara (Punjab)-144411, India.
E-mail: `mishrasam73@gmail.com`

Received: 2024-02-24
Accepted: 2024-05-22