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A Solution of Mathematical Multi-Objective Transportation Problems under Uncertain Environments

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Abstract Transportation systems are fundamental to global supply chains, yet they often face uncertainties arising from fluctuating costs, varying supply-demand dynamics, and unforeseen disruptions. Addressing these challenges, in this research, we aim to propose a mathematical model to unfold traditional transportation problems (TTP) by utilizing fermate fuzzy parameters (FFP). We embody the mathematical model of TTP into a crisp form by employing a new fermatean fuzzy score function (NFFSF) within the fermatean fuzzy environment (FFE). This research study extends and develops a mathematical model of multi-objective transportation problems (MOTP). We then transformed it into crisp form using NFFSF in the same environment. Our mathematical model of MOTP minimizes three substantial objectives: total transportation cost, total transportation time, and deterioration cost during transportation. The findings of the present work spur the parameters involved in the mathematical models, including objective costs, supply, and demand, which are considered FFP. We apply the Fermatean fuzzy programming approach (FFPA) and the neutrosophic goal programming approach (NGPA) to obtain the best compromise solutions to the proposed problem, utilizing solution method-based strategies. This study also provided a simulated numerical example to demonstrate the effectiveness and practicality of the proposed paradigm. The findings offer valuable managerial insights, providing a structured framework for policymakers and industry leaders to enhance transportation efficiency in complex, uncertain environments.

1 Introduction

The transportation problem (TP) is a classic optimization problem in operations research and logistics [1, 2, 3]. It involves determining the most economical method [4] to transport products from suppliers (origines) to destinations (receivers or demand points) while satisfying supply and demand constraints [89]. Efficient resolution of transportation problems is critical for improving business operational effectiveness and profitability and optimizing public services and infrastructure. The MOTP refers to a variant of the TP where multiple conflicting objectives must be optimized simultaneously [5, 6, 7].

Transportation is vital in modern supply chain management, impacting economic growth, environmental sustainability, and overall efficiency [38]. However, real-world transportation systems are often subject to uncertainties arising from fluctuating demand, varying supply levels, unpredictable transportation costs, and deterioration during transit—traditional mathematical models for MOTP struggle to accommodate these uncertainties effectively, leading to suboptimal decision-making[2, 61]. Researchers have increasingly used fuzzy logic and uncertainty-based optimization approaches to address these challenges. Among these, FFS offers a superior method for handling uncertainty due to its higher flexibility in modeling membership and non-membership functions. Moreover, the study explores the potential role of the Metaverse in sustainable transport planning, emphasizing how virtual simulations and digital twins can assist in optimizing transportation strategies before real-world implementation. By integrating artificial intelligence (AI) and data analytics, policymakers can use the Metaverse to evaluate different transportation scenarios, improve decision-making, and reduce the environmental impact of logistics operations[16].

In recent years, the emergence of the Metaverse has introduced new possibilities for sustainable transport planning by leveraging virtual simulations and digital twins. The Metaverse enables urban planners and policymakers to model, analyze, and optimize transportation networks in a risk-free virtual environment before implementing them in the real world. By integrating artificial intelligence, big data analytics, and immersive virtual reality (VR) simulations, transportation planners can evaluate various scenarios, assess traffic patterns, and identify the most efficient and sustainable solutions[16]. Additionally, the Metaverse promotes remote work and virtual tourism, reducing the need for physical travel and minimizing carbon footprints. This innovative approach aligns with sustainable transportation goals by fostering efficient mobility solutions while mitigating environmental impacts in uncertain and dynamic environments.

In contrast to the traditional transportation problem, which focuses on minimizing transportation costs or maximizing efficiency [8]. MOTP considers additional objectives such as minimizing transportation time, deterioration cost during transportation, minimization of inventory levels, maximization of customer satisfaction, carbon emissions, minimization of the number of vehicles used, maximizing resource utilization and minimizing congestion [9, 10, 11]. The solid transportation problem (STP) is an extension of the transportation problem that considers additional constraints related to the transportation capacity of vehicles or modes of transport [3, 14]. In STP, vehicle capacity is limited by weight and other physical characteristics, such as space occupancy [15, 16]. This additional constraint introduces complexities in the optimization process, as it requires considering the number of goods to be transported and their physical properties and compatibility with the vehicles [17, 18, 19]. Various methods, e.g., can solve TP, the North-West Corner Method, the Least Cost Method, Vogel's Approximation Method, and Heuristic and metaheuristic Algorithms (Genetic Algorithms, NSGA-2 3).

In the same manner, the MOTP can be solved by various types of methodologies, e.g., Chance constraint programming [20, 21], fuzzy programming [22, 23], fuzzy goal programming [24, 25], multi-criteria decision-making analysis [26], and interactive evolutionary algorithms [27, 28]. This research study explores solution approaches for mathematical multi-objective transportation problems under uncertain environments. By addressing the complexity and uncertainty inherent in these problems, we aim to contribute to developing more effective, efficient, and sustainable transportation solutions. Traditional transportation problem models often fail to account for the uncertain and imprecise nature of transportation parameters, such as fluctuating supply and demand, varying transportation costs, and uncertain deterioration rates. While existing studies have explored fuzzy and neutrosophic approaches to address uncertainty, limited research has leveraged FFP to model uncertainty in transportation problems. Additionally, most prior works focus either on cost minimization alone or employ single-objective optimization methods, lacking a comprehensive multi-objective approach that optimizes transportation, time, and deterioration costs. Moreover, integrating FFPA and NGPA within a unified framework for transportation problems remains unexplored, leaving a significant gap in the literature. This study fills these gaps by introducing a robust multi-objective optimization framework under the FFE, providing a more realistic and practical solution for uncertain transportation scenarios.

The remaining sections of this article are arranged as follows: We discussed the literature review in section 2. Basic definitions, theorems, and arithmetic operations are presented in Section 3. In Section 4, uncertain and crisp mathematical models of the TTP and MOTP are shown, and in Sections 5 and 6, proposed FFPA and NGPA are for the presented mathematical modeling. Section 7 presents the methodology for the proposed problem. Section 8 presents the numerical example of a proposed mathematical model. Finally, Section 9 discusses the conclusion and the managerial and practical implications.

2 Literature review

Fermatean fuzzy programming approach is a non-linear programming approach that handles the MOTP and other problems. It extends the concept of Pythagorean fuzzy programming [29]. An NFFSF is introduced to convert fuzzy data into crisp form using the fermatean fuzzy technique. FFP provides an alternative approach to solving MOTP [29, 81, 92, 93]. Senapati and Yager [30] introduced fermatean fuzzy sets and compared them to Pythagorean and intuitionistic fuzzy sets. They proposed a score function to rank fermatean fuzzy sets and used Euclidean distance to measure their similarity. Sharma et al. [31] established a score function for grading fermatean fuzzy sets and presented an algorithm for optimizing TP using FFP. Akram et al. [32] proposed a fermatean fuzzy data envelopment analysis (DEA) method to address TP with multiple goals and fuzzy costs. They introduced a new method (multi-criteria decision-making approach) to measure efficiency for each segment of the transportation route using fermatean fuzzy score functions [33, 34, 35] and developed a multi-objective transportation mathematical model under FFE. They transformed this mathematical model into crisp mathematical form using the Trapezoidal FFPA and LR fully Pythagorean fuzzy programming approach. They also provided some numerical examples to justify this approach.

Sirbiladze et al. [36] presented a mathematical model of a fuzzy vehicle routing problem under different environments. They used the multi-criteria decision-making approach to solve this mathematical model and get the best Pareto optimal solution [32, 37, 38] developed a mathematical model for TP that considered multiple goals. They utilized a Type-1 fermatean fuzzy Number in an FFE to minimize the factors that needed consideration. Sahoo [39] proposed a novel method for addressing TP that involves fuzzy parameters, particularly fermatean fuzzy ones. It addresses the uncertainty and imprecision frequently arising in transportation issues, mainly during volatile economic conditions [40].

Apart from this, Sahoo [41] addressed complex TP by utilizing FFP and proposing a novel solution. They introduced a mathematical approach that considers these fuzzy parameters and transforms them into well-defined transportation problems using score and accuracy functions. This method is simple, pragmatic, and can be readily implemented, making it valuable for real-world decision-making scenarios involving FFP. Yazdi et al. [42], Muneeb et al. [43], and Bouraima et al. [44] proposed a novel approach to making informed decisions regarding sustainable urban transportation. They developed a model based on fermatean fuzzy logic that prioritizes various strategies for enhancing city transport sustainability. Chaudhary et al. [45] presented an STP model with multiple objectives using TS-PFHS parameters for sustainable green transportation. They evaluated a TS-PFHS set to address randomness and imprecision in a single framework. Maity et al. [46] investigated the MOTP using uncertain conditions, particularly transportation costs, supply, and demands. They proposed a novel approach to address this uncertainty by incorporating the concept of reliability, which influences the dependability of the costs [47]. Garg and Rizk-Allah [48] proposed a new mathematical model and solved it using the alpha-cult method for addressing the complex MOTP, which aids decision-making when faced with uncertainty. Abd El-Wahed [49] proposed various mathematical models under uncertain parameters and converted these models into a deterministic form fuzzy programming approach. They demonstrated the superiority of the fuzzy approach over the interactive procedure, mainly when dealing with numerous goals and constraints. Fathy Ammar [50] developed a multi-level, multi-objective mathematical model using uncertain conditions and solved this mathematical model using the interval programming technique [51].

Given this importance, the potential research study of Das [52] and Kumar et al. [53] developed an uncertain mathematical model of MOTP using a neutrosophic programming approach and gets efficient solutions. A non-linear hyperbolic membership function is used in this mathematical programming approach. Sharma Chaudhary [54] developed a mathematical model of MOTP utilizing time sequential dual hesitant fuzzy sets. This approach considers hesitation and uncertainty in decision-making over different periods. Mekawy [55] presented a fuzzy multi-objective linear fractional programming problem and converted it into a precise problem. The conversion utilizes real numbers from close interval approximations and the order relations of piecewise quadratic fuzzy numbers. Edalatpanah [56] and Borza et al. [57] presented a novel approach for resolving fuzzy linear programming problems. The approach utilizes horizontal membership functions and multi-dimensional relative-distance-measure fuzzy interval arithmetic. Sheikhi and Ebadi [58] proposed a method for resolving linear fractional programming transportation problems utilizing fuzzy numbers. They focused on linear fractional programming, a specialized area within non-linear programming. Linear programming is also found to assess transportation efficiency concerning the effects it has on the environment [59].

Wang et al. [60] developed a method for solving neutrosophic multi-objective linear programming problems (NMOLP) using triangular neutrosophic numbers. This method addresses real-world decision-making scenarios and proposes a novel strategy for solving NMOLP problems with mixed constraints, where parameters are triangular neutrosophic numbers. Khalifa et al. [61] introduced a method for solving multi-objective transportation problems with fuzzy parameters represented as (α, β) interval-valued fuzzy numbers. They developed a solution procedure for multi-objective fractional programming problems in a hesitant fuzzy decision environment. The utilization of linguistic variables, interactive methods, and goal planning has enabled the creation of effective strategies, which have significantly enhanced decision-making methodologies in this context [62].

Beyond the above consideration, the pivotal research established by Farrokhi-Asl and Tavakkoli-Moghaddam [63] introduced a mathematical model for a bi-objective vehicle routing problem. Salamatbakhsh et al. [64] extended this model to include additional objectives. The model aims to minimize waste collection costs and reduce the environmental risks of transporting hazardous waste. Using a neutrosophic goal programming approach. Veeramani et al. [65] present a compromise solution framework for the multi-objective fractional transportation problem (MOFTP). The framework addresses complex transportation scenarios by considering multiple objectives: cost, time, and environmental and social concerns. Ur Rahman [66] proposed a two-phase parametric approach for flexible fuzzy transportation problems. This approach addressed real-world situations involving uncertain transportation costs and demands. Kané et al. [67] proposed a novel approach to transportation problems that employs trapezoidal fuzzy numbers to represent costs and supply and demand values. A fuzzy linear programming method was developed that converts these fuzzy transportation problems into two interval transportation problems. Pratihar et al. [68] proposed an algorithm to solve fuzzy transportation problems using a modified Vogel's approximation method with costs represented as trapezoidal interval type-2 fuzzy sets. This approach introduces a linear programming problem method for effectively handling transportation parameters such as costs, demand, and supply uncertainty.

Furthermore, Kané et al. [69] identified several advantages of their proposed method compared to existing techniques. A generic ranking index was introduced to compare fuzzy numbers involved in transportation problems with triangular fuzzy numbers. Sheikhi and Ebadi [58] presented a novel approach to solving linear interval fractional transportation problems (ILFTPs) with interval objective functions by transforming the ILFTP into a non-linear programming problem and then converting it into a linear programming problem with additional constraints and variables. Khalifa et al. [70] addressed the challenge of solving piecewise quadratic fuzzy multi-objective de novo programming problems by applying a min-max goal programming approach. Edalatpanah [71] introduced a novel concept in neutrosophic sets: the neutrosophic structured element (NSE). He put forth a decision-making methodology for multi-attribute decision-making problems utilizing NSE information, thereby exemplifying the efficacy of this concept in addressing neutrosophic decision-making issues.

Jaikumar et al. [72] introduced the concept of picture fuzzy soft graphs (PFSGs), a powerful mathematical tool for modeling real-world vagueness. PFSGs extend the scope of fuzzy graphs (FGs) and intuitionistic fuzzy graphs (IFGs) by providing a unified framework for expressing positive, negative, and neutral membership functions. Akram et al. [73] introduced complex q-rung picture fuzzy sets (Cq-RPFSs), which generalize q-RPFSs by including a phase term to handle ambiguity and periodicity. They addressed multi-criteria decision-making (MCDM) problems by proposing complex q-rung picture fuzzy Einstein averaging operators in the Cq-RPFSs environment. Adak Gunjan [74] examined the potential of using fermatean fuzzy numbers to construct profitable portfolios within the financial market. Their approach addresses the uncertainty inherent in decision-making, which often arises from subjective opinions and expressions of decision-makers.

3 Preliminaries and definitions

The basic definitions of the farmatean fuzzy programming, which are used in our proposed work, which is given below:

Definition 3.1 According to Senapati and Yager [29], Farmatean fuzzy sets: Farmatean fuzzy sets (FFSs) can be represented as $F = \{\langle \omega, \alpha_F(\omega), \beta_F(\omega) : \omega \in Y \rangle\}$

Where $\alpha_F(\omega) : Y \longrightarrow [0,1]$ is the degree of satisfaction, and $\beta_F(\omega) : Y \longrightarrow [0,1]$ is the degree of dissatisfaction, including the conditions.

$0 \leq \alpha_F(\omega)^3 + \beta_F(\omega)^3 \leq 1 \forall\ \omega \in Y$. For any fermatean fuzzy set F and $\omega \in Y, \sigma_F(\omega) = \sqrt[3]{1 - (\alpha_F(\omega))^3 - (\beta_F(\omega))^3}$ is identified as the degree of indeterminacy of $\omega \in Y$ to F.The set $F = \{\langle \omega, \alpha_F(\omega), \beta_F(\omega) : \omega \in Y \rangle\}$ is denote as $F = \langle \alpha_F, \beta_F \rangle$.

Definition 3.2 Let $F = \langle \alpha_F, \beta_F \rangle$, $F_1 = \langle \alpha_{F_1}, \beta_{F_1} \rangle$ and $F_2 = \langle \alpha_{F_2}, \beta_{F_2} \rangle$ be three fermatean fuzzy sets on the universal set Y, and $\zeta > 0$ be any scalar. Arithmetic operations of fermatean fuzzy sets is as follows with numerical examples.

$$F_1 \bigoplus F_2 = (\sqrt[3]{\alpha_{F_1}^3 + \alpha_{F_2}^3 - \alpha_{F_1}^3 \alpha_{F_2}^3}, \beta_{F_1}\beta_{F_2}) \tag{3.1}$$

Let $F = \langle 0.4, 0.7 \rangle, F_1 = \langle 0.8, 0.6 \rangle$ and $F_2 = \langle 0.2, 0.9 \rangle$ be three fermatean fuzzy sets and $\zeta = 2$ be any scalar quantity. Then,

$F_1 \bigoplus F_2 = \langle 0.8, 0.6 \rangle \bigoplus \langle 0.2, 0.9 \rangle = (0.8020, 0.54)$

$$F_1 \bigoplus F_2 = (\alpha_{F_1}\alpha_{F_2}, \sqrt[3]{\beta_{F_1}^3 + \beta_{F_2}^3 - \beta_{F_1}^3 \beta_{F_2}^3}) \tag{3.2}$$

$F_1 \bigotimes F_2 = \langle 0.8, 0.6 \rangle \bigoplus \langle 0.2, 0.9 \rangle = (0.16, 0.923)$

$$\zeta \odot F = (\sqrt[3]{1 - (1 - \alpha_F{}^3)^\zeta}), \beta_F{}^\zeta \tag{3.3}$$

$\zeta \bigodot F = 2 \bigodot \langle 0.4, 0.7 \rangle = (0.498, 0.49)$

$$F^\zeta = \alpha_F{}^\zeta, (\sqrt[3]{1 - (1 - \beta_F{}^3)^\zeta}) \tag{3.4}$$

$$F^\zeta = \langle 0.4, 0.7 \rangle^2 = (0.064, 0.828).$$

Definition 3.3 Let $F = \langle \alpha_F, \beta_F \rangle, F_1 = \langle \alpha_{F_1}, \beta_{F_1},$ and $F_2 = \langle \alpha_{F_2}, \alpha_{F_2} \rangle$ be three fermatean fuzzy sets on the universal set Y, and $\zeta > 0$ be any scalar. Their arithmetic operations of fermatean fuzzy set are defined as follows:

$$F_1 \bigcup F_2 = (max\{\alpha_{F_1}, \alpha_{F_2}\}, min\{\beta_{F_1}, \beta_{F_2}\}) \tag{3.5}$$

$F_1 \bigcup F_2 = (max\{\langle 0.8, 0.6\rangle\}, min\{\langle 0.2, 0.9\rangle\}) = (0.8, 0.2)$

$$F_1 \bigcap F_2 = (min\{\alpha_{F_1}, \alpha_{F_2}\}, max\{\beta_{F_1}, \beta_{F_2}\}) \tag{3.6}$$

$F_1 \bigcap F_2 = (min\{\langle 0.8, 0.6\rangle\}, max\{\langle 0.2, 0.9\rangle\}) = (0.2, 0.6)$

$$F^c = (\beta_F, \alpha_F) \tag{3.7}$$

$F^c = \langle 0.4, 0.7\rangle^c = (0.7, 0.4)$

Accuracy function (AF)
Assuming $F = \langle \alpha_F, \beta F\rangle$ is a fermatean fuzzy set, the accuracy function of fermatean fuzzy set can be represented as follows:

$$A_F(F) = ({\alpha_F}^3 + {\beta_F}^3) \tag{3.8}$$

Score function
Theorem 1. Let F be a fermatean fuzzy set $F = \langle \alpha_F, \beta_F\rangle$ then the score function F represented simply proceeds;

$$S_F{}^*(F) = 1/2(1 = {\alpha_F}^3 - {\beta_F}^3).(min(\alpha_F, \beta_F)) \tag{3.9}$$

Property 1. Consider a fermatean fuzzy set $F = \langle \alpha_F, \beta_F\rangle$, then $S_F{}^*(F) \in [0, 1]$.

Proof: According to the ortho-pair definition, $\alpha_F, \beta_F \in [0, 1]$. Then, $min(\alpha_F, \beta_F) \in [0, 1]$, and also ${\alpha_F}^3 \geq 0, {\beta_F}^3 \geq 0, {\alpha_F}^3 \geq 1, {\beta_F}^3 \geq 1$

$$\implies 1 - {\beta_F}^3 \geq 0, \implies 1 + {\alpha_F}^3 - {\beta_F}^3 \geq 0, \because 1/2(1 + {\alpha_F}^3 - {\beta_F}^3).(min(\alpha_F, \beta_F)) \geq 0 \tag{3.10}$$

Again ${\alpha_F}^3 - {\beta_F}^3 \leq 1$, add one both sides $\implies$ 1+$\alpha_F^3 - {\beta_F}^3 \leq 2, (\because {\alpha_F}^3 \geq 0) \implies 1/2(1 + {\alpha_F}^3 - {\beta_F}^3).(min(\alpha_F, \beta_F) \leq 1)$
Hence, $S_F{}^*(F) \in [0, 1]$.

Theorem 2. Let F be a fermatean fuzzy set $F = (\alpha_F, \beta_F)$, then the new fermatean fuzzy score function (NFFSF) F_{1d} represent as follows:

$$S_F{}^*(F_{1d}) = 1/2(1 + \alpha_F - \beta_F).(min(\alpha_F, \beta_F))^2 \tag{3.11}$$

Property 1. Consider a fermatean fuzzy set $F = (\alpha_F, \beta_F)$, then $S_F{}^*(F_{1d}) \in [0, 1]$

proof: According to the ortho-pair definition, $\alpha_F, \beta_F \in [0, 1]$. Then, $min(\alpha_F, \beta_F) \in [0, 1]$ and also $\alpha_F \geq 0, \beta_F \geq 0, \beta_F \leq 1, \implies 1 - \beta_F \geq 0 \implies 1 + \alpha_F - \beta_F \geq 0$.

$\because 1/2(1+\alpha_F-\beta_F).(min(\alpha_F,\beta F))^2 \leq 0$, again, $\alpha_F \leq 1$ and $\beta_F \leq 1, \alpha_F-\beta_F \leq 1$, and add one both sides, $\implies 1+\alpha_F-\beta_F \leq 2 \implies (min(\alpha_F,\beta_F) \leq 1) \implies (min(\alpha_F,\beta_F))^2 \leq 1 \implies 1/2(1+\alpha_F-\beta_F).(min(\alpha_F,\beta F))^2 \leq 1 (\because (min(\alpha_F,\beta_F))^2 \leq 1)$

Hence, $S_F{}^*(F_{1d}) \in [0,1]$.

Theorem 3. Let F be a fermatean fuzzy set $F = \langle\alpha_F,\beta_F\rangle$ then the Type 1 score function F_1 represented as follows:

Type-1 fermatean fuzzy score function $S_F{}^*(F_{11}) = 1/2(1+\alpha_F{}^2-\beta_F{}^2)$.

According to the ortho-pair definition, $\alpha_F,\beta_F \in [0,1]$, and

$\alpha_F{}^2 \geq 0, \beta_F{}^2 \geq 0, \alpha_F{}^2 \leq 1$, and $\beta_F{}^2 \leq 1 \implies 1-\beta_F{}^2 \geq 0 \implies 1+\alpha_F{}^2-\beta_F{}^2 \geq 0 \because 1/2(1+\alpha_F{}^2-\beta_F{}^2) \geq 0$.

Now, again $\alpha_F{}^2-\beta_F{}^2 \leq 1$, add one both sides $\implies 1+\alpha_F{}^2-\beta_F{}^2 \geq 2, (\because \alpha_F{}^2 \geq 0) \implies 1/2(1+\alpha_F{}^2-\beta_F{}^2 \geq 1)(\because (\alpha_F,\beta_F) \leq 1)$

Hence, $S_F{}^*(F_{11}) \in [0,1]$.

Type-2 fermatean fuzzy score function $S_F{}^*(F_{11}) = 1/3(1+2\alpha_F{}^3-\beta_F{}^3)$

Type-3 fermatean fuzzy score function $S_F{}^*(F_{13}) = 1/2(1+\alpha_F{}^2-\beta_F{}^2).|\alpha_F-\beta_F|$

Let $F_1 = \langle\alpha_{F_1},\beta_{F_1}\rangle$, and $F_2 = \langle\alpha_{F_2},\beta_{F_2}\rangle$ be two fermatean fuzzy sets, then the following operations will be satisfied,

$$S_F{}^*(F_1) \geq S_F{}^*(F_2) \text{ with } A_F(F_1) > A_F(F_2) \text{ iff } F_1 > F_2$$
$$S_F{}^*(F_1) \leq S_F{}^*(F_2) \text{ with } A_F(F_1) < A_F(F_2) \text{ iff } F_1 < F_2$$
$$S_F{}^*(F_1) = S_F{}^*(F_2) \text{ with } A_F(F_1) = A_F(F_2) \text{ iff } F_1 = F_2$$

Example 1. Let $F_1 = \langle 0.7, 0.6\rangle$ and $F_2 = \langle 0.8, 0.5\rangle$ be the two fermatean fuzzy sets; then we will see the following operations,

By using the score function $S_F{}^*(F) = 1/2(1+\alpha_F{}^3-\beta_F{}^3).(min(\alpha_F,\beta_F))$.

$$S_F{}^*(F_1) = 1/2(1+0.7^3-0.6^3).(min(0.7,0.6)) = 0.337$$
$$S_F{}^*(F_2) = 1/2(1+0.8^3-0.5^3).(min(0.8,0.5)) = 0.346$$
$$\text{Hence } S_F{}^*(F_1) < S_F{}^*(F_2) \implies F_1 < F_2$$

Example 2. Let $F_1 = \langle 0.9, 0.8\rangle$ and $F_2 = \langle 0.6, 0.5\rangle$ be the two fermatean fuzzy sets; then the following operations are represented,

By using the score function $S_F{}^*(F) = 1/2(1+\alpha_F{}^3-\beta_F{}^3).(min(\alpha_F,\beta_F))$.

$$S_F{}^*(F_1) = 1/2(1+0.9^3-0.8^3).(min(0.9,0.8)) = 0.486$$
$$S_F{}^*(F_2) = 1/2(1+0.6^3-0.5^3).(min(0.6,0.5)) = 0.0.022$$
$$\text{Hence } S_F{}^*(F_1) > S_F{}^*(F_2) \implies F_1 > F_2$$

4 Mathematical model

4.1 Mathematical model of traditional transportation problem

The mathematical model of the traditional transportation problem (TTP) is presented as follows:

$$Minf = \sum_{i=1}^{M}\sum_{i=1}^{N} c_{ij} y_{ij} \tag{4.1}$$

s.t.

$$\sum_{j=1}^{N} y_{ij} \leq s_i, i = 1, 2, ..., M \tag{4.2}$$

$$\sum_{i=1}^{M} y_{ij} \leq d_j, j = 1, 2, ..., N \tag{4.3}$$

$$y_{ij} \geq 0 \,\forall\, i, j. \tag{4.4}$$

The mathematical model of TTP with fermatean fuzzy parameters is represented as follows:

$$Minf^* = \sum_{i=1}^{M}\sum_{i=1}^{N} {c_{ij}}^{F} y_{ij} \tag{4.5}$$

s.t.

$$\sum_{j=1}^{N} y_{ij} \leq {s_i}^{F}, i = 1, 2, ..., M \tag{4.6}$$

$$\sum_{i=1}^{M} y_{ij} \leq {d_j}^{F}, j = 1, 2, ..., N \tag{4.7}$$

such that

$$\begin{aligned} {s_i}^{F} &= (\alpha_{s_i}, \beta_{s_i}) \text{ where } 0 \leq \alpha_{s_i}^3 + \beta_{s_i}^3 \leq 1 \\ {d_i}^{F} &= (\alpha_{d_i}, \beta_{d_i}) \text{ where } 0 \leq \alpha_{d_i}^3 + \beta_{d_i}^3 \leq 1 \\ {c_{ij}}^{F} &= (\alpha_{c_{ij}}, \beta_{c_{ij}}) \text{ where } 0 \leq \alpha_{c_{ij}}^3 + \beta_{c_{ij}}^3 \leq 1, \\ y_{ij} &\geq 0 \,\forall\, i, j. \end{aligned}$$

Now, we convert the above mathematical model into the crisp form using the new fermatean fuzzy score function under the fermatean fuzzy environment.

$$Minf^* = \sum_{i=1}^{M}\sum_{i=1}^{N} S(c_{ij}^{F}) y_{ij} \tag{4.8}$$

s.t.

$$\sum_{j=1}^{N} y_{ij} \leq S({s_i}^{F}), i = 1, 2, ..., M \tag{4.9}$$

$$\sum_{i=1}^{M} y_{ij} \leq S({d_j}^{F}), j = 1, 2, ..., N \tag{4.10}$$

$$y_{ij} \geq \; 0 \;\; \forall\, i, j.$$

4.2 Mathematical model of multi-objective transportation problem (MOTP)

The formulation for the mathematical model of the multi-objective transportation problem with fermatean fuzzy parameters under the fermatean fuzzy environment is represented as follows:

$$Minf_t^* = \sum_{i=1}^{M}\sum_{j=1}^{N} c_{ij_t}{}^{F} y_{ij}, \forall t = 1, 2, ..., T \tag{4.11}$$

s.t.

$$\sum_{j=1}^{N} y_{ij} \leq s_i{}^{F}, i = 1, 2, ..., M \tag{4.12}$$

$$\sum_{i=1}^{M} y_{ij} \leq d_j{}^{F}, j = 1, 2, ..., N \tag{4.13}$$

such that

$$\begin{aligned} s_i{}^{F} &= (\alpha_{s_i}, \beta_{s_i}) \text{ where } 0 \leq \alpha_{s_i}^3 + \beta_{s_i}^3 \leq 1 \\ d_i{}^{F} &= (\alpha_{d_i}, \beta_{d_i}) \text{ where } 0 \leq \alpha_{d_i}^3 + \beta_{d_i}^3 \leq 1 \\ c_{ij}{}^{F} &= (\alpha_{c_{ij}}, \beta_{c_{ij}}) \text{ where } 0 \leq \alpha_{c_{ij}}^3 + \beta_{c_{ij}}^3 \leq 1, \\ & y_{ij} \geq 0 \; \forall \; i, j. \end{aligned}$$

Where $s_i^F = (\alpha_{s_i}, \beta_{s_i})$ units are available at the i^{th} supply node, and $d_j^F = (\alpha_{d_j}, \beta_{d_j})$ units are in demand on the j^{th} demand node. Let the transportation cost $c_{ij}{}^{F} = (\alpha_{c_{ij}}, \beta_{c_{ij}})$ is the unit fermatean fuzzy transportation cost and the i^{th} source node to the j^{th} demand node, and δ_{ij} is the number of items that are carried from the i^{th} source node to the j^{th} demand node.

Now, we convert this MOTP mathematical model with fermatean fuzzy parameters into crisp form using the proposed new fermatean fuzzy score function. The following is a representation of the crisp model for MOTP.

$$Minf_t^* = \sum_{i=1}^{M}\sum_{j=1}^{N} S(c_{ij_t}{}^{F}) y_{ij}, t = 1, 2, ..., T \tag{4.14}$$

s.t.

$$\sum_{j=1}^{N} y_{ij} \leq S(s_i{}^{F}), i = 1, 2, ..., M \tag{4.15}$$

$$\sum_{i=1}^{M} y_{ij} \geq S(d_j{}^{F}), j = 1, 2, ..., N \tag{4.16}$$

$$y_{ij} \geq \forall \; i, j.$$

5 Proposed mathematical modeling for fermatean fuzzy programming approach

The proposed mathematical modeling for the fermatean fuzzy programming approach involves defining membership and non-membership functions that satisfy the properties of fermatean fuzzy sets. These functions are then integrated into the objective function and subject to the constraints of multi-objective optimization problems, allowing for a more flexible and accurate representation of uncertainty and vagueness in decision-making scenarios. Senapati and Yager [39] introduced fermatean fuzzy sets (FFSs) as an extension of Intuitionistic fuzzy sets and compared them comprehensively with Pythagorean and Intuitionistic fuzzy sets when the sum of truth and false grades is greater than 1. However, the truth grade and false grade square sum is less than or equal to 1.

FFSs are considered more realistic and capable of handling more significant uncertainty than Intuitionistic and Pythagorean fuzzy sets. They discussed the fundamental properties of FFSs, including the complement operator and the entire set of operations. Silambarasan [75] examined the algebraic properties of these operators, providing valuable insights into the mathematical foundation of FFSs. They expand upon the theoretical framework of FFSs and provide a deeper understanding of their operational characteristics. Akram et al. [76] evaluated interval-valued FFSs as a robust approach for handling uncertain and incomplete data. They also proposed a novel method for directly solving interval-valued fermatean fuzzy fractional TP, avoiding the need to convert the original problem into a crisp equivalent, streamlining the solution process. It enhances the resilience and efficiency of addressing uncertainties in TP.

Zimmermann [77] applied fuzzy linear programming (FLP) to the linear vector maximum issues. He discusses the effects of using different techniques to combine distinct objective functions to find the best compromise solution. He also provides valuable insights into the effectiveness of FLP in tackling multi-objective transportation problems and offers guidance on selecting appropriate approaches for achieving optimal compromise solutions. Fermatean fuzzy programming approach (FFPA) utilizes linear, exponential, or hyperbolic truth functions to achieve optimal solutions to problems through compromise. Intuitionistic fuzzy programming approaches have been developed for multi-objective transportation problems. This environment allows truth and false grades to be represented as linear, exponential, or hyperbolic functions. The Pythagorean fuzzy programming approach can also solve similar challenges in a fuzzy environment. The non-linear programming method, FFPA, is now presented to find a compromise optimal solution for multi-objective optimization problems in FFE and other contexts. This approach allows simultaneous consideration of all objectives. The FFPA is defined as follows:

For the objective function $f_t^*(y)$, FFPA incorporates upper bounds U_t and lower bounds L_t. Additionally; it involves the membership function $\mu(f_t^*(y))$ and non-membership function $\theta(f_t^*(y))$ for the objective function $f_t^*(y)$. This model aims to optimize decision-making under uncertainty, leveraging FFS to handle imprecision and uncertainty in objective functions. Including upper and lower bounds and membership and non-membership functions allows for a comprehensive representation of uncertainty, enabling robust decision-making in scenarios lacking precise information.

Then, the proposed mathematical model for FFPA is as follows:

$$\text{Max } \delta{\tau_1}^3 - {\tau_2}^3 \tag{5.1}$$

Where,

$$\mu(f_t^*(y)) = \begin{cases} 1, & \text{if} f_t^*(y) \le L_t \\ \dfrac{U_t - f_t^*(y)}{U_t - L_t}, & \text{if} L_t \le f_t^*(\delta) \le U_t \text{ and} \\ 0, & \text{if} f_t^*(y) \ge U_t \end{cases}$$

$$\theta(f_t^*(y)) = \begin{cases} 0, & \text{if} f_t^*(y) \le L_t \\ \dfrac{f_t^*(y) - L_t}{U_t - L_t}, & \text{if} L_t \le f_t^*(\delta) \le U_t \\ 1, & \text{if} f_t^*(y) \ge U_t \end{cases}$$

i.e., $(\mathrm{U}_t - f_t)^3 \ge {d_t}^3{\tau_1}^3, (f_t^*(y) - L_t)^3 \le {d_t}^3{\tau_2}^3$ where $d_t = U_t - L_t$

S.t

$$\mathrm{y}_{11} + y_{12} + \ldots\ldots\ldots\ldots\ldots + y_{1N} \le s_1$$

$$
\begin{aligned}
y_{21} + y_{22} + \ldots\ldots\ldots\ldots\ldots + y_{2N} &\leq s_2 \\
&\vdots \\
y_{M1} + y_{M2} + \ldots\ldots\ldots\ldots\ldots + y_{MN} &\leq s_M \\
y_{11} + y_{21} + \ldots\ldots\ldots\ldots\ldots + y_{M1} &\leq d_1 \\
y_{12} + y_{22} + \ldots\ldots\ldots\ldots\ldots + y_{M2} &\leq d_2 \\
&\vdots \\
y_{1M} + y_{2M} + \ldots\ldots\ldots\ldots\ldots + y_{NM} &\leq d_N
\end{aligned}
$$
$$\sum_{i=1}^{M} s_i = \sum_{j=1}^{N} d_j,\ y_{ij} \geq 0,\ 0 \leq {\tau_1}^3,\ {\tau_2}^3 \leq 1,\ 0 \leq {\tau_1}^3 + {\tau_2}^3 \leq 1,\ {\tau_1}^3 \geq {\tau_2}^3$$

6 Neutrosophic goal programming approach

This section introduces a new strategy for solving the multi-objective transportation problem (MOTP) using a neutrosophic goal programming approach. This approach builds on Zimmermann's [77] neutrosophic extension. The proposed neutrosophic compromise programming technique provides a fresh way to handle uncertainty in optimization problems [78, 79]. It aims to optimize three aspects of a neutrosophic decision: the degree of truth (satisfaction), the degree of falsity (dissatisfaction), and the degree of indeterminacy (partial satisfaction). Bellman and Zadeh [80] introduced three critical concepts for fuzzy sets: the fuzzy decision, the fuzzy goal, and the fuzzy constraints. These concepts have been widely applied in decision-making scenarios involving fuzziness. Here is a brief explanation:

Fuzzy Decision (Fd): A decision that incorporates the fuzziness of the problem's parameters. Fuzzy Goal (Fg): The desired outcome expressed in fuzzy terms. Fuzzy Constraints (Fc): The limitations or restrictions of the problem are described using fuzzy sets. This new methodology leverages these foundational concepts to enhance decision-making where indeterminacy is a significant factor. The fuzzy decision is defined as follows;

$$Fd = Fg \cap Fc \tag{6.1}$$

Accordingly, the neutrosophic decision set $(Fd)^N$, which represents a combination of neutrosophic objectives and constraints, is defined as follows:

$$(Fd)^N = (\cap_{t=1}^{T}(Fg)_K)(\cap_{i=1}^{M}(Fc)_i) = (y, \varphi_{Fd}(y), \theta_F d(y), \phi_F d(y)) \tag{6.2}$$

Where

$$\varphi_{Fd}(y) = min \begin{cases} {\varphi_F g}^1, {\varphi_F g}^2, ..., {\varphi_F g}^t & \forall y \in Y \\ {\varphi_F c}^1, {\varphi_F c}^2, ..., {\varphi_F c}^t & \forall y \in Y \end{cases}$$
$$\theta_{Fd}(y) = min \begin{cases} {\theta_F g}^1, {\theta_F g}^2, ..., {\theta_F g}^t & \forall y \in Y \\ {\theta_F c}^1, {\theta_F c}^2, ..., {\theta_F c}^t & \forall y \in Y \end{cases}$$
$$\phi_{Fd}(y) = min \begin{cases} {\phi_F g}^1, {\phi_F g}^2, ..., {\phi_F g}^t & \forall y \in Y \\ {\phi_F c}^1, {\phi_F c}^2, ..., {\phi_F c}^t & \forall y \in Y \end{cases}$$

Where $\varphi_{Fd}(y)$ represents the truth membership function, $\theta_{Fd}(y)$ denotes the indeterminacy membership function, and $\phi_{Fd}(y)$ signifies the falsity membership function of neutrosophic decision set $(Fd)^N$.

To formulate the membership function for the MOTP, we start by determining the bounds for each objective function. Each objective's lower and upper bounds are denoted by ${f_t}^L$ and ${f_t}^U$, respectively. These bounds are calculated by optimizing each objective as a single objective, subject to the problem's constraints. By solving each T objective independently, we obtain T solutions, $y_1, y_2, ..., y_T$. These solutions are then substituted into each objective function to determine the bounds for each objective as follows:

$$f_t{}^L = min\{f_t{}^*(y)\}_{t=1}^T \tag{6.3}$$

$$f_t{}^U = max\{f_t{}^*(y)\}_{t=1}^T \tag{6.4}$$

Next, the bounds within the neutrosophic environment are determined as follows:

$f_t{}^L(\varphi) = f_t{}^L, f_t{}^U(\varphi) = f_t{}^U$ for the truth membership, $f_t{}^L(\theta) = L(\theta), f_t{}^L(\theta) = f_t{}^U(\varphi) + s_t(f_t{}^U(\varphi) - f_t{}^L(\varphi))$, for the indeterminacy membership, $f_t{}^L(\phi) = f_t{}^L(\varphi) + t_t(f_t{}^U(\varphi) - f_t{}^L(\varphi)), f_t{}^U(\phi) = f_t{}^U(\varphi)$ for the false membership

Where t_t and s_t are predetermined real numbers within the interval (0, 1). Based on these bounds, the membership function can be defined as follows:

$$\varphi_t(f_t{}^*(y)) = \begin{cases} 1, & \text{if} f_t{}^*(y) < f_t{}^L(\varphi) \\ 1 - \dfrac{f_t{}^*(y) - f_t{}^L(\varphi)}{f_t{}^U(\varphi) - f_t{}^L(\varphi)}, & \text{if} f_t{}^L(\varphi) \le f_t{}^*(\varphi) \le f_t{}^U(\varphi) \\ 0, & \text{if} f_t{}^*(y) \ge f_t{}^U(\varphi) \end{cases} \tag{6.5}$$

$$\theta_t(f_t{}^*(y)) = \begin{cases} 1, & \text{if} f_t{}^*(y) < f_t{}^L(\theta) \\ 1 - \dfrac{f_t{}^*(y) - f_t{}^L(\theta)}{f_t{}^U(\theta) - f_t{}^L(\theta)}, & \text{if} f_t{}^L(\theta) \le f_t{}^*(\theta) \le f_t{}^U(\theta) \\ 0, & \text{if} f_t{}^*(y) \ge f_t{}^U(\theta) \end{cases} \tag{6.6}$$

$$\phi_t(f_t{}^*(y)) = \begin{cases} 1, & \text{if} f_t{}^*(y) < f_t{}^U(\phi) \\ 1 - \dfrac{f_t{}^U(\phi) - f_t{}^*(y)}{f_t{}^U(\phi) - f_t{}^L(\phi)}, & \text{if} f_t{}^L(\phi) \le f_t{}^*(\phi) \le f_t{}^U(\phi) \\ 0, & \text{if} f_t{}^*(y) \ge f_t{}^L(\phi) \end{cases} \tag{6.7}$$

Where $f_t{}^U(.) \neq f_t{}^L(.)$ for all objectives. If $f_t{}^U(.) = f_t{}^L(.)$ For any membership, the value of this membership is set to 1. Utilizing equations (6.5)– (6.7) and following the principle outlined by Bellman and Zadeh [80], the neutrosophic optimization model for the MOTP can be expressed as follows:

$$\begin{gathered} MaxMin \sum_{t=1,2,\dots,T} \varphi_t(f_t{}^*(y)) \\ MaxMin \sum_{t=1,2,\dots,T} \theta_t(f_t{}^*(y)) \\ MaxMin \sum_{t=1,2,\dots,T} \phi_t(f_t{}^*(y)) \end{gathered} \tag{6.8}$$

S.t

$$\begin{gathered} \textstyle\sum_{j=1}^N y_{ij} \le s_i, i = 1,2,...,M \\ \textstyle\sum_{i=1}^M y_{ij} \ge d_j, j = 1,2,...,M \end{gathered}$$

$$y_{ij} \ge 0 \,\forall\, i, j$$

Through the utilization of auxiliary parameters, problem (6.8) can be reformulated as follows:

$$Max\alpha, Max\beta, Min\gamma \tag{6.9}$$

$$\varphi_{f_t}(y \geq \alpha), \theta_{f_t}(y) \geq \beta, \phi_{f_t}(y) \geq \gamma$$

S.t

$$\begin{gathered}\sum_{j=1}^{N} y_{ij} \leq s_i, i = 1, 2, ..., M \\ \sum_{j=1}^{N} y_{ij} \leq s_i, i = 1, 2, ..., N\end{gathered}$$

$$y_{ij} \geq \ 0 \,\forall\, i, j;\ \alpha \geq \beta,\ \alpha \geq \gamma,\ \alpha + \beta + \gamma \geq 3,\ \alpha, \beta, \gamma \in [0, 1], t = 1, 2, ..., T.$$

The mathematical problem presented in equation (6.9) can be depicted as follows:

$$Max \alpha - \gamma + \beta \tag{6.10}$$

$$\begin{gathered}f_t^*(y) + (f_t^U(\varphi) - f_t^L(\varphi))\alpha \leq f_t^U(\varphi) \\ f_t^*(y) + (f_t^U(\theta) - f_t^L(\theta))\beta \leq f_t^U(\theta) \\ f_t^*(y) + (f_t^U(\phi) - f_t^L(\phi))\gamma \leq f_t^U(\phi) \\ \text{S.t} \\ \sum_{j=1}^{N} y_{ij} \leq s_i, i = 1, 2, ..., M \\ \sum_{i=1}^{M} y_{ij} \geq d_j, j = 1, 2, ..., N\end{gathered}$$

$$y_{ij} \geq 0 \,\forall\, i, j;\ \alpha \geq \beta, \alpha \geq \gamma, \alpha + \beta + \gamma \geq 3, \alpha, \beta, \gamma \in [0, 1], t = 1, 2, ..., T.$$

The mathematical problem presented in (6.10) can be rewritten as:

$$Max\ \alpha - \gamma + \beta \tag{6.11}$$

$$\begin{gathered}{f_t}^*(y) + ({f_t}^U(\varphi) - {f_t}^L(\varphi))\alpha - {f_t}^U(\varphi) \leq 0 \\ {f_t}^*(y) + ({f_t}^U(\theta) - {f_t}^L(\theta))\beta - {f_t}^U(\theta) \leq 0 \\ {f_t}^*(y) + ({f_t}^U(\phi) - {f_t}^L(\phi))\gamma - {f_t}^U(\phi) \leq 0 \\ \text{S.t} \\ \sum_{j=1}^{N} y_{ij} \leq s_i, i = 1, 2, ..., M \\ \sum_{i=1}^{M} y_{ij} \geq d_j, j = 1, 2, ..., N\end{gathered}$$

$$\text{y}_{ij} \geq 0 \,\forall\, i, j;\ \alpha \geq \beta, \alpha \geq \gamma, \alpha + \beta + \gamma \geq 3, \alpha, \beta, \gamma \in [0, 1], t = 1, 2, ..., T.$$

7 Proposed methodology

We propose a comprehensive methodology for addressing the mathematical model of MOTP within the fermatean fuzzy programming approach (FFPA) framework. The methodology enhances efficiency and robustness in solving MOTP instances. The following essential steps are part of the suggested methodology:

Step 1: Formulate the balance MOTP model within the fermatean fuzzy environment (FFE).

Step 2: Then, convert the MOTP problems into the crisp form using the new fermatean fuzzy score function.

Step 3: At this point, individually deal with this problem for all objectives. We obtain possible primary responses for every objective function.

Step 4: Develop a pay-off matrix to capture objective-performance relationships in the FFE. Calculate upper U_t and lower L_t bounds for each objective ${f_t}^*(y)$ using fermatean fuzzy aggregation techniques applied to the pay-off matrix δ.

Step 5: A problem model will be built using the proposed FFPA and solved using the SciPy library in the Python programming language. In Figure 1, the architecture is displayed. Table 1,2,3,4,5 provide the suggested technique's computations numerically.

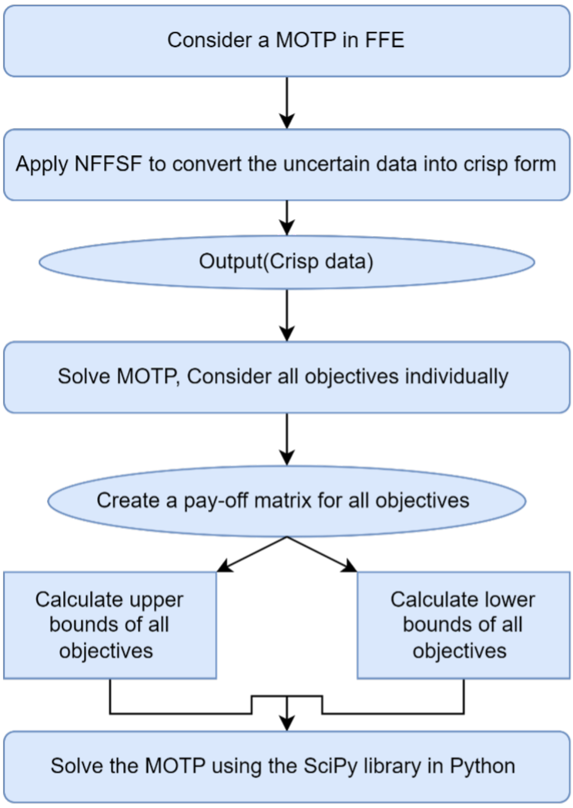

Figure 1. Shows the flow chart of the proposed methodology.

The proposed methodology is significant due to its innovative approach to addressing the complexities and uncertainties inherent in MOTP. Previous research has shown that traditional methods often must catch up in handling the multi-dimensional and uncertain nature of real-world transportation scenarios [81, 82, 83]. The fermatean fuzzy programming approach leverages the advanced capabilities of fermatean fuzzy sets to model and manage uncertainty more effectively than classical fuzzy sets or intuitionistic fuzzy sets [84, 85, 86, 87, 88]. This research approach aligns with prior findings emphasizing the need for more robust and adaptable techniques in MOTP [89, 90]. By incorporating the fermatean fuzzy environment and NFFSF, this approach enhances the precision of solutions and ensures their applicability in dynamic and uncertain contexts. Thus, the research methodology and anticipated results are consistent with and build upon existing literature, advancing knowledge and practice in transportation optimization.

8 Numerical example

In our multi-objective transportation problem (MOTP), we aim to minimize three key objectives: total transportation cost, total transportation time, and deterioration cost during transportation. In this framework, we consider a network of suppliers and demand places where adequate transportation of commodities is required. Each supplier has associated transportation costs, times, and deterioration costs for delivering goods to each demand point. These parameters are expressed as fermatean fuzzy parameters (FFP), representing the uncertainty inherent in real-world transportation scenarios. To facilitate analysis, we employ the NFFSF to convert these fuzzy parameters into crisp values. This conversion enables us to quantify and optimize our objectives effectively. For instance, the total transportation cost objective is calculated by summing the crisp transportation costs from each supplier to each demand point. Similarly, transportation time and deterioration cost during transportation objectives are determined. By integrating fuzzy parameter handling with multi-objective optimization, our approach offers a robust methodology for tackling transportation logistics problems, promoting efficiency and sustainability in the transportation sector.

Table 1. Total transportation cost.

Source	β_1	β_2	β_3	β_4
α_1	(0.8, 0.7)	(0.7, 0.2)	(0.1, 0.6)	(0.2, 0.9)
α_2	(0.5, 0.8)	(0.1, 0.9)	(0.2, 0.6)	(0.2, 0.1)
α_3	(0.3, 0.4)	(0.7, 0.99)	(0.1, 0.8)	(0.7, 0.9)

Table 2. Total transportation time.

Source	β_1	β_2	β_3	β_4
α_1	(0.4, 0.8)	(0.7, 0.5)	(0.2, 0.9)	(0.6, 0.9)
α_2	(0.7, 0.5)	(0.1, 0.99)	(0.6, 0.8)	(0.4, 0.7)
α_3	(0.6, 0.8)	(0.8, 0.6)	(0.5, 0.1)	(0.3, 0.9)

Table 3. Deterioration cost during transportation.

Source	β_1	β_2	β_3	β_4
α_1	(0.5, 0.7)	(0.6, 0.8)	(0.2, 0.7)	(0.8, 0.7)
α_2	(0.4, 0.5)	(0.1, 0.2)	(0.8, 0.1)	(0.4, 0.7)
α_3	(0.8, 0.4)	(0.6, 0.4)	(0.4, 0.9)	(0.5, 0.9)

Table 4. Supply of the transportation problem.

i	α_1	α_2	α_3
$(\alpha_{F_i}, \beta_{F_i})$	(0.3, 0.5)	(0.4, 0.8)	(0.6, 0.4)

Table 5. Demand of the transportation problem.

j	β_1	β_2	β_3	β_4
$(\alpha_{F_j}, \beta_{F_j})$	(0.4, 0.7)	(0.2, 0.5)	(0.6, 0.4)	(0.2, 0.5)

Next, we used the NFFSF to transform the fermatean fuzzy data into the crisp form. The crisp data of the proposed problem are represented as follows.

Table 6. Total transportation cost.

Source	β_1	β_2	β_3	β_4
α_1	(0.2695)	(0.03)	(0.0025)	(0.006)
α_2	(0.0875)	(0.001)	(0.012)	(0.0055)
α_3	(0.0405)	(0.1739)	(0.0015)	(0.196)

Table 7. Total transportation time.

Source	β_1	β_2	β_3	β_4
α_1	(0.048)	(0.15)	(0.006)	(0.126)
α_2	(0.15)	(0.00055)	(0.144)	(0.056)
α_3	(0.144)	(0.216)	(0.007)	(0.018)

Table 8. Deterioration cost during transportation.

Source	β_1	β_2	β_3	β_4
α_1	(0.1)	(0.144)	(0.01)	(0.2695)
α_2	(0.072)	(0.0045)	(0.0085)	(0.056)
α_3	(0.112)	(0.096)	(0.04)	(0.075)

Table 9. Supply of the transportation problem.

i	α_1	α_2	α_3
$(\alpha_{F\,i}, \beta_{F\,i})$	(0.036)	(0.048)	(0.096)

Table 10. Demand of the transportation problem.

j	β_1	β_2	β_3	β_4
$(\alpha_{F\,j}, \beta_{F\,j})$	(0.056)	(0.014)	(0.096)	(0.014)

Since $\sum_{i=1}^{M} S(s_i{}^F) = \sum_{i=1}^{N} S(d_j{}^F) = 0.18$. The best solutions for each objective were then found by solving the three TP problems.

For the first objective function (Total transportation cost),

$$\begin{aligned} f_1{}^*(y) = {} & 0.2695y_{11} + 0.03y_{12} + 0.0225y_{13} + 0.006y_{14} + 0.0875y_{21} + 0.001y_{22} \\ & +0.012y_{23} + 0.0055y_{24} + 0.0405y_{31} + 0.1739y_{32} + 0.0015y_{33} + 0.196y_{34} \end{aligned}$$

S.t

$$\begin{aligned} y_{11} + y_{12} + y_{13} + y_{14} &\leq 0.036, \\ y_{21} + y_{22} + y_{23} + y_{24} &\leq 0.048, \\ y_{31} + y_{32} + y_{33} + y_{34} &\leq 0.096, \\ y_{11} + y_{21} + y_{31} &\leq 0.056, \\ y_{12} + y_{22} + y_{32} &\leq 0.014, \\ y_{13} + y_{23} + y_{33} &\leq 0.096, \\ y_{14} + y_{24} + y_{34} &\leq 0.014, \\ \sum_{i=1}^{M} s_i = \sum_{j=1}^{N} d_j, y_{ij} &\geq 0. \end{aligned}$$

After solving this problem using the SciPy optimization library in Python, we obtain the optimal solutions as follows:

Optimal Value $f_1{}^*(y)$: $1.6563000993102234e - 13$

Optimal solution:

$$y_{11} = 6.701462050627256e-14,$$
$$y_{12} = 5.447537900889634e-13,$$
$$y_{13} = 6.540701226004457e-13$$
$$y_{14} = 2.3433877894449986e-12$$
$$y_{21} = 1.6269086281778882e-13$$
$$y_{22} = 2.8037183688672324e-12$$
$$y_{23} = 1.1917212705073402e-12$$
$$y_{24} = 2.4672960645959706e-12$$
$$y_{31} = 3.4491566715606586e-13$$
$$y_{32} = 8.160054271726014e-14$$
$$y_{33} = 8.95759410927497e-12$$
$$y_{34} = 8.134841315142161e-14.$$

For the second objective function (Total transportation time),

$$f_2{}^*(y) = 0.048y_{11} + 0.15y_{12} + 0.0065y_{13} + 0.126y_{14} + 0.15y_{21} + 0.00055y_{22} \\ +0.144y_{23} + 0.056y_{24} + 0.114y_{31} + 0.216y_{32} + 0.007y_{33} + 0.018y_{34}$$

S.t

$$y_{11} + y_{12} + y_{13} + y_{14} \leq 0.036,$$
$$y_{21} + y_{22} + y_{23} + y_{24} \leq 0.048,$$
$$y_{31} + y_{32} + y_{33} + y_{34} \leq 0.096,$$
$$y_{11} + y_{21} + y_{31} \leq 0.056,$$
$$y_{12} + y_{22} + y_{32} \leq 0.014,$$
$$y_{13} + y_{23} + y_{33} \leq 0.096,$$
$$y_{14} + y_{24} + y_{34} \leq 0.014,$$
$$\sum_{i=1}^{M} s_i = \sum_{j=1}^{N} d_j, y_{ij} \geq 0.$$

After solving this problem using the SciPy library in Python, we obtain the optimal solution as follows: Optimal Value $f_2{}^*(y)$: 4.211826137490883$e-12$

Optimal solution:

$$y_{11} = 8.263438891810004e-12,$$
$$y_{12} = 2.5838370001640527e-12,$$
$$y_{13} = 6.335818003711425e-11,$$
$$y_{14} = 3.0146972331010592e-12,$$
$$y_{21} = 2.5299779571885494e-12,$$
$$y_{22} = 8.790645598327858e-12,$$
$$y_{23} = 2.669450392021306e-12,$$
$$y_{24} = 6.670724613805137e-12,$$
$$y_{31} = 2.6400866542766385e-12,$$
$$y_{32} = 1.800596343315708e-12,$$
$$y_{33} = 5.36625998899776e-11,$$
$$y_{34} = 2.1142895443476957e-11.$$

For the last objective function (Deterioration cost during transportation),

$$f_3{}^*(y) = 0.1y_{11} + 0.144y_{12} + 0.01y_{13} + 0.2695y_{14} + 0.072y_{21} + 0.0045y_{22}$$
$$+0.0085y_{23} + 0.056y_{24} + 0.112y_{31} + 0.096y_{32} + 0.04y_{33} + 0.075y_{34}$$

S.t

$$y_{11} + y_{12} + y_{13} + y_{14} \leq 0.036,$$
$$y_{21} + y_{22} + y_{23} + y_{24} \leq 0.048,$$
$$y_{31} + y_{32} + y_{33} + y_{34} \leq 0.096,$$
$$y_{11} + y_{21} + y_{31} \leq 0.056,$$
$$y_{12} + y_{22} + y_{32} \leq 0.014,$$
$$y_{13} + y_{23} + y_{33} \leq 0.096,$$
$$y_{14} + y_{24} + y_{34} \leq 0.014,$$
$$\sum_{i=1}^{M} s_i = \sum_{j=1}^{N} d_j, y_{ij} \geq 0.$$

After solving this problem using the SciPy optimization library in Python, we obtain the optimal solution as follows:

Optimal Value $f_2{}^*(y)$: $7.110957493321067e - 12$

Optimal solution:

$$y_{11} = 6.1746254985444975e - 12,$$
$$y_{12} = 4.317295196854511e - 12,$$
$$y_{13} = 6.194365813830885e - 11,$$
$$y_{14} = 2.30864320123096e - 12,$$
$$y_{21} = 8.587199756168197e - 12,$$
$$y_{22} = 7.54519853118348e - 11,$$
$$y_{23} = 6.803357681972516e - 11,$$
$$y_{24} = 1.0962059083018053e - 11,$$
$$y_{31} = 5.5603385632255506e - 12,$$
$$y_{32} = 6.461011159919284e - 12,$$
$$y_{33} = 1.5425548584548254e - 11,$$
$$y_{34} = 8.269047323003344e - 12.$$

Once we have a solution to each objective individually, we can create the pay-off matrix this way, see .

Table 11. Pay-off matrix.

	f_1^*	f_2^*	f_3^*
f_1^*	0.0000037438	0.000888743	0.00134174
f_2^*	0.0018528783	0.000025878	0.00995687
f_3^*	0.0115596912	0.014590691	0.00004369

Finding the upper and lower bounds for every objective function and $d_t = U_t - L_t$, which are as follows:

$$L_i = 0.0000037438382, U_1 = 0.001341743, d_1 = 0.001337991618$$
$$L_2 = 0.0000258783, U_2 = 0.00995687, d_2 = 0.0099309917$$
$$L_3 = 0.00004369123, U_3 = 0.014590691, d_3 = 0.01454699977$$

Now, we solved the mathematical model using the proposed FFPA.

Where,

$$\mu({f_t}^*(y))^3 \geq \tau_1^3, \forall t, \theta({f_t}^*(y))^3 \leq \tau_2^3, \forall t$$

i.e.,

$$(U_t - f_t^*(y))^3 \geq d_t^3 \tau_1^3, (f_t^*(y) - L_t)^3 \leq d_t^3 \tau_2^3 \text{ where } d_t = U_t - L_t$$

For upper bound: $\implies (0.001341743 - f_1^3)^3 \geq 0.0000000023897975\tau_1^3, (0.00995687 - f_2^*)^3 \geq 0.000000979146657\tau_1^3, (0.014590691 - f_3^*)^3 \geq 0.000003077731643\tau_1^3$

For lower bound: $\implies (f_1^* - 0.0000037438382)^3 \geq .0000000023897975\tau_2^3, (f_2^* - 0.0000258783)^3 \geq 0.000000979146657\tau_2^3, (f_3^* - 0.00004369123)^3 \geq 0.000003077731643\tau_2^3$

S.t

$$\begin{aligned}
y_{11} + y_{12} + \cdots + y_{1N} &\leq s_1, \\
y_{21} + y_{22} + \cdots + y_{2N} &\leq s_2, \\
&\vdots \\
y_{M1} + y_{M2} + \cdots + y_{MN} &\leq s_M \\
y_{11} + y_{21} + \cdots + y_{M1} &\leq d_1 \\
y_{12} + y_{22} + \cdots + y_{M2} &\leq d_2 \\
&\vdots \\
y_{1M} + y_{2M} + \text{.........} + y_{NM} &\leq d_N
\end{aligned}$$

$$\sum_{i=1}^{M} s_i = \sum_{j=1}^{N} d_j, y_{ij} \ \geq \ 0, 0 \ \ \leq \tau_1^3, \tau_2^3 \leq 1, 0 \leq \tau_1^3 + \tau_2^3 \leq 1, \tau_1^3 \geq \tau_2^3$$

To solve this problem using the SciPy library, we obtain the optimal values of all objective functions, such that $f_1^* = 0.004191344499158313$, $f_2^* = 0.003977749589394637$, $f_3^* = 0.002018492386056$, $\tau_1 = 0.001289240882763909$, $\tau_2 = 0.0006606981979325261$, $y_{11} = 0.30573288027448287$, $y_{12} = 0.02845428401361542$, $y_{13} = 0.0011005037929001924$, $y_{14} = 0.0010085155145783693$, $y_{21} = 0.017421950162707155$, $y_{22} = 0.0010101271115978064$, $y_{23} = 0.0011462979936144556$, $y_{24} = 0.0010634351469773963$, $y_{31} = 0.01747142198034052$, $y_{32} = 0.0010542754473920062$, $y_{33} = 0.02057583566682344$, $y_{34} = 0.0009997035453720312$.

Solutions by NGPA

$$\begin{aligned}
f_1^*(y) &= 0.2695y_{11} + 0.03y_{12} + 0.0225y_{13} + 0.006y_{14} + 0.0875y_{21} + 0.001y_{22} \\
&\quad + 0.012y_{23} + 0.0055y_{24} + 0.0405y_{31} + 0.1739y_{32} + 0.0015y_{33} + 0.196y_{34} \\
f_2^*(y) &= 0.048y_{11} + 0.15y_{12} + 0.006y_{13} + 0.126y_{14} + 0.15y_{21} + 0.00055y_{22} \\
&\quad + 0.144y_{23} + 0.056y_{24} + 0.144y_{31} + 0.216y_{32} + 0.007y_{33} + 0.018y_{34} \\
f_3^*(y) &= 0.1y_{11} + 0.144y_{12} + 0.01y_{13} + 0.2695y_{14} + 0.072y_{21} + 0.0045y_{22} \\
&\quad + 0.0085y_{23} + 0.056y_{24} + 0.112y_{31} + 0.096y_{32} + 0.04y_{33} + 0.075y_{34}
\end{aligned}$$

S.t

$$
\begin{aligned}
y_{11} + y_{12} + y_{13} + y_{14} &\leq 0.036, \\
y_{21} + y_{22} + y_{23} + y_{24} &\leq 0.048, \\
y_{31} + y_{32} + y_{33} + y_{34} &\leq 0.096, \\
y_{11} + y_{21} + y_{31} &\leq 0.056, \\
y_{12} + y_{22} + y_{32} &\leq 0.014, \\
y_{13} + y_{23} + y_{33} &\leq 0.096, \\
y_{14} + y_{24} + y_{34} &\leq 0.014, \\
\sum_{i=1}^{M} s_i = \sum_{j=1}^{N} d_j, y_{ij} &\geq 0, i = 1, 2, 3 \ \ j = 1, 2, 3, 4.
\end{aligned}
$$

Using the proposed neutrosophic goal programming approach, we have solved the above numerical example in each objective function individually as a single objective transportation problem. The individual solution of the problem is presented as:

$$
\begin{aligned}
Y_1 = (&6.701462050627256e - 14, 5.447537900889634e - 13, 6.540701226004457e - 13, \\
&\text{2.3433877894449986e-12, 1.6269086281778882e-13, 2.8037183688672324e-12,} \\
&\text{1.1917212705073402e-12, 2.4672960645959706e-12, 3.4491566715606586e-13,} \\
&\text{8.160054271726014e-14, 8.95759410927497e-12, 8.134841315142161e-14),} \\
Y_2 = (&8.263438891810004e - 12, 2.5838370001640527e - 12, 6.335818003711425e - 11, \\
&\text{3.0146972331010592e-12, 2.5299779571885494e-12, 8.790645598327858e-12,} \\
&\text{2.669450392021306e-12, 6.670724613805137e-12, 2.6400866542766385e-12,} \\
&\text{1.800596343315708e-12, 5.36625998899776e-11, 2.1142895443476957e-11),} \\
Y_3 = (&6.1746254985444975e - 12, 4.317295196854511e - 12, 6.194365813830885e - 11, \\
&\text{2.30864320123096e-12, 8.587199756168197e-12, 7.54519853118348e-11,} \\
&\text{6.803357681972516e-11, 1.0962059083018053e-11, 5.5603385632255506e-12,} \\
&\text{6.461011159919284e-12, 1.5425548584548254e-11, 8.269047323003344e-12).}
\end{aligned}
$$

We are using the pay-off matrix from Table 11 to obtain the solutions of each objective function and calculate the lower and upper bounds for each objective function. These bounds are assigned using the following formula: $f_t^L = min\{f_t^*(y)\}_{t=1}^3, f_t^U = max\{f_t^*(y)\}_(t = 1)^3$. The bounds of each objective function are determined by $0.0000037 \leq f_1^* \leq 0.002743, 0.0000258 \leq f_2^* \leq 0.009735, 0.0000436 \leq f_3^* \leq 0.00531$. Define the membership function of each objective function using the NGPA.

For the first objective function f_1^*:

$f_1^L(\varphi) = 0.0000037, f_1^U(\varphi) = 0.002743$, for the truth membership, $f_1^L(\theta) = 0.0000037, f_1^U(\theta) = f_1^U(\varphi) + s_1(f_1^U(\varphi) - f_1^L(\varphi)) = 0.0000037 + s_1$, for the indeterminacy membership, $f_1^L(\phi) = f_1^L(\varphi) + t_1(f_1^U(\varphi) - f_1^L(\varphi)) = 0.0000037 + t_1, f_1^U(\phi) = 0.002743$, for the falsity membership, where t_1 and s_1 are predetermined real numbers within the interval (0, 1). Based on these bounds, the membership function can be defined as follows:

$$
\varphi_1(f_1^*(y)) = \begin{cases} 1, & \text{if } f_1^*(y) < 0.0000037 \\ 1 - \dfrac{f_1^*(y) - 0.0000037}{0.0027}, & \text{if } 0.0000037 \leq f_1^*(y) \leq 0.002743 \\ 0, & \text{if } f_1^*(y) > 0.002743 \end{cases}
$$

$$
\theta_1(f_1^*(y)) = \begin{cases} 1, & \text{if } f_1^*(y) < 0.0000037 \\ 1 - \dfrac{f_1^*(y) - 0.0000037}{s_1}, & \text{if } 0.0000037 \leq f_1^*(y) \leq 0.002743 + s_1 \\ 0, & \text{if } f_1^*(y) > 0.002743 + s_1 \end{cases}
$$

$$\phi_1(f_1^*(y)) = \begin{cases} 1, & \text{if } f_1^*(y) > 0.002743 \\ 1 - \dfrac{0.002743 - f_1(y)}{0.0027 - t_1}, & \text{if } 0.0000037 + t_1 \leq f_1^*(y) \leq 0.002743 \\ 0, & \text{if } f_1^*(y) > 0.0000037 + t_1 \end{cases}$$

For the second objective function f_2^*:

$f_2^L(\varphi) = 0.0000258, f_2^U(\varphi) = 0.00973$, for the truth membership, $f_2^L(\theta) = 0.0000258, f_2^U(\theta) = f_2^U(\varphi) + s_2(f_2^U(\varphi) - f_2^L(\varphi)) = 0.0000258 + s_2$, for the indeterminacy membership, $f_2^L(\phi) = f_2^L(\varphi) + t_2(f_2^U(\varphi) - f_2^L(\varphi)) = 0.0000258 + t_2, f_2^U(\phi) = 0.00973$, for the falsity membership, where t_2 and s_2 are predetermined real numbers within the interval (0, 1). Based on these bounds, the membership function can be defined as follows:

$$\varphi_2(f_2^*(y)) = \begin{cases} 1, & \text{if } f_2^*(y) < 0.000025 \\ 1 - \dfrac{f_2^*(y) - 0.000025}{0.0094}, & \text{if } 0.000025 \leq f_2^*(y) \leq 0.00973 \\ 0, & \text{if } f_2^*(y) > 0.00973 \end{cases}$$

$$\theta_2(f_2^*(y)) = \begin{cases} 1, & \text{if } f_2^*(y) < 0.0000025 \\ 1 - \dfrac{f_2^*(y) - 0.000025}{s_2}, & \text{if } 0.000025 \leq f_2^*(y) \leq 1.0982 + s_2 \\ 0, & \text{if } f_2^*(y) > 0.000025 + s_2 \end{cases}$$

$$\phi_2(f_2^*(y)) = \begin{cases} 1, & \text{if } f_2^*(y) > 0.00973 \\ 1 - \dfrac{0.00973 - f_2(y)}{0.0094 - t_2}, & \text{if } 0.000025 + t_2 \leq f_2^*(y) \leq 0.00973 \\ 0, & \text{if } f_2^*(y) < 0.000025 + t_2 \end{cases}$$

For the third objective function f_3^*:

$f_3^L(\varphi) = 0.000043, f_3^U(\varphi) = 0.00531$, for the truth membership $f_3^L(\theta) = 0.000043, f_3^U(\theta) = f_3^U(\varphi) + s_3(f_3^U(\varphi) - f_3^L(\varphi)) = 0.000043 + s_3$, for the indeterminacy membership, $f_3^L(\phi) = f_3^L(\varphi) + t_3(f_3^U(\varphi) - f_3^L(\varphi)) = 0.000043 + t_3, f_3^U(\phi) = 0.00531$, for the falsity membership, where t_3 and s_3 are predetermined real numbers within the interval (0, 1). Based on these bounds, the membership function can be defined as follows:

$$\varphi_3(f_3^*(y)) = \begin{cases} 1, & \text{if } f_3^*(y) < 0.000043 \\ 1 - \dfrac{f_3^*(y) - 0.000043}{0.00526}, & \text{if } 0.000043 \leq f_3^*(y) \leq 0.00531 \\ 0, & \text{if } f_3^*(y) > 0.00531 \end{cases}$$

$$\theta_3(f_3^*(y)) = \begin{cases} 1, & \text{if } f_3^*(y) < 0.000043 \\ 1 - \dfrac{f_3^*(y) - 0.000043}{s_3}, & \text{if } 0.000043 \leq f_3^*(y) \leq 0.000043 + s_3 \\ 0, & \text{if } f_3^*(y) > 0.000043 + s_3 \end{cases}$$

$$\phi_3(f_3^*(y)) = \begin{cases} 1, & \text{if } f_3^*(y) > 0.00531 \\ 1 - \dfrac{0.00531 - f_3(y)}{0.00526 - t_3}, & \text{if } 0.000043 + t_3 \leq f_3^*(y) \leq 0.00531 \\ 0, & \text{if } f_3^*(y) < 0.000043 + t_3 \end{cases}$$

The construction of an equivalent neutrosophic programming model for the proposed problem is presented as follows;

$$Max\alpha - \gamma + \beta$$

S.t

$$
\begin{aligned}
y_{11}+y_{12}+y_{13}+y_{14} &\leq 0.036,\\
y_{21}+y_{22}+y_{23}+y_{24} &\leq 0.048,\\
y_{31}+y_{32}+y_{33}+y_{34} &\leq 0.096,\\
y_{11}+y_{21}+y_{31} &\leq 0.056,\\
y_{12}+y_{22}+y_{32} &\leq 0.014,\\
y_{13}+y_{23}+y_{33} &\leq 0.096,\\
y_{14}+y_{24}+y_{34} &\leq 0.014,
\end{aligned}
$$

$0.048\mathrm{y}_{11}+0.15y_{12}+0.006y_{13}+0.126y_{14}+0.15y_{21}+0.00055y_{22}+0.144y_{23}+0.056y_{24}+0.144y_{31}+0.216y_{32}+0.007y_{33}+0.018y_{34}+0.0094\alpha\leq\ 0.009735$

$0.1\mathrm{y}_{11}+0.144y_{12}+0.01y_{13}+0.2695y_{14}+0.072y_{21}+0.0045y_{22}+0.0085y_{23}+0.056y_{24}+0.112y_{31}+0.096y_{32}+0.04y_{33}+0.075y_{34}+0.00526\alpha\leq\ 0.00531$

$0.2695\mathrm{y}_{11}+0.03y_{12}+0.0225y_{13}+0.006y_{14}+0.0875y_{21}+0.001y_{22}+0.012y_{23}+0.0055y_{24}+0.0405y_{31}+0.1739y_{32}+0.0015y_{33}+0.196y_{34}+s_1\gamma-s_1\leq\ 0.0000037$

$0.048\mathrm{y}_{11}+0.15y_{12}+0.006y_{13}+0.126y_{14}+0.15y_{21}+0.00055y_{22}+0.144y_{23}+0.056y_{24}+0.144y_{31}+0.216y_{32}+0.007y_{33}+0.018y_{34}+s_2\gamma-s_2\leq\ 0.0000258$

$0.1\mathrm{y}_{11}+0.144y_{12}+0.01y_13+0.2695y_{14}+0.072y_{21}+0.0045y_{22}+0.0085y_{23}+0.056y_{24}+0.112y_{31}+0.096y_{32}+0.04y_{33}+0.075y_{34}+s_3\gamma-s_3\leq\ 0.0000436$

$0.2695\mathrm{y}_{11}+0.03y_{12}+0.0225y_{13}+0.006y_{14}+0.0875y_{21}+0.001y_{22}+0.012y_{23}+0.0055y_{24}+0.0405y_{31}+0.1739y_{32}+0.0015y_{33}+0.196y_{34}-(0.0027-t_1)\beta-t_1\leq\ 0.0000037$

$0.048\mathrm{y}_{11}+0.15y_{12}+0.006y_{13}+0.126y_{14}+0.15y_{21}+0.00055y_{22}+0.144y_{23}+0.056y_{24}+0.144y_{31}+0.216y_{32}+0.007y_{33}+0.018y_{34}-(0.0094-t_2)\beta-t_2\leq\ 0.0000258$

$0.1\mathrm{y}_{11}+0.144y_{12}+0.01y_{13}+0.2695y_{14}+0.072y_{21}+0.0045y_{22}+0.0085y_{23}+0.056y_{24}+0.112y_{31}+0.096y_{32}+0.04y_{33}+0.075y_{34}-(0.00526-t_3)-t_3\leq\ 0.0000436$

$\sum_{i=1}^{M}s_i=\sum_{j=1}^{N}d_j, y_{ij}\geq\ 0, i=1,2,3,\ j=1,2,3,4,\ \alpha\geq\beta, \alpha\geq\gamma, \alpha+\beta+\gamma\leq 3, \alpha,\beta,\gamma\in[0,1], t=1,2,3.$

Now, we obtain the best compromise solutions using the SciPy optimization library in Python. The compromise solutions are presented in Table12.

Table 12. Compromise solution of the proposed problem using FFPA and NGPA.

Methods	f_1^*	f_2^*	f_3^*
FFPA	0.004191344	0.00397775	0.002018492
NGPA	0.00325317	0.00314763	0.001723742

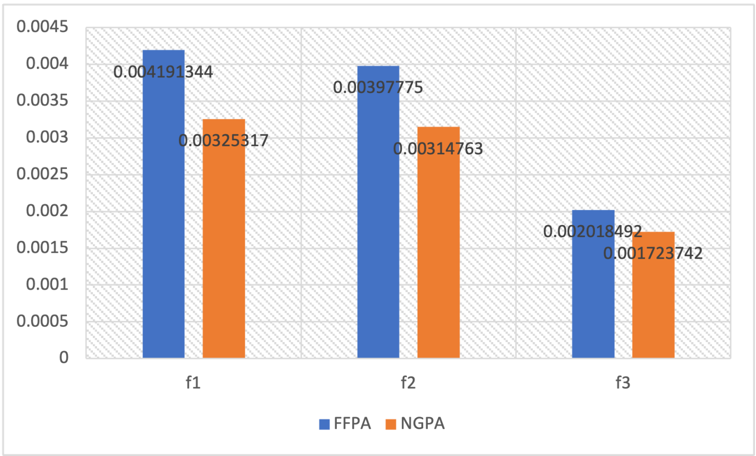

Figure 2. . Shows that the compromise solutions for three objective functions using FFPA and NGPA.

Table 12 presents the compromise solutions for multi-objective transportation using the FFPA and the NGPA. The results indicate that NGPA outperforms FFPA across all three objective functions. Furthermore, NGPA achieves the minimum values for total transportation time, total transportation cost, and deterioration cost during transportation, suggesting that NGPA provides a more efficient and cost-effective solution for the given muti-objective transportation problem.

9 Conclusion

This study introduced the fermatean fuzzy programming approach (FFPA) and neutrosophic goal programming approach (NGPA) to solve the multi-objective transportation problem. Initially, we developed the mathematical model of TTP and MOTP, converting these models into crisp form using NFFSF within FFE. Various methods exist for converting fuzzy data into crisp data, extending fuzzy data to Intuitionistic, Pythagorean, fermatean, and other uncertain data. Among these methods, the NGPA has proven to be more effective in solving MOTP than FFPA. Nonetheless, the proposed FFPA provides a robust alternative solution approach for multi-objective decision-making in the FFE. A numerical example was provided to validate the effectiveness of these approaches in solving the MOTP using the SciPy library in Python. The results demonstrate that the proposed approaches can effectively identify compromise optimal solutions for multi-objective optimization problems.

Furthermore, FFPA and NGPA show great potential for application in solving muti-objective decision-making problems in other fuzzy environments, offering versatile and adaptive solutions to complex and uncertain scenarios. This research advances optimization techniques, providing valuable insights and methodologies in multi-objective optimization under uncertainty.

9.1 Managerial and Practical Implications

The proposed fermatean fuzzy programming approach (FFPA) and neutrosophic goal programming approach (NGPA) offer significant advancements for addressing traditional transportation problems under uncertainty. These methods enhance decision-making by accurately modeling multiple objectives, such as minimizing costs and transit times and optimizing resource utilization and operational efficiency. Their versatility across various industries, including logistics, manufacturing, and public transportation, allows for more precise and adaptive planning, crucial in dynamic environments with fluctuating demand and supply chain disruptions [91]. By integrating these techniques, managers can develop resilient transportation plans, leading to sustainable and long-term benefits [92]. Moreover, the practical implementation of these methods using tools like the SciPy library in Python, LINGO, and GAMS ensures their scalability and ease of integration into existing systems. It enables organizations to adopt advanced optimization techniques without significant infrastructure changes, transforming their approach to transportation problem-solving [93]. Consequently, FFPA and NGPA provide a comprehensive and robust framework for improving efficiency, adaptability, and overall effectiveness in transportation management.

9.2 Limitations and future research

While presenting innovative approaches with FFPA and NGPA for solving multi-objective transportation problems, this study has several limitations. First, the computational complexity of these methods can be high, potentially making them less practical for large-scale problems without further optimization. Second, the reliance on specific fuzzy aggregation techniques may limit the ability to generalize the results to different types of fuzzy environments. Finally, the proposed methods were tested in controlled scenarios, which might need to capture the variability and unpredictability of real-world applications [94, 95]. Future research could focus on developing more efficient algorithms to reduce computational complexity, exploring alternative fuzzy aggregation techniques to enhance robustness, and conducting extensive empirical studies across diverse real-world cases to validate and refine the proposed methodologies.

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