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# VALUE DISTRIBUTION OF MEROMORPHIC AND ENTIRE FUNCTIONS CONCERNING DIFFERENTIAL POLYNOMIALS SHARING A FINITE SET

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**Abstract** *In this literature survey, we deal with the uniqueness problems of meromorphic and entire functions that concerning differential polynomials sharing a finite set and obtain a theorems it generalizes the recent results due to V. Husna .*

## 1 Introduction, Definitions

*Let $f(z)$ and $g(z)$ be two meromorphic and entire functions in the open complex plane $\mathbb{C}$. For $a \in \overline{\mathbb{C}} = \mathbb{C} \cup \{\infty\}$, if $f(z) - a$ and $g(z) - a$ have the same zeros with the same multiplicities then we say that $f(z)$ and $g(z)$ share $a$ CM, if we do not consider the multiplicities then we say that $f(z)$ and $g(z)$ share $a$ IM. It is assumed that readers are known about the notations of Nevanlinna's value distribution theory such as $T(r, f)$, $m(r, f)$, $N(r, f)$ and so on (see [21], [7], [22]).*

*Let $f(z)$ be a non-constant meromorphic function and $\alpha \in \tilde{S}(f) = S(f) \cup \{\infty\}$ and $S$ be a subset of $\tilde{S}(f)$. We define*

$$E(S, f) = \bigcup_{\alpha \in S} \{z : f(z) - \alpha = 0,\ counting\ multiplicity\},$$

$$\overline{E}(S, f) = \bigcup_{\alpha \in S} \{z : f(z) - \alpha = 0,\ ignoring\ multiplicity\}.$$

*If $E(S, f) = E(S, g)$, then we say that $f(z)$ and $g(z)$ share the set $S$ CM; if $\overline{E}(S, f) = \overline{E}(S, g)$, then we say that $f(z)$ and $g(z)$ share the set $S$ IM. Especially, if $S = \{\alpha\}$ and $E(S, f) = E(S, g)$, then we say that $f(z)$ and $g(z)$ share $\alpha$ CM; and we say that $f(z)$ and $g(z)$ share $\alpha$ IM if $\overline{E}(S, f) = \overline{E}(S, g)$.*

*Set "$E(a, f) = \{z : f(z) - a = 0\}$", where $a$ zero point with multiplicity $k$ is counted $k$ times in the set. If the zero points are only counted once ,then we denote the set by $\overline{E}(a, f)$. Let $f$ and $g$ be two non constant meromorphic and entire functions. If $E(a, f) = E(a, g)$, then we say that $f$ and $g$ share the value $a$ $CM$. If $\overline{E}(a, f) = \overline{E}(a, g)$, then we say that $f$ and $g$ share the value $a$ $IM$. We denote by $E_{k)}(a, f)$ the set of all $a$ points of $f$ with multiplicities not exceeding "$k$", where an $a$ point is counted according to its multiplicity, $\overline{E_{k)}}(a, f)$ is the set of distinct $a$ points of $f$ with multiplicities not greater than k. Here, we will define the counting function as $N_{(k}\left(r, \frac{1}{f(z)-a}\right)$ is the counting function of zeros of $f(z) - a$ with multiplicity greater than or equal to k. $N_{k)}\left(r, \frac{1}{f(z)-a}\right)$ is the counting function of zeros of $f(z) - a$ with multiplicity less*

*than or equal to* $k$.
*Here, again we define the reduced counting function as* $\overline{N}_{(k}\left(r, \frac{1}{f(z)-a}\right)$ *is the reduced counting function of zeros of* $f(z)-a$ *in which multiplicity is not counted.* $\overline{N}_{k)}\left(r, \frac{1}{f(z)-a}\right)$ *is the reduced counting function of zeros of* $f(z)-a$ *in which multiplicity is not counted.*
*In 2001, Indrajit Lahiri [11] introduced the notion of weighted sharing, which measures how close a shared value is to being shared CM or to being shared IM.*

**Definition 1.1.** [11] For a complex number $a \in \overline{\mathbb{C}} = \mathbb{C} \cup \{\infty\}$, we denote by $E_k(a, f)$ the set of all $a$ points of "$f(z)$", where an $a$- point with multiplicity $m$ is counted $m$ times if $m \leq k$ and $k+1$ times if $m > k$. For a complex number $a \in \overline{\mathbb{C}}$, such that $E_k(a, f) = E_k(a, g)$, then we say that $f(z)$ and $g(z)$ share the value $a$ with weight $k$.

*The definition implies that if* $f(z),\ g(z)$ *share a value* $a$ *with weight* $k$*, then* $z_0$ *is a zero of* $f(z)-a$ *with multiplicity* $m(\leq k)$ *if and only if it is a zero of* $g(z)-a$ *with multiplicity* $m(\leq k)$ *and* $z_0$ *is a zero of* $f(z)-a$ *with multiplicity* $m(> k)$ *if and only if it is a zero of* $g(z)-a$ *with multiplicity "*$n(> k)$*", where* $m$ *is not necessarily equal to* $n$*. We write* $f(z),\ g(z)$ *share* $(a, k)$ *to mean that* $f(z),\ g(z)$ *share the value* $a$ *with weight* $k$*. Clearly, if* $f(z),\ g(z)$ *share* $(a, k)$ *then* $f(z),\ g(z)$ *share* $(a, p)$ *for all integers* $p,\ 0 \leq p < k$*. Also we note that* $f(z),\ g(z)$ *share a value* $a$ *IM or CM if and only if* $f(z),\ g(z)$ *share* $(a, 0)$ *or* $(a, \infty)$ *respectively.*

**Definition 1.2.** [11] Let $S$ be a set of distinct elements of $\overline{\mathbb{C}}$ and $k$ be a non-negative integer or $\infty$. We denote by $E_f(S, k)$ the set $\bigcup_{a \in S} E_k(a, f)$. Clearly, $E_f(S) = E_f(S, \infty)$ and $\overline{E}_f(S) = E_f(S, 0)$.

*W. K Hayman [7] proposed the following well-known conjecture.*
***Hayman's Conjecture*** *[7]*
*If an entire function satisfies* $f^n f' \neq 1$ *for all positive integers* $n \in \mathbb{N}$*, then* $f$ *is a constant.*
*In 1997, corresponding to the above famous conjecture of Hayman, Yang and Hua studied the unicity of differential monomials and obtained the following theorem.*

**Theorem 1.3.** *[23] Let* $f(z)$ *and* $g(z)$ *be two non-constant entire functions,* $n \geq 6$ *a positive integer. If* $f^n f'$ *and* $g^n g'$ *share 1 CM, then either "*$f(z) = c_1 e^{cz}$*,* $g(z) = c_2 e^{-cz}$*", where* $c_1,\ c_2$*,* $c$ *are three constants satisfying* $(c_1 c_2)^{n+1} c^2 = -1$*, or* $f(z) = tg(z)$ *for a constant* $t$ *such that* $t^{n+1} = 1$*.*

*In 2018, V. H. An and H. H. Khoai [2] considered the set of roots of unity of degree* $d$ *and studied the relations of* $f(z)$ *and* $g(z)$ *when* $E_{(f^n)^{(k)}}(S) = E_{(g^n)^{(k)}}(S)$*. Infact, they proved the following result.*

**Theorem 1.4.** *[2] Let* $f(z)$ *and* $g(z)$ *be two non-constant meromorphic functions, and let* $n,\ d,\ k$ *be positive integers with* $n > 2k + \frac{2k+8}{d}$*,* $d \geq 2$*, and* $S = \{a \in \mathbb{C} : a^d = 1\}$*. If* $E_{(f^n)^{(k)}}(S) = E_{(g^n)^{(k)}}(S)$*, then one of the following two cases holds: (i)* $f(z) = c_1 e^{cz}$ *and* $g(z) = c_2 e^{-cz}$ *for three non-zero constants* $c_1,\ c_2$ *and* $c$ *such that* $(-1)^{kd}(c_1 c_2)^{nd}(nc)^{2kd} = 1$*; (ii)* $f(z) = tg(z)$ *with* $t^{nd} = 1$*,* $t \in \mathbb{C}$*.*

*In 2020, Chao Meng and Xu Li [12] proved the following results.*

**Theorem 1.5.** *[12] Let* $f(z)$ *and* $g(z)$ *be two non-constant meromorphic functions, and let* $n,\ d,\ k$ *be positive integers with* $n > 2k + \frac{3k+9}{d}$*,* $d \geq 2$ *and* $S = \{a \in \mathbb{C} : a^d = 1\}$*. If* $E_{(f^n)^{(k)}}(S, 1) = E_{(g^n)^{(k)}}(S, 1)$ *then one of the following two cases holds: (i)* $f(z) = c_1 e^{cz}$ *and* $g(z) = c_2 e^{-cz}$ *for three non-zero constants* $c_1,\ c_2$ *and* $c$ *such that* $(-1)^{kd}(c_1 c_2)^{nd}(nc)^{2kd} = 1$*; (ii)* $f(z) = tg(z)$ *with* $t^{nd} = 1$*,* $t \in \mathbb{C}$*.*

**Theorem 1.6.** *[12] Let* $f(z)$ *and* $g(z)$ *be two non-constant meromorphic functions, and let* $n,\ d,\ k$ *be positive integers with* $n > 2k + \frac{8k+14}{d}$*,* $d \geq 2$ *and* $S = \{a \in \mathbb{C} : a^d = 1\}$*. If* $E_{(f^n)^{(k)}}(S, 0) = E_{(g^n)^{(k)}}(S, 0)$ *then one of following the two cases holds: (i)* $f(z) = c_1 e^{cz}$ *and* $g(z) = c_2 e^{-cz}$ *for three non-zero constants* $c_1,\ c_2$ *and* $c$ *such that* $(-1)^{kd}(c_1 c_2)^{nd}(nc)^{2kd} = 1$*; (ii)* $f(z) = tg(z)$ *with* $t^{nd} = 1$*,* $t \in \mathbb{C}$*.*

*In 2021,V. Husna [8] proved some theorems by the relationship between two meromorphic and entire functions $f(z)$ and $g(z)$ by considering $(f^n(f-1)^s)^{(k)}$ by taking $n(\geq 1)$, $s(\geq 1)$ are integers.*

**Theorem 1.7.** *[8] Let $f(z)$ and $g(z)$ be two non-constant meromorphic functions, and let $n, d, k, s,$ be a positive integers with $n > 2k - s + \frac{3k+9}{d}$, $d \geq 2$ and $S = \{a \in \mathbb{C} : a^d = 1\}$. If $E_{\left(f^n(f-1)^s\right)^{(k)}}(S,1) = E_{\left(g^n(g-1)^s\right)^{(k)}}(S,1)$ then one of the following two cases holds: (i) $f(z) = c_1e^{cz}$ and $g(z) = c_2e^{-cz}$ for three non-zero constants $c_1,\ c_2$ and $c$ such that $(-1)^{2kd}(c_1c_2)^{(n+s)d}((n+s)c)^{2kd} = 1$; (ii) $f = tg$ with $t^{(n+s)d} = 1$, $t \in \mathbb{C}$.*

**Theorem 1.8.** *[8] Let $f(z)$ and $g(z)$ be two non-constant meromorphic functions, and let $n, d, k, s,$ be a positive integers with $n > 2k - s + \frac{8k+14}{d}$, $d \geq 2$ and $S = \{a \in \mathbb{C} : a^d = 1\}$. If $E_{\left(f^n(f-1)^s\right)^{(k)}}(S,0) = E_{\left(g^n(g-1)^s\right)^{(k)}}(S,0)$ then one of the following two cases holds: (i) $f = c_1e^{cz}$ and $g = c_2e^{-cz}$ for three non-zero constants $c_1,\ c_2$ and $c$ such that $(-1)^{kd}(c_1c_2)^{(n+s)d}((n+s)c)^{2kd} = 1$; (ii) $f = tg$ with $t^{nd} = 1$, $t \in \mathbb{C}$.*

**Theorem 1.9.** *[8] Let $f(z)$ and $g(z)$ be two non-constant entire functions, and let $n, d, k, s,$ be a positive integers with $n > 2k - s + \frac{k+6}{2d}$, $d \geq 2$ and $S = \{a \in \mathbb{C} : a^d = 1\}$. If $E_{\left(f^n(f-1)^s\right)^{(k)}}(S,1) = E_{\left(g^n(g-1)^s\right)^{(k)}}(S,1)$ then one of the following two cases holds: (i) $f(z) = c_1e^{cz}$ and $g(z) = c_2e^{-cz}$ for three non-zero constants $c_1,\ c_2$ and $c$ such that $(-1)^{2kd}(c_1c_2)^{(n+s)d}((n+s)c)^{2kd} = 1$; (ii) $f = tg$ with $t^{(n+s)d} = 1$, $t \in \mathbb{C}$.*

**Theorem 1.10.** *[8] Let $f(z)$ and $g(z)$ be two non-constant entire functions, and let $n, d, k, s,$ be a positive integers with $n > 2k - s + \frac{k+11}{2d}$, $d \geq 2$ and $S = \{a \in \mathbb{C} : a^d = 1\}$. If $E_{\left(f^n(f-1)^s\right)^{(k)}}(S,0) = E_{\left(g^n(g-1)^s\right)^{(k)}}(S,0)$ then one of the following two cases holds: (i) $f = c_1e^{cz}$ and $g = c_2e^{-cz}$ for three non-zero constants $c_1,\ c_2$ and $c$ such that $(-1)^{kd}(c_1c_2)^{(n+s)d}((n+s)c)^{2kd} = 1$; (ii) $f = tg$ with $t^{nd} = 1$, $t \in \mathbb{C}$.*

## 2 Main Results

**Theorem 2.1.** *Let $f(z)$ and $g(z)$ be two non-constant meromorphic functions, and let us define a equation $P(\omega) = a_m\omega^m + a_{m-1}\omega^{m-1} + \cdots + a_1\omega + a_0$ is a polynomial where $a_0 \neq 0$ and $a_1, \cdots, a_m = 0$ are complex constants, where $n, d, k, s, m$ be a positive integers with $n > 2k - ms + \frac{3k+9}{d}$, $d \geq 2$ and $S = \{a \in \mathbb{C} : a^d = 1\}$ . If $E_{\left(f^nP(f)^s\right)^{(k)}}(S,1) = E_{\left(g^nP(g)^s\right)^{(k)}}(S,1)$ then one of the following two cases holds: (i) $f(z) = c_1e^{cz}$ and $g(z) = c_2e^{-cz}$ for three non-zero constants $c_1,\ c_2$ and $c$ such that $(-1)^{2kd}(c_1c_2)^{(n+ms)d}((n+ms)c)^{2kd} = 1$; (ii) $f = tg$ with $t^{(n+ms)d} = 1$, $t \in \mathbb{C}$.*

**Theorem 2.2.** *Let $f(z)$ and $g(z)$ be two non-constant meromorphic functions, and let us define a equation $P(\omega) = a_m\omega^m + a_{m-1}\omega^{m-1} + \cdots + a_1\omega + a_0$ is a polynomial where $a_0 \neq 0$ and $a_1, \cdots, a_m = 0$ are complex constants, where $n, d, k, s, m$ be a positive integers with $n > 2k - ms + \frac{8k+14}{d}$, $d \geq 2$ and $S = \{a \in \mathbb{C} : a^d = 1\}$. If $E_{\left(f^nP(f)^s\right)^{(k)}}(S,0) = E_{\left(g^nP(g)^s\right)^{(k)}}(S,0)$ then one of the following two cases holds: (i) $f(z) = c_1e^{cz}$ and $g(z) = c_2e^{-cz}$ for three non-zero constants $c_1,\ c_2$ and $c$ such that $(-1)^{2kd}(c_1c_2)^{(n+ms)d}((n+ms)c)^{2kd} = 1$; (ii) $f = tg$ with $t^{nd} = 1$, $t \in \mathbb{C}$.*

**Theorem 2.3.** *Let $f(z)$ and $g(z)$ be two non-constant entire functions, and let us define a equation $P(\omega) = a_m\omega^m + a_{m-1}\omega^{m-1} + \cdots + a_1\omega + a_0$ is a polynomial where $a_0 \neq 0$ and $a_1, \cdots, a_m = 0$ are complex constants, $n, d, k, s, m$ be a positive integers with $n > 2k - ms + \frac{k+6}{2d}$, $d \geq 2$ and $S = \{a \in \mathbb{C} : a^d = 1\}$. If $E_{\left(f^nP(f)^s\right)^{(k)}}(S,1) = E_{\left(g^nP(g)^s\right)^{(k)}}(S,1)$ then one of the following two cases holds: (i) $f(z) = c_1e^{cz}$ and $g(z) = c_2e^{-cz}$ for three non-zero constants $c_1,\ c_2$ and $c$ such that $(-1)^{2kd}(c_1c_2)^{(n+ms)d}((n+ms)c)^{2kd} = 1$; (ii) $f = tg$ with $t^{(n+ms)d} = 1$, $t \in \mathbb{C}$.*

**Theorem 2.4.** *Let $f(z)$ and $g(z)$ be two non-constant entire functions, and let us define a equation $P(\omega) = a_m\omega^m + a_{m-1}\omega^{m-1} + \cdots + a_1\omega + a_0$ is a polynomial where $a_0 \neq 0$ and $a_1, \cdots, a_m = 0$ are complex constants, $n,\ d,\ k,\ s,\ m$ be a positive integers with $n > 2k - ms + \frac{k+11}{2d}$, $d \geq 2$ and $S = \{a \in \mathbb{C} : a^d = 1\}$. If $E_{\left(f^nP(f)^s\right)^{(k)}}(S, 0) = E_{\left(g^nP(g)^s\right)^{(k)}}(S, 0)$ then one of the following two cases holds: (i) $f = c_1e^{cz}$ and $g = c_2e^{-cz}$ for three non-zero constants $c_1,\ c_2$ and $c$ such that $(-1)^{kd}(c_1c_2)^{(n+ms)d}((n+ms)c)^{2kd} = 1$; (ii) $f = tg$ with $t^{nd} = 1$, $t \in \mathbb{C}$.*

## 3 Some Lemmas

*Let $F$ and $G$ be two non-constant meromorphic functions defined in $\mathbb{C}$. We will denote by $H$ the following function:*

$$H = \left(\frac{F''}{F'} - \frac{2F'}{F-1}\right)\left(\frac{G''}{G'} - \frac{2G'}{G-1}\right), \tag{3.1}$$

**Lemma 3.1.** *[21] Let $f$ be a non-constant meromorphic function, and p,k be positive integers. Then*

$$N_p\left(r, \frac{1}{f^{(k)}}\right) \leq T\left(r, f^{(k)}\right) - T(r, f) + N_{p+k}(r, \frac{1}{f}) + S(r, f),$$

$$N_p\left(r, \frac{1}{f^{(k)}}\right) \leq k\overline{N}(r, f) + N_{p+k}(r, \frac{1}{f}) + S(r, f).$$

**Lemma 3.2.** *[4] Let $F$ and $G$ be two non-constant meromorphic functions sharing $(1, 1)$ and $H \not\equiv 0$. Then*

$$\begin{aligned} T(r, F) \leq & N_2(r, \frac{1}{F}) + N_2(r, \frac{1}{G}) + N_2(r, F) + N_2(r, G) + \frac{1}{2}\overline{N}(r, \frac{1}{F}) \\ & + \frac{1}{2}\overline{N}(r, F) + S(r, F) + S(r, G). \end{aligned}$$

**Lemma 3.3.** *[4] Let $F$ and $G$ be two non-constant meromorphic functions sharing $(1, 0)$ and $H \not\equiv 0$. Then*

$$\begin{aligned} T(r, F) \leq & N_2(r, \frac{1}{F}) + N_2(r, \frac{1}{G}) + N_2(r, F) + N_2(r, G) + 2\overline{N}(r, \frac{1}{F}) \\ & + \overline{N}(r, \frac{1}{G}) + 2\overline{N}(r, F) + \overline{N}(r, G) + S(r, F) + S(r, G). \end{aligned}$$

**Lemma 3.4.** *[25] Let $f(z)$ and $g(z)$ be two non-constant entire functions and $n,\ k$ be positive integers, $n > k$. If $(f^n)^{(k)}(g^n)^{(k)} = h$, $h \in \mathbb{C}$, $h \neq 0$, then $f(z) = l_1e^{lz}$ and $g(z) = l_2e^{-lz}$ for three non-zero constants $l_1,\ l_2$ and $l$ such that $(-1)^k(l_1l_2)^n(nl)^{2k} = h$.*

**Lemma 3.5.** *Let $f(z)$ be a non-constant meromorphic function on complex plane $\mathbb{C}$ and $n,\ k,\ s \in \mathbb{Z}^+$, $n + s > 2k$. Then*

$$(n + ms - 2k)T(r, f) + kN(r, f) + N\left(r, \frac{f^{n+ms-k}}{\left(f^nP(f)^s\right)^{(k)}}\right) \leq T\left(r, \left(f^nP(f)^s\right)^{(k)}\right) + S(r, f).$$

*Proof.* Using the same as in Lemma 2.6 [2], we can easily obtain Lemma 3.5. □

## 4 Proof of Main Results

***Proof of Theorem 2.1.***
*Let*

$$F = \left(\left(f^nP(f)^s\right)^{(k)}\right)^d, \quad G = \left(\left(g^nP(g)^s\right)^{(k)}\right)^d. \tag{4.1}$$

$$F_1=\left(\left(f^nP(f)^s\right)^{(k)}\right),\ \ G_1=\left(\left(g^nP(g)^s\right)^{(k)}\right). \tag{4.2}$$

*Since $E_{(f^nP(f)^s)^{(k)}}(S,1)=E_{(g^nP(g)^s)^{(k)}}(S,1)$ and we see that $F$ and $G$ share $(1,1)$ .*
*If $H\not\equiv 0$ then by Lemma 3.2*

$$\begin{aligned} T(r,F)\le N_2(r,\frac{1}{F})+N_2(r,\frac{1}{G})+N_2(r,F)+N_2(r,G)+\frac{1}{2}\overline{N}(r,\frac{1}{F}) \\ +\frac{1}{2}N_2(r,\frac{1}{F})+\frac{1}{2}\overline{N}(r,\frac{1}{F})+S(r,F)+S(r,G). \end{aligned} \tag{4.3}$$

*By Lemma 3.5, we obtain*

$$(n+ms-2k)T(r,f)\le T\Big(r,(f^nP(f)^s)^{(k)}\Big)+S(r,f)\le (k+1)(n+ms)T(r,f)+S(r,f). \tag{4.4}$$

*Similarly,*

$$(n+ms-2k)T(r,g)\le T\Big(r,(g^nP(g)^s)^{(k)}\Big)+S(r,g)\le (k+1)(n+ms)T(r,g)+S(r,g). \tag{4.5}$$

*Since,*

$$T\left(r,\left(\left(f^nP(f)^s\right)^{(k)}\right)^d\right)=dT\left(r,\left(f^nP(f)^s\right)^{(k)}\right)+S\left(r,\left(f^nP(f)^s\right)^{(k)}\right), \tag{4.6}$$

$$T\left(r,\left(\left(g^nP(g)^s\right)^{(k)}\right)^d\right)=dT\left(r,\left(g^nP(g)^s\right)^{(k)}\right)+S\left(r,\left(g^nP(g)^s\right)^{(k)}\right). \tag{4.7}$$

*It is easy to see that,*

$$S\left(r,\left(\left(f^nP(f)^s\right)^{(k)}\right)^d\right)=S\left(r,\left(f^nP(f)^s\right)^{(k)}\right)=S(r,f), \tag{4.8}$$

$$S\left(r,\left(\left(g^nP(g)^s\right)^{(k)}\right)^d\right)=S\left(r,\left(g^nP(g)^s\right)^{(k)}\right)=S(r,g). \tag{4.9}$$

*Again we know,*

$$N_2(r,F)=N_2(r,\left(\left(f^nP(f)^s\right)^{(k)}\right)^d)=2\overline{N}(r,f), \tag{4.10}$$

$$N_2(r,G)=N_2(r,\left(\left(g^nP(g)^s\right)^{(k)}\right)^d)=2\overline{N}(r,g). \tag{4.11}$$

*By Lemma 3.1 we have,*

$$\begin{aligned} N_2\Big(r,\frac{1}{F}\Big) &= N_2\left(r,\frac{1}{\left(\left(f^nP(f)^s\right)^{(k)}\right)^d}\right), \\ &= 2\overline{N}\left(r,\frac{1}{\left(f^nP(f)^s\right)^{(k)}}\right), \\ &\le 2N_{k+1}\left(r,\frac{1}{f^nP(f)^s}\right)+2k\overline{N}\Big(r,f^nP(f)^s\Big)+S(r,f^n), \\ &\le 2(k+1)\overline{N}\Big(r,\frac{1}{f}\Big)+2k\overline{N}(r,f)+S(r,f). \end{aligned} \tag{4.12}$$

$$\begin{aligned}\frac{1}{2}\overline{N}\Big(r,\frac{1}{F}\Big) &= \frac{1}{2}\overline{N}\left(r,\frac{1}{\left(\left(f^nP(f)^s\right)^{(k)}\right)^d}\right),\\ &= \frac{1}{2}\overline{N}\left(r,\frac{1}{\left(f^nP(f)^s\right)^{(k)}}\right),\\ &\le \frac{1}{2}N_{k+1}\left(r,\frac{1}{f^nP(f)^s}\right)+\frac{k}{2}\overline{N}\Big(r,f^nP(f)^s\Big)+S(r,f^nP(f)^s),\\ &\le \frac{k+1}{2}\overline{N}\Big(r,\frac{1}{f}\Big)+\frac{k}{2}\overline{N}(r,f)+S(r,f). \end{aligned} \tag{4.13}$$

*On the other hand,*

$$\begin{aligned}N_2\Big(r,\frac{1}{G}\Big) &= 2\overline{N}\left(r,\frac{1}{\left(f^nP(g)^s\right)^{(k)}}\right),\\ &\le 2\left(\overline{N}\left(r,\frac{1}{g^{n+ms-k}}\right)+N\left(r,\frac{g^{n+ms-k}}{\left(g^nP(g)^s\right)^{(k)}}\right)\right), \qquad (4.14)\\ &\le 2\left(\overline{N}\left(r,\frac{1}{g}\right)+N\left(r,\frac{g^{n+ms-k}}{\left(g^nP(g)^s\right)^{(k)}}\right)\right).\end{aligned}$$

*Similarly*

$$N_2\Big(r,\frac{1}{F}\Big) \le 2\left(\overline{N}\left(r,\frac{1}{f}\right)+N\left(r,\frac{f^{n+ms-k}}{\left(f^nP(f)^s\right)^{(k)}}\right)\right). \tag{4.15}$$

*Also,*

$$\overline{N}(r,F)+\overline{N}(r,G)=2\overline{N}(r,f). \tag{4.16}$$

*On combining all the above equations from (4.1), (4.8) - (4.12) we get,*

$$\begin{aligned}T\left(r,\left(\left(f^nP(f)^s\right)^{(k)}\right)^d\right) &\le 2(k+1)\overline{N}\Big(r,\frac{1}{f}\Big)+(2k+2)\overline{N}(r,f)\\ &\quad +2\left(\overline{N}\left(r,\frac{1}{f}\right)+N\left(r,\frac{f^{n+ms-k}}{\left(f^nP(f)^s\right)^{(k)}}\right)\right)+S(r,f).\end{aligned} \tag{4.17}$$

*On combining all the above equations from (4.1), ( 4.8) - (4.11), (4.13) we get,*

$$\begin{aligned}T\left(r,\left(\left(g^nP(g)^s\right)^{(k)}\right)^d\right) &\le 2(k+1)\overline{N}\Big(r,\frac{1}{g}\Big)+(2k+2)\overline{N}(r,g)\\ &\quad +2\left(\overline{N}\left(r,\frac{1}{g}\right)+N\left(r,\frac{g^{n+ms-k}}{\left(g^nP(g)^s\right)^{(k)}}\right)\right)+S(r,g).\end{aligned} \tag{4.18}$$

*Adding the inequalities (4.17) and (4.18) we obtain,*

$$\begin{aligned}T\left(r,\left(\left(f^nP(f)^s\right)^{(k)}\right)^d\right) &\le 2(k+1)\overline{N}\Big(r,\frac{1}{f}\Big)+(2k+2)\overline{N}(r,f)+2\overline{N}(r,\frac{1}{g})\\ &\quad +2N\left(r,\frac{g^{n+ms-k}}{\left(g^nP(g)^s\right)^{(k)}}\right)+2\overline{N}(r,g)+\overline{N}(r,f)+S(r,f)+S(r,g),\\ &\le (3k+5)T(r,f)+2k\overline{N}(r,f)+4T(r,g)+2N\left(r,\frac{g^{n+ms-k}}{\left(g^nP(g)^s\right)^{(k)}}\right)\\ &\quad +S(r,f)+S(r,g).\end{aligned} \tag{4.19}$$

*Similarly for G*

$$T\left(r, \left(\left(g^nP(g)^s\right)^{(k)}\right)^d\right) \leq (3k+5)T(r,g) + 2k\overline{N}(r,g) + 4T(r,g) + 2N\left(r, \frac{f^{n+ms-k}}{\left(f^nP(f)^s\right)^{(k)}}\right)$$
$$+ S(r,f) + S(r,g). \tag{4.20}$$

*By Lemma 3.1 we have,*

$$(n+ms-2k)dT(r,f)+kdN(r,f)+dN\left(r, \frac{f^{n+ms-k}}{\left(f^nP(f)^s\right)^{(k)}}\right) \leq dT(r,(f^nP(f)^s)^{(k)})+S(r,f). \tag{4.21}$$

$$(n+ms-2k)dT(r,g)+kdN(r,g)+dN\left(r, \frac{g^{n+ms-k}}{\left(g^nP(g)^s\right)^{(k)}}\right) \leq dT(r,(g^nP(g)^s)^{(k)})+S(r,g). \tag{4.22}$$

*From (4.19), (4.20), (4.21), (4.22) we have,*

$$\begin{aligned}&(n+ms-2k)dT(r,f)+kdN(r,f)+dN\left(r, \frac{f^{n+ms-k}}{\left(f^nP(f)^s\right)^{(k)}}\right)\\ &+(n+ms-2k)dT(r,g)+kdN(r,g)+dN\left(r, \frac{g^{n+ms-k}}{\left(g^nP(g)^s\right)^{(k)}}\right)\\ &\leq (3k+9)\{T(r,f)+T(r,g)\}+2N\left(r, \frac{f^{n+ms-k}}{\left(f^nP(f)^s\right)^{(k)}}\right)\\ &+2N\left(r, \frac{g^{n+ms-k}}{\left(g^nP(g)^s\right)^{(k)}}\right)+S(r,f)+S(r,g).\end{aligned} \tag{4.23}$$

*Since* $d \geq 2$*,*

$$dN\left(r, \frac{f^{n+ms-k}}{\left(f^nP(f)^s\right)^{(k)}}\right) \geq 2N\left(r, \frac{f^{n+ms-k}}{\left(f^nP(f)^s\right)^{(k)}}\right), \tag{4.24}$$

$$dN\left(r, \frac{g^{n+ms-k}}{\left(g^nP(g)^s\right)^{(k)}}\right) \geq 2N\left(r, \frac{g^{n+ms-k}}{\left(g^nP(g)^s\right)^{(k)}}\right), \tag{4.25}$$

$$kdN(r,f) \geq 2k\overline{N}(r,f), \tag{4.26}$$

$$kdN(r,g) \geq 2k\overline{N}(r,g). \tag{4.27}$$

*Therefore,*

$$(nd+msd-2kd-3k-9)\{T(r,f)+T(r,g)\} \leq S(r,f)+S(r,g),$$

*which is absurd with*

$$n > 2k - ms + \frac{3k+9}{d}, \tag{4.28}$$

*hence* $H \equiv 0$.
*By integration we get*

$$\frac{1}{G-1} = \frac{A}{F-1} + B, \tag{4.29}$$

*where* $A \neq 0$ *and* $B$ *are constants. Thus*

$$G = \frac{(B+1)F + (A-B-1)}{BF + (A-B)}, \tag{4.30}$$

*and*

$$F = \frac{(B-A)G + (A-B-1)}{BG - (B+1)}. \tag{4.31}$$

***Case 1.*** $B \neq 0, -1$ *,then from (4.31)*

$$\overline{N}\left(r, \frac{1}{G - \frac{B+1}{B}}\right) = \overline{N}(r, F). \tag{4.32}$$

*By using Nevanlinna Second fundamental theorem and (4.14)*

$$\begin{aligned} T(r,G) \leq & \overline{N}(r,G) + \overline{N}\left(r, \frac{1}{G}\right) + \overline{N}\left(r, \frac{1}{G - \frac{B+1}{B}}\right) + S(r,G), \\ \leq & \overline{N}(r,G) + N_2\left(r, \frac{1}{G}\right) + \overline{N}(r,F) + S(r,G), \\ \leq & \overline{N}(r,g) + 2\left(\overline{N}\left(r, \frac{1}{g}\right) + N\left(r, \frac{g^{n+ms-k}}{(g^nP(g)^s)^{(k)}}\right)\right) \\ & + \overline{N}(r,f) + S(r,g). \end{aligned} \tag{4.33}$$

*If* $A - B - 1 \neq 0$*, then it follows (4.30) from that*

$$\overline{N}\left(r, \frac{1}{F - \frac{B+1-A}{B}}\right) = \overline{N}\left(r, \frac{1}{G}\right). \tag{4.34}$$

*Again by Nevanlinna second fundamental theorem and (4.15)*

$$\begin{aligned} T(r,F) \leq & \overline{N}(r,F) + \overline{N}\left(r, \frac{1}{F}\right) + \overline{N}\left(r, \frac{1}{F - \frac{B+1-A}{B+1}}\right) + S(r,F), \\ \leq & \overline{N}(r,F) + N_2\left(r, \frac{1}{F}\right) + \overline{N}\left(r, \frac{1}{G}\right) + S(r,f), \\ \leq & \overline{N}(r,f) + 2\left(\overline{N}\left(r, \frac{1}{F}\right) + N\left(r, \frac{f^{n+ms-k}}{(f^nP(f)^s)^{(k)}}\right)\right) \\ & + N_{k+1}\left(r, \frac{1}{g^nP(g)^s}\right) + k\overline{N}(r,g) + S(r,f), \\ \leq & \overline{N}(r,f) + 2\left(\overline{N}\left(r, \frac{1}{F}\right) + N\left(r, \frac{f^{n+ms-k}}{(f^nP(f)^s)^{(k)}}\right)\right) \\ & + (k+1)\overline{N}\left(r, \frac{1}{g}\right) + k\overline{N}(r,g) + S(r,f). \end{aligned} \tag{4.35}$$

*From (4.21)-(4.22), (4.33) and (4.35), we get*

$$\begin{aligned} & (n-2k)dT(r,f) + kdN(r,f) + dN\left(r, \frac{f^{n+ms-k}}{(f^nP(f)^s)^{(k)}}\right) \\ & + (n+ms-2k)dT(r,g) + kdN(r,g) + dN\left(r, \frac{g^{n+ms-k}}{(g^nP(g)^s)^{(k)}}\right) \\ \leq & \overline{N}(r,f) + 2\left(\overline{N}\left(r, \frac{1}{f}\right) + N\left(r, \frac{f^{n+ms-k}}{(f^n)^{(k)}}\right)\right) + (k+1)\overline{N}\left(r, \frac{1}{g}\right) \\ & + k\overline{N}(r,g) + \overline{N}(r,g) + 2\left(\overline{N}\left(r, \frac{1}{g}\right) + N\left(r, \frac{g^{n+ms-k}}{(g^nP(g)^s)^{(k)}}\right)\right) \\ & + \overline{N}(r,f) + S(r,f) + S(r,g). \end{aligned} \tag{4.36}$$

*Since* $d \geq 2$

$$dN\left(r, \frac{f^{n+ms-k}}{(f^nP(f)^s)^{(k)}}\right) \geq 2N\left(r, \frac{f^{n+ms-k}}{(f^nP(f)^s)^{(k)}}\right), \tag{4.37}$$

$$dN\left(r, \frac{g^{n+ms-k}}{(g^nP(g)^s)^{(k)}}\right) \geq 2N\left(r, \frac{g^{n+ms-k}}{(g^nP(g)^s)^{(k)}}\right), \tag{4.38}$$

$$kdN(r, f) \geq 2\overline{N}(r, f), \tag{4.39}$$

$$kdN(r, g) \geq (k+1)\overline{N}(r, g). \tag{4.40}$$

*Using (4.37), (4.38), (4.39) and (4.40)*

$$(nd + msd - 2kd - 2)T(r, f) + (nd + msd - 2kd - k - 3)T(r, g) \leq S(r, f) + S(r, g), \tag{4.41}$$

*which contradicts with* $n > 2k - ms + \frac{3k+9}{d}$*. Hence* $A - B - 1 = 0$*. Then by (4.30)*

$$\overline{N}\left(r, \frac{1}{F + \frac{1}{B}}\right) = N(r, G). \tag{4.42}$$

*Again by Nevanlinna Second Fundamental theorem*

$$\begin{aligned} T(r, F) \leq& \overline{N}(r, F) + \overline{N}\left(r, \frac{1}{F}\right) + \overline{N}\left(r, \frac{1}{F + \frac{1}{B}}\right) + S(r, f), \\ &\leq \overline{N}(r, F) + N_2\left(r, \frac{1}{F}\right) + \overline{N}(r, G) + S(r, f), \\ &\leq \overline{N}(r, f) + 2\left(\overline{N}\left(r, \frac{1}{f}\right) + N\left(r, \frac{f^{n+ms-k}}{(f^nP(f)^s)^{(k)}}\right)\right) \\ &+ \overline{N}(r, g) + S(r, f). \end{aligned} \tag{4.43}$$

*Combine (4.21), (4.22), (4.33) and (4.43), we have,*

$$\begin{aligned} (n + ms - 2k)dT(r, f) + kdN(r, f) + dN\left(r, \frac{f^{n+ms-k}}{(f^nP(f)^s)^{(k)}}\right) + (n + ms - 2k)dT(r, g) \\ + kdN(r, g) + dN\left(r, \frac{g^{n+ms-k}}{(g^nP(g)^s)^{(k)}}\right) \\ \leq 2\overline{N}(r, f) + 2\left(\overline{N}\left(r, \frac{1}{f}\right) + N\left(r, \frac{f^{n+ms-k}}{(f^nP(f)^s)^{(k)}}\right)\right) \\ + 2\overline{N}(r, g) + 2\left(\overline{N}\left(r, \frac{1}{g}\right) + N\left(r, \frac{g^{n+ms-k}}{(g^nP(g)^s)^{(k)}}\right)\right) \\ + S(r, f) + S(r, g). \end{aligned} \tag{4.44}$$

*Since* $d \geq 2$

$$dN\left(r, \frac{f^{n+ms-k}}{(f^nP(f)^s)^{(k)}}\right) \geq 2N\left(r, \frac{f^{n+ms-k}}{(f^nP(f)^s)^{(k)}}\right), \tag{4.45}$$

$$dN\left(r, \frac{g^{n+ms-k}}{(g^nP(g)^s)^{(k)}}\right) \geq 2N\left(r, \frac{g^{n+ms-k}}{(g^nP(g)^s)^{(k)}}\right), \tag{4.46}$$

$$kdN(r, f) \geq 2\overline{N}(r, f), \tag{4.47}$$

$$kdN(r, g) \geq 2\overline{N}(r, g). \tag{4.48}$$

*Using (4.45), (4.46), (4.47) and (4.48),*

$$(nd+msd-2kd-2)\{T(r,f)+T(r,g)\} \le S(r,f)+S(r,g), \tag{4.49}$$

*which violates our given assumption.*
***Case 2.*** $B=-1$. *Then*

$$G=\frac{A}{A+1-F},$$

*and*

$$F=\frac{(1+A)G-A}{G}.$$

*If* $A+1\neq 0$. *We obtain*

$$\overline{N}\left(r,\frac{1}{F-A-1}\right)=\overline{N}(r,G),$$
$$\overline{N}\left(r,\frac{1}{G-\frac{A}{A+1}}\right)=\overline{N}\left(r,\frac{1}{F}\right).$$

*By similar arguments we can obtain a contradiction. Therefore* $A+1=0$, *then* $FG\equiv 1$, *that is*

$$\left((f^nP(f)^s)^{(k)}\right)^d\left((g^nP(g)^s)^{(k)}\right)^d=1,$$

*we have* $(f^nP(f)^s)^{(k)}(g^nP(g)^s)^{(k)}=h$, *where* $h^d=1$.
*Suppose* $z_0$ *is a zero of* $f$ *with multiplicity* $p$, *then* $z_0$ *is a pole of* $g$ *with multiplicity* $q$ *such that* $np-k=nq+k$. *So* $n(p-q)-2k=0$. *Since* $n>2k-ms+\frac{3k+9}{d}$, *we can deduce a contradiction. So* $f(z)\neq 0$. *Similarly, we can prove* $f(z)\neq\infty, g(z)\neq 0$ *and* $g(z)\neq\infty$. *So* $f(z)$ *and* $g(z)$ *are two non constant entire functions. According to Lemma 3.4, we obtain* $f(z)=c_1e^{cz}$ *and* $g(z)=c_2e^{-cz}$ *for three non zero constants* $c_1, c_2$ *and* $c$ *such that* $(-1)^{kd}(c_1c_2)^{(n+ms)d}((n+ms)c)^{2kd}=1$.
***Case 3.*** $B=0$. *Then (4.30) and (4.31) gives* $G=\frac{F+A-1}{A}$ *and* $F=AG+1-A$. *If* $A-1\neq 0$, *then*

$$\overline{N}\left(r,\frac{1}{F+A-1}\right)=\overline{N}\left(r,\frac{1}{G}\right),$$

*and*

$$\overline{N}\left(r,\frac{1}{G+\frac{1-A}{A}}\right)=\overline{N}\left(r,\frac{1}{F}\right).$$

*Proceeding similarly as in case 1 , we get a contradiction. Therefore* $A-1=0$, *then* $F\equiv G$, *that is,* $\left((f^nP(f)^s)^{(k)}\right)^d=\left((g^nP(g)^s)^{(k)}\right)^d$. *We have* $(f^n(P(f))^s)^{(k)}=h\,(g^nP(g)^s)^{(k)}$ *with* $h^d=1$. *This completes the proof of Theorem 2.1.*

***Proof of Theorem 2.2.***
*By using* $F$ *and* $G$ *as defined in Theorem 2.1. Since* $E_{(f^nP(f)^s)^{(k)}}(S,0)=E_{(g^nP(g)^s)^{(k)}}(S,0)$ , *we see that* $F$ *and* $G$ *share* $(1,0)$,*if* $H\not\equiv 0$, *then by Lemma 3.3*

$$\begin{aligned}T(r,f)\le& N_2\left(r,\frac{1}{F}\right)+N_2(r,F)+N_2\left(r,\frac{1}{G}\right)+2\overline{N}\left(r,\frac{1}{F}\right)+2\overline{N}(r,F)\\&+\overline{N}\left(r,\frac{1}{G}\right)+\overline{N}(r,G)+S(r,F)+S(r,G).\end{aligned}\tag{4.50}$$

*By Lemma 3.5 we obtain*

$$(n+ms-2k)T(r,f)\le T\left(r,(f^n)^{(k)}\right)+S(r,f)\le(k+1)(n+ms)T(r,f)+S(r,f), \tag{4.51}$$

$$(n+ms-2k)T(r,g)\le T\left(r,(g^n)^{(k)}\right)+S(r,f)\le(k+1)(n+ms)T(r,g)+S(r,g), \tag{4.52}$$

$$2\overline{N}(r,F) = 2\overline{N}(r,f), \tag{4.53}$$

$$2\overline{N}(r,G) = 2\overline{N}(r,g). \tag{4.54}$$

$$\begin{aligned} 2\overline{N}\left(r,\frac{1}{F}\right) &= 2\overline{N}\left(r,\frac{1}{\left((f^nP(f)^s)^{(k)}\right)^d}\right) = 2\overline{N}\left(r,\frac{1}{\left((f^nP(f)^s)^{(k)}\right)}\right), \\ &\leq 2N_{k+1}\left(r,\frac{1}{f^nP(f)^s}\right) + 2k\overline{N}\left(r,f^n\right) + S\left(r,f^nP(f)^s\right), \\ &= 2(k+1)\overline{N}\left(r,\frac{1}{f}\right) + 2k\overline{N}(r,f) + S(r,f). \end{aligned} \tag{4.55}$$

$$\begin{aligned} 2\overline{N}\left(r,\frac{1}{G}\right) &= 2\overline{N}\left(r,\frac{1}{\left((g^nP(g)^s)^{(k)}\right)^d}\right) = 2\overline{N}\left(r,\frac{1}{\left((g^nP(g)^s)^{(k)}\right)}\right), \\ &= 2(k+1)\overline{N}\left(r,\frac{1}{g}\right) + 2k\overline{N}(r,g) + S(r,g). \end{aligned} \tag{4.56}$$

*Combining (4.50), (4.6), (4.9)-(4.10), (4.53)-(4.56) and (4.14) we have*

$$\begin{aligned} T\left(r,\left((f^nP(f)^s)^{(k)}\right)^d\right) \leq &(6k+8)T(r,f) + (2k+6)T(r,g) + 2k\overline{N}(r,f) \\ &+ 2N\left(r,\frac{g^{n+ms-k}}{(g^nP(g)^s)^{(k)}}\right) + S(r,f) + S(r,g). \end{aligned} \tag{4.57}$$

*Similarly for G*

$$\begin{aligned} T\left(r,\left((g^nP(g)^s)^{(k)}\right)^d\right) \leq &(6k+8)T(r,g) + (2k+6)T(r,f) + 2k\overline{N}(r,g) \\ &+ 2N\left(r,\frac{f^{n+ms-k}}{(f^nP(f)^s)^{(k)}}\right) + S(r,f) + S(r,g). \end{aligned} \tag{4.58}$$

*By Lemma 3.5 we have,*

$$(n+ms-2k)dT(r,f) + kdN(r,f) + dN\left(r,\frac{f^{n+ms-k}}{(f^nP(f)^s)^{(k)}}\right) \leq dT\left(r,(f^n)^{(k)}\right). \tag{4.59}$$

$$(n+ms-2k)dT(r,g) + kdN(r,g) + dN\left(r,\frac{g^{n+ms-k}}{(g^nP(g)^s)^{(k)}}\right) \leq dT\left(r,(g^n)^{(k)}\right). \tag{4.60}$$

*From (4.57)-(4.60), we have*

$$\begin{aligned} &(n+ms-2k)dT(r,f) + kdN(r,f) + dN\left(r,\frac{f^{n+ms-k}}{(f^nP(f)^s)^{(k)}}\right) \\ &\quad + (n+ms-2k)dT(r,g) + kdN(r,g) + dN\left(r,\frac{g^{n+ms-k}}{(g^nP(g)^s)^{(k)}}\right) \\ &\leq (6k+8)T(r,f) + (2k+6)T(r,g) + 2k\overline{N}(r,f) + 2N\left(r,\frac{g^{n+ms-k}}{(g^nP(g)^s)^{(k)}}\right) \\ &\quad + (6k+8)T(r,g) + (2k+6)T(r,f) + 2k\overline{N}(r,g) + 2N\left(r,\frac{f^{n+ms-k}}{(f^nP(f)^s)^{(k)}}\right) \\ &\quad + S(r,f) + S(r,g). \end{aligned} \tag{4.61}$$

*Since $d \geq 2$*

$$dN\left(r, \frac{f^{n+ms-k}}{(f^nP(f)^s)^{(k)}}\right) \geq 2N\left(r, \frac{f^{n+ms-k}}{(f^nP(f)^s)^{(k)}}\right), \tag{4.62}$$

$$dN\left(r, \frac{g^{n+ms-k}}{(g^nP(g)^s)^{(k)}}\right) \geq 2N\left(r, \frac{g^{n+ms-k}}{(g^nP(g)^s)^{(k)}}\right), \tag{4.63}$$

$$kdN(r,f) \geq 2k\overline{N}(r,f), \tag{4.64}$$

$$kdN(r,g) \geq 2k\overline{N}(r,g). \tag{4.65}$$

*Therefore, on combining above all (4.62)-(4.65) equations*

$$(nd + msd - 2kd - 8k - 14)\{T(r,f) + T(r,g)\} \leq S(r,f) + S(r,g), \tag{4.66}$$

*which contradicts with $n > 2k - ms + \frac{8k+14}{d}$. Hence $H \equiv 0$. Similar to the arguments in Theorem 2.1., we see that Theorem 2.2. holds.*

***Proof of Theorem 2.3.***

*Since $F$ and $G$ are entire functions, we have $\overline{N}(r,f) = \overline{N}(r,g) = 0$. Proceeding as in the proof of Theorem 2.1. and applying Lemma 3.5 we shall obtain that Theorem 2.3. holds.*

***Proof of Theorem 2.4.***

*Since $F$ and $G$ are entire functions, we have $\overline{N}(r,f) = \overline{N}(r,g) = 0$. Proceeding as in the proof of Theorem 2.2. and applying Lemma 3.5 we shall obtain that Theorem 2.4. holds.*

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