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LEFT ANNIHILATOR OF IDENTITIES WITH DERIVATION

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Abstract *The main motive of this paper is to study the commutativity of prime and semiprime rings. Let R be a prime ring, $a \neq 0 \in R$ and I a non zero ideal of R. If d is a derivation on R satisfying the identities (i) $a\Big([d(x), d(y)]_m - [x, y]^n\Big) = 0$, (ii) $a\Big(d([x, y])^m - [x, y]_n\Big) = 0$ for all $x, y \in I$, then R is a commutative ring.*

1 Introduction

Throughout this paper, unless specifically stated, R always denotes a prime ring with center $Z(R)$ and with extended centroid C, Q its two sided Martindale quotient ring. For any a, $b \in R$, a ring R is said to be prime if whenever $aRb = 0$ implies $a = 0$ or $b = 0$ and is semiprime if for any $a \in R$, $aRa = 0$ implies $a = 0$. A mapping f is called an additive mapping on R if $f(x + y) = f(x) + f(y)$ holds for all x, $y \in R$. By d, we mean a derivation of R. An additive mapping $d : R \to R$ is called a derivation of R, if $d(xy) = d(x)y + xd(y)$ holds for all $x, y \in R$. Let R be a ring and $S \subseteq R$. A mapping $f : R \to R$ is called strong-commutativity preserving (scp) on S, if $[f(x), f(y)] = [x, y]$ for all $x, y \in S$. Given $x, y \in R$, we set $[x, y]_0 = x,\ [x, y]_1 = [x, y] = xy - yx$ and $[x, y]_k = [[x, y]_{k-1}, y]$ for $k > 1$.

The Engel type identity with derivation appeared first time in the well known paper of Posner [28]. Posner [28] proved that for a nonzero derivation d of R, if $[d(x), x] \in Z(R)$ for all $x \in R$, then R is commutative. Daif and Bell [12] proved that if R is a semiprime ring and I is a two-sided ideal of R such that $d([x, y]) = \pm[x, y]$ for all $x, y \in I$, then I is central ideal.

In [7], Ashraf and Rehman proved that if I is a nonzero ideal of R such that R satisfies any one of the following conditions: (i) $d([x, y]) = [x, y]$, (ii) $d(xy + yx) = xy + yx$, (iii) $d(x)d(y) + d(y)d(x) = xy + yx$; for all $x, y \in I$, then R is commutative. In 2009, Argaç and Inceboz [6] proved the result by considering $d(xy + yx)^n = xy + yx$ for all $x, y \in I$, where I is a nonzero ideal of R and $n \geq 1$ fixed integer, and then obtained that R is commutative. On the other hand, Herstein [19] proved that if char $(R) \neq 2$ and R admits a nonzero derivation d on R such that $[d(x), d(y)] = 0$ for all $x, y \in R$, then R is commutative. For semiprime ring R, Bell and Daif [9] proved that the nonzero right ideal ρ of R is central, if $[d(x), d(y)] = [x, y]$ for all $x, y \in \rho$. In [29], Sharma and Dhara proved that if char $(R) \neq 2$ and d a nonzero derivation of R such that $[[d(x), d(y)]_n, [y, x]_m] = 0$ for all $x, y \in R$, where $m, n \geq 0$ are fixed integers, then R must be commutative. Further, number of algebraists study the structure of rings via derivations. For example, we refer the reader to ([5, 1, 2, 3, 4]), where further references can be found).

In [13], Filippis showed that if R is of characteristic different from 2 and I a nonzero ideal of R such that $[[d(x), x], [d(y), y]] = 0$ for all $x, y \in R$, then R is commutative. Dhara proved the same conclusion in [17] when $[[d(x), x]_n, [y, d(y)]_m]^t = 0$ for all $x, y \in R$, where $n \geq 0, m \geq 0, t \geq 1$ are fixed integers.

There are many papers which study the identities involving derivations with left annihilator conditions. In [16], Dhara and Sharma studied the case when $a[[d(x), x]_n, [y, d(y)]_m]^t = 0$ for all $x, y \in R$, $0 \neq a \in R$ fixed and proved that if $d \neq 0$, then prime ring R must be commutative.

In [15], Dhara et al. proved that if $0 \neq a \in R$ such that $a((d(xy+yx))^n - (xy+yx))^m = 0$ for all $x, y \in I$, where I is a nonzero ideal of R and $m \geq 1, n \geq 1$ are fixed integers, then R must be commutative. Further, Huang [21] proved the following:

Let R be a prime ring with a derivation d, I a nonzero ideal of R and m, n fixed positive integers. (i) If $(d[x, y])^m = [x, y]_n$ for all $x, y \in I$, then R is commutative. (ii) If char $(R) \neq 2$ and $[d(x), d(y)]_m = [x, y]^n$ for all $x, y \in I$, then R is commutative.

Note that in the second result, it is assumed that char $(R) \neq 2$. In the present paper, our aim is to study the same situation of Huang [21] with left annihilator condition in a prime ring. In our result, the characteristic assumption is omitted. We prove that if R is a prime rings with a derivation d, I a nonzero ideal of R and $0 \neq a \in R$ such that (i) $a((d[x, y])^m - [x, y]_n) = 0$ for all $x, y \in I$, or (ii) $a([d(x), d(y)]_m - [x, y]^n) = 0$ for all $x, y \in I$, where m, n are fixed positive integers, then R must be commutative. Moreover, we also study the situations in semiprime rings.

It is well known that any derivation of R can be uniquely extended to a derivation of Q, and so any derivation of R can be defined on the whole of Q. Moreover Q is a prime ring as well as R and the extended centroid C of R coincides with the center of Q. We refer to [8, 26] for more details.

Denote by $Q *_C C\{X, Y\}$ the free product of the C-algebra Q and $C\{X, Y\}$, the free C-algebra in noncommuting indeterminates X, Y.

We mention a very important result which will be used quite frequently as follows:

Theorem 1.1. *[23, Kharchenko] Let R be a prime ring, d a nonzero derivation on R and I a nonzero ideal of R. If I satisfies the differential identity*

$$f(r_1, r_2, \ldots, r_n, d(r_1), d(r_2), \ldots, d(r_n)) = 0$$

for any $r_1, r_2, \ldots, r_n \in I$ then any one of following holds:
(i) I satisfies the generalized polynomial identity $f(r_1, r_2, \ldots, r_n, x_1, x_2, \ldots, x_n) = 0$;
(ii) d is Q-inner i.e., for some $q \in Q$, $d(x) = [q, x]$ and I satisfies the generalized polynomial identity $f(r_1, r_2, \ldots, r_n, [q, r_1], [q, r_2], \ldots, [q, r_n]) = 0$.

2 Main Results

Theorem 2.1. *Let R be a prime ring, I a nonzero ideal of R and $0 \neq a \in R$. Suppose that d is a derivation of R and $n \geq 1, m \geq 1$ fixed integers such that $a\Big([d(x), d(y)]_m - [x, y]^n\Big) = 0$ for all $x, y \in I$. Then R is commutative.*

Proof. If R is commutative, then we have our conclusion. So we assume that R is noncommutative and then we show a number of contradictions. If $d = 0$, then our hypothesis reduces to

$$a[x, y]^n = 0 \tag{2.1}$$

for all $x, y \in I$. By [11, Theorem 2], this generalized polynomial identity (GPI) is also satisfied by Q and hence by R. Let $w = [x, y]^n$. Then $aw = 0$. From (2.1) we can write $a[p, wqa]^n = 0$ for all $p, q \in R$. Since $aw = 0$, it reduces to $a(pwqa)^n = 0$. This can be written as $(wqap)^{n+1} = 0$ for all $p, q \in R$. By Levitzki's lemma [20, Lemma 1.1], $wqa = 0$ for all $q \in R$. Since R is prime, either $a = 0$ or $w = 0$. Since $a \neq 0$, $w = [x, y]^n = 0$ for all $x, y \in R$. This is a polynomial identity. By [24, lemma 1], there exists a field F such that $R \subseteq M_m(F)$ with $m > 1$ and R and $M_m(F)$ satisfy the identity $[x, y]^n = 0$. But by choosing $x = e_{12}$ and $y = e_{21}$, we get $[x, y]^n = e_{11} + (-1)^n e_{22}$, which is a contradiction.

Now we assume $d \neq 0$. Now we divide the proof in two parts:

Case-1: If d is not Q-inner, then by Kharchenko's Theorem [23] we have from the assumption that

$$a\Big([u, v]_m - [x, y]^n\Big) = 0 \tag{2.2}$$

for all $x, y, u, v \in R$. In particular, for $u = 0$, we have

$$a[x, y]^n = 0$$

for all $x, y \in R$. Then by same argument as before, we get $a = 0$, a contradiction.

Case-2: We assume the case when d is Q-inner derivation, for some $b \in Q$ let $d(x) = [b, x]$ for all $x \in R$. Since $d \neq 0$, $b \notin C$. Set $f(x, y) = a\Big([[b, x], [b, y]]_m - [x, y]^n\Big) = 0$. Then $f(x, y)$ is a nontrivial generalized polynomial identity (GPI) for R. By Chuang [11, Theorem 2], $f(x, y)$ is also a GPI for Q. Denote by F either the algebraic closure of C or C according as C is either infinite or finite, respectively. Then, by a standard argument (see for instance, [25, Proposition], $f(x, y)$ is also a GPI for $Q \otimes_C F$. Since $Q \otimes_C F$ is centrally closed prime F-algebra [18, Theorem 2.5 and 3.5], by replacing R, C with $Q \otimes_C F$ and F, respectively, we may assume R is centrally closed and C is either finite or algebraically closed. By Martindale's Theorem [27], R is then a primitive ring having nonzero socle H with C as the associated division ring. Hence by Jacobson's Theorem [22, p.75] R is isomorphic to a dense ring of linear transformations of some vector space V over C, and H consists of the linear transformations in R of finite rank. If V is finite dimensional over C then the density of R on V implies that $R \cong M_k(C)$, where $k = \dim_C V$.

Since R is noncommutative, $\dim_C V \geq 2$.

We show that for any $v \in V$, v and bv are linearly C-dependent. Suppose that v and bv are linearly independent for some $v \in V$. By density there exist $x, y \in R$ such that

$$\begin{aligned} xv &= bv, \quad xbv = b^2v, \\ yv &= 0, \quad ybv = -v. \end{aligned}$$

Then $[[b, x], [b, y]]_m v = 0$ and $[x, y]^n v = v$. Hence

$$0 = a\Big([[b, x], [b, y]]_m - [x, y]^n\Big)v = av.$$

This implies that if $av \neq 0$, then v and bv are linearly C-dependent. Now suppose that $av = 0$. Since $a \neq 0$, there exists $w \in V$ such that $aw \neq 0$ and then $a(v + w) = aw \neq 0$. By the previous argument we have that w, bw are linearly C-dependent and $(v + w), b(v + w)$ are also. Thus there exist $\alpha, \beta \in C$ such that $bw = w\alpha$ and $b(v + w) = (v + w)\beta$. Moreover, v and w are clearly C-independent and so by density there exist $x, y \in R$ such that

$$\begin{aligned} xw = 0, \quad xv = v + w, \\ yw = v + w, \quad yv = v. \end{aligned}$$

Then $[x, y]w = v + w$, $[x, y]^2 w = w$, $[[b, x], [b, y]]w = 0$, $[[b, x], [b, y]]v = 0$ and hence by using $av = 0$, we get

$$0 = a\Big([[b, x], [b, y]]_m - [x, y]^n\Big)w = -aw,$$

a contradiction. Hence for each $v \in V$, $bv = v\alpha_v$ for some $\alpha_v \in C$. It is very easy to prove that α_v is independent of the choice of $v \in V$. Thus we can write $bv = v\alpha$ for all $v \in V$ and $\alpha \in C$ fixed.

Now let $r \in R$, $v \in V$. Since $bv = v\alpha$,

$$[b, r]v = (br)v - (rb)v = b(rv) - r(bv) = (rv)\alpha - r(v\alpha) = 0.$$

Thus $[b, r]v = 0$ for all $v \in V$ i.e., $[b, r]V = 0$. Since $[b, r]$ acts faithfully as a linear transformation on the vector space V, $[b, r] = 0$ for all $r \in R$. Therefore, $b \in Z(R)$ implies $d = 0$, a contradiction. □

Theorem 2.2. *Let R be a prime ring and I a nonzero ideal of R. Suppose that d is a derivation of R and $0 \neq a \in R$ such that $a\Big((d([x, y]))^m - [x, y]_n\Big) = 0$ for all $x, y \in I$, $n \geq 1, m \geq 1$ fixed integers, then R is commutative.*

Proof. We assume that R is noncommutative and then we show a number of contradictions. If $n = 1$, then by [14, Corollary 2.5], R must be commutative, a contradiction. Hence assume $n \geq 2$.

First, we consider $d = 0$, then by our hypothesis we have for all $x, y \in I$,

$$a[x, y]_n = 0. \tag{2.3}$$

By [11, Theorem 2], this GPI is also satisfied by Q and hence by R. Let $v = [x, y]_n$. Then $av = 0$. From (2.3) we can write $a[t, v]_n = 0$ for all $t \in R$. Since $av = 0$, it reduces to $atv^n = 0$ for all $t \in R$. Since R is prime and $a \neq 0$, we get $v^n = ([x, y]_n)^n = 0$ for all $x, y \in R$. This is a polynomial identity and hence there exists a field F such that $R \subseteq M_m(F)$ with $m > 1$ and R and $M_m(F)$ satisfy the same polynomial identity [24, lemma 1]. But by choosing $x = e_{12} + e_{21}$ and $y = e_{11}$, we get $[x, y]_n = e_{21} + (-1)^n e_{12}$ and hence $0 = ([x, y]_n)^n$ $= e_{11} + e_{22}$ or $= \pm(e_{21} - e_{12})$, which is a contradiction.

Next, we assume $d \neq 0$. In light of Kharchenko's Theorem [23], we divide the proof into two parts:

Case-1: Let d be Q-outer. Since I satisfies $a\Big((d([x, y]))^m - [x, y]_n\Big) = 0$, that is $a\Big(([d(x), y] + [x, d(y)])^m - [x, y]_n\Big) = 0$, by Kharchenko's Theorem [23], I satisfies $a\Big(([s, y] + [x, t])^m - [x, y]_n\Big) = 0$. In particular, for $s = t = 0$, I satisfies the blended component

$$a[x, y]_n = 0. \tag{2.4}$$

By same argument as before, it yields a contradiction.

Case-2: Next, we assume the case when d is Q-inner derivation, for some $b \in Q$ let $d(x) = [b, x]$ for all $x \in R$. Since $d \neq 0$, $b \notin C$. Set $f(x, y) = a\Big([b, [x, y]]^m - [x, y]_n\Big)$. Then $f(x, y)$ is a nontrivial generalized polynomial identity (GPI) for R. By Chuang [11, Theorem 2], $f(x, y)$ is also a GPI for Q. Denote by F either the algebraic closure of C or C according as C is either infinite or finite, respectively. Then, by a standard argument (see for instance, [25, Proposition]), $f(x, y)$ is also a GPI for $Q \otimes_C F$. Since $Q \otimes_C F$ is centrally closed prime F-algebra [18, Theorem 2.5 and 3.5], by replacing R, C with $Q \otimes_C F$ and F, respectively, we may assume R is centrally closed and C is either finite or algebraically closed. By Martindale's Theorem [27], R is then a primitive ring having nonzero socle H with C as the associated division ring. Hence by Jacobson's Theorem [22, p.75], R is isomorphic to a dense ring of linear transformations of some vector space V over C, and H consists of the linear transformations in R of finite rank. If V is finite dimensional over C then the density of R on V implies that $R \cong M_k(C)$ where $k = \dim_C V$.

Since R is noncommutative, $\dim_C V \geq 2$.

We show that for any $v \in V$, v and bv are linearly C-dependent. Suppose that v and bv are linearly independent for some $v \in V$. By density there exist $x, y \in R$ such that

$$xv = 0, \quad xbv = v,$$
$$yv = v, \quad ybv = 0.$$

Then $([b, [x, y]])^m v = v$ and $[x, y]_n v = 0$. Hence

$$0 = a\Big([b, [x, y]]^m - [x, y]_n\Big)v = av.$$

This implies that if $av \neq 0$, then v and bv are linearly C-dependent. Now suppose that $av = 0$. Since $a \neq 0$, there exists $w \in V$ such that $aw \neq 0$ and then $a(v + w) = aw \neq 0$. By the previous argument we have that w, bw are linearly C-dependent and $(v + w), b(v + w)$ are also. Thus there exist $\alpha, \beta \in C$ such that $bw = w\alpha$ and $b(v + w) = (v + w)\beta$. Moreover, v and w are clearly C-independent and so by density there exist $x, y \in R$ such that

$$xw = 0, \quad xv = v + w,$$
$$yw = v + w, \quad yv = v.$$

Then $[x,y]w = v+w$, $[x,y]_2 w = -2v$, $[x,y]_3 w = 0$, $[b,[x,y]]w = (v+w)(\beta-\alpha)$, $[b,[x,y]]v = -(v+2w)(\beta-\alpha)$ and $[b,[x,y]]^2 w = -w(\beta-\alpha)^2$.

Hence for $n \geq 2$, $0 = a\Big([b,[x,y]]^m - [x,y]_n\Big)w = \pm aw(\beta-\alpha)^m$. Since $aw \neq 0$, $\alpha = \beta$ and so $bv = v\alpha$ contradicting the independent of v and bv. Hence for each $v \in V$, $bv = v\alpha_v$ for some $\alpha_v \in C$. It is very easy to prove that α_v is independent of the choice of $v \in V$. Thus we can write $bv = v\alpha$ for all $v \in V$ and $\alpha \in C$ fixed.

Now let $r \in R$, $v \in V$. Since $bv = v\alpha$,

$$[b,r]v = (br)v - (rb)v = b(rv) - r(bv) = (rv)\alpha - r(v\alpha) = 0.$$

Thus $[b,r]v = 0$ for all $v \in V$ i.e., $[b,r]V = 0$. Since $[b,r]$ acts faithfully as a linear transformation on the vector space V, $[b,r] = 0$ for all $r \in R$. Therefore $b \in Z(R)$ implies $d = 0$, which is a contradiction. □

Our next theorem is on semiprime rings. Let R be a semiprime ring and U be its left Utumi quotient ring. Then $C = Z(U)$ is the extended centroid of R ([10, p-38]). Let $M(C)$ be the set of all maximal ideals of C.

Theorem 2.3. *Let R be a noncommutative semiprime ring, U the left Utumi quotient ring of R, $C = Z(U)$ the extended centroid of R, d a derivation on R and $0 \neq a \in R$. If any one of the following holds:*

(i) $a\Big([d(x),d(y)]_m - [x,y]^n\Big) = 0$ for all $x,y \in R$,

(ii) $a\Big((d([x,y]))^m - [x,y]_n\Big) = 0$ for all $x,y \in R$, where $n \geq 1, m \geq 1$ fixed integers, then there exists central idempotent $e \in U$ such that eU is commutative and $(1-e)a = 0$.

Proof. First we consider the case $a\Big([d(x),d(y)]_m - [x,y]^n\Big) = 0$ for all $x,y \in R$. Other case is similar. We known the fact that any derivation of a semiprime ring R can be uniquely extended to a derivation of its left Utumi quotient ring U and so any derivation of R can be defined on the whole of U [26, Lemma 2]. Moreover R and U satisfy the same GPIs as well as same differential identities. Thus

$$a\Big([d(x),d(y)]_m - [x,y]^n\Big) = 0$$

for all $x,y \in U$. Let $M(C)$ be the set of all maximal ideals of C and $P \in M(C)$. Now by the standard theory of orthogonal completions for semiprime rings (see [26, p.31-32]), we have PU is a prime ideal of U invariant under all derivations of U. Moreover, $\bigcap\{PU \mid P \in M(C)\} = 0$. Set $\overline{U} = U/PU$. Then derivation d canonically induces a derivation $\overline{d}$ on $\overline{U}$ defined by $\overline{d}(\overline{x}) = \overline{d(x)}$ for all $x \in U$. Therefore,

$$\overline{a}\Big([\overline{d}(\overline{x}),\overline{d}(\overline{y})]_m - [\overline{x},\overline{y}]^n\Big) = 0$$

for all $\overline{x},\overline{y} \in \overline{U}$. By the prime ring case of Theorem 2.1, we have either $[\overline{U},\overline{U}] = 0$ or $\overline{a} = 0$. In any case we have $a[U,U] \subseteq PU$ for all $P \in M(C)$. Since $\bigcap\{PU \mid P \in M(C)\} = 0$, $a[U,U] = 0$. By using the theory of orthogonal completion for semiprime rings (see [8, Chapter 3]), there exists central idempotent $e \in U$ such that eU is commutative and $(1-e)a = 0$. Hence the theorem is proved. □

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