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ON POWER DOMINATOR CHROMATIC NUMBER OF CERTAIN CLASSES OF GRAPHS

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Abstract The power dominator chromatic number χ_{pd} (G) of a simple connected graph G = (V, E) is a proper coloring of G such that every vertex of V power dominates every vertex of at least one color class of G. In this paper, we obtain power dominator chromatic number of Broom graph. We also derive the power dominator chromatic number of middle, line and total graphs of broom graph, olive tree and double star.

1 Introduction

A mathematical structure known as a graph G = (V, E) is made up of a finite number of elements, known as vertices, and a finite number of pairs of vertices, known as edges. The theory of domination in graph theory is a significant and actively researched area of research with applications in many areas. Graph coloring and theory of domination are two of the most significant problems related to graph theory, algorithms and combinatorial optimizations. Both areas have an extensive range of research opportunities in the fields of applied sciences. In the concept of graph coloring, it is necessary to color the vertices using different colors in such a way that both the end vertices of the edges receive different colors. Some graph-theoretic properties of the line graph associated with unit graph is investigated in [6]. Rainbow dominator chromatic number for some standard graphs were introduced in [5]. The domination theory is concerned with a dominating set, which requires the minimum set of vertices such that every vertex of a graph not in the set has a neighbor in it. Coloring ([1], [9], [11]) and domination in graphs are well studied areas in graph theory. Electric power providers must continuously check the state of their system according to a number of state variables such as the magnitude of the voltage at loads and the generator machine phase angle. Placing phase measuring units (PMU's) at certain system locations is one way to monitor these variables. Because of a PMU's high cost, it is preferable to keep the number of them to a minimum while yet having the ability to observe (monitor) the entire system. The well-known vertex coloring and domination problems are strongly related to the graph theory problem of determining the minimal number of PMU's to monitor the entire system. As a result, this area of study is relevant not just to the power system sector but also to graph theory as a whole. Haynes et al.[13] introduced a variant of domination in graphs, called power domination, studying the problem of monitoring the state of an electric power system.
A subset S is a dominating set [11] of G with vertex set V, if every node in V-S has at least one neighbor in S. A subset of V is a power dominating set [11] of G = (V, E), if all the vertices of V can be observed by the following observation rules:

- Any vertex that is incident to an observed edge is observed.
- Any edge joining two observed vertices is observed.
- If a vertex is incident to a total of $k \geq 1$ edges and if $k-1$ of these edges are observed, then all k of these edges are observed.

Combine the concepts of power domination and dominator coloring to create the new concept called power dominator coloring, requiring for each vertex to power dominate every other vertex in a color class. The concept of a power dominator chromatic number was recently introduced by Sathish Kumar et al. [7]. For extensive results of power dominator coloring in graphs refer([3], [4], [7]). In this paper, only connected, finite and undirected graphs are taken into consideration.

2 Power Dominator Chromatic Number of Graphs

A power dominator coloring of a simple connected graph G = (V, E) is a proper coloring of G such that every vertex of V power dominates every vertex of some color class of G. The power dominator chromatic number χ_{pd} (G) is the minimum number of colors required for a power dominator coloring of G. In Figure 1 , vertices can be colored by at least 4 colors C_1, C_2,C_3 and C_4 to power dominate all the vertices.

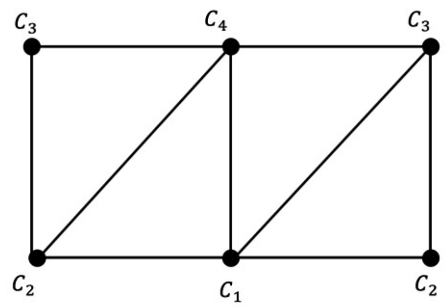

Figure 1. $\chi_{pd}(G) = 4$

A broom graph $B_{n,m}$ is a specific kind of graph on *n* vertices, having a path P with *m* vertices and *n-m* pendant vertices, all of these being adjacent to either the origin u or the terminus *v* of the path P. If d (*v*) = 1, then *v* is called a pendant vertex. A tree containing exactly two non-pendant vertices is called a double-star. A double-star $S_{m,n}$ with *m* vertices adjacent to one non-pendant vertex and *n* vertices adjacent to the other non-pendant vertex has degree sequence. Suppose that the non-pendant vertices $u_1, u_2 \in V(S_{m,n})$ and $d(u_1) = m + 1$ and $d(u_2) = n + 1$ without loss of generality, then $e = u_1u_2$ is called the central edge of the double-star. Power dominator chromatic number for double star has been studied in [2].

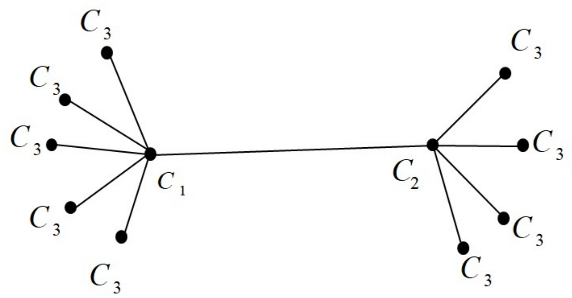

Figure 2. $\chi_{pd}(S_{5,4}) = 3$

Olive tree T_k is a rooted tree consisting of *k* branches where the i^{th} branch is a path of length *i*. The power dominator chromatic number of olive tree has been studied in [2]. The Figure 3 is an Olive tree of order 4.

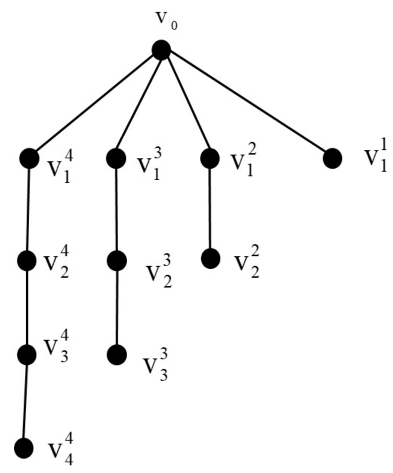

Figure 3. Olive tree T_4

Theorem 2.1. *For the broom graph $B_{n,m}$ the power dominator chromatic number,* $\chi_{pd}(B_{n,m}) = 3$.

Proof. Let $B_{n,m}$ be the broom graph. Let $V(B_{n,m}) = \{v_1, v_2, ..., v_m, v_{m+1}, ..., v_n\}$ be the vertex set, the vertices $v_1, v_2, v_3, ..., v_m$ are in a path and the vertices $v_{m+1}, ..., v_n$ are the pendant vertices that are adjacent to the vertex v_m. The vertex v_m will power dominate all the vertices in $B_{n,m}$. Assign color C_1 to the vertex v_m , C_2 to the vertices $v_{2i+1}(0 \leq i < m)$ and C_3 to the vertices $v_{2i}(1 \leq i < m)$. Further, all the pendant vertices will assigned either by the color classes C_2 or C_3. All the vertices of $B_{n,m}$ power dominates the vertex v_m and hence the vertices power dominates the color class C_1. Thus $\chi_{pd}(B_{n,m}) = 3$. □

3 Power Dominator Chromatic number of line graphs

In this section, we give Power Dominator Chromatic Number for line graphs of double star, broom Graph and olive tree. The line graph L (G) of a connected graph G is a graph such that

- each vertex of L(G) corresponds to an edge of G
- two vertices of L(G) are adjacent iff their corresponding edges share a common end point.

Theorem 3.1. *The power dominator chromatic number of the line graph of double star $S_{m,n}$,* $\chi_{pd}(L(S_{m,n})) = max(m, n) + 1$.

Proof. The line graph of double star $L(S_{m,n})$ is a graph that has two cliques of sizes $m + 1$ and $n + 1$ with a common vertex. By assigning a color to the common vertex, the remaining vertices can be colored with m distinct colors if $m \geq n$; otherwise with n distinct colors. The common vertex power dominates all the remaining vertices. Thus $\chi_{pd}(L(S_{m,n})) = max(m, n) + 1$. □

Example 3.2. The power dominator chromatic number of $L(S_{5,4})$ is presented in Figure 4.

Theorem 3.3. *[8] For the Lollipop graph $L_{m,n}$, $(n \geq 2, m \geq 1)$, the power dominator chromatic number is n. (i.e)* $\chi_{pd}(L_{m,n}) = n$.

Theorem 3.4. *The power dominator chromatic number of line graph of broom graph,* $\chi_{pd}(L(B_{n,m})) = n - m + 1$.

Proof. By the definition of line graph, the edge set $e_1, e_2, ..., e_{n-1}$ of $B_{n,m}$ forms the vertices for $L(B_{n,m})$. The vertices $e_{m-1}, e_m, e_{m+1}, ..., e_{n-1}$ are adjacent with each other and those vertices forms a clique. The remaining vertices $e_1, e_2, ..., e_{m-2}$ forms a path and connected with the vertex e_{m-1}. This is a lollipop graph having $n - m + 1$ vertices in the clique with path of length $m - 1$. By theorem 3.3, the power dominator chromatic number is $n - m + 1$. Thus $\chi_{pd}(L(B_{n,m})) = n - m + 1$. □

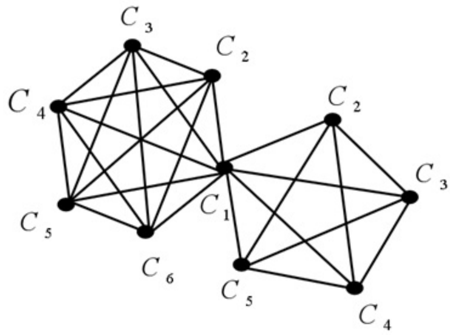

Figure 4. $\chi_{pd}(L(S_{5,4})) = 6$

Example 3.5. The power dominator chromatic number of line graph of broom graph, $L(B_{9,5})$ is given in Figure 5.

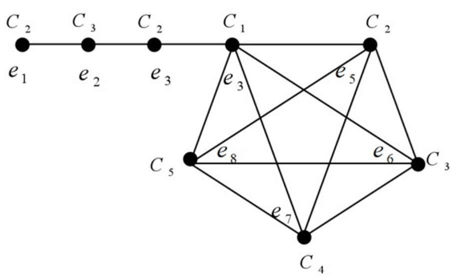

Figure 5. $\chi_{pd}(L(B_{9,5})) = 5$

Theorem 3.6. *For an olive tree T_k, the power dominator chromatic number of line graph of the olive tree,*
$\chi_{pd}(L(T_k)) = k$.

Proof. In $L(T_k)$, the *k* edges incident at the common vertex of T_k forms a clique of order *k* and each vertex except one vertex of the clique is having an end with new path $P_i, i = 2, 3, ...k$. Assign *k* different colors to the vertices on the clique, vertex in the path $P_i, i = 2, 3, ...k$ can be accommodated alternatively by the colors C_1 and $C_i, i = 2, 3, ..., k$, where as the vertex adjacent to the clique can take color C_1. Thus every vertex in the graph will power dominates at least one color class. □

Example 3.7. The power dominator chromatic number of $L(T_5)$ is presented in Figure 6.

4 Power Dominator Chromatic number of Middle graphs

In this section, we give Power Dominator Chromatic Number for middle graphs of double star, broom Graph and olive tree. The middle graph of a connected graph G denoted by $M(G)$ is defined as follows: The collection of vertices in M(G) is $V(G) \cup E(G)$, where two vertices are adjacent if

- they are adjacent edges of G or
- one is a vertex of G and the other is an edge incident with it.

Theorem 4.1. *The power dominator chromatic number of middle graph of a double star,*
$\chi_{pd}(M(S_{m,n})) = m + n + 2$.

Proof. By the definition of middle graph, the vertices$\{v_1, e_1, e_2, ..., e_m\}$ and $\{e_1, v_2, e'_1, e'_2, ..., e'_n\}$ forms two different cliques having e_1 as a common vertex. The vertices $\{e_2, e_3, ..., e_m\}$ and

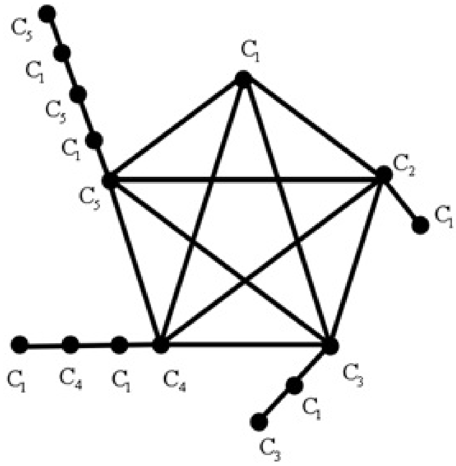

Figure 6. $\chi_{pd}(L(T_5)) = 5$

$\{e'_1, e'_2, ..., e'_n\}$ in $M(S_{m,n})$ having pendant vertices $\{v_3, v_4, ..., v_{m+2}\}$ and $\{v'_1, v'_2, ..., v'_n\}$ respectively. So, the vertices $\{v_3, v_4, ..., v_{m+2}\}$ and $\{v'_1, v'_2, ..., v'_n\}$ will power dominates respectively the vertices $\{e_2, e_3, ..., e_{m+1}\}$ and $\{e'_1, e'_2, ..., e'_n\}$ only. The vertices $\{e_2, e_3, ..., e_{m+1}\}$ and $\{e'_1, e'_2, ..., e'_n\}$ can be assigned by $m+n$ color classes and two more color classes are needed for the set of vertices $\{v_1, v_2\}$ and $\{e_1\}$ for making all vertices of the graph to power dominates at least one color class. □

Example 4.2. The middle graph of a double star $S_{5,4}$ is presented in Figure 7.

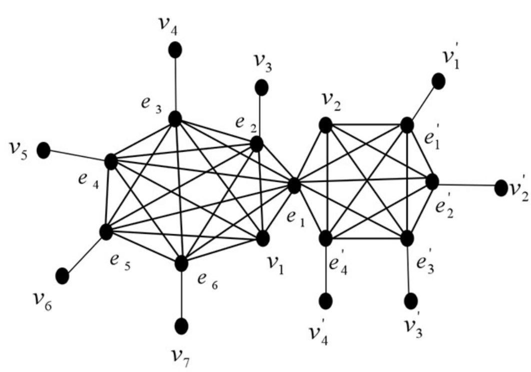

Figure 7. $\chi_{pd}(M(S_{5,4})) = 11$

The colors assign to the vertices to power dominate the graph are as follows:
$C_1 = \{v_1, v_2, v_3, v_4, v_5, v_6, v_7, v'_1, v'_2, v'_3, v'_4\}$; $C_2 = \{e_2\}$; $C_3 = \{e_3\}$; $C_4 = \{e_4\}$; $C_5 = \{e_5\}$; $C_6 = \{e_6\}$; $C_7 = \{e_1\}$; $C_8 = \{e'_1\}$; $C_9 = \{e'_2\}$; $C_{10} = \{e'_3\}$; $C_{11} = \{e'_4\}$.

Theorem 4.3. *The power dominator chromatic number of middle graph of the broom graph* $B_{n,m}$,

$$\chi_{pd}(M(B_{n,m})) = \begin{cases} n - m + 2 + \lceil \frac{m-1}{2} \rceil, & m \text{ is odd} \\ (\frac{2n-m}{2}) + 2, & m \text{ is even.} \end{cases}$$

Proof. In a broom graph $B_{n,m}$, the vertex set is $\{v_1, v_2, ..., v_m, v_{m+1}, ..., v_n\}$ and the edge set is $\{e_1, e_2, ..., e_{n-1}\}$. Among all the *n* vertices, *m* vertices are in the path and *n-m* vertices are connected as pendant vertices with v_m. From the definition of middle graph, in $M(B_{n,m})$ the vertex v_m and the vertices $e_{m-1}, e_m, e_{m+1}, ..., e_{n-1}$ are adjacent to each other and those vertices form a clique. Also the vertices $v_{m+1}, v_{m+2}, ..., v_n$ are pendant vertices of $e_m, e_{m+1}, e_{m+2}, ..., e_{n-1}$ respectively. Assign *n-m+2* different color classes to power dominate the clique formed by the vertex v_m and the vertices $\{e_{m-1}, e_m, e_{m+1}, ..., e_{n-1}\}$. Length of the horizon is m-1. It is enough to assign $\lceil\frac{m-1}{2}\rceil$ color classes to power dominate all the m-1 vertices in the horizon when *m* is odd and we have to assign $\frac{m}{2}$ color classes when *m* is even. So the power dominator chromatic number of $M(B_{n,m})$ is $n-m+2+\lceil\frac{m-1}{2}\rceil$ when *m* is odd and $(\frac{2n-m}{2})+2$ when *m* is even. □

Example 4.4. The middle graph of a broom graph, $B_{9,5}$ is presented in Figure 8.

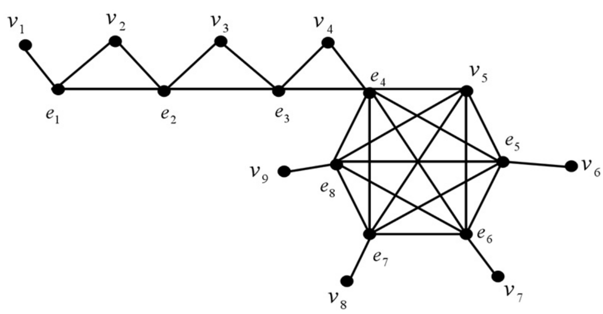

Figure 8. $\chi_{pd}(M(B_{9,5})) = 8$

The colors assign to the vertices to power dominate the graph in Figure 8 are as follows:
$C_1 = \{v_1, v_2, v_3, v_4, v_5, v_6, v_7, v_8, v_9\}$; $C_2 = \{e_1\}$; $C_3 = \{e_3\}$; $C_4 = \{e_2, e_4\}$; $C_5 = \{e_5\}$; $C_6 = \{e_6\}$; $C_7 = \{e_7\}$;$C_8 = \{e_8\}$.

Theorem 4.5. *The power dominator chromatic number for middle graph of olive tree,*

$$\chi_{pd}(M(T_k)) = \begin{cases} k+1+\lceil\frac{k^2-1}{4}\rceil, & k \text{ is even} \\ (\frac{k^2+4k+3}{2})+4, & k \text{ is odd.} \end{cases}$$

Proof. By the definition of middle graph, the root vertex v_0 and the first internal nodes $\{v_1^1, v_1^2, v_1^3, ..., v_1^k\}$ are connected with each other. The vertex set $\{v_1^1, v_1^2, v_1^3, ..., v_1^k\}$ are connected with the vertices and edges in the path $P_i, i = 2, 3, ..., k$ of the olive tree T_k. First we have to assign $k+1$ different colors to v_0 and $v_1^1, v_1^2, v_1^3...v_1^k$, The remaining vertices in the graph $M(T_k)$ can be colored by $(\frac{k^2-1}{4})$ color classes when k is odd or $\lceil\frac{k^2-1}{4}\rceil$ color classes when k is even to make all the vertices to power dominates at least one color class. □

Example 4.6. The power dominator chromatic number of middle graph of olive tree, $M(T_4)$ is presented in Figure 9.

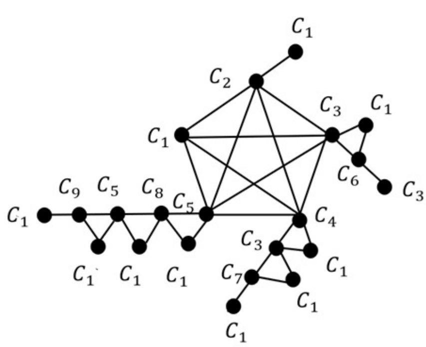

Figure 9. $\chi_{pd}(M(T_4)) = 9$

5 Power Dominator Chromatic number of Total graphs

In this section, we give Power Dominator Chromatic Number for total graphs of double star, broom Graphs and olive tree . The total graph $T(G)$ of a graph G is a graph such that the vertex set of $T(G)$ corresponds to the vertices and edges of G and two vertices are adjacent in $T(G)$ iff their corresponding elements are either adjacent or incident in G. Total graphs are generalization of line graphs.

Theorem 5.1. *The power dominator chromatic number of total graph of double star ,* $\chi_{pd}(T(S_{m,n})) = max(m,n) + 3$.

Proof. By the definition of total graph, the pendant edges of v_1 and v_2 in $S_{m,n}$ adjacent with each other along with e_1. All the vertices in the graph are connected with at least one of the above vertices. By assigning three different color classes to those vertices and maximum of (m,n) colors can be used for remaining vertices will make all the vertices of the graph to power dominate at least one color class.

□

Example 5.2. The total graph of double star $T(S_{5,4})$ is presented in Figure 10.

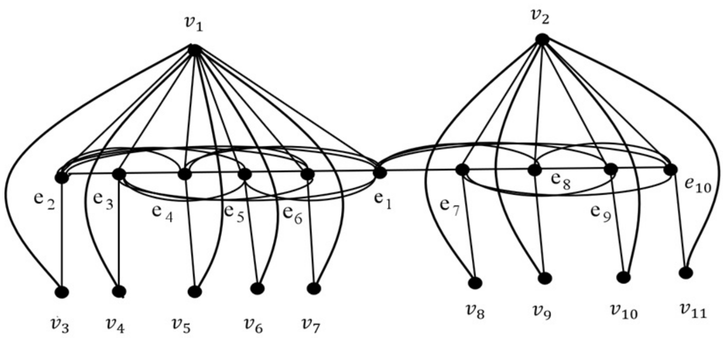

Figure 10. $\chi_{pd}(T(S_{5,4})) = 8$

The colors assign to the vertices to power dominate the graph in Figure 10 are as follows:
$C_1 = \{v_1\}; C_2 = \{v_2\}; C_3 = \{e_1\}; C_4 = \{e_2, e_7, v_7, v_{11}\}; C_5 = \{e_3, e_8, v_6, v_{10}\}; C_6 = \{e_4, e_9, v_4, v_9\}; C_7 = \{e_5, e_{10}, v_5, v_8\}; C_8 = \{e_6, v_3\}$.

Theorem 5.3. *The power dominator chromatic number of total graph of broom graph,* $\chi_{pd}(T(B_{n,m})) = n - m + 2$.

Proof. The vertex set of $T(B_{n,m})$ is $\{v_1, v_2, ..., v_m, ..., v_n, e_1, e_2, ..., e_{n-1}\}$. From the definition of total graph, the vertices $\{e_{m-1}, e_m, ..., e_{n-1}\}$ along with v_m are adjacent to each other with $n-m+2$ vertices. Assign color class C_1 to the vertex v_m and the color classes $\{C_2, C_3, ..., C_{n-m+2}\}$ to the vertices $\{e_{m-1}, e_m, ..., e_{n-1}\}$. All the vertices of $T(B_{n,m})$ will power dominate the vertex v_m. The color classes $\{C_2, C_3, ..., C_{n-m+2}\}$ can be repeated for the vertices $\{v_1, v_2, ..., v_{m-1}, v_{m+1}, ..., v_n, e_1, e_2, ..., e_{m-2}\}$ without violating the concept of coloring. By the above assignment of colors, every vertex of $T(B_{n,m})$ will power dominates all the vertices of at least one color class. □

Example 5.4. The total graph of broom graph $T(B_{9,5})$ is presented in Figure 11.
$C_1 = \{v_5\}; C_2 = \{e_4, v_9, v_1, e_2, v_4\}; C_3 = \{e_5, e_1, v_3\}; C_4 = \{e_6, v_6, v_2, e_3\}; C_5 = \{e_7, v_7\}; C_6 = \{e_8, v_8\}$.

Theorem 5.5. *The power dominator chromatic number for total graph of olive tree,*

$$\chi_{pd}(T(T_k)) = \begin{cases} k + 1 + \lceil \frac{k^2-1}{4} \rceil, & k \text{ is even} \\ (\frac{k^2+4k+3}{2}) + 4, & k \text{ is odd.} \end{cases}$$

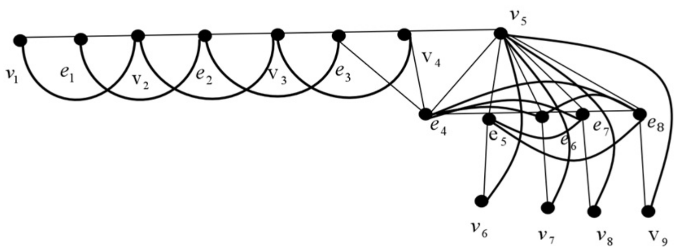

Figure 11. $\chi_{pd}(T(B_{9,5})) = 6$

Proof. In total graph of olive tree, the root vertex v_0 and first internal edges $\{e_1, e_2, ..., e_k\}$ in T_k are connected with each other. Assigning $k+1$ different colors will make these vertices to power dominates at least one color class. In $T(T_k)$ all the vertices and edges of $P_i, i = 2, 3, ..., k$ of the olive tree T_k will appear. Assigning $(\frac{k^2-1}{4})$ color classes to remaining vertices of the graph $T(T_k)$ when k is odd and $\lceil \frac{k^2-1}{4} \rceil$ color classes when k is even to make all the vertices power dominates at least one color class. □

Example 5.6. The power dominator chromatic number of total graph of olive tree, $T(T_4)$ is given in Figure 12 .

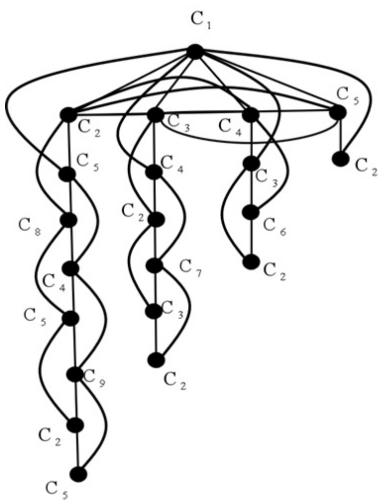

Figure 12. $\chi_{pd}(T(T_4)) = 9$

6 Conclusion

In this paper, the power dominator chromatic number of broom graph and its middle, line and total graph are investigated. Also power dominator chromatic number of middle, line and total graph of olive tree and double star has been investigated. This work can be extended to identifying graph families for which Power dominator chromatic number of middle and total graphs are equal.

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