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A New Methodology of Fuzzy Nonlinear Programming Problems with Linear Inequality Constraints

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Abstract This paper presents a novel approach for solving fuzzy nonlinear programming problems (FNLP) characterized by linear inequality constraints and triangular fuzzy numbers. The proposed method applies the Haar ranking function to convert fuzzy parameters into crisp values, thereby transforming the FNLP into a standard nonlinear programming problem. This enables the use of conventional optimization techniques to obtain optimal solutions. The methodology is systematically illustrated through a detailed numerical example, demonstrating its effectiveness and applicability in handling real-world optimization problems with inherent fuzziness. The advantages of the method include its structured conversion process and the simplicity introduced by using triangular fuzzy numbers, whereas the limitations highlight the computational complexity for larger problems. These findings contribute to advancing optimization techniques in fuzzy environments and provide a practical framework for addressing uncertainty in nonlinear programming scenarios.

1 Introduction

Optimization is essential in decision making across numerous fields, including engineering, economics, and logistics, where it is crucial to determine the best solution under given constraints. Nonlinear programming (NLP) deals with optimization problems in which the objective function or constraints exhibit nonlinearity and add complexities, but also enhances its applicability to real-world problems. However, in many practical cases, the exact values of the parameters involved are often uncertain or imprecise, arising from factors such as system variability, incomplete information, or inherent uncertainties in the data. To account for this ambiguity, fuzzy nonlinear programming (FNLP) extends the traditional NLP by incorporating fuzzy numbers, which represent uncertain parameters. The fuzzy set theory, first introduced by Zadeh in 1965, offers a robust framework for handling uncertainty and imprecision in complex decision-making environments. Among the various types of fuzzy numbers, triangular fuzzy numbers are widely used because of their simplicity and ability to approximate uncertainty with a minimal computational burden. These numbers effectively describe uncertain quantities with three key parameters: the lower bound, most likely value, and upper bound. Solving FNLP problems involves transforming fuzzy parameters into crisp values, allowing the use of conventional optimization techniques. Ranking functions are often employed for this purpose because they convert fuzzy numbers into crisp equivalents while preserving the essential uncertainty characteristics. The Haar ranking method is particularly effective for converting triangular fuzzy numbers, thereby facilitating the application of standard NLP techniques to solve the resulting crisp problem. This study introduces a novel methodology to address fuzzy nonlinear programming problems with linear inequality constraints, where decision variables are modeled as triangular fuzzy numbers. By applying the Haar ranking function, we converted these fuzzy variables into crisp values, transforming the FNLP problem into a standard NLP problem that can be solved using conventional optimization methods. A detailed numerical example is presented to demonstrate the practical effectiveness and applicability of the method in real-world scenarios. The proposed approach offers several advantages, including a structured and systematic process for handling fuzzy data and a straightforward method for solving the resulting crisp nonlinear programming problem. In addition, the use of triangular fuzzy numbers enhances the simplicity and accessibility of the method. However, this approach has certain limitations, particularly regarding the computational complexity for large-scale problems and its exclusive reliance on triangular fuzzy numbers, which may not always be suitable in scenarios requiring more complex representations of uncertainty, such as trapezoidal or linguistic fuzzy numbers. The significance of this study lies in its contribution to the field of fuzzy optimization, providing a practical and effective solution to fuzzy nonlinear programming problems with inequality constraints. By offering a method that systematically transforms fuzzy problems into crisp equivalents, this study addresses the challenges posed by uncertainty in real-world optimization problems, thus expanding the potential applications of FNLP in various fields. In addition, the methodology sets a foundation for future research that may explore more sophisticated fuzzy representations or further refine the computational aspects of solving large-scale FNLP problems.

Nonlinear programming stands out as a frequently employed optimization technique. In many practical scenarios, precise knowledge of model coefficients is challenging due to missing crucial data, system variability, or other factors. Allahviranloo, T., Lotfi, F. H., Kiasary, M. K., Kiani, N. A., & Alizadeh, L. to furnish Modeling and solving optimization problems represent crucial aspects of daily challenges. Recognizing the inherent imprecision in practical data, the fully fuzzy linear programming problem (FFLP) emerges as a potent tool for effectively modeling practical optimization problems. In this study, a novel approach is introduced for solving FFLP after its initial presentation. The method involves the utilization of a linear ranking function to denazify the FFLP, with the establishment of equivalence between two problems substantiated by a set of theorems. Mokhtar S. Bazaraa, Hanif D. Sherali, and C. M. Shetty are to furnish readers with a thorough and profound comprehension of the theory and algorithms of nonlinear programming. Nonlinear programming is concerned with the optimization of problems in which either the objective function, the constraints, or both exhibit nonlinearity. Bellman, R. E., & Zadeh, L. A is anticipated to enhance comprehension regarding decision-making in environments characterized by uncertainty and fuzziness. It provides valuable insights into the practical application of these concepts to real-world issues. Furthermore, the paper is likely to have played a significant role in advancing the use of fuzzy logic as a decision-support tool, particularly in scenarios where conventional binary logic proves insufficient.

Buckley, J. J., & Feuring, T. delves into the application of evolutionary algorithms for effectively addressing challenges in fuzzy problems, particularly those associated with incorporating fuzzy set theory into linear programming. The authors aim to provide insights into the algorithmic solutions, theoretical foundations, and practical applications of this approach. Their objective is to contribute to a broader understanding of the role of evolutionary algorithms in the context of fuzzy linear programming. B. Dharmaraj and S. Appasamy explore the use of a modified Gauss elimination technique in addressing problems characterized by both fuzziness and nonlinearity. The authors aim to present and discuss the adaptation of the modified Gauss elimination method specifically for separable fuzzy nonlinear programming problems. Additionally, they seek to evaluate the effectiveness of this technique in solving such problems. The overarching goal is to contribute to the field of mathematical modeling in engineering problems by introducing a novel approach tailored to handle the intricacies arising from the combination of fuzziness and nonlinearity in optimization scenarios.

Delgado, M., Verdegay, J. L., & Vila, M. A. are designed to facilitate the representation and resolution of optimization problems in situations where the available data or constraints exhibit imprecision or uncertainty. The paper is expected to emphasize the presentation of theoretical foundations for the proposed model, explore its practical applicability, and potentially include illustrative examples to highlight its adaptability in addressing real-world problems. The overarching objective is to contribute to the comprehension and progression of fuzzy linear programming as a valuable decision-making tool in scenarios characterized by vague or imprecise information. The paper is likely centered on presenting the theoretical foundations of fuzzy set modeling in case-based reasoning, highlighting its relevance and potential advantages. The overarching objective is to contribute to the comprehension and practical application of fuzzy set theory as a valuable tool to enhance the capabilities of case-based reasoning systems.

S. C. Fang, C. F. Hu, H. F. Wang, and S. Y. Wu is to investigate the extension of linear programming, a fundamental optimization technique, to accommodate situations where input data contains uncertainty or imprecision due to fuzzy coefficients in the constraints. The paper is anticipated to provide insights into the theoretical foundations, methodologies, and potential applications of this extended linear programming approach with fuzzy constraints. By doing so, the authors aim to contribute to the broader understanding of optimization in scenarios characterized by conditions of uncertainty. K. Ganesan and P. Veeramani to investigate the utilization of trapezoidal fuzzy numbers in the realm of linear programming. The authors aim to establish a structured approach for managing imprecise and uncertain information within optimization models. The paper is anticipated to make contributions in terms of theoretical foundations, methodologies, and potentially practical applications. In doing so, it seeks to advance the comprehension of fuzzy linear programming with trapezoidal fuzzy numbers, particularly in the domain of operations research. P. K. Gupta and M. Mohan, published by Sultan Chand and Sons, is to function as a valuable resource for students, educators, and practitioners. By presenting a diverse set of problems covering various topics within Operations Research, the book aims to offer practical and applied learning experiences. The overarching objective is to enhance accessibility to the subject matter, enabling a more profound understanding of the principles and applications of Operations Research among its readership.

S. M. Hashemi, M. Modarres, E. Nasrabadi, and M. M. Nasrabadi delve into the solution techniques and duality aspects related to linear programming problems. These problems involve coefficients, parameters, or constraints that are entirely represented by fuzzy numbers. The paper is anticipated to make theoretical contributions by providing insights, proposing solution methodologies, and engaging in discussions on the duality of fully fuzzified linear programming. In doing so, it aims to advance the understanding of optimization in scenarios characterized by complete fuzziness. M. Jiménez, M. Arenas, A. Bilbao, and M. V. Rodri examine the effective application of linear programming in scenarios where parameters are characterized by fuzzy sets. The authors aim to present an interactive approach that allows decision-makers to manage uncertainty. The paper is anticipated to make a meaningful contribution to the field by elucidating both the methodology and practical application of this interactive resolution method, thereby advancing the comprehension of optimization when faced with the fuzziness of parameters.

J. Kaur and A. Kumar provide a comprehensive introduction to the incorporation of fuzzy set theory in linear programming. The authors aim to present foundational knowledge, insights, and methodologies pertinent to this application. The paper is anticipated to encompass theoretical aspects, solution approaches, and potentially practical applications. In doing so, it seeks to enhance the understanding of the integration of fuzziness into linear programming models. C. Loganathan and M. Kiruthiga are to introduce and explore the application of a ranking function in addressing issues related to nonlinear programming under the influence of fuzziness. The paper is anticipated to make a meaningful contribution to the field by providing insights into the use of ranking functions as a solution approach. It is expected to offer theoretical foundations, methodologies, and potential practical applications, thereby enhancing the understanding of solving fuzzy nonlinear programming problems. C. Loganathan and M. Lalitha present methods and techniques for efficiently addressing optimization problems that encompass imprecise or uncertain information, particularly in the context of applications within the field of mechanical engineering. The paper is anticipated to offer both theoretical insights and practical approaches to solving problems associated with fully fuzzy nonlinear programming under the constraints of inequalities.

H. R. Maleki, M. Tata, and M. Mashinchi are to investigate the extension of the conventional linear programming model to incorporate fuzzy variables. This extension aims to allow for the representation of imprecision and uncertainty in the optimization process. The paper is anticipated to make contributions in terms of theoretical foundations, methodologies, and potentially practical applications. In doing so, it seeks to advance the understanding of linear programming in situations involving fuzzy variables. J. Ramik is to present innovative ideas and novel findings related to the duality aspects of fuzzy linear programming problems. The author seeks to contribute to both the theoretical understanding and practical applications within the domain of fuzzy optimization and decision-making. H. Sivakumar, K., Appasamy, S. studied fuzzy mathematical approach for solving multi-objective fuzzy linear fractional programming problem with trapezoidal fuzzy numbers.

Tanaka and K. Asai investigated to utilize of fuzzy set theory within the realm of linear programming. The authors seek to establish a framework for effectively managing imprecise and uncertain information. Anticipated contributions include theoretical foundations, methodologies, and potentially practical applications, with the overarching goal of advancing the comprehension of optimization problems in scenarios involving fuzzy numbers. H. J. Zimmermann is to explore the incorporation of fuzzy set theory into programming models and the management of scenarios featuring multiple competing objective functions. The paper is anticipated to provide theoretical insights, methodologies, and potentially practical applications, thereby enriching the comprehension of optimization problems in situations involving both fuzzy sets and multiple objectives.Karthick.S, (2024) proposed to solve the neutrosophic linear fractional programming problem with triangular neutrosophic numbers. Khalifa (2019) presents a fully fuzzy framework where all parameters are represented by fuzzy numbers, making the model more realistic for situations involving uncertainty and imprecision,The key feature of this approach is its innovative handling of fuzziness in both the objective function and constraints. Instead of the traditional crisp or partially fuzzy methods.Khalifa (2021) introduces the use of normalized heptagonal fuzzy numbers (NHFNs) to approximate the uncertainty inherent in fractional programming. By adopting close interval approximations for these fuzzy numbers, the authors provide a more refined approach to capturing uncertainty in both the objective function and constraints. Abdelkebir (2024) investigated the existence and uniqueness of solutions for nonlinear fractional Volterra integro-differential equations with non-local boundary conditions. This work extends classical methods to fractional calculus, offering valuable insights into solving complex boundary value problems in various applied fields.

The objective of the work

- Propose a new methodology for solving fuzzy nonlinear programming (FNLP) problems with linear inequality constraints.
- Decision variables modeled as triangular fuzzy numbers to represent uncertainty and imprecision.
- Utilize the Haar ranking function to transform fuzzy variables into crisp values.
- Convert the fuzzy problem into a standard nonlinear programming (NLP) problem. Solve the resulting NLP problem using conventional optimization techniques. Offer a structured and practical approach for addressing uncertainty in optimization problems.
- Illustrate the method effectiveness through a detailed numerical example. Discuss potential limitations of the method, including:
- Handling complex representations of uncertainty.
- Scalability to large-scale problem-solving.
- Explore future research directions, focusing on improving the approach for broader applications.

2 Preliminaries

2.1 Fuzzy Sets

Let R be the real line, then a fuzzy set A in R is defined to be a set of ordered pairs $\tilde{A} = \{(x, \mu_{\tilde{A}}(x)) \mid x \in R\}$ where $\mu_{\tilde{A}}(x)$ is called the membership function for the fuzzy set. The membership function maps each element of R to a membership value between 0 and 1.

2.2 Support

The support of a fuzzy set $\tilde{A}$ is defined as $supp(\tilde{A}) = \{x \in R \mid \mu_{\tilde{A}}(x) > 0\}$.

2.3 Core

The core of a fuzzy set is the set of all points x in R with $\mu_{\tilde{A}}(x) = 1$.

2.4 Normal

A fuzzy set A is called normal if its core is non-empty. In other words, there is at least one point x in R with $\mu_{\tilde{A}}(x) = 1$.

2.5 α cut

The α cut or α level set of a fuzzy set is a crisp set defined by
$\tilde{A}_a = \{x \in R \mid \mu_{\tilde{A}}(x) > \alpha\}$.

2.6 Triangular Fuzzy Number

A fuzzy number $A = (a, b, c)$ on R is said to be a triangular fuzzy number if its membership function is given by:

$$A(x) = \begin{cases} \frac{x-a}{b-a} & \text{if } x \in [a, b], \\ \frac{c-x}{c-b} & \text{if } x \in [b, c], \\ 0 & \text{otherwise.} \end{cases}$$

2.7 Haar Ranking

A ranking function is a function $\Re : F(R) \to R$ is a set of fuzzy numbers defined on the set of real numbers, which maps each fuzzy number into the real line, where a natural order exists. Let $(\tilde{A}) = (a, b, c)$ be a triangular fuzzy number then
R($\tilde{A}$) = $\frac{(a+2b+c)}{4}$

2.8 Arithmetic operations

Let $\widetilde{a} = (a, b, c)$ and $\tilde{b} = (e, f, g)$ be either two triangular fuzzy numbers identified on the real R set. And there is
$(\tilde{a} + \tilde{b} = (a + e, b + f, c + g))$
$(-\tilde{a} = (-c, -b, -a)$
$\tilde{a} - \tilde{b} = (a - g, b - f, c - e)$
$\tilde{a} \leq \tilde{b}$ if and only if $a \leq e, b \leq f, c \leq g$ and for any $\tilde{b} = (e, f, g) \ \leq 0$
we define

$$\tilde{a} \times \tilde{b} = \begin{Bmatrix} (ae, bf, cg), a \geq 0 \\ (ag, bf, cg), a < 0, c \geq 0 \\ (ag, bf, ce), c < 0 \end{Bmatrix}$$

2.9 Nonlinear Programming Problem

Maximize (or minimize) $Z = \sum_{j=1}^{n} C_j x_j^n$ ($n \geq 2$ as the objective function is nonlinear)
Subject to the constraints
$\sum_{j=1}^{n} a_{ij} x_j^n \leq = (\geq) b_i$, $i = 1, 2, \ldots, m$
and $x_j \geq 0$.
We find programming problem is Nonlinear.
Maximize (or minimize) $\widetilde{C}^T \times \widetilde{X}^a$
Subject to the constraints
1.$\widetilde{A} \times \widetilde{X}$
2. $\widetilde{X}$ is a non-negative fuzzy number

3. Where $\widetilde{C}^T = [\widetilde{C}]^T_{n\times 1}$, $\widetilde{A} = [\widetilde{a}_{ij}]_{m\times n}$, $\widetilde{b} = [\widetilde{b}_i]_{m\times 1}$, $\widetilde{X} = [\widetilde{x}_i]_{n\times 1}$

3 Fuzzy Nonlinear Programming Problems

In this section, we propose a new method to find the fuzzy optimal solution to the FNLP problem.

3.1 Proposed Algorithm

Step 1: Let Maximize (or Minimize) $(\tilde{C}^T \times \tilde{X}^{\alpha j})$ Subject to $\tilde{A}\tilde{X} =, =, = \tilde{b}$, $\widetilde{X}$ is non negative triangular fuzzy number, be converted into
Maximize (or Minimize)
$(\tilde{C}^T \times \tilde{X}^{\alpha j})$
Subject to $\tilde{A}\tilde{X} = \tilde{b}$
where $\tilde{X}$ is a non-negative triangular fuzzy number

$$\tilde{C}^T = [\tilde{C}_j]^T{}_{n\times 1},$$

$$\tilde{A} = [a_{ij}]_{m\times n}, \quad \tilde{X} = [\tilde{x}_j]_{n\times 1}, \quad \tilde{b} = [\tilde{b}_j]_{n\times 1}$$

Step 2: Substituting $\tilde{C}^T = [\tilde{C}_j]^T{}_{n\times 1}, \tilde{A} = [\tilde{a}_{ij}]_{m\times n}, \tilde{b} = [\tilde{b}_j]_{n\times 1}$ and $\tilde{X} = [\tilde{x}_j]_{n\times 1}$ the FNLP problem may be written as follows
Max (or min) $\sum_{j=0}^{n} \widetilde{C}_j \times \widetilde{x}^{a_j}$
Subject to

$$\sum_{lj=1}^{n} \widetilde{a}_{ij} \times \widetilde{x}_j = \widetilde{b}_j, \forall_i = 1, 2, \ldots, m$$

$\tilde{x}_j$ is a non-negative fuzzy number.

Step 3: If all the parameters $\tilde{C}_j, \tilde{a}_{ij}, \tilde{b}_i$ and $\tilde{x}_j$ are represented by triangular fuzzy numbers (p_j, q_j, r_j),(a_{ij}, b_{ij}, c_{ij}), (b_i, g_i, h_i), and (x_j, y_j, z_j) respectively then the FNLP problem obtained in step2, may be written as:
Max (or min) $\sum_{j=1}^{n} (p_j, q_j, r_j) \times (x_j, y_j, z_j)\alpha^j$

Subject to

$$\sum_{j=1}^{n} (a_{ij},\ b_{ij},\ c_{ij}) \times (x_j, y_j, z_j) = (\ b_i, g_i, h_i)\ , \forall_i = 1, \ldots, m$$

(x_j, y_j, z_j) is non$-$negative fuzzy number.

Step 4: Assuming $(a_{ij},\ b_{ij},\ c_{ij}) \times (x_j, y_j, z_j) = (\ m_{ij}, n_{ij}, o_{ij})$, the FNLPP obtained in step 3, may be written as follows:
Max (or min) $\Re \sum_{j=1}^{n} (p_j, q_j, r_j) \times (x_j, y_j, z_j)\alpha^j$
Subject to

$$\sum_{j=1}^{n} (\ m_{ij}, n_{ij}, o_{ij}) = (\ b_i, g_i, h_i)\ , \forall_i = 1, \ldots, m$$

(x_j, y_j, z_j) is non$-$negative fuzzy number.

Step 5: The fuzzy linear programming issue produced in step 4 is converted into the following problem using the arithmetic operations stated in subsection 3 and Definition 2.4.
Max (or min) $\Re \sum_{j=1}^{n} (p_j, q_j, r_j) \times (x_j, y_j, z_j)\alpha^j$
Subject to

$$\sum_{j=1}^{n} m_{ij} = \ b_i, \forall_i = 1, \ldots, m$$

$$\sum_{j=1}^{n} n_{ij} = g_i, \forall_i = 1, \ldots, m$$

$$\sum_{j=1}^{n} o_{ij} = h_i, \forall_i = 1, \ldots, m$$

$$y_j - x_j = 0, z_j - y_j = 0$$

Step 6: Find the optimal solution $x_j, y_j,$ and z_j by solving the problem in step 5.

Step 7: Find the fuzzy optimal solution by putting the values of $x_j, y_j,$ and z_j in

$$\tilde{x}_j = (x_j, y_j, z_j)$$

Step 8: Determine the fuzzy optimal value by substituting $\tilde{x}_j$ into the equation

$$\sum_{j=1}^{n} \tilde{C}^T \times \tilde{X}^{\alpha j}$$

4 Numerical Example

Consider the FNLP problem and solve it using the proposed method.
Maximize $\tilde{Z} = (-1, 2, 3) \times \tilde{x}_1 + (2, 3, 4) \times \tilde{x}_2^2$
Subject to

$$(0, 1, 2,) \times \tilde{x}_1 + (1, 2, 3) \times \tilde{x}_2 \leq (6, 10, 25)$$

$$(1, 2, 3) \times \tilde{x}_1 + (0, 1, 2) \times \tilde{x}_2 \leq (2, 11, 26)$$

Let $\tilde{x}_1 = x_1, y_1, z_1 \; and \; \tilde{x}_2 = x_2, y_2, z_2$,
Where $\tilde{x}_1$ and $\tilde{x}_2$ are non- negative fuzzy numbers.
Then the given FNLP problem may be written as follows:

$$\mathrm{M}ax\tilde{Z} = \left((-1, 2, 3) \times (x_1, y_1, z_1) + (2, 3, 4) \times (x_2, y_2, z_2)^2\right)$$

Subject to

$$(0, 1, 2) \times (x_1, y_1, z_1) + (1, 2, 3) \times (x_2, y_2, z_2) \leq (6, 10, 25)$$

$$(1, 2, 3) \times (x_1, y_1, z_1) + (0, 1, 2) \times (x_2, y_2, z_2) \leq (2, 11, 26)$$

By Step 4 of the proposed methodology, the aforementioned problem can be expressed as follows:

$$\mathrm{Max}\tilde{Z} = \Re \left(-x_1 + 2{x_2}^2, 2y_1 + 3{y_2}^2, 3z_1 + 4{z_2}^2\right)$$

Subject to

$$(0, y_1, 2z_1) + (x_2, 2y_2, 3z_2) \leq (6, 10, 25)$$

$$(x_1, 2y_1, 3z_1) + (0, y_2, 2z_2) \leq (2, 11, 26)$$

By applying Step 5, the aforementioned Fuzzy Nonlinear Programming (FNLP) problem is transformed into the corresponding Crisp Nonlinear Programming Problem.

$$\mathrm{Max}\ \tilde{Z} \left(\frac{1}{4}(-x_1 + 2{x_2}^2 + 4y_1 + 6{y_2}^2 + 3z_1 + 4{z_2}^2)\right)$$

Subject to

$$x_2 \leq 6, y_1 + 2y_2 \leq 10, 2z_1 + 3z_2 \leq 25,$$

$$x_1 \leq 2, 2y_1 + y_2 \leq 11, 3z_1 + 2z_2 \leq 26$$

The optimal solution of the above problem is x_1 =2, y_1=4, z_1 28/5, x_2 =6, y_2=3, z_2 = 23/5.
Using step 7, the fuzzy optimal solution is given by $\tilde{x}_1$=(2, 4, 28/5), $\tilde{x}_2$=(6, 3, 23/5).

Thus, by employing Step 8, the fuzzy optimal value for the specified Fuzzy Nonlinear Programming (FNLP) problem is determined Max $\tilde{Z}$=(10,17,35.2).

4.1 Comparison between the Proposed solution and the Existing solution

This paper compares the proposed calculation method with existing approaches. Previous methods prioritized minimizing computations to enhance efficiency. In contrast, the proposed method achieves better computational performance by maximizing outcomes.

Existing method Calculation	Proposed method Calculation
The optimal solution is $Max\ \tilde{Z} = (10, 17, 25)$	The optimal solution is $Max\ \tilde{Z} = (10, 17, 35.2)$

Figure 1. Comparison Table

4.2 Managerial and Practical Implications

- By converting fuzzy data into precise values, this method helps managers make informed decisions under uncertainty, thereby improving clarity and reliability.
- This approach also enhances resource allocation in scenarios with incomplete or ambiguous data, such as in manufacturing or logistics.
- Additionally, it can be applied across various industries and complex optimization challenges, supporting both strategic planning and operational efficiency.
- The use of triangular fuzzy numbers simplifies the model, making it easier to implement without requiring extensive knowledge of the fuzzy theory.
- By transforming fuzzy parameters into crisp values, the method reduces calculation complexity and increases efficiency.
- This study's numerical examples further demonstrate its practical applicability, making it suitable for addressing optimization challenges in areas such as supply chain management and risk analysis.

4.3 Advantages

The proposed method for solving fuzzy nonlinear programming problems (FNLP) with linear inequality constraints offers several advantages. By converting fuzzy problems into crisp nonlinear problems using a linear ranking function, conventional optimization techniques can be applied. The use of triangular fuzzy numbers further simplifies the formulation, making the approach more intuitive and easier to implement. Its effectiveness was validated through numerical examples, thereby demonstrating its practical applicability to real-world problems. This step-by-step process ensures clarity and facilitates smooth implementation. Additionally, the simplicity of the method, combined with illustrative examples, makes it accessible even to those with limited knowledge of the fuzzy set theory. Overall, these attributes underscore the practicality, clarity, and broad accessibility of the method in addressing FNLP problems.

4.4 Limitations

First, the computational complexity of converting fuzzy numbers into crisp equivalents can be significant for large-scale problems, requiring substantial resources and potentially impacting the efficiency and scalability. Second, the exclusive reliance on triangular fuzzy numbers may restrict flexibility; other representations, such as trapezoidal or Gaussian fuzzy numbers, can better capture uncertainty in certain applications.Finally, the assumption that the Haar ranking function accurately reflects decision-maker risk preferences may not always hold true in practice. Alternative ranking methods can provide a more nuanced approach to address fuzziness. Although this methodology presents a structured solution to FNLP problems, its computational demands and reliance on triangular fuzzy numbers highlight areas for further research and improvements.

5 Conclusion and Future works

This study introduces a new approach for solving Fuzzy Nonlinear Programming (FNLP) problems with linear inequality constraints by utilizing the Haar ranking method to convert triangular fuzzy numbers into crisp values. Traditional optimization techniques can be applied to obtain solutions by transforming fuzzy problems into standard nonlinear programming problems. The practicality of the method was demonstrated through a numerical example, confirming its usefulness in real-world scenarios where uncertainty is present in the data. Overall, this approach offers an effective and computationally manageable solution to FNLP problems, making it valuable for addressing ambiguities in optimization models.

Future Work

Future studies can expand this method by applying it to more complex mathematical models and exploring different fuzzy representations, such as trapezoidal fuzzy numbers or linguistic variables. Research could also investigate the scalability and efficiency of the method, particularly for larger or more complex problems. The development of algorithms or software tools based on this approach would improve accessibility and ease of use. Comparative analyses with other methods and sensitivity studies will further highlight the strengths and limitations of this technique. In addition, testing the method for real-world applications in various fields would validate its practical effectiveness and broaden its range of use.

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