Palestine Journal of Mathematics

Vol 14(2)(2025) , 551–561

WEAKLY $(m, n)-$PRIME IDEALS IN AN ALMOST DISTRIBUTIVE LATTICE

Natnael Teshale Amare

Communicated by Ayman Badawi

MSC 2010 Classifications: 06D72, 06F15, 08A72.

Keywords and phrases:Almost Distributive Lattice; Prime ideal; weakly Prime ideal; $(m, n)-$prime Ideal; Weakly $(m, n)-$prime Ideal.

The authors would like to thank the reviewers and editor for their constructive comments and valuable suggestions that improved the quality of our paper.

Corresponding Author: T.A. Natnael

Abstract *The conditions under which $(m, n)-$prime ideals in an ADL become weakly $(m, n)-$prime ideals are studied. Additionally, we explore the direct product of weakly $(m, n)-$prime ideals. Finally, we investigate whether the homomorphic images and inverse images of $(m, n)-$prime (or weakly $(m, n)-$prime) ideals retain their status as $(m, n)-$prime (or weakly $(m, n)-$prime) ideals.*

1 Introduction

The study of prime ideals is motivated by the desire to generalize prime numbers, understand localization, develop representation theorems, and explore order theory, all of which contribute to a deeper and more elegant understanding of mathematical structures. Prime ideals help in studying the properties of partially ordered sets (posets). They provide a way to understand the ordering relations within a poset and contribute to the development of a more comprehensive theory of ordered structures. According to Anna et al.[2] and Beddani and Messirdi [5], a proper ideal I of a ring R is a prime ideal in R if $ab \in I$, then $a \in I$ or $b \in I$, for all $a, b \in R$. Generalizing this concept, a proper ideal I of a ring R is a weakly prime ideal in R if $0 \neq ab \in I$, then $a \in I$ or $b \in I$, for all $a, b \in R$ introduced in [3, 4, 6, 10]. Koc et al.[9] introduced the concepts of 1-absorbing prime ideals which is the generalizations of weakly 1-API introduced by [15]. Let $m, n \in Z^+$ with $m > n$ and I a proper ideal in R. Khashan and Celikel [7] and [8] introduced $(m, n)-$prime (or, weakly $(m, n)-$prime) ideals in R if for $a, b \in R, a^m b \in I$ (or, $0 \neq a^m b \in I$) implies either $a^n \in I$ or $b \in I$. Furthermore, the concepts of weakly 2-absorbing ideals and $(m, n)-$absorbing ideals have introduced by [14] and [1].

The concept of Almost Distributive Lattices (ADLs) was later introduced by Swamy and Rao [13]. Building on this, Natnael [11] have proposed the concepts of weakly 2-absorbing ideals of an ADL. In this paper, we introduce the notion of $(m, n)-$prime ideals in an Almost Distributive Lattice. We prove every a 1-API (or, a prime ideal) is an $(m, n)-$prime ideal and $(m, n)-$prime ideal is an n-absorbing ideal, and the converse of these results are not true, justified by counter examples. We note that $(m, n)-$maximal ideal is an $(m, n)-$prime ideal. Mainly, we defined and characterized the notions of weakly $(m, n)-$prime ideals in an ADL L. A proper ideal H is a weakly $(m, n)-$prime ideal in L if $0 \neq \bigwedge_{i=1}^{m} h_i \wedge g \in H$ implies that either $\bigwedge_{i=1}^{n} h_i \in H$ or $g \in H$, for all $h_1, h_2, ..., h_m, g \in L$. We establish the relation between weakly $(m, n)-$prime ideal in L and $(m, n)-$prime ideal in L. Also, we prove that the intersection of weakly $(m, n)-$prime ideals is a $(m, n)-$prime ideal. Additionally, we discuss the direct product of weakly $(m, n)-$prime ideals and we prove their equivalent conditions. Finally, we prove that the homomorphic images and inverse homomorphic images of weakly $(m, n)-$prime ideals are again $(m, n)-$prime ideal.

2 Preliminaries

In this portion, we revisit certain definitions and fundamental findings primarily sourced from [7], [8] and [13].

Definition 2.1. An algebra $R = (R, \wedge, \vee, 0)$ of type $(2, 2, 0)$ is referred to as an ADL if it meets the subsequent conditions for all r, s and t in R.

(i) $0 \wedge r = 0$

(ii) $r \vee 0 = r$

(iii) $r \wedge (s \vee t) = (r \wedge s) \vee (r \wedge t)$

(iv) $r \vee (s \wedge t) = (r \vee s) \wedge (r \vee t)$

(v) $(r \vee s) \wedge t = (r \wedge t) \vee (s \wedge t)$

(vi) $(r \vee s) \wedge s = s$.

Every distributive lattice with a lower bound is categorized as an ADL.

Example 2.2. For any nonempty set A, it's possible to transform it into an ADL that doesn't constitute a lattice by selecting any element 0 from A and fixing an arbitrary element $u_0 \in R$. For every $u, v \in R$, define $\wedge$ and $\vee$ on R as follows:

$$u \wedge v = \begin{cases} v & \text{if } u \neq u_0 \\ u_0 & \text{if } u = u_0 \end{cases} \quad \text{and} \quad u \vee v = \begin{cases} u & \text{if } u \neq u_0 \\ v & \text{if } u = u_0 \end{cases}$$

Then $(A, \wedge, \vee, u_0)$ is an ADL (called the **discrete ADL**) with u_0 as its zero element.

Definition 2.3. Consider $R = (R, \wedge, \vee, 0)$ be an ADL. For any r and $s \in R$, establish $r \leq s$ if $r = r \wedge s$ (which is equivalent to $r \vee s = s$). Then $\leq$ is a partial order on R with respect to which 0 is the smallest element in R.

Theorem 2.4. *The following conditions are valid for any r, s and t in an ADL R.*

(1) $r \wedge 0 = 0 = 0 \wedge r$ *and* $r \vee 0 = r = 0 \vee r$

(2) $r \wedge r = r = r \vee r$

(3) $r \wedge s \leq s \leq s \vee r$

(4) $r \wedge s = r$ *iff* $r \vee s = s$

(5) $r \wedge s = s$ *iff* $r \vee s = r$

(6) $(r \wedge s) \wedge t = r \wedge (s \wedge t)$ *(in other words, $\wedge$ is associative)*

(7) $r \vee (s \vee r) = r \vee s$

(8) $r \leq s \Rightarrow r \wedge s = r = s \wedge r$ $\big($*iff* $r \vee s = s = s \vee r\big)$

(9) $(r \wedge s) \wedge t = (s \wedge r) \wedge t$

(10) $(r \vee s) \wedge t = (s \vee r) \wedge t$

(11) $r \wedge s = s \wedge r$ *iff* $r \vee s = s \vee r$

(12) $r \wedge s = \inf\{r, s\}$ *iff* $r \wedge s = s \wedge r$ *iff* $r \vee s = \sup\{r, s\}$.

Definition 2.5. Let R and G be ADLs and form the set $R \times G$ by
$R \times G = \{(r, g) : r \in R \text{ and } g \in G\}$. Define $\wedge$ and $\vee$ in $R \times G$ by,
$(r_1, g_1) \wedge (r_2, g_2) = (r_1 \wedge r_2, g_1 \wedge g_2)$ and $(r_1, g_1) \vee (r_2, g_2) = (r_1 \vee r_2, g_1 \vee g_2)$, for all $(r_1, g_1), (r_2, g_2) \in R \times G$. Then $(R \times G, \wedge, \vee, 0)$ is an ADL under the pointwise operations and $0 = (0, 0)$ is the zero element in $R \times G$.

Definition 2.6. Let R and G be ADLs. A mapping $g : R \to G$ is called a homomorphism if the following are satisfied, for any $r, s, t \in R$.
(1). $f(r \wedge s \wedge t) = f(r) \wedge f(s) \wedge f(t)$
(2). $f(r \vee s \vee t) = f(r) \vee f(s) \vee f(t)$
(3). $f(0) = 0$.

Definition 2.7. A non-empty subset, denoted as I in an ADL R is termed an ideal in R if it satisfies the conditions: if u and v belong to I, then $u \vee v$ is also in F, and for every element r in R, the $u \wedge r$ is in F.

Definition 2.8. A proper ideal I in R is a prime ideal if for any u and v belongs R, $u \wedge v$ belongs F, then either u belongs F or v belongs F.

Theorem 2.9. *Let I be an ideal in R. Let F be a non-empty subset in R such that $r \wedge s \in F$, for all r and $s \in F$. Assume $I \cap F$ is empty set. Then there exists a prime ideal P in R containing I and $P \cap F$ is empty set.*

Theorem 2.10. *Let P be an ideal in R. Then P a weakly prime ideal in R only if P is a prime ideal in R.*

Definition 2.11. Let R be a ring and m, n be positive integers. A proper ideal I of R is called a $(m,n)-$prime in R if for $a, b \in R, a^m b \in I$ implies either $a^n \in I$ or $b \in I$..

Definition 2.12. Let R be a ring and m, n be positive integers. A proper ideal I of R is called weakly $(m,n)-$prime in R if for $a, b \in R, 0 \neq a^m b \in I$ implies either $a^n \in I$ or $b \in I$..

3 $(m,n)-$Prime Ideals

In this section, we define and characterize the concept of $(m,n)-$prime Ideals (in short, $(m,n)-$PIs) in an ADL L and their properties. In particular, we study on the direct product of $(m,n)-$PIs and their homomorphic images.

Definition 3.1. Let $m, n \in Z^+$ with $m > n$. A proper ideal H in L is an $(m,n)-$prime ideal (in short $(m,n)-$PI) in L if for all $h_1, h_2, ..., h_m, g \in L$ such that $\bigwedge\limits_{i=1}^{m} h_i \wedge g \in H \Rightarrow \bigwedge\limits_{i=1}^{n} h_i \in H$ or $g \in H$.

Let us recall that, $\langle r] = \{r \wedge s : s \in L\}$.

Lemma 3.2. *Let $h_i, k_i \in L$, for all $1 \leq i \leq m$. Then the following hold.*
(1). $\bigcap\limits_{i=1}^{m} \langle h_i] = \langle \bigwedge\limits_{i=1}^{m} \langle h_i]$
(2). $\langle \bigwedge\limits_{i=1}^{m} \langle h_i] \cap \langle \bigwedge\limits_{i=1}^{m} \langle k_i] = \langle \bigwedge\limits_{i=1}^{m} (h_i \wedge k_i)] = \langle \bigwedge\limits_{i=1}^{m} (k_i \wedge h_i)]$
(3). $\langle \bigwedge\limits_{i=1}^{m} \langle h_i] \vee \langle \bigwedge\limits_{i=1}^{m} \langle k_i] = \langle \bigwedge\limits_{i=1}^{m} (h_i \vee k_i)] = \langle \bigwedge\limits_{i=1}^{m} (k_i \vee h_i)]$.

Let H be an ideal in L. We note that, $H \cap \langle r] = \{s \in L : r \wedge s \in H\}$.

Lemma 3.3. *Let H be a proper ideal in L and $m, n \in Z^+$ with $m > n$. Then the following assertion hold.*
(1). H is prime iff H is a $(1,1)-$PI
(2). If H is a 1-absorbing prime ideal, then H is an $(m,n)-$PI, for all $n \geq 2$
(3). If H is prime, then H is an $(m,n)-$PI
(4). If H is an $(m,n)-$PI, then H is n-absorbing ideal
(5). If H is an $(m,n)-$PI, then H is an $(m^*, n^*)-$PI, where $m^* \leq m$ and $n \leq n^*$
(6). H is an $(m,n)-$PI iff $H \cap (r]$ is an $(m,n)-$PI, for all $r \in L - H$.

Proof. (1). For $m = n = 1$, it is clear.
(2). Suppose that H is a 1-absorbing prime ideal. Let $h_1, h_2, ..., h_m, g \in L$ with $\bigwedge\limits_{i=1}^{m} h_i \wedge g \in H$ and $g \notin H$. If $g = 1$, then $\bigwedge\limits_{i=1}^{m} h_i = h_1 \wedge \bigwedge\limits_{i=2}^{m-1} h_i \wedge h_m \in H$ and since H is a 1-absorbing prime ideal, we have $\bigwedge\limits_{i=1}^{n} h_i = h_1 \wedge \bigwedge\limits_{i=2}^{n-1} h_i \in H$ or $h_n \in H$. Continue this process to get $h_{n-1} \wedge h_n \in H$

and so, $\bigwedge_{i=1}^{n} h_i \in H$, for all $n \geq 2$. Thus, H is an $(m,n)-$PI.

(3). Assume that H is prime, and let $h_1, h_2, ..., h_m, g \in L$ with $\bigwedge_{i=1}^{m} h_i \wedge g \in H$ and $\bigwedge_{i=1}^{n} h_i \notin H$. Since H is prime, we have $g \in H$. Hence the result.

(4). Suppose H is an $(m,n)-$PI and $\bigwedge_{i=1}^{m} h_i \wedge h_{m+1} \in H$, for all $h_1, h_2, ..., h_m, h_{m+1} \in L$. Then $\bigwedge_{i=1}^{m-1} h_i \wedge h_m \wedge h_{m+1} \in H$. Thus, either $\bigwedge_{i=1}^{n-1} h_i \in H$ or $h_n \wedge h_{n+1} \in H$. Continue this process to get $\bigwedge_{i=1}^{n-3} h_i \in H$ or $h_{n-2} \wedge h_{n-1} \in H$ or $h_n \wedge h_{n+1} \in H$. Thus there are n of h_i's whose meet is in H. Hence the result.

(5). Assume that H is an $(m,n)-$PI and $m^* \leq m$ and $n \leq n^*$, for all $m, n, m^*, n^* \in Z^+$ with $m > n$ and $m^* > n^*$. Let $h_1, h_2, ..., h_{m^*}, g \in L$ with $\bigwedge_{i=1}^{m^*} h_i \wedge g \in H$. Since H is an ideal in L and $h_m \in L$, we have $h_m \wedge \bigwedge_{i=1}^{m^*} h_i \wedge g \in H$ and hence $\bigwedge_{i=1}^{m} h_i \wedge g \in H$, since $m \geq m^*$. Again, since H is an $(m,n)-$PI, we get $\bigwedge_{i=1}^{n} h_i \in H$ or $g \in H$. Consequently, $\bigwedge_{i=1}^{n^*} h_i \in H$ or $g \in H$, since $n^* \geq n$; for, $\bigwedge_{i=1}^{n^*} h_i = \bigwedge_{i=1}^{n} h_i \wedge h_{n^*}$ and if $\bigwedge_{i=1}^{n} h_i \in H$, then clearly $\bigwedge_{i=1}^{n^*} h_i \in H$.

(6). Suppose H is an $(m,n)-$PI. Let $h_1, h_2, ..., h_m, g \in L$. Now,

$$\bigwedge_{i=1}^{m} h_i \wedge g \in H \cap \langle r] \Rightarrow r \wedge \bigwedge_{i=1}^{m} h_i \wedge g \in H$$

$$\Rightarrow r \wedge r \wedge \bigwedge_{i=1}^{m} h_i \wedge g \in H$$

$$\Rightarrow r \wedge \bigwedge_{i=1}^{m} h_i \wedge r \wedge g \in H \quad \text{(by 2.4(9))}$$

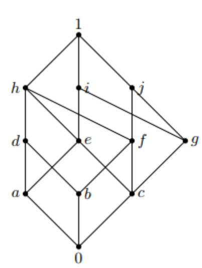
$$\Rightarrow r \wedge \bigwedge_{i=1}^{n} h_i \in H \text{ or } r \wedge g \in H \quad \text{(by assumption)}$$

$$\Rightarrow \bigwedge_{i=1}^{n} h_i \in H \cap \langle r] \text{ or } g \in H \cap \langle r].$$

Hence the result. Conversely suppose $H \cap \langle r]$ is an $(m,n)-$PI. Let $h_1, h_2, ..., h_m, g \in L$ such that $\bigwedge_{i=1}^{m} h_i \wedge g \in H$. Since H is an ideal in L, we have $r \wedge \bigwedge_{i=1}^{m} h_i \wedge g \in H$, for all $r \in L - H$. So, $\bigwedge_{i=1}^{m} h_i \wedge g \in H \cap \langle r]$ and by assumption to get $\bigwedge_{i=1}^{n} h_i \in H \cap \langle r]$ or $g \in H \cap \langle r]$. It follows that, $r \wedge \bigwedge_{i=1}^{n} h_i \in H$ or $r \wedge g \in H$. By the property of ideal and $r \in L - H$, we get $\bigwedge_{i=1}^{n} h_i \in H$ or $g \in H$.Thus H is an $(m,n)-$PI. □

The converse of the above results (2-5) are not true; consider the following example.

Example 3.4. Let Let $D = \{0, u, v\}$ be a discrete ADL with 0 as its zero element defined in 2.2 and $L = \{0, a, b, c, d, e, f, g, h, i, j, 1\}$ be a lattice whose Hasse diagram is given below:



Consider $D \times L = \{(t, s) : t \in D \text{ and } s \in L\}$. Then $(D \times L, \wedge, \vee, 0)$ is an ADL (note that

$D \times L$ is not a lattice) under the point-wise operations $\wedge$ and $\vee$ on $D \times L$ and $0 = (0, 0)$, the zero element in $D \times L$.
(2). Put $K = \{(0,0),(u,a),(v,b),(v,d)\}$. Let $(0,g),(u,f),(v,e),(u,d) \in D \times L$ such that $(0,g)\wedge(u,f)\wedge(v,e)\wedge(u,d) \in K$ implies $(v,e)\wedge(u,d) \in K$ but $(0,g)\wedge(u,f)\wedge(v,e) = (0,c) \notin K$ and $(u,d) \notin K$. From this we can conclude that K is a $(3,2)-$PI but not a $1-$absorbing prime ideal.
(3). K is a $(3,2)-$PI but not prime ideal in $D \times L$, where K is defined in above (2). Since, $(0,g) \wedge (u,f) \wedge (v,e) \wedge (u,d) \in K$ implies $(0,g) \wedge (u,f) \wedge (v,e) = (0,c) \notin K$ and $(u,d) \notin K$, or $(u,f) \wedge (v,e) \wedge (u,d) = (u,0) \notin K$ and $(0,g) \notin K$.
(4). Put $Q = \{(0,0),(0,b),(u,c),(u,f)\}$. Then $(0,d)\wedge(u,e)\wedge(v,f) \in Q \Rightarrow (0,d)\wedge(v,f) \in Q$. Thus Q is a $2-$absorbing ideal. On the other hand, consider $(0,d) \wedge (u,e) \wedge (v,f) \wedge (1,g) \in Q$ implies $(0,d) \wedge (u,e) = (0,a) \notin Q$, $(v,f) \wedge (1,g) = (v,c) \notin Q$ and $(1,g) \notin Q$. Thus Q is not a $(3,2)-$PI in $D \times L$.
(5). Let us defined Q in above (4). Let $(0,d),(u,e),(v,f),(1,g),(v,h) \in D \times L$. Then $(0,d) \wedge (u,e) \wedge (v,f) \wedge (1,g) \wedge (v,h) \in Q$ implies $(0,d) \wedge (u,e) \wedge (v,f) = (0,0) \in Q$. Thus Q is a $(4.3)-$PI. But Q is not a $(5,2)-$PI, since $(0,d) \wedge (u,e) \wedge (v,f) \wedge (1,g) \wedge (v,h) \wedge (u,i) \in Q$ implies $(0,d) \wedge (u,e) = (o,a) \notin Q$, $(v,f) \wedge (1,g) = (v,c) \notin Q$ and $(v,h) \wedge (u,i) = (u,e) \notin Q$.

Theorem 3.5. *Let H be a proper ideal in L and $m, n \in Z^+$ with $m > n$. Then the following are equivalent.*
(1). H is an $(m,n)-$PI
(2). For any ideal $I_1, I_2, ..., I_m, J \in L$ such that $\bigcap_{i=1}^{m} I_i \cap J \subseteq H \Rightarrow \bigcap_{i=1}^{n} I_i \subseteq H$ or $J \subseteq H$
(3). For any ideal $I_1, I_2, ..., I_m, J \in L$ such that $H = \bigcap_{i=1}^{m} I_i \cap J \Rightarrow H = \bigcap_{i=1}^{n} I_i$ or $H = J$.

Definition 3.6. Let $h \in L$. Then h is said to be an $(m,n)-$meet irreducible element in L if, for any ideals $h_1, h_2, ..., h_m, g \in L$ such that $\bigwedge_{i=1}^{m} h_i \wedge g \leq h \Rightarrow \bigwedge_{i=1}^{n} h_i \leq h$ or $g \leq h$.

Theorem 3.7. *Let H be a proper ideal in L. Then H is an $(m,n)-$PI iff H is an $(m,n)-$meet irreducible element in the lattice of ideals in L.*

Proof. Suppose H is an $(m,n)-$PI. Let $I_1, I_2, ..., I_m, J \in L$ such that $\bigcap_{i=1}^{n} I_i \nsubseteq H$ and $J \nsubseteq H$. Then we can choose h_i and g such that $\bigwedge_{i=1}^{n} h_i \in \bigcap_{i=1}^{n} I_i$, $\bigwedge_{i=1}^{n} h_i \notin H, g \in J$ and $g \notin H$. Then $\bigwedge_{i=1}^{m} h_i \wedge g \in \bigcap_{i=1}^{m} I_i \cap J$ and $\bigwedge_{i=1}^{m} h_i \wedge g \notin H$. Therefore, $\bigcap_{i=1}^{m} I_i \cap J \nsubseteq H$. Thus, H is an $(m,n)-$meet irreducible. Conversely suppose H is an $(m,n)-$meet irreducible element in the lattice of ideals in L. We are already given that H is a proper ideal in L. Let $h_1, h_2, ..., h_m, g \in L$ such that $\bigwedge_{i=1}^{n} h_i \notin H$ and $g \notin H$. Consider the ideals $\langle \bigwedge_{i=1}^{n} h_i]$ and $\langle g]$. Clearly, $\langle \bigwedge_{i=1}^{n} h_i] \nsubseteq H$ and $\langle g] \nsubseteq H$. By assumption, we get $\langle \bigwedge_{i=1}^{n} h_i] \cap \langle g] \nsubseteq H$ and hence $\langle \bigwedge_{i=1}^{n} h_i \wedge g] \nsubseteq H$. Therefore, H is an $(m,n)-$PI. □

In the following, we extend Stone Theorem[12] on prime ideals of ADLs to $(m,n)-$PI.

Theorem 3.8. *Let K be an ideal and G a non-empty subset in L such that $\bigwedge_{i=1}^{m} h_i \wedge g \in G$ implies $\bigwedge_{i=1}^{n} h_i \in G$ or $g \in G$, for all $h_1, h_2, ..., h_m, g \in L$ and $K \cap G = \emptyset$. Then there exists an $(m,n)-$PI H in L such that $K \subseteq H$ and $H \cap G = \emptyset$.*

Corollary 3.9. *Let K be an ideal and $h_1, h_2, ..., h_m, g \in L$ such that $\bigwedge_{i=1}^{n} h_i \notin K$. Then there exists an $(m,n)-$PI H in L such that $K \subseteq H$ and $\bigwedge_{i=1}^{n} h_i \notin H$.*

Next, we introduce the notion of the direct product of $(m,n)-$PI in $L_1 \times L_2$, where L_1 and L_1 are ADLs. Let H and G be ideals in L_1 and L_2, respectively. Let $(a,b),(c,d) \in H$. Then $(a,b)\vee(c,d) = (a\vee c, b\vee d) \in H\times L_2$, since $a\vee c \in H$. Also, $(a,b)\wedge(r,s) = (a\wedge r, b\wedge s) \in H\times L_2$, since $a\wedge r \in H$. Thus $H\times L_2$ is an ideal. Similarly, $L_1\times G$ is an ideal. In the case of $(m,n)-$PI, we have the following.

Theorem 3.10. *Let $L = L_1 \times L_2$. Then the following assertion hold. If H is an $(m,n)-$PI in L_1, then $H \times L_2$ is an $(m,n)-$PI in L. Also, if G is an $(m,n)-$PI in L_2, then $L_1 \times G$ is an $(m,n)-$PI in L.*

Proof. Suppose H is an $(m,n)-$PI in L_1 and $h_1,h_2,...,h_m,g \in L_1$ such that $\bigwedge_{i=1}^{m}(h_i,h_i^*) \wedge (g,g^*) \in H\times L_2$, for all $h_1,h_2,...,h_m^*,g \in L_2$. Then $\bigwedge_{i=1}^{m}(h_i,h_i^*)\wedge(g,g^*) = \bigwedge_{i=1}^{m}(h_i\wedge g, h_i^*\wedge g^*) \in H\times L_2$ and by assumption to get $\bigwedge_{i=1}^{m}(h_i,h_i^*) \in H\times L_2$ or $(g,g^*) \in H\times L_2$, since $\bigwedge_{i=1}^{m} h_i\wedge g \in H$ implies either $\bigwedge_{i=1}^{n} h_i \in H$ or $g \in H$. Thus, $H \times L_2$ is an $(m,n)-$PI in L. Similarly, $L_1 \times G$ is an $(m,n)-$PI in L if G is an $(m,n)-$PI in L_2. □

In the following, we establish that both the image and pre-image of any $(m,n)-$PI is again $(m,n)-$PI.

Theorem 3.11. *Let L_1 and L_2 be ADLs and $k : L_1 \to L_2$ be a lattice homomorphism. then the following hold. Let k be a monomorphism and if G is an $(m,n)-$PI in L_2, then $k^{-1}(G)$ is an $(m,n)-$PI in L_1. Also, if H is an $(m,n)-$PI in L_1, then $k(H)$ is an $(m,n)-$PI in L_2 if K is an epimorphism.*

Proof. (1). Suppose G is an $(m,n)-$PI in L_2. Let $h_1,h_2,...,h_m,g \in L_1$ such that $\bigwedge_{i=1}^{m} h_i\wedge g \in k^{-1}(G)$. Then $k(\bigwedge_{i=1}^{m} h_i\wedge g) \in G$, and hence $k(\bigwedge_{i=1}^{m} h_i)\wedge k(g) \in G$. It follows that, $\bigwedge_{i=1}^{m} k(h_i)\wedge k(g) \in G$. By assumption to get $\bigwedge_{i=1}^{m} k(h_i) \in G$ or $k(g) \in G$. So, $\bigwedge_{i=1}^{m} h_i \in k^{-1}(G)$ or $g \in k^{-1}(G)$. Thus, $k^{-1}(G)$ is an $(m,n)-$PI in L_1.
(2). Suppose $k^{-1}(G)$ is an $(m,n)-$PI in L_1. Let $h_1,h_2,...,h_m,g \in L_1$ such that $k(h_1) = a_1$, $k(h_2) = a_2$, ..., $k(h_m) = a_m$, $k(g) = b$, for some $a_1,a_2,...,a_m,b \in L_2$. Let $\bigwedge_{i=1}^{m} k(h_i)\wedge k(g) \in k(H)$. As k is a lattice homomorphism, $k(\bigwedge_{i=1}^{m} h_i\wedge g) = \bigwedge_{i=1}^{m} k(h_i)\wedge k(g) \in k(H)$ and hence $k(\bigwedge_{i=1}^{m} h_i\wedge g) \in k(H)$. So, $\bigwedge_{i=1}^{m} h_i\wedge g \in k^{-1}(k(H))$. By assumption to get $\bigwedge_{i=1}^{n} h_i \in k^{-1}(k(H))$ or $g \in k^{-1}(k(H))$. Which implies that, $k(\bigwedge_{i=1}^{n} h_i) = \bigwedge_{i=1}^{n} k(h_i) \in k(H)$ or $k(g) \in k(H)$. Hence the result. □

Definition 3.12. Let $m,n \in Z^+$ with $m > n$. A proper ideal H in L is an $(m,n)-$maximal ideal in L if, for any $h_1,h_2,...,h_m \in L$ such that $H \subseteq \bigcap_{i=1}^{m} h_i \Rightarrow H = \bigcap_{i=1}^{n} h_i$ or $\bigcap_{i=1}^{n} h_i = L$.

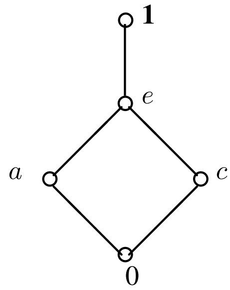
Lemma 3.13. *Let H be a proper ideal in L and $m,n \in Z^+$ with $m > n$. Then H is an $(m,n)-$maximal ideal iff $\bigwedge_{i=1}^{n} h_i \in L - H$ and $g \in L \Rightarrow g = (\bigwedge_{i=1}^{m} h_i \wedge g)\vee r$, for some $r \in L$.*

Finally, we discuss the relationship between $(m,n)-$PI and $(m,n)-$maximal ideal.

Theorem 3.14. *Every $(m,n)-$maximal ideal is an $(m,n)-$PI, where $m,n \in Z^+$ with $m > n$.*

Proof. Suppose H is an $(m, n)-$maximal ideal. Let $h_1, h_2, ..., h_m, g \in L$ and $\bigwedge_{i=1}^{m} h_i \wedge g \in H$. Assume $\bigwedge_{i=1}^{n} h_i \in L - H$. By the above lemma, we get $g = (\bigwedge_{i=1}^{m} h_i \wedge g) \vee r$, for some $r \in L$. Since $\bigwedge_{i=1}^{m} h_i \wedge g \in H$, it follows that, $g \in H$. Therefore, H is an $(m, n)-$PI. □

Example 3.15. Let $L = \{0, a, c, e, 1\}$ be the lattice represented by the Hasse diagram given below:



Put $Q = \{0, a\}$. Clearly Q is a $(1, 1)-$PI but not $(1, 1)-$maximal ideal.

4 Weakly $(m, n)-$Prime Ideals

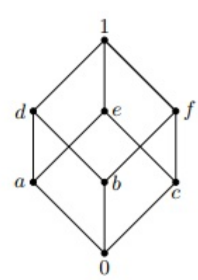
In this section, we introduce the concepts of weakly $(m, n)-$PI, generalize the notion of weakly prime ideals and $(m, n)-$PIs. We justify several properties and characterizations of weakly $(m, n)-$PIs with supportive examples. Furthermore, we investigate the direct product, homomorphic images and pre-images of weakly $(m, n)-$PIs.

Definition 4.1. Let $m, n \in Z^+$ with $m > n$. A proper ideal H in L is an weakly $(m, n)-$PI in L if for all $h_1, h_2, ..., h_m, g \in L$ such that $0 \neq \bigwedge_{i=1}^{m} h_i \wedge g \in H \Rightarrow \bigwedge_{i=1}^{n} h_i \in H$ or $g \in H$.

In the following, we introduce the relationship between $(m, n)-$PI and weakly $(m, n)-$PI.

Theorem 4.2. *Every $(m, n)-$PI is a weakly $(m, n)-$PI and the converse of this is not true.*

Example 4.3. Let $L = \{0, a, b, c, d, e, f, 1\}$ be a lattice whose Hasse diagram is given below:



Put $G = \{0\}$. Clearly G is a weakly $(m, n)-$PI but not an $(m, n)-$PI, since $d \wedge e \wedge f \in G$ implies $d \wedge e \notin G, e \wedge f \notin G, d \notin G$ and $f \notin G$.

Lemma 4.4. *Let H be a proper ideal in L and $m, n \in Z^+$ with $m > n$. Then the following assertion hold.*
(1). H is a weakly prime ideal iff H is a weakly $(1, 1)-$PI
(2). If H is a weakly 1-absorbing prime ideal, then H is a weakly $(m, n)-$PI
(3). If H is a weakly prime ideal, then H is a weakly $(m, n)-$PI
(4). If H is a weakly $(m, n)-$PI, then H is a weakly n-absorbing ideal
(5). H is a weakly $(m, n)-$PI iff $H \cap \langle r]$ is a weakly $(m, n)-$PI, for all $r \in L - H$.

Theorem 4.5. *Let H be a proper ideal in L with $H = \langle \bigwedge_{i=1}^{k} h_i]$, where $n \geq k$. Then H is a weakly $(m,n)-$PI iff $n \geq k$. Also, H is a weakly $(m,n)-$PI iff H is $(m,n)-$PI.*

Lemma 4.6. *Let $S \subseteq L$, we define $S = \{g \in L : \bigwedge_{i=1}^{m} h_i \wedge g = 0,\}$, for all $h_1, h_2, ..., h_m \in L$. Then S is an ideal in L.*

Proof. Clearly $S \neq \emptyset$, since $0 \in S$. Let $r, s \in S$. Then $\bigwedge_{i=1}^{m} h_i \wedge r = 0$ and $\bigwedge_{i=1}^{m} h_i \wedge s = 0$, for all $h_1, h_2, ..., h_m \in L$. Consider, $\bigwedge_{i=1}^{m} h_i \wedge (r \vee s) = (\bigwedge_{i=1}^{m} h_i \wedge r) \vee (\bigwedge_{i=1}^{m} h_i \wedge s)$ (by 2.1(3))
$= 0$.
Thus, $r \vee s \in S$. Also, for all $a \in L$, by 2.4(4), we have $\bigwedge_{i=1}^{m} h_i \wedge (r \wedge a) = (\bigwedge_{i=1}^{m} h_i \wedge r) \wedge a = 0 \wedge a = 0$ and hence $r \wedge a \in S$. Thus, S is an ideal. □

Lemma 4.7. *Let H be an ideal in L. Define $H^* = \{g \in L : \bigwedge_{i=1}^{m} h_i \wedge g \in H\}$. Then H^* is an ideal in L.*

Proof. Clearly $H^* \neq \emptyset$, since $0 \in H^*$ and H is an ideal. Let $g, k \in G$. Then $\bigwedge_{i=1}^{m} h_i \wedge g \in H$ and $\bigwedge_{i=1}^{m} h_i \wedge \in H$, for all $h_1, h_2, ..., h_m \in L$. Consider, $\bigwedge_{i=1}^{m} h_i \wedge (g \vee k) = (\bigwedge_{i=1}^{m} h_i \wedge g) \vee (\bigwedge_{i=1}^{m} h_i \wedge k) \in H$, (by 2.1(3)) and since H is an ideal. Thus, $g \vee k \in H^*$. Also, for all $t \in L$, by by 2.4(4) to get $\bigwedge_{i=1}^{m} h_i \wedge (g \wedge t) = (\bigwedge_{i=1}^{m} h_i \wedge g) \wedge t \in H$, since H is an ideal. So, $g \wedge t \in H^*$. Therefore, H^* is an ideal. □

Next, we characterize weakly $(m,n)-$PIs in the following.

Theorem 4.8. *Let H be a proper ideal in L, and H^* and S are defined above. Then the following are equivalent.*
(1). H is a weakly $(m,n)-$PI
(2). $H^* \subseteq H \vee S$, for all $h_1, h_2, ..., h_m \in L$ such that $\bigwedge_{i=1}^{n} h_i \notin H$
(3). $H^* = H$ or $H^* = S$, for all $h_1, h_2, ..., h_m \in L$ such that $\bigwedge_{i=1}^{n} h_i \notin H$
(4). Whenever $h_1, h_2, ..., h_m \in L$ and G is an ideal in L with $0 \neq \langle \bigwedge_{i=1}^{m} h_i] \cap G \subseteq H$, then $\bigwedge_{i=1}^{n} h_i \in H$ or $G \subseteq H$.

Proof. $(1) \Rightarrow (2)$: Suppose H is a weakly $(m,n)-$PI. Let $h_1, h_2, ..., h_m \in L$ such that $\bigwedge_{i=1}^{n} h_i \notin H$. Let $g \in H^*$. Then $\bigwedge_{i=1}^{m} h_i \wedge g \in H$. By assumption to get $g \in H$ and clearly $g \in H \vee S$, since $g = g \vee 0 = g \vee (\bigwedge_{i=1}^{m} h_i \wedge g)$. Thus, $H^* \subseteq H \vee S$.
$(2) \Rightarrow (3)$: Assume (2) hold. Then $H^* \subseteq H$ or $H^* \subseteq S$. Next, we prove that either $H \subseteq H^*$ or $S \subseteq H^*$. Assume $H \nsubseteq H^*$ and $S \nsubseteq H^*$. Then there exists $h \in H$, $h \notin H^*$, $g \in S$ and $g \notin H^*$. As, $h \notin H^*$, then $\bigwedge_{i=1}^{m} h_i \wedge h \notin H$, which gives a contradiction, since H is an ideal, $h \in H$ and $\bigwedge_{i=1}^{n} h_i \notin H$. Therefore, $H \subseteq H^*$. Also, if $g \notin H^*$, then $\bigwedge_{i=1}^{m} h_i \wedge g \notin H$, gives a contradiction, since $g \in S$ and hence $\bigwedge_{i=1}^{m} h_i \wedge g = 0 \in H$. Hence the result.
$(3) \Rightarrow (4)$: Assume (3) hold. Let $h_1, h_2, ..., h_m \in L$ and G is an ideal with $0 \neq \langle \bigwedge_{i=1}^{m} h_i] \cap G \subseteq H$

and $\bigwedge\limits_{i=1}^{n} h_i \notin H$. Then $G \subseteq H^* - S$ and by hypothesis, we have $G \subseteq H^* = H$. Thus, $G \subseteq H$.

$(4) \Rightarrow (1)$: Let $h_1, h_2, ..., h_m, g \in L$ such that $0 \neq \bigwedge\limits_{i=1}^{m} h_i \wedge g \in H$. Put $G = \langle g]$. Then $0 \neq \langle \bigwedge\limits_{i=1}^{m} h_i] \cap G \subseteq H$ and by (4), we have $\bigwedge\limits_{i=1}^{n} h_i \in H$ or $g \in G \subseteq H$. Thus, H is a weakly $(m,n)-$PI in L. □

Theorem 4.9. *Let $\{H_\alpha\}_{\alpha\in\Delta}$ be a family of weakly $(m,n)-$PI. Then $\bigcap\limits_{\alpha\in\Delta} H_\alpha$ is a weakly $(m,n)-$PI.*

Proof. Suppose $\{H_\alpha\}_{\alpha\in\Delta}$ is a family of weakly $(m,n)-$PI. Let Let $h_1, h_2, ..., h_m, g \in L$ with $0 \neq \bigwedge\limits_{i=1}^{m} h_i \wedge g \in \bigcap\limits_{\alpha\in\Delta} H_\alpha$. Thus, $\bigwedge\limits_{i=1}^{m} h_i \wedge g \in H_\alpha$, for all $\alpha \in \Delta$. By assumption, we have $\bigwedge\limits_{i=1}^{n} h_i \in H_\alpha$ or $g \in H_\alpha$, for all $\alpha \in \Delta$. Thus, $\bigwedge\limits_{i=1}^{n} h_i \in \bigcap\limits_{\alpha\in\Delta} H_\alpha$ or $g \in \bigcap\limits_{\alpha\in\Delta} H_\alpha$. Hence the result. □

Next, we characterize weakly $(m,n)-$PIs in direct product of ADLs.

Theorem 4.10. *Let $H(\neq \{0\})$ be a proper ideal in $L = L_1 \times L_2$. Then H is a weakly $(m,n)-$PI in L iff H is an $(m,n)-$PI in L.*

Theorem 4.11. *Let H and G be proper ideals of L_1 and L_2. If $H \times G$ is a weakly $(m,n)-$PI in $L_1 \times L_2$, then H are G are weakly $(m,n)-$PIs in L_1 and L_2, respectively.*

Proof. Suppose $H \times G$ is a weakly $(m,n)-$PI in $L_1 \times L_2$. Let $h_1, h_2, ..., h_m, g \in L_1$ and $h_1^*, h_2^*, ..., h_m^*, g^* \in L_2$ such that $0 \neq \bigwedge\limits_{i=1}^{m} h_i \wedge g \in H$ and $0 \neq \bigwedge\limits_{i=1}^{m} h_i^* \wedge g^* \in G$. Then

$(0,0) \neq \big(\bigwedge\limits_{i=1}^{m} h_i \wedge g, \bigwedge\limits_{i=1}^{m} h_i^* \wedge g^*\big) \in H \times G \Rightarrow \big(\bigwedge\limits_{i=1}^{m} h_i, \bigwedge\limits_{i=1}^{m} h_i^*\big) \wedge \big(g, g^*\big) \in H \times G$

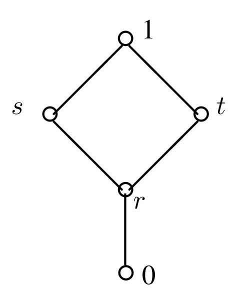
$\Rightarrow \big(\bigwedge\limits_{i=1}^{m} h_i, \bigwedge\limits_{i=1}^{m} h_i^*\big) \in H \times G$ or $(g, g^*) \in H \times G$

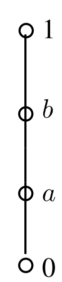
$\Rightarrow \bigwedge\limits_{i=1}^{m} h_i \in H$ or $g \in H$, and $\bigwedge\limits_{i=1}^{m} h_i^* \in G$ or $g^* \in G$.

Thus, H are G are weakly $(m,n)-$PIs. □

If there are weakly $(m,n)-$PIs, then their direct product may not weakly $(m,n)-$PI; consider the following example.

Example 4.12. Let $L_1 = \{0, r, s, t, 1\}$ be the lattice and $L_2 = \{0, a, b, 1\}$ be a chain respectively represented by the Hasse diagram given below:





Consider $L_1 \times L_2 = \{(x,y) : x \in L_1$ and $y \in L_2\}$. Put $H = \{0\}$ and $G = \{0, a\}$. Then $H \times G = \{(0,0), (0,a)\}$. Clearly H and G are weakly $(m,n)-$PIs in L_1 and L_2, respectively. But, $H \times G$ is not a weakly $(m,n)-$PI in $L_1 \times L_2$, since $(0,0) \neq (0,1) \wedge (r,b) \wedge (s,a) \in H \times G$ implies $(0,1) \wedge (r,b) \notin H \times G$ and $(s,a) \notin H \times G$.

Theorem 4.13. *Let L_1 and L_2 be ADLs and $H(\neq \{0\})$ be a proper ideal in L_1. Then the following are equivalent.*
(1). $H \times L_2$ is a weakly $(m,n)-$PI in $L_1 \times L_2$
(2). $H \times L_2$ is an $(m,n)-$PI in $L_1 \times L_2$
(3). H is an $(m,n)-$PI in L_1.

Proof. $(1) \Leftrightarrow (2)$: It is clear.
$(2) \Rightarrow (3)$: Assume (2) hold. Let $h_1, h_2, ..., h_m, g \in L_1$ with $\bigwedge\limits_{i=1}^{m} h_i \wedge g \in H$. Since $H \times L_2$ is an $(m,n)-$PI in $L_1 \times L_2$, we have $(\bigwedge\limits_{i=1}^{m} h_i \wedge g, r) \in H \times L_2$, for some $r \in L_2$, implies that $(\bigwedge\limits_{i=1}^{m} h_i, r) \wedge (g, r) \in H \times L_2$ and hence $(\bigwedge\limits_{i=1}^{m} h_i, r) \in H \times L_2$ or $(g, r) \in H \times L_2$. Thus, $\bigwedge\limits_{i=1}^{m} h_i \in H$ or $g \in H$. Hence the result.
$(3) \Rightarrow (2)$: Assume (3) hold. Let $h_1, h_2, ..., h_m, g \in L_1$ and $u \in L_2$ with $(\bigwedge\limits_{i=1}^{m} h_i \wedge g, u) \in H \times L_2$. Then $(\bigwedge\limits_{i=1}^{m} h_i \wedge g, u) = (\bigwedge\limits_{i=1}^{m} h_i, u) \wedge (g, u) \in H \times L_2$. Which implies that $(\bigwedge\limits_{i=1}^{m} h_i, u) \in H \times L_2$ or $(g, u) \in H \times L_2$, by assumption. Thus, $H \times L_2$ is an $(m,n)-$PI in $L_1 \times L_2$.
$(3) \Rightarrow (1)$: Assume (3) hold. Let $h_1, h_2, ..., h_m, g \in L_1$ such that $(0,0) \neq (\bigwedge\limits_{i=1}^{m} h_i \wedge g, t) \in H \times L_2$, for some $t \in L_2$. Then $(0,0) \neq (\bigwedge\limits_{i=1}^{m} h_i, t) \wedge (g, t) \in H \times L_2$ and hence $(\bigwedge\limits_{i=1}^{m} h_i, t) \in H \times L_2$ or $(g, t) \in H \times L_2$. Hence the result.
$(3) \Rightarrow (1)$: It is clear. □

The following Theorem is an immediate consequence of 4.11 and 4.13.

Theorem 4.14. *Let $H(\neq \{0\})$ and $G(\neq \{0\})$ be proper ideals in L_1 and L_2, respectively. Then the following are equivalent.*
(1). $H \times G$ is a weakly $(m,n)-$PI in $L_1 \times L_2$
(2). $G = L_2$ and H is an $(m,n)-$PI in L_1, or G is an $(m,n)-$PI in L_2 and H is an $(m,n)-$PI in L_1
(3). $H \times G$ is an $(m,n)-$PI in $L_1 \times L_2$.

If H and G are weakly $(m,n)-$PI in L_1 and L_2, respectively, then $H \times G$ is weakly $(m,n)-$PI in $L_1 \times L_2$, where $H \neq \{0\}$ and $G \neq \{0\}$. In general, we have the following characterization.

Theorem 4.15. *Let $L = L_1 \times L_2 \times ... \times L_k$ and $H(\neq \{0\})$ be proper ideal in L. Then the following are equivalent.*
(1). H is a weakly $(m,n)-$PI in L
(2). $H = L_1 \times L_2 \times ... \times H_j \times ... \times L_k$, where H_j is an $(m,n)-$PI in L_j, for some $j \in \{1, 2, ..., k\}$
(3). H is an $(m,n)-$PI in L.

Finally, we discuss the homomorphism of weakly $(m,n)-$PIs.

Theorem 4.16. *Let L_1 and L_2 be ADLs and $k : L_1 \to L_2$ be a lattice homomorphism. Then the following hold.*
(1). If k is a monomorphism and G is a weakly $(m,n)-$PI in L_2, then $k^{-1}(G)$ is a weakly $(m,n)-$PI in L_1
(2). If k is an epimorphism and H is a weakly $(m,n)-$PI in L_1 containing $ker(k)$, then $k(H)$ is a weakly $(m,n)-$PI in L_2.

Proof. (1). Suppose G is a weakly $(m,n)-$PI in L_2. Let $h_1, h_2, ..., h_m, g \in L_1$ such that $0 \neq \bigwedge\limits_{i=1}^{m} h_i \wedge g \in k^{-1}(G)$ and $g \notin k^{-1}(G)$. Since $ker(k) = 0$, we have $0 \neq k(\bigwedge\limits_{i=1}^{m} h_i \wedge g) = \bigwedge\limits_{i=1}^{m} k(h_i) \wedge k(g) \in G$ and $k(g) \notin G$ implies that $\bigwedge\limits_{i=1}^{n} k(h_i) = k(\bigwedge\limits_{i=1}^{n} h_i) \in G$ and hence $\bigwedge\limits_{i=1}^{n} h_i \in k^{-1}(G)$. Thus $k^{-1}(G)$ is an $(m,n)-$PI in L_1.
(2). Suppose H is a weakly $(m,n)-$PI in L_1. Let $b_1, b_2, ..., b_m, s \in L_2$ such that $k(a_1) = b_1$, $k(a_2) = b_2$,...,$k(a_m) = b_m$ and $k(r) = s$, for some $a_1, a_2, ..., a_m, r \in L_1$. Suppose $0 \neq \bigwedge\limits_{i=1}^{m} b_i \wedge s \in k(H)$ and $s \notin k(H)$. Then $0 \neq k(\bigwedge\limits_{i=1}^{m} a_i \wedge r) \in k(H)$ and since $k(H) \subseteq H$, we conclude $0 \neq \bigwedge\limits_{i=1}^{m} a_i \wedge r \in H$. By assumption to get $\bigwedge\limits_{i=1}^{n} a_i \in H$ or $t \in H$. Thus, $\bigwedge\limits_{i=1}^{n} b_i = k(\bigwedge\limits_{i=1}^{n} a_i) \in k(H)$ or $s = k(r) \in k(H)$. Hence the result. □

5 Conclusion

We define the notions of $(m, n)-$PIs and weakly $(m, n)-$PIs in an ADL and discuss their properties. Also, we introduce the concept of weakly $(m, n)-$PIs, generalizing weakly prime ideals and $(m, n)-$PIs. Furthermore, we explore the properties of $(m, n)-$PIs and weakly $(m, n)-$PIs for various lattice-theoretic construction such as direct products, homomorphism images, and homomorphic inverse images. In future work, we plan to focus on the concepts of L-fuzzy $(m, n)-$PIs and their prime spectrum.

References

[1] B.J. Abadi and H.F. Moghimi, "On (m,n)- absorbing ideals in commutative rings", Proc. Indian Acad. Acad. (Math.Sci), Vol. 127, No. 2, pp. 251-261, 2017.

[2] M. D. Anna, C. A. Finocchiaro and M. Fontana, Properties of chains of prime ideals in an amalgamated algebras along an ideal, J. Pure Appl. Algebra 214 (2010), 1633-1641.

[3] D.D. Anderson and E. Smith, Weakly Prime Ideals, Houston Journal of Math, 29(4), (2003) 831–840. https://iro.uiowa.edu/esploro/outputs/9984230419402771.

[4] A. Badawi, On weakly semi-prime ideals of commutative rings, Beitr. Algebra Geom. 51(4) (2014), 1163-1173.

[5] C. beddani and W. Messirdi, "2-prime ideals and their applications," Journal of Algebra and Its Application, Vol. 15, No. 3, (2016) (11 pages). DOI: 10.1142/S0219498816500511.

[6] M.K. Dubey, "prime and weakly prime ideals," Quasi-groups and Related Systems, Vol. 20, (2012) pp. 197-202. http://www.quasigroups.eu/contents/20.php.

[7] H. A. Khashan and E.Y. Celikel, (m, n)-prime Ideals of Commutative Rings, Preprints 2024, 2024010472. https://doi.org/10.20944/preprints202401.0472.v1.

[8] H. A. Khashan and E.Y. Celikel, On Weakly (m, n)-prime Ideals of Commutative Rings, Bull. Korean Math. Soc. 61 (2024), No. 3, pp. 717–734. https://doi.org/10.4134/BKMS.b230319

[9] S. Koc, U. Tekir, and E. Yildiz, On weakly 1-absorbing prime ideals, Ric. Mat. 2021 (2021), 1–16.

[10] N. Mahdou, M.A.S. Moutui and Y. Zahir, Weakly prime ideals issued from an amalgamated algebra, Hacettepe J. Math. Stat., 49 (3) (2020), 1159-1167.

[11] Natnael Teshale A., Weakly 2-absorbing Ideals in Almost Distributive Lattices, Journal of Mathematics (Hindawi), Volume (2022), Article ID 9252860, 1-9. DOI: 10.1155/2022/9252860.

[12] M.H. Stone, Topological representation of distributive lattices and Brouwerian logic, Casopis. Mat. Fyz., 67, (1937), 1 - 25.

[13] U.M. Swamy and G.C. Rao, Almost Distributive Lattices, J. Australian Math. Soc., (Series A), Vol. 31 (1981), 77–91. DOI: 10.1017/S1446788700018498.

[14] M.P. Wasadikar and K.T. Gaikevad, "On 2-absorbing ideals and weakly 2-absorbing ideals of lattice," Mathematical Sciences International Research Journal, Vol. 4, No. 2, pp. 82-85, 2015.

[15] A. Yassine, M.J. Nikmehr and R. Nikandish, "On 1-absorbing prime ideals of commutative rings," Journal of Algebra and Its Application (World Scientific), (Accepted on May 22), 2020.

Author information

Natnael Teshale Amare, Department of Mathematics, College of NCS, University of Gondar, Ethiopia.
E-mail: yenatnaelteshale@gmail.com

Received: 2024-08-23
Accepted: 2024-10-27